A NOVEL OPTIMIZATION MODEL FOR THE SIMULTANEOUS BATCHING, PRODUCTION AND DISTRIBUTION SCHEDULING IN PROCESS INDUSTRIES

Aldana S. Tibaldo†, Yanina Fumero† and Jorge M. Montagna†

†Instituto de Desarrollo y Diseño, CONICET-UTN, Avellaneda 3657, S3002GJC Santa Fe,
aldanatibaldo@santafe-conicet.gov.ar, yfumero@santafeconicet.gov.ar, mmontagna@santafe-conicet.gov.ar,
http://www.ingar.santafe-conicet.gov.ar/

Abstract: Traditionally, the short-term production and distribution activities have been addressed with a decoupled and sequential methodology. Although this approach simplifies the problem, there are several environments in which this methodology generates inefficiencies. Consequently integrated management of these activities is essential. This paper presents a novel mixed-integer linear optimization model to manage production and distribution decisions simultaneously at minimum operating cost. An effective strategy is proposed where all feasible routes that vehicles can travel are generated beforehand. This approach is very valuable mainly for two reasons: infeasible routes are not considered in the problem formulation (reduction of the search space) and several important decisions are made simultaneously when a route is selected, such as the allocation and sequencing of the customers visited in each route. The performance of the proposed model is compared against a first approach that, unlike the current formulation, presents difficulties in large problems.

Key words: batching, production scheduling, distribution, batch industries, MILP
2000 AMS Subjects Classification: 21A54 - 55P5T4

1. INTRODUCTION

Since production and distribution are two interrelated supply chain operations, the interest to simultaneously solve short-term production and distribution scheduling has been increased in recent years [1], [2]. The integration of both problems is very valuable in a variety of industrial applications, especially in industries where final products must be delivered to customers shortly after production. Perishable or shelf-life-limited products are at the top of the list, some examples are ready-mixed concrete [3], the catering industry [4], drugs for treatments such as cancer [5], etc. On the other hand, due to the emergence of Industry 4.0 technologies and the high variability of markets, there are companies that have changed their systems from make-to-stock (MTS) to make-to-order operation (MTO) in order to remain competitive. Due to the products are customized in the MTO manufacturing system, these companies have practically zero inventory, and, consequently, orders must be shipped once they are produced. Despite the above, in real-world applications, taking into account that production and distribution decisions are of combinatorial complexity, and the development of representations can lead to computationally intractable models due to the large number of decision variables involved, both problems are often decoupled. Although this hierarchical approach simplifies the problem, it can lead to suboptimal solutions for the integrated problem. In fact, the integration of production and distribution scheduling decisions at the operational level shows economic savings ranging from 5% to 20% compared to an uncoordinated approach [1].

In a previous work [6], the authors present a mixed-integer linear programming (MILP) model (TS-IP) that solves the integrated short-term batching, production and distribution problem in single-stage multi-product batch facilities with multiple units of different sizes operating in parallel. They develop a first approach to solve the integrated problem and although it has shown good performance, it has difficulties for large-scale problems. In the present paper, an effective strategy is proposed to solve the problem addressed in [6], which allows to achieve optimal solutions in reduced computational times for large-size instances. In this novel approach, a preprocessing algorithm, based on domain information, is developed to generate the feasible routes, which means that all possible infeasible combinations are not considered in the new model formulation, thus reducing the search space. In addition, when a route is selected, several strong decisions are made simultaneously, such as the sequence of customers visited on each route together to the corresponding time window. Although route departure times and delivery times to customers are not

decisions that are determined when the route is selected, taking into account that orders must be delivered within a time window, good bounds for such variables can be obtained. Several cases of study are solved and compared to demonstrate the effectiveness of the proposed model.

Section 2 describes the addressed problem. The main characteristics of the model formulation are presented in Section 3. Section 4 compares examples of different scales and shows the optimal detailed solution for one of them. Finally, Section 5 summarizes the conclusions and future works.

2. PROBLEM DESCRIPTION

At the beginning of the planning horizon, each customer $i \in IC$ places several orders. Each of them may consist of different products $p \in P$ and it is associated with a time window $d \in D$. Each order must be delivered within the limits $[a_d, c_d]$ of the time window d for which it was requested. At the production stage, the demands for each product are consolidated into a single demand for each product. Then, the demand of product p is partitioned into batches $b \in B_p$ for its production. Each of these batches must be processed by one of the multiple parallel available units $u \in U$ in the plant, which may have different capacities, times and processing costs depending on the product to be processed. The number and size of batches required to process all the orders is not known a priori, so appropriate bounds must be proposed in order to facilitate the solution of the problem and ensure optimality.

The plant has a homogeneous fleet of vehicles for shipping orders. The vehicles have a minimum required capacity and a maximum capacity that must not be exceeded. Travel time and distance between the different customers and the plant are parameters of the problem. The loading and unloading times of the products depend on the order sizes to be transported. In the time horizon, each vehicle can only make one trip, and is initially parked at the manufacturing facility. Orders of different customers can be loaded in each vehicle, but a customer's order for a given time window must be delivered completely with a single vehicle, and, therefore, partial deliveries are not allowed. In each vehicle, the batches or parts of batches necessary to complete all orders to be transported are consolidated.

3. MODEL FORMULATION

In order to optimally integrate batch sizing (or batching), production scheduling and transportation decisions, a MILP mathematical model (denoted by GP-R) is developed. For batching decisions, the continuous variable BS_{bp}, which represents the size of batch b of product p, and the binary variable x_{bp}, which indicates if batch b of product p is processed, are defined. The assignment of batches to units is modeled through the binary variable X_{bpu}, which is 1 if batch b of product p is processed in unit u, otherwise it takes value 0. For the batch sequencing, the notion of global general precedence is used. This type of representation is based on the concept that one task, operation or job precedes another if it is executed earlier in the same equipment or processing unit. In effect, the binary variable $XX_{bpb'p'}$ only takes the value 1 if batch b of product p is processed earlier than batch b' of product p', when both are assigned to the same unit. For timing decisions, non-negative variables, ST_{bp} and FT_{bp}, which symbolize the starting and completion times of batch b of product p, respectively, are introduced.

For distribution decisions, this paper presents a novel approach in which the possible vehicle routes are known a priori. A route r is defined as the sequence of *customer-time window* pairs visited by a vehicle. For example, the route $r = [(i_1, d_1), (i_3, d_2), (i_5, d_2)]$ is interpreted as: the vehicle departs from the plant and travels to customer i_1 to deliver the order required by him for time window d_1, then travels to customer i_3 and delivers the order demanded for d_2, and finally delivers the order of customer i_5 for time window d_2, i.e., each route shows the assignment and sequencing of customer-time window pairs to be visited but does not determine the exact times at which deliveries are scheduled. All feasible routes are generated beforehand considering the parameters of the problem (customer demands, time window limits, travel times between the plant and the customers, and vehicle capacity). This allows leaving out unfeasible routes and consequently reduces the search space. Furthermore, when a route is selected, several assignments are simultaneously defined. Then, to attain the set of optimal routes, the binary variable W_r is introduced, which is equal to 1 if a vehicle travels along route r. Since the start time of the vehicle that travel along a route depends on the completion time of the batches assigned to it, two continuous time variables representing the departure time of the vehicle that travel along route r and the delivery time to each customer i visited along route r, DT_r and DET_{ir}, respectively, are defined. As the sequence of customer-time window pairs of each route is known a priori, good bounds for variable DT_r can be obtained. It is

important to note that vehicles cannot have idle waiting, this means that, once a vehicle delivers to a customer, the vehicle must depart to the next customer immediately after the unloading is completed.

Batches must be assigned to vehicles to be delivered and fulfill the demand requirements. Each batch can be partially assigned to different vehicles or, as a special case, fully assigned to a single vehicle. For this purpose, the binary variable R_{bpr} indicates if batch b of product p is assigned to the vehicle that travels along route r, while the variable QT_{bpr} determines the size (total or partial) of batch b of product p that is assigned to the vehicle that travels along route r.

The model considers production constraints: *i)* demand satisfaction, *ii)* batch sizing, *iii)* allocation of batches to units, *iv)* batch sequencing, and *v)* timing; and distribution constraints: *vi)* availability of products to load the vehicles, *vii)* route selection, and *viii)* departure and arrival times to customers. The objective function minimizes production and distribution costs. The problem consists of determining the number and size of batches processed for each product (batching), their allocation and sequencing in the units, and timing of these batches, as well as selection of routes, allocation of batches to routes, departure time of the vehicle that travels along each route and delivery times to customers. For reasons of space, the detailed formulation is not provided in this manuscript, but readers can request it from the authors.

4. COMPUTATIONAL STUDY

To evaluate the performance of the proposed formulation, six instances of different sizes were solved. A CPU time limit of 3600 seconds was established for each case. The examples were implemented in Python 3.9.12 and solved with GUROBIPY 10.0.0 solver on a PC with an Intel Core i7 processor, 3.60 GHz and 32 GB of RAM. Table 1 presents for each instance: the number of customers and time windows, the total placed orders, the number of production units, considered products, and, for each formulation, the optimality Gap and CPU time. For reasons of space, the data of the instances are not provided.

Instance	Customers	Time windows	Orders	Units	Products	TS-IP		GP-R	
						Gap [%]	CPU time [sec]	Gap [%]	CPU time [sec]
I0	4	2	8	2	3	0	20	0	5
I1	5	2	10	2	3	0	109	0	7
I2	6	2	12	2	3	0	2600	0	24
I3	8	2	16	2	3	-	-	0	72
I4	6	3	18	3	5	-	-	0	2097
I5	10	3	30	4	4	-	-	0	1695

Table 1: Comparative results of both formulations for all instances

I0, I1 and I2 instances show that the proposed model has a better performance than the one previously presented in [6]. In addition, the last three instances evidence that larger study cases could be optimally solved, while no feasible solution was found using the TS-IP formulation in 3600 seconds.

Finally, Figure 1 shows the Gantt chart that illustrates the optimal batching, production and distribution scheduling for instance I4. The diagram is divided into two parts: the upper part shows the assignment and sequencing of the batches in the three considered units, as well as the assignment of batches to vehicles. In the middle, the three time windows are delimited in color red and. In the bottom, the routes of the seven vehicles required are detailed. To understand the information shown in the picture, unit u_1 and route r_{801} are explained as example. A total of twenty batches are processed in unit u_1: seven of product p_1, twelve of product p_3 and only one of product p_5. The batch b_1 of product p_1 is the first processed in unit u_1 and is completely assigned to route r_{1046} (route referenced with blue color), then batch b_1 of product p_3 is consecutively processed and completely assigned to the same route. In the case of batch b_2 of product p_3, it is assigned partially to routes r_{1046} and r_{417} (vehicle referenced with emerald green color). For distribution, the lower part shows the route of each of the seven vehicles used in the optimal solution. The route r_{801} (colored in light blue) travels with its 98.3% of full vehicle capacity. Once the batches assigned to this route are processed, 25 minutes are required to load the products. The vehicle leaves from the plant at time 11.7 hours, delivers the order to customer i_4 for time window d_2 at time 13 hours, travels to

customer i_1 and unloads the order for time window d_3 at time 14.1 hours and finishes its route visiting customer i_6 and delivering the order for time window d_3 at time 15.1 hours.

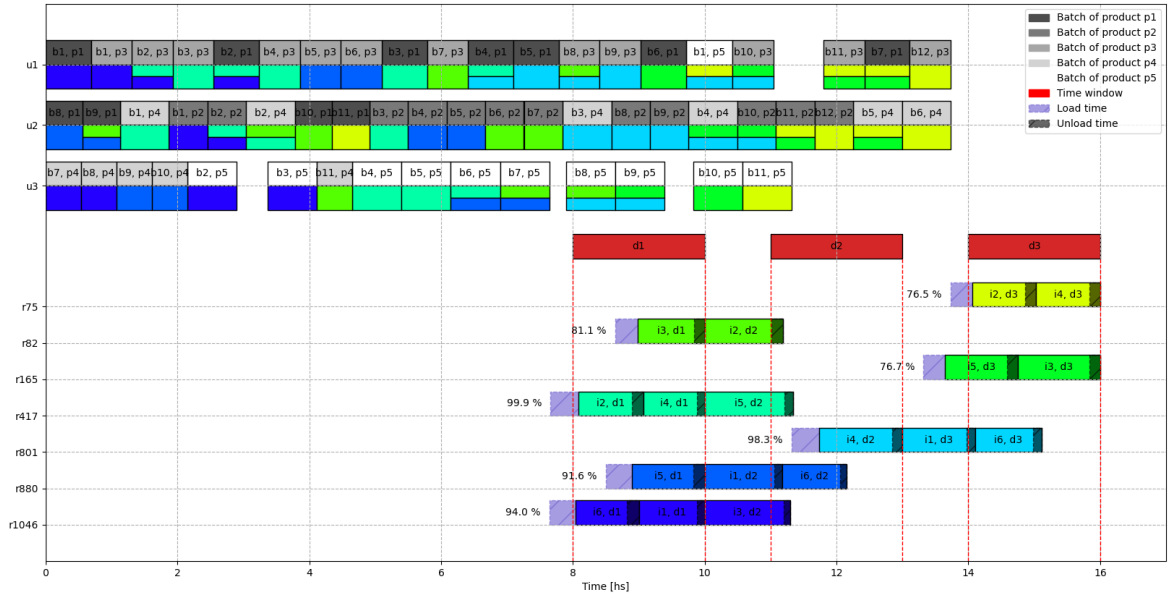


Figure 1: Production and distribution schedule for instance I4

5. Conclusions

A significant contribution of this paper is the development of a new MILP model that allows the simultaneous integration of batching, production scheduling and distribution decisions in a single-stage multi-product batch plant with multiple non-identical units operating in parallel. The most novel feature of the proposed approach is that unfeasible routes are eliminated before solving the MILP. Then, the model selects the optimal set of routes to satisfy all orders considering only among feasible ones. Based on cases of study of different scales, the new formulation is more efficient compared to [6]. In addition, cases of study of medium-large scale were addressed with good CPU times.

As future work, the possibility of including a multi-plant structure, as well as a heterogeneous fleet of vehicles will be analyzed. In addition, given that many plants have their own fleet of vehicles, the possibility that the same vehicle can make more than one trip will be considered in future research.

References

[1] S. Moons, K. Ramaekers, A. Caris and Y. Arda, *Integrating production scheduling and vehicle routing decisions at the operational decision level: A review and discussion*, Computers & Industrial Engineering, 104 (2017), pp. 224–245.

[2] R. Kumar, L. Ganapathya, R. Gokhaleb and M. K. Tiwari, *Quantitative approaches for the integration of production and distribution planning in the supply chain: a systematic literature review*, International Journal of Production Research, 58 (2020), pp. 3527-3553.

[3] D. Naso, M. Surico, B. Turchiano, U. Kaymak, *Genetic algorithms for supply-chain scheduling: A case study in the distribution of ready-mixed concrete*, European Journal of Operational Research, 177(3) (2007), pp. 2069-2099.

[4] P. Farahani, M. Grunow, H. O. Günther, *Integrated production and distribution planning for perishable food products*, Flexible Services and Manufacturing Journal, 24(1) (2012), pp. 28-51.

[5] A. Robbes, Y. Kergosien, V. André, J. C. Billaut, *Efficient heuristics to minimize the total tardiness of chemotherapy drug production and delivery*, Flexible Services and Manufacturing Journal, 34 (2022), pp. 785–820.

[6] A. S. Tibaldo, J. M. Montagna, Y. Fumero, *Programación integrada de la producción y la distribución en una planta batch multiproducto*, Proc. VII Matemática Aplicada, Computacional e Industrial, 8 (2021), pp. 485-488.