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Magnetic seed and cosmology as quantum hall effect

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ABSTRACT

In the framework of a bimetric model, we discuss a relation between the (modified) Friedmann equations and a mechanical system similar to the quantum Hall effect problem. Firstly, we show how these modified Friedmann equations are mapped to an anisotropic two-dimensional charged harmonic oscillator in the presence of a constant magnetic field, with the frequencies of the oscillator playing the role of the cosmological constants. This problem has two energy scales leading to the identification of two different regimes, namely, one dominated by the cosmological constants, with exponential expansions for the scale factors, and the other dominated by a magnetic seed, which would be responsible for both a component of dark energy and a primordial magnetic field. The latter regime would be described by a (nonperturbative) mapping between the cosmological evolution and the quantum Hall effect.

The standard description of the Universe rests on the cosmological principle, which states that, on large scales, space-time is homogeneous and isotropic. The observations are consistent with this hypothesis for distances above 100 Mpc [1,2]. But this mathematical idealization, which greatly simplifies the physical interpretation of the model, has limitations for lower scales. In particular, the formation of structures can only be understood after the occurrence of some gravitational instability due to tiny deviations from a homogeneous distribution [3,4].

These departures from the cosmological principle can be observed, for example, in the spectrum of the cosmic microwave background (CMB), which presents temperature fluctuations of the order of 10^{-5}, showing that corrections to classical cosmology can be incorporated via perturbations [5].

However, one might wonder if there are other phenomena of cosmological interest that might require a non-perturbative analysis. This possibility is particularly relevant since, in many fields of physics, there are problems that are perturbative or non-perturbative depending on the range of parameters one is considering. As an example, one can consider a gas of charged particles subject to a magnetic field perpendicular to the plane. If the magnetic field is strong enough, the system presents the quantum Hall effect, with a Hamiltonian spectrum that can not be perturbatively obtained from that of the free case.

This simple example could also be translated into the cosmological regime by noting that in the center of galaxies there are strong magnetic fields which are observed through the Zeeman's splitting they produce. Although the origin of these magnetic fields is at present unknown, the idea that a very small magnetic seed was formed in an early epoch of the universe evolution and that, after a dynamo mechanism, the field grew up to what is observed today in galaxies is widely accepted [6–10,25]. Our present knowledge does not allow us to determine when these magnetic seeds were created, but one can speculate that they might have been originated in the small inhomogeneities existing before the recombination epoch.

Very probably the primordial magnetic fields did not produce any relevant effect after the recombination, but these could be important in the first 100.000-years and eventually to affect the big-bang nucleosynthesis, the dynamics of the phase transitions and even baryogenesis and leptogenesis [11].

The magnetic seed must satisfy two consistency requirements. The first one is that the coherence length is not larger than about 10 kpc, and the second one is that the field in the magnetic seed must be between 10^{-19} and 10^{-22} G. In the analogous Hall system we discuss below, the coherence length corresponds to the magnetic depth ℓ_B [1] [12], that is,

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[1] Here we use natural units and $e = 1$ [13].

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$$\ell_B = \frac{1}{\sqrt{B}} < 10 \text{ kpc}. \quad (1)$$

The second condition is necessary for the stability of the dynamo mechanism [8–10].

A central issue not solved so far is how to provide the cosmological standard model with a mechanism that incorporates a magnetic seed as a fundamental element [14]. Any possible answer to this question requires extra new ideas in a model that satisfies all constraints known so far and that incorporates the magnetic field as a central element.

In this direction and using arguments coming from the formation of primordial magnetic fields [8–10] (we say for $t \sim 10^6$ years), the mechanism proposed here should work.

The purpose of this paper consists in investigating the possible emergence of magnetic seeds in a model with two metrics with an effective interaction between them. This interaction can be considered as a relic of a causal primordial connection between sectors in a very early epoch of the Universe. This problem is considered in the context of a simple mechanical system that nevertheless reproduces the Friedmann's equations of the two interacting sectors. We emphasize that the important issue is not only the existence of a mapping between these apparently unrelated systems but also that the same mechanism contributes to the production of dark energy.

Interestingly, no matter how different the dark energy and magnetic seed scales might be since in the present approach both are linked through a dynamical mechanism which (see Eq. (11)) allows to fix them in a rather independent way.

In order to develop this idea let us consider the Lagrangian[2]

$$L = \frac{1}{2N}\left(\dot{x}_1^2 + \dot{x}_2^2\right) - \frac{N}{2}\left(\omega_1^2 x_1^2 + \omega_2^2 x_2^2\right) - \frac{\theta}{2}\left(x_1 \dot{x}_2 - \dot{x}_1 x_2\right). \quad (2)$$

Here x_1 and x_2 are the dynamical variables, the coefficients ω_1, ω_2 and θ are constants, and $N = N(t)$ is an auxiliary variable that transforms as $N(t) \to t'(s)N(t(s))$ when $t \to t(s)$, thus ensuring the invariance of the action under time reparametrizations. This Lagrangian yields the following Hamiltonian

$$H = \frac{N}{2}\left[p_1^2 + p_2^2 + \left(\omega_1^2 + \frac{\theta^2}{4}\right)x_1^2 + \left(\omega_2^2 + \frac{\theta^2}{4}\right)x_2^2 + \theta\left(x_1 p_2 - x_2 p_1\right)\right]. \quad (3)$$

This Hamiltonian describes an anisotropic two-dimensional charged harmonic oscillator with frequencies ω_1 and ω_2, interacting with a constant magnetic field.

The Hamiltonian equations of motion for (3) are

$$\dot{x}_i = [x_i, H], \quad \dot{p}_i = [p_i, H],$$

where [,] is the Poisson bracket, with the standard structure for the canonical variables, that is $\left[x_i, p_j\right] = \delta_{ij}$ and zero for the remaining brackets. Alternatively, one can define the new variables $\pi_i = p_i - \frac{\theta}{2}\epsilon_{ij}x_j$ and rewrite the Hamiltonian $H = H(x_i, \pi_j)$ in order to obtain the equations of motion

$$\dot{x}_i = [x_i, H], \quad \dot{\pi}_i = [\pi_i, H], \quad (4)$$

but with the following Poisson brackets

$$\left[x_i, x_j\right] = 0, \quad \left[x_i, \pi_j\right] = \delta_{ij}, \quad \left[\pi_i, \pi_j\right] = \epsilon_{ij}\theta. \quad (5)$$

The equations of motion, once the momenta are eliminated, reduce to

$$\ddot{x}_1 + \omega_1^2 x_1 + \theta \dot{x}_2 = 0, \quad (6)$$

$$\ddot{x}_2 + \omega_2^2 x_2 - \theta \dot{x}_1 = 0, \quad (7)$$

$$\dot{x}_1^2 + \dot{x}_2^2 + \omega_1^2 x_1^2 + \omega_2^2 x_2^2 = 0. \quad (8)$$

The constraint (8) is a consequence of time reparametrization invariance and, at the end of the derivation, the gauge $N \equiv 1$ has been chosen.

Notice that this constraint – from the point of view of the second order differential equations (6) and (7) – is in fact a relation between initial conditions since the left hand side is a constant of the motion. Indeed, multiplying (6) by $\dot{x}_1$ and (7) by $\dot{x}_2$, and adding both equations, we immediately find that

$$\frac{d}{dt}\left[\dot{x}_1^2 + \dot{x}_2^2 + \omega_1^2 x_1^2 + \omega_2^2 x_2^2\right] = 0. \quad (9)$$

The physical solutions correspond to those for which the constant $\dot{x}_1^2 + \dot{x}_2^2 + \omega_1^2 x_1^2 + \omega_2^2 x_2^2$ vanishes.

One of the goals of this paper is to point out the following remarkable mapping. If we redefine the variables x_1, x_2 as follows,

$$x_1 = \frac{2}{3}a^{3/2}(t), \qquad x_2 = \frac{2}{3}b^{3/2}(t), \quad (10)$$

and replace them in (6)–(7), the resulting equations turn out to be

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{4}{3}\omega_1^2 = -2\theta\sqrt{a\,b}\,\frac{\dot{b}}{a^2}, \quad (11)$$

$$2\frac{\ddot{b}}{b} + \left(\frac{\dot{b}}{b}\right)^2 + \frac{4}{3}\omega_2^2 = 2\theta\sqrt{a\,b}\,\frac{\dot{a}}{b^2}, \quad (12)$$

$$a^3\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{2}{3}\omega_1\right)^2\right] = -b^3\left[\left(\frac{\dot{b}}{b}\right)^2 + \left(\frac{2}{3}\omega_2\right)^2\right]. \quad (13)$$

These equations are identical to the Friedmann equations for a cosmology with two metrics[3] if we identify their respective cosmological constants Λ_1 and Λ_2 as

$$-\omega_1^2 \longleftrightarrow \frac{3}{4}\Lambda_1, \qquad -\omega_2^2 \longleftrightarrow \frac{3}{4}\Lambda_2. \quad (14)$$

In fact, Eqs. (11)–(13) form a coupled system of nonlinear second order differential equations for the scale factors $a(t)$ and $b(t)$, where the right hand sides of (11)–(12) can be considered as sources of dark energy (see [17] for a discussion on a similar system and for string theory see [18]). Moreover, from these equations one can read off the effective pressure and density contributions induced by the coupling between scale factors. Indeed, expressing the Friedmann equations for the scale factor $a(t)$ in terms of the pressure p_b and energy density ρ_b of an additional component of "dark energy", from Eqs. (11) and (13) one obtains the equivalence

$$8\pi G\, p_b = -2\theta\sqrt{a\,b}\,\frac{\dot{b}}{a^2},$$

$$\frac{8\pi G}{3}\rho_b = -\frac{1}{a^3}\left(\frac{4}{9}\omega_2^2 b^3 + \dot{b}^2 b\right).$$

This leads to the following equation of state for the effective component of dark energy,

[2] The approach proposed here is valid for any number of patches, however for simplicity in the presentation we will restrict ourselves to two of them.

[3] The literature of cosmology with two metrics is very extensive, see for example [15] and [16].

$$\rho_b + \frac{6\pi G}{\theta^2} p_b^2 = \frac{\Lambda_2}{8\pi G}\left(\frac{b}{a}\right)^3 . \tag{15}$$

For the case $\Lambda_2 = 0$, the dark energy so described turns out to be a generalized Chaplygin gas [19–23]. Notice that this is a non-perturbative result, valid for any $\theta \neq 0$.

Now we can use the mapping (10) to solve the Friedmann equations. Equations (6)–(7) form a system of two coupled linear second order differential equations which consequently have four linearly independent solutions. The latter have the form

$$\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} i\Omega\theta \\ \Omega^2 - {\omega_1}^2 \end{pmatrix} e^{i\Omega t}, \tag{16}$$

where the frequency Ω takes one of the four values

$$\Omega_{\pm,\pm} = \pm\frac{1}{\sqrt{2}}\left\{\omega_1^2 + \omega_2^2 + \theta^2 \pm \sqrt{\left[\omega_1^2 + \omega_2^2 + \theta^2\right]^2 - 4\omega_1^2\omega_2^2}\right\}^{1/2} . \tag{17}$$

The general solution of Eqs. (6)–(8) is an arbitrary linear combination of these four functions with coefficients c_1, c_2, c_3, c_4.

To get a solution of our problem, we must also impose the constraint (8). Since the left hand side of (8) is proportional to the Hamiltonian (written in terms of coordinates and velocities), it is a constant real symmetric quadratic form in c_1, c_2, c_3, c_4 (but not positive definite for the ω's given in Eq. (14)). The constrained solutions we are looking for correspond to the isotropic vectors of this quadratic form.[4]

The mapping (10) allows us first, to understand the evolution of the scale factors $(a(t), b(t))$ under the previously described interaction through the knowledge of the evolution of a mechanical system of two degrees of freedom $(x_1(t), x_2(t))$, and second, to describe the system by means of the Hamiltonian in Eq. (3).

It is interesting to note that the limit $\omega_{1,2}^2 \to 0$ does not eliminate the causal connection between metrics since, in this case, the Hamiltonian (3) reduces to

$$H = \frac{N}{2}\left(p_1^2 + p_2^2 + \frac{\theta^2}{4}\left(x_1^2 + x_2^2\right) + \theta\,(x_1 p_2 - x_2 p_1)\right).$$

This can also be written as

$$H = \frac{1}{2}\left[\frac{\theta^2}{4}\left(\bar p_1^2 + \bar p_2^2\right) + \left(\bar x_1^2 + \bar x_2^2\right) - \theta\,(\bar x_1 \bar p_2 - \bar x_2 \bar p_1)\right]. \tag{18}$$

Here we have rescaled variables as $x_i = 2\bar x_i/\theta$, and $p_i = \theta\,\bar p_i/2$, with $i = 1, 2$, and changed $\theta \to -\theta$ to obtain the Hamiltonian considered in [24] in the context of noncommutative quantum mechanics.

Let us remark that the region where

$$2|\omega_{1,2}| \ll |\theta| \tag{19}$$

is similar to the strong magnetic field regime in the quantum Hall effect. In terms of cosmological constants this region corresponds to

$$\sqrt{3|\Lambda_{a,b}|} \ll |\theta|, \tag{20}$$

which can be called the *cosmological Hall regime*.

From the quantum mechanical point of view, the system described by the quantized Hamiltonian (3) is particularly interesting because this implies replacing the Poisson brackets (5) by the commutators (with $\hbar = 1$)

$$\left[\hat x_i, \hat x_j\right] = 0, \quad \left[\hat x_i, \hat\pi_j\right] = i\,\delta_{ij}, \tag{21}$$

$$\left[\hat\pi_i, \hat\pi_j\right] = i\,\epsilon_{ij}\theta, \tag{22}$$

where $\hat\pi_i = \hat p_i - \frac{\theta}{2}\epsilon_{ij}\hat x_j$, with $\hat p_i$ the canonical momentum operator. For $\theta \neq 0$, the commutator in Eq. (22) induces entangled states for $(x_1(t), x_2(t))$ and, therefore, for the two metrics of our model, represented by the scale factors $(a(t), b(t))$.

The commutator (22) implements non-local communication between different spacetime regions, equivalently, entanglement states.

In addition, we note that in the problem at hand we have three energy scales, $\sqrt{|\Lambda_a|}$, $\sqrt{|\Lambda_b|}$, and $\sqrt{|\theta|}$. This allows us to identify two regimes of cosmological interest, namely

(i) $\sqrt{|\Lambda_i|} \gg \sqrt{|\theta|}$, which corresponds to a cosmological-constant dominated era in which each metric evolves independently with no effective interaction, showing an exponential behavior and making the corresponding side of Eq. (13) to vanish;

(ii) $\sqrt{|\Lambda_i|} \ll \sqrt{|\theta|}$ which, by analogy, could be interpreted as the magnetic-seed dominated era, which eventually would be responsible for the existence of the magnetic fields in the universe.

The quantum description of these two regimes, which could be relevant in a very early epoch of the Universe evolution, is very different. In regime (i) the system is formally described (in the gauge $N \equiv 1$) by the Hamiltonian operator of an anisotropic harmonic oscillator. On the other hand, in regime (ii) the equivalent mechanical system is exactly a Landau problem, whose eigenstates are given by

$$\psi_{n_+,n_-}(x_1, x_2) = e^{\frac{\theta^2}{4}(x_1^2+x_2^2)}\left(\frac{\partial}{\partial x_1} + i\frac{\partial}{\partial x_2}\right)^{n_+} \times \left(\frac{\partial}{\partial x_1} - i\frac{\partial}{\partial x_2}\right)^{n_-} e^{-\frac{\theta^2}{4}(x_1^2+x_2^2)}.$$

The corresponding energy eigenvalues

$$\psi_{n_+,n_-} = \theta\,(2n_- + 1)$$

do not depend of n_+, leading to an infinitely degenerate Hamiltonian spectrum.

This regime would be responsible for inducing both a component of dark energy [17] and traces of magnetic fields that would subsequent grow. In this sense, one might attribute both effects to a quantum origin of the Universe.

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