Structural Safety 77 (2019) 10–17

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Dynamic behaviour of a timber footbridge with uncertain material properties under a single deterministic walking load

Diego A. García[a,b,*], Marta B. Rosales[c,d], Rubens Sampaio[e]

[a] Department of Civil Engineering, Universidad Nacional de Misiones (UNaM), Juan Manuel de Rosas 325, 3360 Oberá, Argentina
[b] Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina
[c] Department of Engineering, Universidad Nacional del Sur (UNS), Av. Alem 1253, 8000 Bahía Blanca, Argentina
[d] Instituto de Física del Sur (IFISUR), UNS-CONICET, Av. Alem 1253, 8000 Bahía Blanca, Argentina
[e] Department of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro (PUC-Rio), Rua Marquês de São Vicente, 225 22453-900 Rio de Janeiro, Brazil

ARTICLE INFO

Keywords:
Timber footbridge
Uncertain properties
Walking load models
Structural dynamics

ABSTRACT

The dynamic study of a short span timber footbridge with uncertain mechanical properties under the action of a deterministic walking load model which represents one individual is presented. The aim of the work is to quantify the influence of the uncertain material properties in the natural and forced vibration problems of a short span timber footbridge. These structural systems made of timber are increasingly employed due to the high stiffness/weight ratio that this material exhibits. This feature can lead to lightweight structural systems in which the acceleration levels can exceed the human comfort limits. The assumed sources of uncertainties of this structural model are the Modulus of Elasticity (MOE) and the mass density. Also, the geometrical design of the boards that compose the laminated timber beams supporting the floor involves variability of the distances between finger joints. Probability Density Functions (PDFs) of the timber properties are formulated from the Principle of Maximum Entropy (PME). The finger joints distance generates the lengthwise variability of the MOE and the mass density in each board of the laminated beams. The PDFs of the natural frequencies of the structure, the mode shapes and the structural response are numerically obtained through the Finite Element Method (FEM) and statistic tools. In order to carry out this analysis, plate and laminated beam elements derived from the First Order Shear Deformation Theory (FSDT) are employed. The variation of the effective stiffness and mass along the elements due to the union of boards with different properties produces an important reduction of the standard deviation of the natural frequencies. The conditions in which the stochastic properties of the structure lead to unacceptable acceleration levels are studied. In some scenarios, the variation of the material properties can lead to unacceptable serviceability performances of the structure and a reduction in the pedestrian comfort, according to the codes in force. However, new design guidelines replace the line limits by comfort regions which are also discussed here.

1. Introduction

Footbridges are one of the most common timber structures mainly due to the high stiffness/weight ratio that this material presents in comparison to other construction materials and the possibility of covering long spans owing to the development and implementation of laminated beams. This type of beams has become greatly employed, and has allowed the construction of slender timber structures. In this work, the complete structure is made of Argentinian *Eucalyptus grandis*, one of the most important renewable species cultivated in Argentina. A simple method for visually strength grading sawn timber of these species has been developed by Piter [1]. As reported in this thesis, the presence of pith and knots are considered the most important visual characteristic for strength grading this material by the Argentinian standard IRAM 9662-2 [2]. Experimental studies related with the bending strength and stiffness in *Eucalyptus grandis* laminated beams have been presented by Piter et al. [3] and Saviana et al. [4]. In these works, the high stiffness/weight ratio of the structural elements is highlighted.

Due to its natural origin, structural timber is characterized by considerable variability of its mechanical properties. Then, it is apparent that a stochastic approach becomes desirable in order to attain a more realistic structural model. The stochastic approaches employed

* Corresponding author.
E-mail address: garciadiego@fio.unam.edu.ar (D.A. García).

https://doi.org/10.1016/j.strusafe.2018.11.001
Received 3 April 2018; Received in revised form 23 August 2018; Accepted 5 November 2018

for the modeling of timber mechanical properties are derived from the probabilistic theories of random variables and processes. They allow simulating the timber mechanical properties within a structural analysis. A probabilistic model of timber structures where the MOE is represented as a random variable with a lognormal PDF and the mass density through a random variable with normal distribution, both assuming a homogeneous value within a structural element, was presented in Köhler et al. [5]. In Casciati and Domaneschi [6], a random field of the timber beam imperfections is introduced into the finite element model of beams. Then, the tensile strength and stiffness properties of timber boards and finger joint connections are modeled through a probabilistic approach in Fink and Köhler [7]. Brandner and Schickofer [8] use probabilistic models for the MOE and the shear modulus considering serial and parallel systems to represent timber elements. The stochastic modeling of Argentinean *Eucalyptus grandis* timber beams is presented in García et al. [9] and García and Rosales [10]. In these works, the random field of the Modulus of Elasticity (MOE) and the mass density are modeled through a gamma random variable based on tests of fit and the application of the Principle of Maximum Entropy (PME).

Timber footbridges must satisfy strength and serviceability requirements. Generally, due to their low weight, the serviceability requirements in terms of peak accelerations constitute the most restrictive condition in their design. An extensive literature review and state of the art report of the dynamic behavior of footbridges is presented in Živanović et al. [11]. Among other topics, the loads models, standards requirements and studies of the walk of people and crowns are reported. The study of the vertical vibrations in existing footbridges of concrete and steel simulated in a Finite Element Method (FEM) software are presented in Da Silva et al. [12] and Figueiredo et al. [13]. Two deterministic load models were employed in these works, the standard one through a Fourier series and a model that includes the heel impact effect over the deck. An evaluation of the serviceability of eight footbridges with the most recent codes in practice is presented in Van Nimmen et al. [14]. In this work, a modified load model that leads to a more robust serviceability evaluation is reported. Numerical works related to the study of the dynamic behavior of timber footbridges are not frequent despite being a common timber structure. However and related with timber floor vibration, recently in Casagrande et al. [15] the effectiveness of different assessments (analytical, numerical and experimental) for the evaluation of vibration performance is presented. In Casciati et al. [16], a model for the stochastic fields that represent the forces induced by the walking of a small group of pedestrians, is presented. A numerical example is solved with a finite element model of an existing wood footbridge with deterministic material properties. Analogously, few works address the uncertainty in the material properties of the footbridges. The dynamic analysis of a footbridge with uncertain structural parameters (density and modulus of elasticity), subjected to stochastic impact load is presented in Rama Rao et al. [17]. The effectiveness of the direct optimization approach based on the fuzzy finite element method and adaptive Taylor methods in the evaluation of the dynamic response of structures with multiple uncertainties, is demonstrated.

The aim of the present work is to quantify the influence of the uncertain material properties in the natural and forced vibration problems of a short span, simply supported timber footbridge made of Argentinian *Eucalyptus grandis*. To the authors' best knowledge, this structural configuration is extensively employed but not adequately studied in the literature, and in particular, the uncertainties of the timber material properties are not adequately considered. The sources of uncertainties in the structural model herein presented are assumed in the timber mechanical and physical properties, and in the geometrical design of the layers that compose the laminated timber beams that support the floor of the footbridge. This geometric uncertainty involves the distances between finger joints which were obtained from visual survey of structural size *Eucalyptus grandis* laminated beams. Then, the Probability Mass Function of the distance between finger joints is constructed. The Probability Density Functions (PDFs) of the MOE and the mass density are obtained through the application of the Principle of Maximum Entropy (PME) proposed by Shannon [18] and Jaynes [19]. In order to measure the fit between the experimental and theoretical PDFs of the MOE and the mass density, tests of fit were used. Between the pieces of timber, defined by the distance among fingers joints, the properties vary stochastically and in a non-correlated way. According to the Argentinean standard IRAM 9662-2 [2], each board of the laminated beams comes from a specific strength class (C1 or C2). Within this quality class, the properties vary stochastically. For the propagation of the uncertainties, a numerical model of the structure is obtained through the Finite Element Method (FEM) employing laminated beams and plate elements [20] with the transversal isotropy assumption for the timber boards. Then, the propagation of these sources of uncertainty in the first natural frequency of the footbridge that constitute one of the evaluation parameters of the serviceability performance, is studied. To accomplish this, the PDF of the first natural frequencies is found via Monte Carlo simulations [21]. An important reduction in the standard deviation of the natural frequency is reported due to the implementation of random finger union distances. The second important parameter in the study of the serviceability performance is the acceleration. Two deterministic load models [12,13] are employed for the study of the peak accelerations. It is expected that the uncertainty introduced by loads on structures be higher than the uncertainty in the material properties. However, in this work, the focus is placed in the uncertain material properties of structural elements made of Argentinean *Eucalyptus grandis*, which have a higher degree of randomness in comparison with other structural materials. For this reason, deterministic load models are employed. The conditions in which the stochastic properties of the structure lead to unacceptable acceleration levels are reported according to the codes in force. In some scenarios, the variation of the material properties can lead to an unacceptable serviceability performance of the structure and a reduction in the pedestrian comfort, according to the codes in force. However, new design guidelines replace the line limits by comfort regions which are also discussed here.

2. Problem statement

The study of the vertical dynamical behavior of a short span timber footbridge with stochastic mechanical properties under walking loads is presented in this work. The structure is composed of three laminated beams simply supported with a span-wise length of 13.2 m and a separation of 0.6 m in the transversal direction of the structure, five transversal laminated beams with a separation of 3.3 m in the longitudinal direction of the structure and a deck of timber boards. The width of the laminated beams and the timber boards of the deck was fixed in 0.15 m, and the height of each lamina of the beams and of the timber boards, in 0.0375 m. The total height of the beams was dimensioned for strength resistance according to the Argentinean standard CIRSOC 601 [22] and for serviceability requirements. The latter constitutes the main factor of the structural design analyzed later in this work. The structure dimensions constitute reference values which are frequent in short span timber footbridges. The number of layers of the beams was considered equal to 16, laminated beams with 0.6 m of height (Fig. 1).

3. Timber Material

3.1. Elastic model

The material model derived from the general assumption of orthotropy in which each board has a longitudinal direction parallel to the main material fibres direction associated with the x axis, a tangential direction with respect to the growth rings of the transversal section

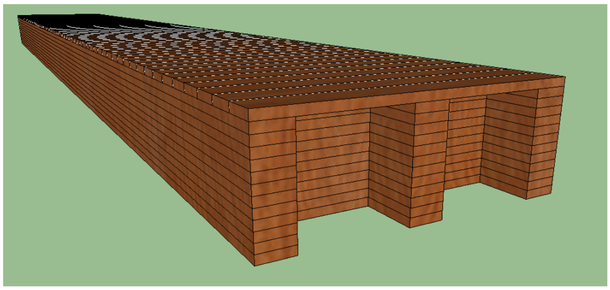


Fig. 1. Timber footbridge model.

associated with the y axis, and finally a radial direction with respect to the growth rings of transversal section associated with the z axis (Eq. (1)).

$$\begin{Bmatrix} \varepsilon_L \\ \varepsilon_R \\ \varepsilon_T \\ \gamma_{LR} \\ \gamma_{LT} \\ \gamma_{RT} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{RL}}{E_R} & -\frac{\nu_{TL}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LR}}{E_L} & \frac{1}{E_R} & -\frac{\nu_{TR}}{E_T} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & -\frac{\nu_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{LR}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} \end{bmatrix} \begin{Bmatrix} \sigma_L \\ \sigma_R \\ \sigma_T \\ \tau_{LR} \\ \tau_{LT} \\ \tau_{RT} \end{Bmatrix} \tag{1}$$

In this work, the orthotropic model (Fig. 2) is reduced to the transversal isotropic model with two main directions: the longitudinal (L) also named parallel to the main fibres direction $E_L = E_x$, and the perpendicular direction that includes the radial (R) and tangential (T) material direction $E_R = E_T = E_{yz}$. The basic stochastic properties proposed in this work are the longitudinal MOE (E_x) and the mass density (ρ). In what follows, these stochastic properties will be noted as $\mathbb{E}_x$ and $\mathbb{P}$, respectively. For a transversally isotropic material, the elastic and shear modulus are defined as $\mathbb{E}_{zy} = \frac{\mathbb{E}_x}{15}$, $\mathbb{G}_{xy} = \mathbb{G}_{xz} = \frac{\mathbb{E}_x}{16}$ [23]. Meanwhile, in a general form the Poisson coefficients for hardwood are $\nu_{RT} = 0.67$, $\nu_{LT} = 0.46$ and $\nu_{LR} = 0.39$ [24]. For a transversally isotropic formulation $\nu_{zy} = \nu_{RT} = 0.67$ and $\nu_{xzy} = \frac{\nu_{LT} + \nu_{LR}}{2} = 0.425$ and $\mathbb{G}_{zy} = \frac{\mathbb{E}_{zy}}{2(1+\nu_{zy})}$.

3.2. MOE and mass density stochastic representation

If a stochastic approach is applied to this problem, first a PDF should be chosen for the random variable. A statistical concept of entropy was introduced by Shannon [18] and its maximization by Jaynes [19]. The Principle of Maximum Entropy (PME) states that, subjected to known constrains, the PDF which best represents the current state of knowledge is the one with largest entropy. The measure of uncertainties of a continuous random variable X is defined by the following expression:

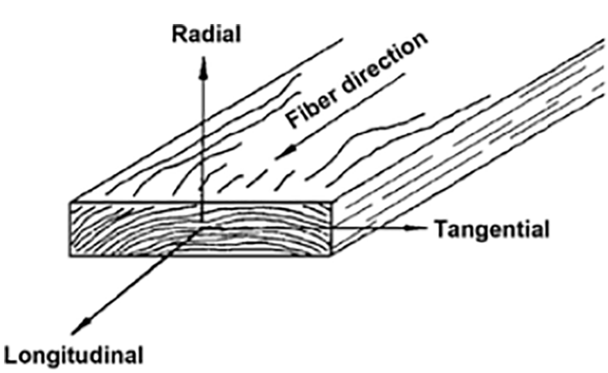


Fig. 2. Orthotrophy assumption of a wood board [25].

$$S(f_X) = -\int_D f_X(x) ln(f_X(x))dx \tag{2}$$

in which f_X stands for the PDF of the random variable X and D is its domain. It is possible to demonstrate that the application of the principle under the constraints of positiveness and bounded second moment, leads to a gamma PDF. The PME conduces to this PDF due to the fact that the domain of both the MOE and the mass density is real and positive.

To find the parameters of the marginal PDF of the MOE and mass density, experimental data presented by Piter [1] were employed. These values were obtained by means of two point load bending tests, performed with 349 sawn beams of Argentinean *Eucalyptus grandis* with structural dimensions and density measurements. Bending tests and density measurement were carried out according to the standard UNE-EN 408 [23]. Using these data, the parameters of the gamma marginal PDFs of the MOE and density are estimated with the help of the Maximum Likelihood Method (MLM). Then, the Kolmogorov-Smirnov (K-S) and the Andersson-Darling (A-D) tests of fit are used, (*e.g.* Benjamin and Cornell [26]). The PDF that best fits with the experimental values of the MOE is the gamma one, in agreement with the PME result. The test of fit was also carried out with the lognormal, normal and truncated normal PDFs, the first one proposed by Köhler et al. [5] to model the MOE and the second PDF, very often employed to represent mechanical properties. The normal PDF fits the experimental data best. However, the use of this PDF in the model would occasionally lead to negative values of the MOE which are physically unacceptable unless a truncated form is used. Thus, the gamma and lognormal PDF seem to be more suitable. On the other hand, for the density, the four PDFs fulfill the critical value, but the lognormal and gamma fit best with respect to the experimental values. Here, following the PME and due to the small difference found among the lognormal and gamma, the last PDF is adopted in order to introduce the mass density uncertainty in the stochastic model. The gamma marginal CDF of the MOE and mass density (García et al. [9]) is:

$$F(x|a, b) = \frac{1}{b^a\Gamma(a)} \int_0^x t^{a-1} e^{-\frac{t}{b}} dt \tag{3}$$

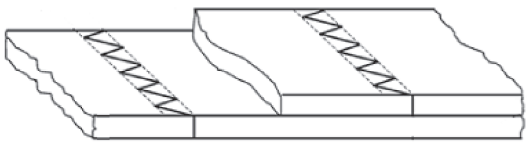
where a and b denote the shape and scale parameters, respectively. For the MOE, the parameters are $a = 34.582$ and $b = 0.402$ with a mean value of the MOE equal to 13.902 GPa and a standard deviation of 1.498 GPa. In the case of the mass density, $a = 72.179$ and $b = 7.659$, the mean value of the mass density equal to 552.819 kg/m^3 and a standard deviation of 65.069 kg/m^3.

3.3. Laminated beams

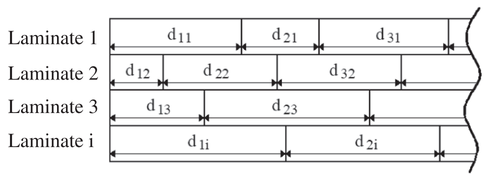
Laminated beams are composed of several layers formed by the union of boards with different mechanical properties. Upper and lower faces of the boars are glued to the superior and inferior continuous board. Previously, the boards of each lamina are assembled by finger joints union. The influence of the finger joints configuration in the natural frequencies will be studied assuming randomness. Distances between finger joint obtained from a visual survey of laminated beams were employed in order to simulate the different boards that conform a laminated beam. With the results of the survey, the Probability Mass Function (PMF) of the distance between fingers joints was constructed. The mean value and standard deviation of the distance between joints are 0.865 m and 0.247 m, respectively. The finger joint union is shown in Fig. 3a and an illustration of the distances between consecutive unions in each laminate obtained from the PMF in Fig. 3b. The simulated distances fulfill the requirements of the Argentinean standard IRAM:9660-1 [27].

4. Load models

Two deterministic load models that represent the effect of an



(a)Fingerjoints,IRAM:9660-1[27].



(b)Fingerjointunionsdistributionaccordingtothe PMF.

Fig. 3. Laminated beams design.

individual walking on the footbridge, are considered. These models were adopted because the aim of the present work is to quantify the influence of the uncertain material properties in the natural and forced vibration problems of a timber footbridge. For this reason deterministic load models are employed. Then, in future works, stochastic load models and diverse transit conditions will be considered.

In both models, the load position is changed according to the individual location along the footbridge. The first walking load model (WL1) is composed of the static load of the pedestrian weight and a combination of harmonic forces represented by a Fourier series:

$$F(t) = P[1 + \sum \alpha_i \cos(2\pi i f_s t + \Phi_i)] \quad (4)$$

where P is the pedestrian weight, here considered equal to 700 N, α_i is the dynamic coefficient of the harmonic force also called Dynamic Load Factor (DLF), i is the harmonic order, f_s is the step frequency and Φ_i the harmonic phase angle as listed in Table 1. This load model is frequently employed in the structural design standards for the verification of serviceability requirements [11,13,28].

The second load model (WL2) incorporates the human heel impact effect over the floor [13]. The formulation of this model is presented in the following equation:

$$F(t) = \begin{cases} \left(\frac{f_{mi}F_m - P}{0.04T_p}\right)t + P & \text{if} \quad 0 \leqslant t < 0.04T_p \\ f_{mi}F_m\left[\frac{C_1(t-0.04T_p)}{0.02T_p} + 1\right] & \text{if} \quad 0.04T_p \leqslant t < 0.06T_p \\ F_m & \text{if} \quad 0.06T_p \leqslant t < 0.15T_p \\ P\left\{1 + \sum_{i=1}^{nh} \alpha_i \sin[2\pi i f_s(t+ \right. & \\ \left. 0.1T_p) - \Phi_i]\right\} & \text{if} \quad 0.15T_p \leqslant t < 0.90T_p \\ 10(P - C_2)\left(\frac{t}{T_p} - 1\right) + P & \text{if} \quad 0.90T_p \leqslant t < T_p \end{cases} \quad (5)$$

Table 1
Forcing frequencies f_s, dynamic coefficients α_i and harmonic phase angle Φ_i. Deterministic load models 1 and 2, Figueiredo et al. [13].

Harmonic i	if_s Hz	α_i	Φ_i	
			WL1	WL2
1	1.6–2.2	0.5	0	0
2	3.2–4.4	0.2	$\pi/2$	$\pi/2$
3	4.8–6.6	0.1	$\pi/2$	π
4	6.4–8.8	0.05	$\pi/2$	$3\pi/2$

Table 2
Human walking characteristics.

Activity	Step distance (m)	Step frequency (Hz)
Slow walking	0.6	1.60–1.85
Normal walking	0.75	1.85–2.15
Fast walking	0.9	2.15–2.30

$$F_m = P\left(1 + \sum_{i=1}^{nh} \alpha_i\right), \quad C_1 = \left(\frac{1}{f_{mi}} - 1\right),$$
$$C_2 = \begin{cases} P(1 - \alpha_2) & \text{if} \quad nh = 3 \\ P(1 - \alpha_2 + \alpha_4) & \text{if} \quad nh = 4 \end{cases} \quad (6)$$

where F_m is the maximum value of the Fourier series, f_{mi} is the heel impact factor (here, 1.12), T_p is the step period, C_1 and C_2 are force coefficients. This model has the same parameters than the first model, but changes the harmonic phase angle Φ_i (Table 1). The walking velocity, step distance and frequency considered in this work are depicted in Table 2. Time and frequency representations of both load models are shown in Fig. 4.

5. Finite element discretization

The application of the Hamilton's principle and the discretization of the system lead to the usual matricial expression:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (7)$$

in which $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the global matrices of mass, damping and stiffness, respectively; $\mathbf{f}(t, x)$ is the global nodal forces vector and $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are the global vectors of nodal accelerations, velocities and displacements, respectively. The equation of motion is discretized for laminated beams using Timoshenko beam elements with two nodes and three degrees of freedoms per node. Cubic and quadratic shape functions for the translational and rotational degrees of freedom and a lineal shape functions for the torsional one, are employed [20]. The torsional stiffness are obtained from Swanson [29] considering the transversal isotropy of the layers. Rotational, translational and torsional inertial terms are taken into account. The equation of motion is discretized for the footbridge deck by rectangular bilinear plate elements with four nodes and three degrees of freedom per node [20]. The damping matrix in Eq. (7) is considered proportional to the stiffness matrix, with damping ratios equal to 5% and 7% in the first natural vibration mode. These values have been suggested by Chopra [30] for wood structures with nailed or bolted joints. Natural frequencies and mode shapes are obtained through the following equation:

$$[\mathbf{K} - \Omega_n^2\mathbf{M}]\Phi_n = 0 \quad (8)$$

in which Ω_n is the n^{th} natural circular frequency and Φ_n is the associated mode shape. Then and in order to obtain the dynamic response, the Modal Superposition Method is applied [30]. The nodal displacements vector is expressed as the product between the mode shape matrix and the vector of modal amplitudes $\mathbf{x}(t) = \Phi\mathbf{y}(t)$. Then, replacing the nodal displacements vector in Eq. (7):

$$\mathbf{M}\Phi\ddot{\mathbf{y}}(t) + \mathbf{C}\Phi\dot{\mathbf{y}}(t) + \mathbf{K}\Phi\mathbf{y}(t) = \mathbf{f}(t) \quad (9)$$

in which the components of nodal forces vector are defined as:

$$f_i(t) = \begin{cases} F(t) & \text{for} \quad t_0 \leqslant t \leqslant t_0 + t_d \\ 0 & \text{for} \quad t < t_0 \quad \text{y} \quad t > t_0 + t_d \end{cases}$$

where $F(t)$ is equal to the load function in the time domain, t_0 is the arrival time to the node i, $t_d = Le/v_p$ is the step time between nodes, v_p is the pedestrian velocity and L_e is the distance between nodes. For the adopted damping model, the damping relationship for each one of the considered vibration modes is expressed as $\zeta_n = (a/2)\Omega_n$ where a is

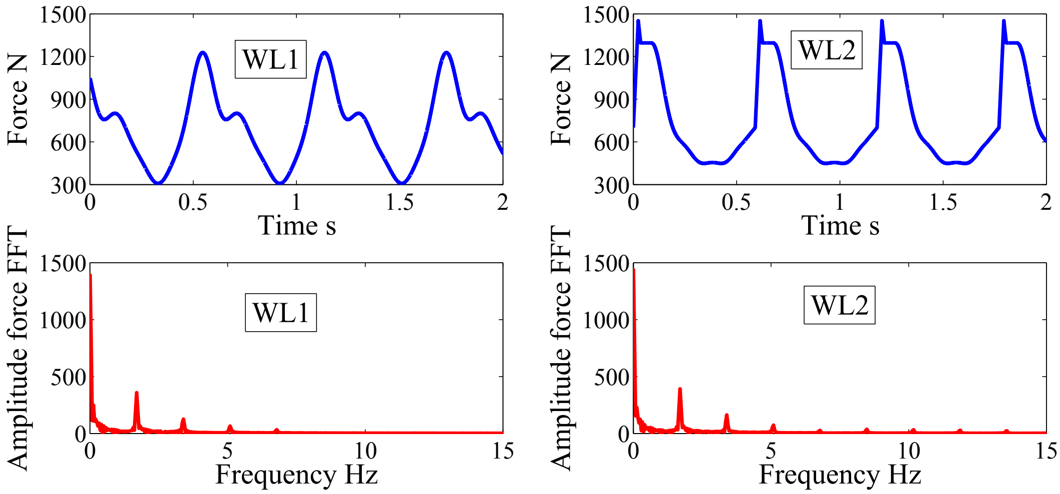


Fig. 4. Time and frequency representations of the walking force models. Walking load model 1 (WL1) and walking load model 2 (WL2).

obtained assuming ζ_n equal to 5–7% for the first vibration mode. Replacing $\mathbf{C} = a\mathbf{K}$ and multiplying the Eq. (9) for the transpose of the modal shape vector Φ_n and due to the orthogonality property of the mode shapes, we can obtain the equation of motion in generalized coordinates for each mode *n*:

$$M_n\ddot{y}_n(t) + aK_n\dot{y}_n(t) + K_n y_n(t) = f_n(t) \tag{10}$$

in which $f_n(t) = \Phi_n(v_p t)F(t)$ for the time between $t_0 \leqslant t \leqslant t_0 + t_d$. Applying the principle of effects superposition, the total Mid-Span (*MS*) accelerations induced for the total number of pedestrians (*NP*) considering *N* vibration modes is obtained from:

$$\ddot{x}_{MS}(t) = \sum_{i=1}^{NP}\left(\sum_{n=1}^{N}\Phi_{MS,n}\ddot{x}_n(t)\right) \tag{11}$$

6. Numerical results

6.1. Modal analysis

A convergence study of the first natural frequency was carried out due to the fact that the step harmonic, the step frequency and the step distance of the load model are considered in this section as functions of the first natural frequency of the footbridge. A number of 3000 independent Monte Carlo Simulations (MCS) is adopted for the stochastic study. The PDF of the first natural frequency F_1, $f(F_1)$ is shown in Fig. 5 for laminated beams with and without finger joint unions. As can be observed, the mean value remains equal in both cases, $\mu(F_1) = 6.78$ Hz while the standard deviation decreases in the first case, $\sigma(F_1) = 0.06$ Hz and $\sigma(F_1) = 0.12$ Hz respectively. This effect is based in the variation of the effective stiffness and mass properties along the beams, introduced by the stochastic model with finger joints union. This behavior was already reported by Brandner and Schickhofer [8] who represented the laminated beams as a system with serial and parallel elements.

6.2. Forced Vibration

According to the Argentinian standard CIRSOC 601 [22], when the first natural frequency of a timber structure is lower than 8 Hz, a dynamical study is needed for ensuring the serviceability behavior. Hence, the serviceability performance of this type of structures can be evaluated through the study of the peak accelerations and the first natural frequencies, according to the international normative [11,31–34].

6.2.1. Dynamic response

The dynamic response registered in the mid-span of the footbridge in terms of displacements and accelerations are presented in Fig. 6 for the load model 1 (WL1) and in Fig. 7, for the load model 2 (WL2). The time variation of the displacements and accelerations are shown in the upper plots while the corresponding Fast Fourier Transform (FFT), in the lower plots of each figure. As can be observed, the shapes of the time functions change with the applied load model. The heel impact effect in the second load model (WL2) is clearly observed in the peaks of

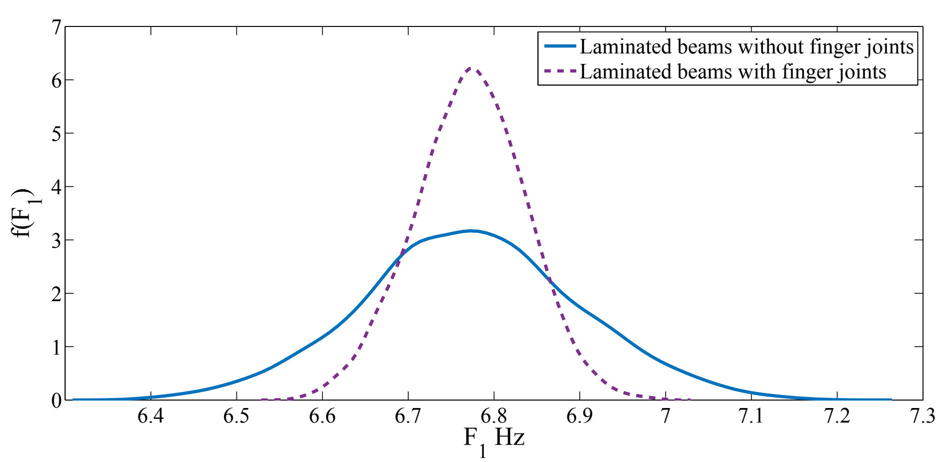


Fig. 5. PDFs of the first natural frequencies. Laminated beams with and without finger joints.

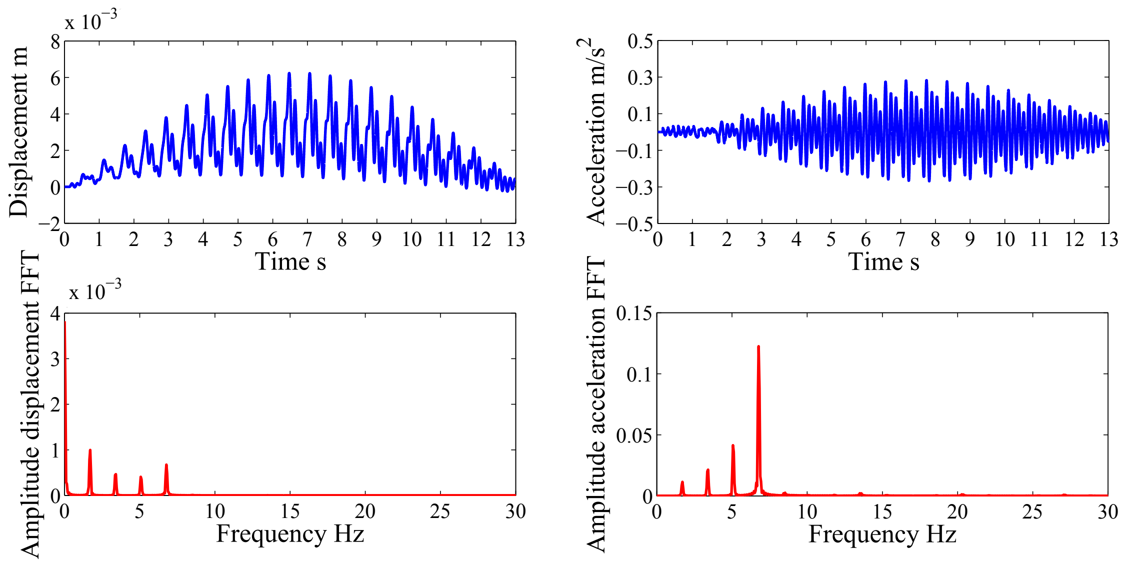


Fig. 6. Displacement results (left) and acceleration results (right) at the midspan point of the footbridge. Upper plots: time register, lower plots: Fast Fourier Transform (FFT). First load model (WL1).

the displacement and acceleration functions (Fig. 7). WL2 model produces structural responses with more frequency components and higher values of displacements and accelerations are observed in comparison with the results obtained when the WL1 model is applied. In comparison with footbridge structures with similar natural frequencies and different materials [12,13], the herein obtained accelerations are larger and the displacements smaller. This can be explained due to the high ratio stiffness/weight of the timber material. In what follows, the study of the serviceability performance of the structure will be presented in order to show the influence of the material uncertainties in the accelerations levels.

6.2.2. Serviceability performance

First, in this section, the excitation frequency of the load is adopted in such a way that if_s (second column of Table 1) be equal to the first natural frequency of the footbridge (F_1). Thus, the step frequency f_s becomes a random variable related with the values of F_1. Depending on the value that this variable adopts (third column Table 2) the activity and the step distance of the force model applied to the footbridge are defined. For the load models applied at the center line of the deck, the bending vibration modes have the higher contribution in the response. The serviceability performance of the results obtained through a stochastic study is presented in Fig. 8 for the first load model (WL1) and, in Fig. 9 for the second load model (WL2). For the serviceability evaluation, the first natural frequency of the footbridge (F_1) and the peak acceleration (a_{max}) constitute the main variables that relate to a limit value of acceleration. In both cases, the left plots were obtained for 5% of damping and the right plots for 7% in the first natural mode. The joint probability of the random variables peak acceleration (a_{max}) and the first natural frequency (F_1) is shown in the color bar at the right of the figure. In an illustrative form, two classic limit values of peak accelerations according to the standards EUROCODE 5 [31] (0.7 m/s^2) and ISO 10137 [32] (0.49 m/s^2) for outdoor footbridges were considered. In the most recent structural guidelines, the pedestrian comfort levels are indicated considering ranges instead of specific limit values of (a_{max}) [33,34]. Comfort situations for vertical vibrations in footbridges

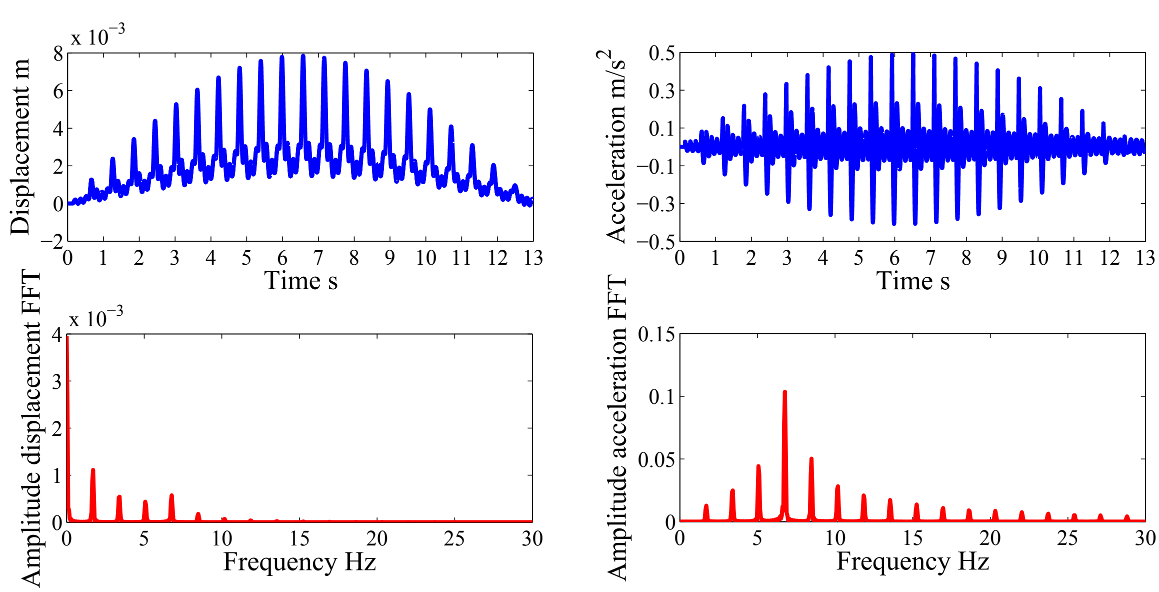


Fig. 7. Displacement results (left) and acceleration results (right) at the midspan point of the footbridge. Upper plots: time register, lower plots: Fast Fourier Transform (FFT). Second load model (WL2).

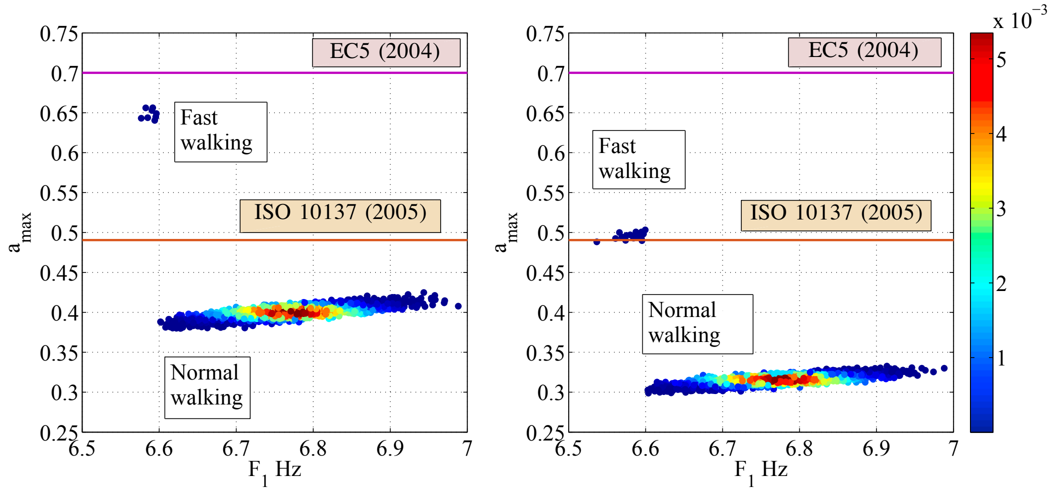


Fig. 8. Serviceability performance of the timber footbridge. First load model (WL1) (damping: 5 % left and 7 % right).

in function of the acceleration levels are depicted in Table 3.

For the results herein presented, the third and fourth harmonic of the force models (WL1 and WL2) are coincident with the first natural frequencies F_1; this aspect is defined due to the frequency range of this random variable (Fig. 5). As can be observed, when the structure is crossed with a fast walking with the third harmonic component of the forces in concordance with the fist natural mode of the footbridge, the peak accelerations in some samples result higher than the EC5 and ISO 10137 limits. In this cases, the dynamic coefficient α_i is higher than for the fourth harmonic, Table 1. The second load model is clearly the most demanding for the structure. The slope of the group of results indicates situations of timber bridges with higher natural frequencies due to the reduction of the effective mass of the structure and the increment of the effective stiffness but with higher accelerations levels produced mainly due to the mass reduction. As can be observed, the range of the natural frequencies variation is higher than the amplitude of the peak accelerations change. Hence, the traditional analysis with a mean model would be inadequate in the frequency zone in which the footbridge could be excited by a fast walking with the third harmonic or by a normal walking with the fourth harmonic of the force in coincidence with the first natural frequency of the footbridge. According to the numerical results, the variation of the material properties could lead to an unacceptable serviceability performance of the structure and a reduction in the pedestrian comfort due to the excitation level or the change in the structural effective properties. In view of the codes in force with line limits, the above presented results are either acceptable or unacceptable. If the guidelines depicted in Table 3 are applied, the outcomes would lead to either regions of maximum or mean comfort, which, in the authors opinion is a more rational assessment.

Table 3
Human comfort levels for vertical vibrations of footbridges [33,34].

Comfort level	Acceleration range (m/s^2)
Maximum	0–0.5
Mean	0.5–1
Minimum	1–2.5
Inadmissible	>2.5

7. Conclusions

A stochastic study of the serviceability performance of a short span timber footbridge made of Argentinean *Eucalyptus grandis* was presented. To the authors' knowledge, this structural configuration is extensively employed but not adequately studied in the literature. The stochastic analysis allows us to extend the range of the response starting from a model with mean properties. In comparison with footbridges structures with similar natural frequencies and different materials, the

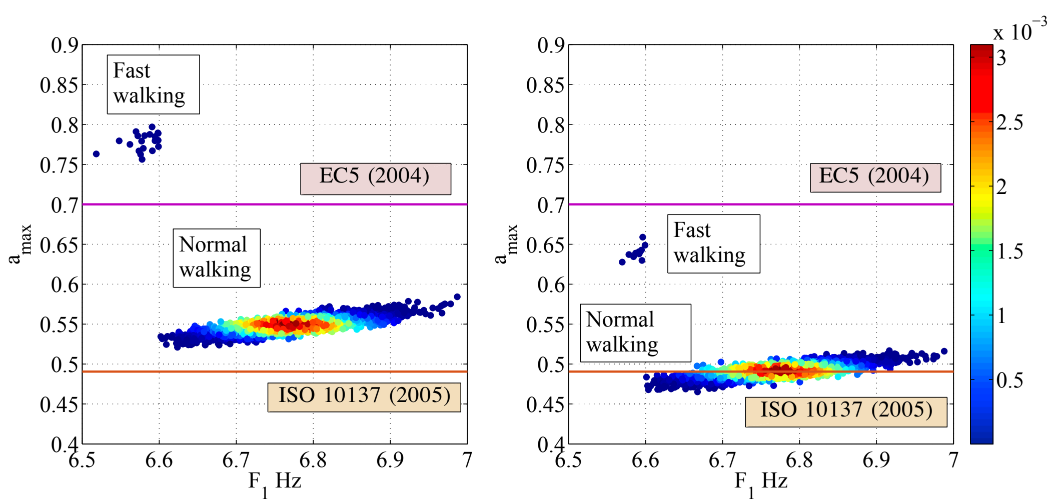


Fig. 9. Serviceability performance of the timber footbridge. Second load model (WL2) (damping: 5 % left and 7 % right).

obtained values of accelerations are significant and the displacements, small. This fact can be explained due to the high stiffness/weight ratio of the timber material.

The influence of the finger joints distances and the stochastic model of the material properties in the natural frequencies were presented. From a visual survey carried out in structural size laminated beams, the PMF of the distance between finger joints was constructed. The variation in the effective stiffness and mass along the elements producing by the union of boards with different properties produces a lower standard deviation in the natural frequencies than when the finger joints unions are not considered. The mean value of the natural frequency remains equal and is not influenced by the laminated beam model.

Through the application of two deterministic walking load models, it has been shown that the variation of the stochastic material properties could lead to an unacceptable serviceability performance and a reduction in the pedestrian comfort due to the excitation level or due to the change in the structural effective properties. New design guidelines provide more comprehensive tools to assess the comfort levels by means of regions. The results were also discussed under these recommendations. Generally, the stochastic variability of the timber material properties is not taken into account in the structural evaluation. The stochastic analysis allows to obtain a larger range of results than a deterministic study, and the effect of the mechanical properties in the results is better understood.

Future works will include the stochastic variability in the induced human load and the consideration of multiple pedestrian transit scenarios. It is expected that the uncertainty introduced by loads on structures be higher than the uncertainty in the material properties. However, in this work, the focus was placed in the uncertain material properties of structural elements made of Argentinean *Eucalyptus grandis*, which have a higher degree of randomness in comparison with other structural materials.

Acknowledgments

The authors are grateful for the financial support of CONICET, SGCyT-UNS, MINCyT from Argentina and CAPES, CNPq, and FAPERJ from Brazil. The experimental data provided by J.C. Piter, E. A. Torrán and co-workers from FRCU-UTN (Argentina) is greatly acknowledged.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.strusafe.2018.11.001.

References

[1] Piter JC. Strength grading of sawn timber as structural material: development of a method for the Argentinean Eucalyptus grandis (in Spanish). La Plata: Universidad Nacional de la Plata; 2003. [PhD thesis].
[2] IRAM:9662-2. Structural glued laminated timber. Visual strenght grading of boards. Part 2: Boards of eucalipto (Eucalyptus grandis) (in Spanish). Instituto Argentino de Normalización y Certificación (IRAM), Buenos Aires; 2006.
[3] Piter JC, Cotrina AD, Sosa Zitto MA, Stefani PM, Torrán EA. Determination of characteristic strength and stiffness values in glued laminated beams of Argentinean Eucalyptus grandis according to European standards. Holz als Roh-und Werkstoff 2007;65(4):261–6.
[4] Saviana J, Sosa Zitto MA, Piter JC. Bending strength and stiffness of structural laminated veneer lumber manufactured from fast-growing Argentinean Eucalyptus grandis. Maderas, Ciencia y Tecnologa 2009;11(3):183–90.
[5] Köhler J, Sørensen JD, Faber MH. Probabilistic modeling of timber structures. Struct Saf 2007;29(4):255–67.
[6] Casciati S, Domaneschi M. Random imperfection fields to model the size effect in laboratory wood specimens. Struct Saf 2007;29(4):308–21.
[7] Fink G, Köhler J. Probabilistic modelling of the tensile related material properties of timber boards and finger joint connections. Eur J Wood Prod 2015;73(3):335–46.
[8] Brandner R, Schickhofer G. Probabilistic models for the modulus of elasticity and shear in serial and parallel acting timber elements. Wood Sci Technol 2015;49(1):121–46.
[9] García DA, Rosales MB, Sampaio R. Eigenproblems in timber structural elements with uncertain properties. Wood Sci Technol 2016;50(4):807–32.
[10] García DA, Rosales MB. Deflections in sawn timber beams with stochastic properties. Eur J Wood Prod 2017;75(5):683–99.
[11] Živanović S, Pavić A, Reynolds P. Vibration serviceability of footbridges under human-induced excitation: a literature review. J Sound Vib 2005;279(1).
[12] Da Silva JGS, Vellasco PDS, De Andrade SAL, De Lima LRO, Figueiredo FP. Vibration analysis of footbridges due to vertical human loads. Comput Struct 2007;85(21):1693–703.
[13] Figueiredo FP, Da Silva JGS, De Lima LRO, Vellasco PDS, De Andrade SALA. parametric study of composite footbridges under pedestrian walking loads. Eng Struct 2008;30(3):605–15.
[14] Van Nimmen K, Lombaert G, De Roeck G, Van den Broeck P. Vibration serviceability of footbridges: Evaluation of the current codes of practice. Eng Struct 2014;59:448–61.
[15] Casagrande D, Giongo I, Pederzolli F, Franciosi A, Piazza M. Analytical, numerical and experimental assessment of vibration performance in timber floors. Eng Struct 2018;168:748–58.
[16] Casciati F, Casciati S, Faravelli L. A contribution to the modelling of human induced excitation on pedestrian bridges. Struct Saf 2017;66:51–61.
[17] Rama Rao MV, Pownuk A, Vandewalle S, Moens D. Transient response of structures with uncertain structural parameters. Struct Saf 2010;32(6):449–60.
[18] Shannon CA. mathematical theory of communication. Bell Tech J 1948;27:379–423.
[19] Jaynes E. Information theory and statistical mechanics. Phys Rev 1957;106(4):620–30.
[20] Reddy JN. An introduction to the finite element method 2. New York: McGraw-Hill; 1993. p. 2.
[21] Rubinstein RY. Simulation and the Monte Carlo method. John Wiley and Sons, Inc; 1981.
[22] CIRSOC 601. Argentinean standard of timber structures (In Spanish). Instituto Nacional de Tecnología Industrial (INTI)- Centro de Investigación de los Reglamentos Nacionales de Seguridad para las Obras Civiles (CIRSOC), Buenos Aires; 2013.
[23] UNE-EN. 408. Timber structures. Structural timber and glued laminated timber. Determination of some physical and mechanical properties. Madrid: Asociación Española de Normalización y Certificación (AENOR); 1996. [in Spanish].
[24] Argüelles Álvarez R, Arriaga Martitegui F, Martínez Calleja JJ. Timber structures. Design and calculation. Madrid: Asociación de Investigación Técnica de las Industrias de Madera (AITIM); 2000. [in Spanish].
[25] Forest Products Laboratory. Wood handbook: Wood as an engineering material. General Technical Report FPL-GTR-190. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory; 2010.
[26] Benjamin RJ, Cornell CA. Probability, Statistics and Decision for Civil Engineers. McGraw-Hill; 1970.
[27] IRAM:9660-1. Structural glued laminated timber. Part 1: Strenght classes and manufacture and control requirements (in Spanish). Instituto Argentino de Normalización y Certificación (IRAM), Buenos Aires; 2006.
[28] Piccardo G, Tubino F. Simplified procedures for vibration serviceability analysis of footbridges subjected to realistic walking loads. Comput Struct 2009;87(13):890–903.
[29] Swanson SR. Torsion of laminated rectangular rods. Compos Struct 1998;42(1):23–31.
[30] Chopra AK. Dynamics of structures vol. 3. New Jersey: Prentice Hall; 1995.
[31] EUROCODE 5. Design of timber structures, Part 2: Bridges. European Committee for Standardization, London, UK; 2004.
[32] ISO 10137. Bases for design of structures: Serviceability of buildings and pedestrian walkways against vibration. International Organization for Standardization, Geneva, Switzerland; 2007.
[33] International Federation for Structural Concrete (FIB). Guidelines for the design of footbridges: Guide to good practice. Ecole Polytechnique Fédérale de Lausanne (EPFL); 2005.
[34] Technical Department for Transport, Roads and Bridges Engineering and Road Safety (Sétra)/French Association of Civil Engineering (AFGC). Footbridges: Assessment of vibrational behavior of footbridges under pedestrian loading, Technical guide; 2006.