Dynamical evolution of a fictitious population of binary Neptune Trojans

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Abstract

We present numerical simulations of the evolution of a synthetic population of Binary Neptune Trojans, under the influence of the solar perturbations and tidal friction (the so called Kozai cycles and tidal friction evolution). Our model includes the dynamical influence of the four giant planets on the heliocentric orbit of the binary centre of mass. In this paper we explore the evolution of initially tight binaries around the Neptune L4 Lagrange point. We found that the variation of the heliocentric orbital elements due to the libration around the Lagrange point introduces significant changes in the orbital evolution of the binaries. Collisional processes would not play a significant role in the dynamical evolution of Neptune Trojans. After 4.5 $\times 10^9$ y of evolution, $\sim$ 50 % of the synthetic systems end up separated as single objects, most of them with slow diurnal rotation rate. The final orbital distribution of the surviving binary systems is statistically similar to the one found for Kuiper Belt Binaries when collisional evolution is not included in the model. Systems composed by a primary and a small satellite are more fragile than the ones composed by components of similar sizes.

KEYWORDS: Solar System - Kuiper Belt: General.

1 INTRODUCTION

Minor bodies orbiting close to the equilateral Lagrange points of a planet, leading (L4) or trailing (L5) it by nearly 60°, are known as Trojan objects. The first Trojan object was discovered around Jupiter L4 stability point in 1906. Today, about 6300 Jupiter Trojans are known orbiting in both, L4 and L5[1]. For more than a century, Jupiter Trojans were considered as single objects. However, very recently, it was discovered that four of them have a companion (Marchis et al. 2014). Three of them are systems with components of similar size, as most known binary Trans Neptunian Objects. The binary Trojan asteroid (624) Hektor is an exception, possessing a small satellite (Marchis et al. 2014).

Trojan objects are also present in other solar system planets. After the discovery of the first Neptune Trojan in 2001, 10 more objects have been detected (Chiang et al. 2003; Sheppard & Trujillo 2006, 2010). They are very important objects, because their orbits and physical characteristics may serve to impose constraints to possible formation histories of the outer solar system (Morbidelli et al. 2005; Tsiganis et al. 2005, Likauka et al. 2010, Yuan-Yuan et al. 2016).

The observational surveys conducted during the last years, suggest that about 10-30 % of the known objects in the trans neptunian region are binary systems (Stephens & Noll 2006, Noll et al. 2008, Grundy et al. 2009, 2011, Parker et al. 2011), although none of the 11 known Neptune Trojans have a detected companion. Nevertheless, given the small number of Neptune Trojans detected so far, we cannot discard that a similar fraction of binary objects could exist in this population. If so, they could be very useful to impose additional constraints to the origin of the outer solar system.

In this paper, we explore how a putative population of primordial Binary Neptune Trojans could survive for the age of the solar system.

The paper is organized al follows: In section 2 we present the method we used to perform the numerical simulations. In section 3 we present the main results, and the last section is devoted to the conclusions.

2 NUMERICAL SIMULATIONS

2.1 Dynamical model

The numerical simulations were performed by means of a generalization of the numerical code already used in Brunini & Zanardi (2016). This code

[1](http://www.minorplanetcentre.org/iau/lists/Trojans.html

was developed to study the dynamical evolution of Binary Trans Neptunian Objects, and in its original version it models the secular hamiltonian theory of Kozai oscillations induced on the binary orbit by the Sun, and the dissipation due to mutual tides (Eggleton & Kiseleva-Eggleton 2001). Both effects acting together constitute the so called Kozai Cycles and Tidal Friction (KCTF) evolution (Fabrycky & Tremaine 2007).

The contribution of tidal forces on the binary orbit by the presence of additional bodies (like Neptune in this case), has been studied in the frame of planetary systems in binary stars (Innanen et al. 1997; Wu & Murray 2003), but the space of free parameters in the general secular four-body problem is vast and complex, and it has not yet been explored in detail. In all previous works related to TNO binary evolution, the centre of mass of the binary was always considered evolving on a Keplerian orbit. It is well known that Trojan populations in general have a complex dynamical behavior. Trojan objects experience large amplitude libration in semi major axis, eccentricity and orbital inclination with a characteristic period of (Garfinkel 1977)

$$T_{lib} = P_{\odot} / \sqrt{\frac{27}{4}\mu}, \tag{1}$$

where $P_{\odot}$ is the period of the heliocentric orbit of the Trojan, and μ is the mass of the planet in solar masses. For Neptune Trojans, eq. (1) gives a libration period of about $\sim 10^4$ y. Kozai oscillations, on the other hand, have a characteristic period which is of the order of (Innanen et al. 1997)

$$\tau = 2\frac{P_{\odot}^2}{3\pi P_{bin}}(1 - e_{\odot}^2)^{3/2}, \tag{2}$$

where $e_{\odot}$ is the eccentricity of the heliocentric orbit of he binary centre of mass, and P_{bin} is the orbital period of the binary pair. For our synthetic sample of Binary Neptune Trojans (see bellow for the initial conditions), τ ranges from ~ 1000 y to ~ 50000 y, bracketing the libration period T_{lib}.

In this situation, it would be difficult to anticipate the consequences that variations of the heliocentric orbit would have in the dynamical evolution of Binary Neptune Trojans. Therefore we decided to consider this effect in our simulations, by means of the numerical integration of the heliocentric orbit of the binary centre of mass, performed by adapting the quasi symplectic code EVORB (Fernández, Gallardo & Brunini 2002) as a subroutine in our KCTF evolution program. A step size of 0.5 y was used for this integration. We have included in the dynamical model the mutual perturbations of the four giant planets of the Solar System. The centre of mass of Binary Trojans were considered as test particles: they are perturbed by the four giant planets, but they do not perturb the planets. We used barycentric coordinates and velocities for this numerical integration.

In addition, we have also investigated the magnitude of the tidal perturbation of Neptune on the mutual orbit of the binary objects. To accomplish this task, we used the secular theory developed by Kozai (1973), where the orbit of the binary is given in terms of their keplerian elements, whereas that of the perturber is given in terms of its polar coordinates. In his paper, Kozai (1973) furnished the secular equations up to the octupolar terms.

We performed a number of test simulations, including and not including both effects. We did not find any noticeable effect when the tidal perturbation on the binary orbit was included. This might be due to the fact that the strength of the perturbation is proportional to the mass of the perturber. So the relative importance of Neptune with respect to the Sun is of the order of μ. Therefore, in the numerical simulations presented in this paper, we have not included this effect, which is very expensive from the computational point of view.

On the contrary, we noted that the inclusion of the perturbations due to the four giant planets on the the heliocentric orbit of the binary centre of mass can induce, in some cases, strong modifications to the KCTF evolution of the binaries. Fig. 1 illustrates this effect. We can see that the binary orbit including the planets diverges noticeably from the one that does not include them. It is worth noting that in not all the cases the difference between both evolutions is so evident, but fig. 1 suffices to demonstrate that it is necessary to include this effect in our simulations.

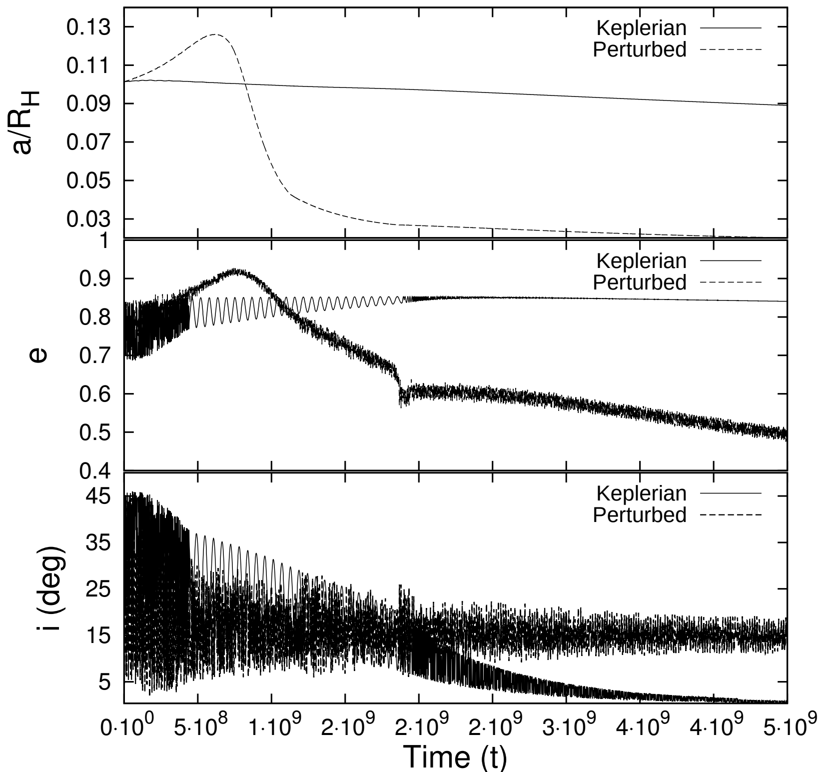

Figure 1: ***Different orbital evolutions of a binary with and without including the perturbations on the heliocentric orbit of its centre of mass.***

2.2 Collisional evolution

In a previous paper (Brunini & Zanardi 2016) we have shown that the exchange of impulse when the components of Kuiper Belt Binaries and the population of Classical Kuiper Belt objects collide, plays an important role in sculpting the orbital distribution of the present population of Trans Neptunian Binaries, this effect being responsible for the existence of a population of ultra wide trans neptunian binary pairs. To analyze the relevance of this effect on Binary Neptune Trojans, the first problem we have is the lack of computed intrinsic probability of collision P_i for them in the literature. Dell'Oro et al. (2013) have shown that P_i among Neptune Trojans and the entire population of Trans Neptunian Objects is almost ten times smaller than among TNOs. On the other hand, when studying possible mechanisms to explain the origin of the Neptune Trojan population, Chiang & Lithwick (2005), using a very simplified model to compute collision rates, concluded that collisional lifetime for this population is longer than the age of the solar system, and therefore large Neptune Trojans have probably not suffered collisional attrition.

An additional important parameter in this problem is N, the number of objects of the population, because the collision rate is

$$\frac{dN_{col}}{dt} = \pi r^2 N v_{col} P_i, \tag{3}$$

where v_{col} is the typical collision velocity and $r^2 = r^2_{prim} + r^2_{sec}$ is the combined cross section of the binary components. Sheppard & Trujillo (2010) presented results of an ultra-deep sky survey using the Subaru 8.2 m and the Magellan 6.5 m telescopes. They concluded that there should be about 400 Neptune Trojans larger than 50 km in radius, which is a factor of about 375 less than the number of Kuiper Belt objects of the same characteristics. They also estimate that at large radii, Neptune Trojans seem to follow a steep power-law slope with $q \sim 5$, similar to the brightest objects in the Kuiper Belt. Alexandersen et al. (2014) conducted a 32 square degree survey detecting one temporary Neptune Trojan and one stable Neptune Trojan, deriving a population of 150^{+600}_{140} objects, with $H_r \leq 10.0$, corresponding to a diameter of about $D = 60$ km, if 5% albedo is assumed. Even if the intrinsic collisional probability among Neptune Trojans were similar to the one found for Classical KBOs, the population is relatively small, and there should be too few collisions among them to have left any important signature in their dynamical evolution. Therefore we have not included this effect in our simulations.

2.3 Initial conditions

The initial conditions were generated as follows. Tello, Di Sisto & Brunini (2015) computed a total of 31,380 orbits of fictitious Trojans around each Lagrange stability point, using the numerical code EVORB (Fernández et al. 2002), under the gravitational influence of the four giant planets. The initial conditions for both Trojan clouds were obtained following Zhou, Dvorak & Sun (2009). In the present paper, we picked at random initial conditions of 500 stable orbits from the results of Tello et al. (2015), around the L4 Lagrange point, using these orbits as the ones of the centre of mass of the binaries. These 500 heliocentric orbits were used for all our numerical simulations. The similarity between the stability maps obtained by Zhou et al. (2009) and also by Tello et al. (2015) for the leading and trailing triangular Lagrange points L4 and L5, is indicative of the dynamical symmetry between the leading and trailing Trojan groups. This is the main reason to restrict our numerical experiments to the L4 Trojan cloud. With these initial conditions, and including the four giant planets in the simulations, it is guaranteed that the centre of mass of our binary systems remain in the Trojan cloud for the total time of the simulation.

The size of the components of each pair was determined following the size distribution of Sheppard & Trujillo (2010) for bright objects, corresponding to a slope of $q = 5$. They also found that for Neptune Trojans the break in the power index occurs at a diameter $D \simeq 90$ km. It is expected that for smaller radii, the population should behave like the other TNO population, with an index similar to that of Centaurs ($q \sim 3.1$, Adams et al. 2014, Fraser et al. 2014). We used this size distribution to generate at random the radius of each component of our synthetic binaries, in the range $30 \leq r \leq 100$ km. We have not imposed any constraint on the primary to secondary mass ratio.

The mutual orbits of the binaries were generated in the same way as in Brunini (2014). In all the simulations the components of the binary system start with random separations between 2 and 10 % of their mutual Hill radius

$$R_H = a_\odot (1 - e_\odot) \left(\frac{M_{bin}}{3M_\odot} \right)^{1/3}, \tag{4}$$

where $a_\odot$ is the semi major axis of the heliocentric orbit, $M_{bin} = M_{prim} + M_{sec}$ is the combined mass of the primary and secondary components. The inclination of the orbital plane were chosen at random between -90° and 90° measured from the plane of the heliocentric orbit of the Binary. The eccentricity of the mutual orbit was also taken at random between 0 and 0.9. As Neptune Trojan Binaries, if they exist, have not been yet discovered, these initial conditions are generated to cover a wide area in phase space, and are similar to the ones explored for the Trans Neptunian and Centaur populations (Brunini 2014; Brunini & Zanardi 2016). The spin period of each component was taken at random in the interval 2 h $\leq T_{spin} \leq$ 48 h, being the orientation of the spin axes also chosen at random.

Our model also includes evolution due to mutual tides, either on the binary orbit as on the diurnal rotation rate and obliquity of the binary components. For the tidal evolution model we adopted two values of Q, the tidal dissipation function of the binary members, and K_L, the second tidal Love number. In any case. both components have the same values of Q and K_L. We used the same definitions as in Porter & Grundy (2012) for them: for half of the cases, we adopted the canonical values for icy homogeneous solid bodies of $Q = 100$, density $\rho = 1$ g cm^{-3} and

$$K_L = \frac{3}{2} \left(1 + \frac{19 \mu_r r}{2 G M_{bin} \rho} \right), \tag{5}$$

with the rigidity $\mu_r = 4 \times 10^9$ N/m^2. For the other half of the simulated cases we have assumed that the binary is composed by two icy rubble piles, with $\rho = 0.5$ g cm^{-3}, $Q = 10$ and

$$K_L = r/10^5\text{km}. \tag{6}$$

In addition, we performed a third set of simulations assuming binaries composed by a primary component with $D = 100$ km, and a smaller secondary of $D = 1km$. There is no precise boundary to distinguish between satellites and binaries, but based on the mass ratio required for stability of oscillation about the second Lagrangian point in the Restricted Three Body Problem, for a size ratio $D_{Prim}/D_{sec} < 3$ we would have a primary with a satellite rather than a binary pair (de Pater and Lissauer 2010). Although being a rather controversial definition, we call this last set of objects "Satellites". For this set we used $Q = 100$ and $\rho = 1$ g cm^{-3}.

The possible end-states we considered for the binary evolution are:

- Survival during the age of the solar system.
- Separation of the components, either because the orbit becomes hyperbolic or the apocentric distance becomes larger then the Hill's radius.
- Collision between the components, when the pericentric distance becomes shorter than the mutual Roche radius, defined as $R_{Roche} = 1.26(r_{prim} + r_{sec})$, and the eccentricity of the binary orbit is high.
- Circularization, when the orbital eccentricity $e \leq 10^{-4}$.
- Contact binary, when the separation becomes smaller than the Roche distance, but with a small eccentricity $e \leq 10^{-2}$.

Regarding the equations for the Kozai evolution, we note that the heliocentric orbit being eccentric, it would be necessary to include in the model the octupole terms of the perturbing function (Lithwick & Naoz 2011). Nevertheless, the larger $e_\odot$ we found in our initial conditions is of the order of $e_\odot \sim 0.2$ (Tello et al. 2015). In this situation, as the relative "strength"of the octupolar terms in relation to the quadrupolar ones is of the order of $(\frac{a_{bin}}{a_\odot}) \times (\frac{e_\odot}{1-e_\odot})$, and as we have $a_{bin} << a_\odot$ by several orders of magnitude, we decided to neglect these terms in our model.

3 Results

In Table 1 we show the bulk statistics of the simulations.

Table 1: ***Statistics of the runs. Surv: objects surviving the entire simulation as binaries. Circ: circularized orbits with*** $e < 10^{-4}$ ***(they are also included in the class survivors). Cont: objects reaching the Roche separation with almost circular orbit. Sep: binaries with apocentric distance greater than the mutual Hill distance. Col: components colliding mutually.***

CLASS	Surv	Circ	Cont	Sep	Col	Tot
Q=10	386	299	0	101	13	500
Q=100	113	23	0	313	74	500
Sat Q=100	29	0	0	387	84	500

The objects with $Q = 10$ are more affected by tides, and as it is natural, they are much more stable than those with $Q = 100$, and also more prone to end up circularized. In fact the fraction of circularized systems for $Q = 10$ is 77% of the survivors or 59 % of the total sample, in agreement with the results that Porter and Grundy (2012) shown for KCTF evolution of Kuiper Belt Binaries not including collisional evolution. Disrupted systems represent in this case $\sim$ 23% of the sample, a fraction almost 10 times higher than in Porter and Grundy (2012). Tidal evolution seems to be less effective when the perturbation of the heliocentric orbit is considered.

Only 22% of the systems survive for $Q = 100$, indicating that tides are less effective in this case. For the same reason, mutual collisions between the components are frequent for objects with $Q = 100$. One interesting speculation is regarding the possibility that systems with a small satellite are originated from mutual collisions among this class of binary systems.

Porter & Grundy (2012) found a fraction of binary destruction ranging from 1% to 15% of their initial sample. We found a much larger fraction (23 % for Q=10, 37% for Q=100, and 94 % for satellites). This is one of the noticeable difference appearing when the oscillations around the L4 Lagrange point of the orbit of the centre of mass of the binary is included in the model.

In fig. 2 and fig. 3 we show the final orbital distribution for both classes of objects. Statistically, they are very similar to the ones already shown for Trans Neptunian Binaries (Brunini & Zanardi 2016; Porter & Grundy 2012). The distribution in i shows a lack of objects with large inclinations, which is consistent with the Kozai Mechanism. Objects with inclination smaller than the critical value, corresponding to $\sin(i_C) = \sqrt{2/5}$ ($i_c \sim 39°.3$ or $140°.7$; Kozai 1962), experience small variations in orbital eccentricity. In the classical Kozai mechanism, the inclination cannot cross the critical value. As $\sqrt{(1-e^2)}\cos(i)$ is nearly conserved, the maximum eccentricity is attained at the minimum of inclination, that is always smaller than the critical value. On the contrary if $39°, 3 < i_0 < 140°, 7$, the eccentricity could attain very large values, making operative the tidal decay, mutual collisions, or the separation of the binary pair.

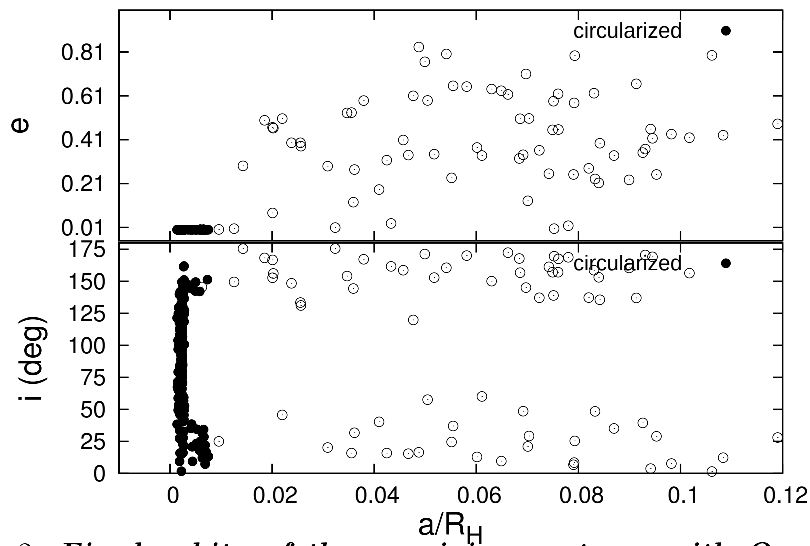

Figure 2: ***Final orbits of the surviving systems with Q=10, ρ=0.5 g cm^{-3}.***

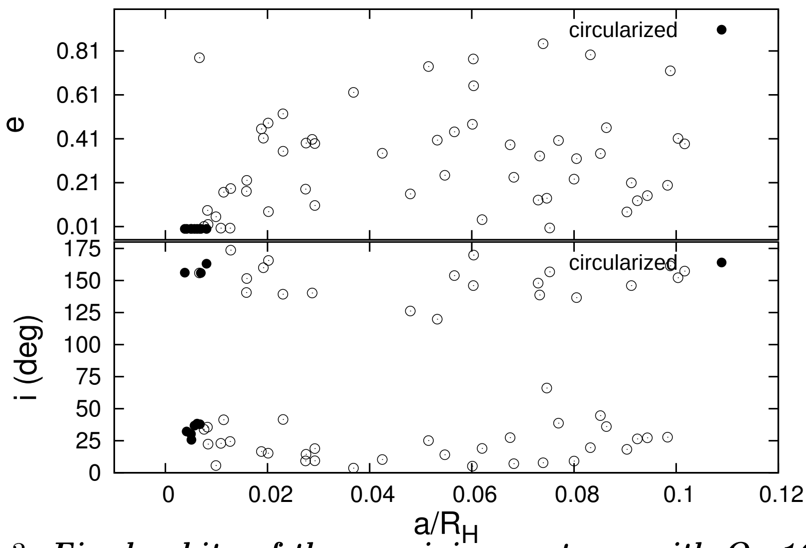

Figure 3: ***Final orbits of the surviving systems with Q=100, ρ=1.0 g cm^{-3}.***

We note that in most cases this is true also in our simulations but in some other few cases the coupling between Trojan libration and Kozai oscillation allows objects to cross the critical inclination. One case is shown in fig. 4. As we already mentioned, the dynamics of Binary Trojans is complex, and a careful dynamical study is out of the scope of the present paper. In these cases, Kozai oscillation does not act as a protection mechanism against binary disruption.

In general, the resulting systems also preserve the direction of their initial mutual orbit relative to the plane of their heliocentric orbit, and so the initial prograde / retrograde ratio in the initial conditions was almost preserved. Although for $Q = 100$ there is a certain excess of direct orbits, in the $Q = 10$ case there is an excess of retrograde ones. We attribute these differences to the small number of the sample of surviving binaries. The differences are consistent with a Poisson statistics up to 99% of confidence level.

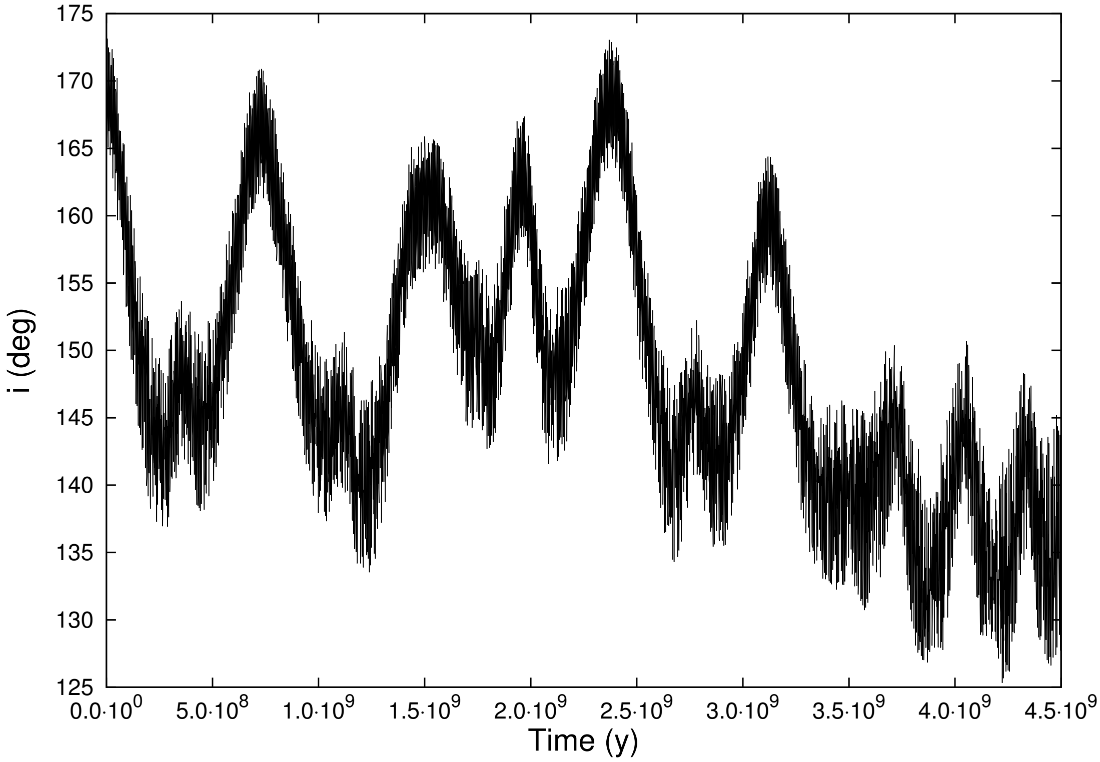

Figure 4: ***The heliocentric orbital elements of one of our synthetic binary centre of mass, experience large amplitude libration, allowing the crossing of the forbidden Kozai inclination*** $i_c \sim 143°$***.***

Our results for satellite systems show some peculiarities. Only very few objects survived the entire simulation. We did not find any surviving satellite with circularized orbit, mostly because tides are completely inoperative in this case. In fig. 5 we show the distribution of initial orbital inclinations for different end states we found in the simulations. The distribution of mutual inclinations and eccentricities of the surviving systems (gray squares in fig. 5) also shows the typical gap at intermediate i values. In this case, satellites with this initial range of orbital inclination, end up colliding onto the primary or being unbounded.

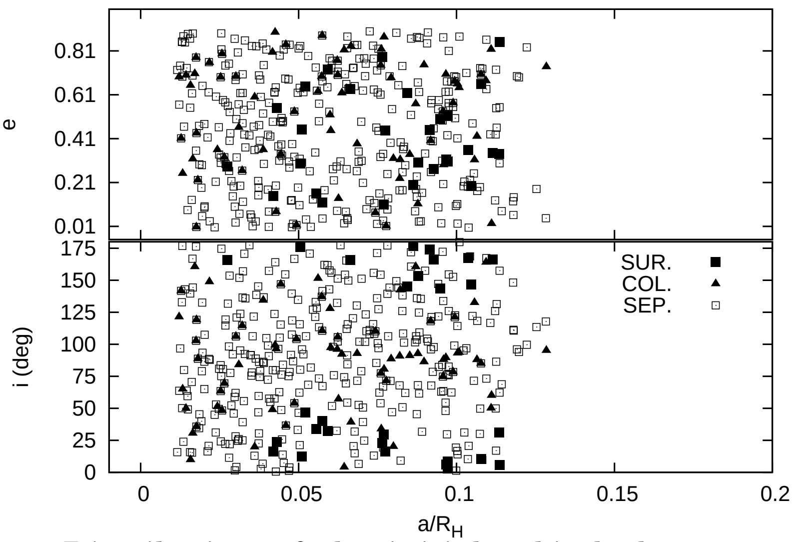

Figure 5: ***Distribution of the initial orbital elements of systems composed by a primary and a satellite (see our definition of satellite) for each one of possible end states.***

Fig. 6 displays the difference of the obliquity of the primary and secondary components for the surviving objects (excluding the circularized systems).

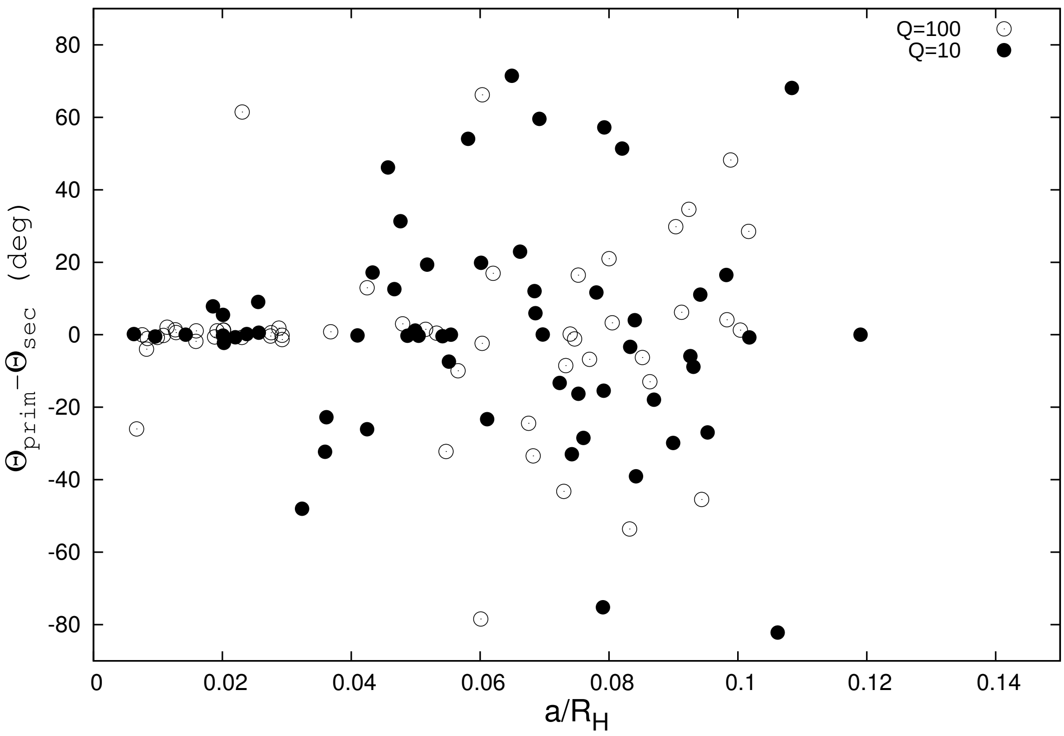

Figure 6: ***Difference between the obliquity of both components for the surviving systems.***

It is clear that tides operate only up to $a < 1\%\ R_H$. The spin periods of those objects also show that tight systems end in synchronous rotation (fig. 7). The spin period of the circularized systems are shown in fig. 8. Some systems did not reach synchronous rotation rate. They are those reaching circularization very fast and were removed from the simulation before the end of the run.

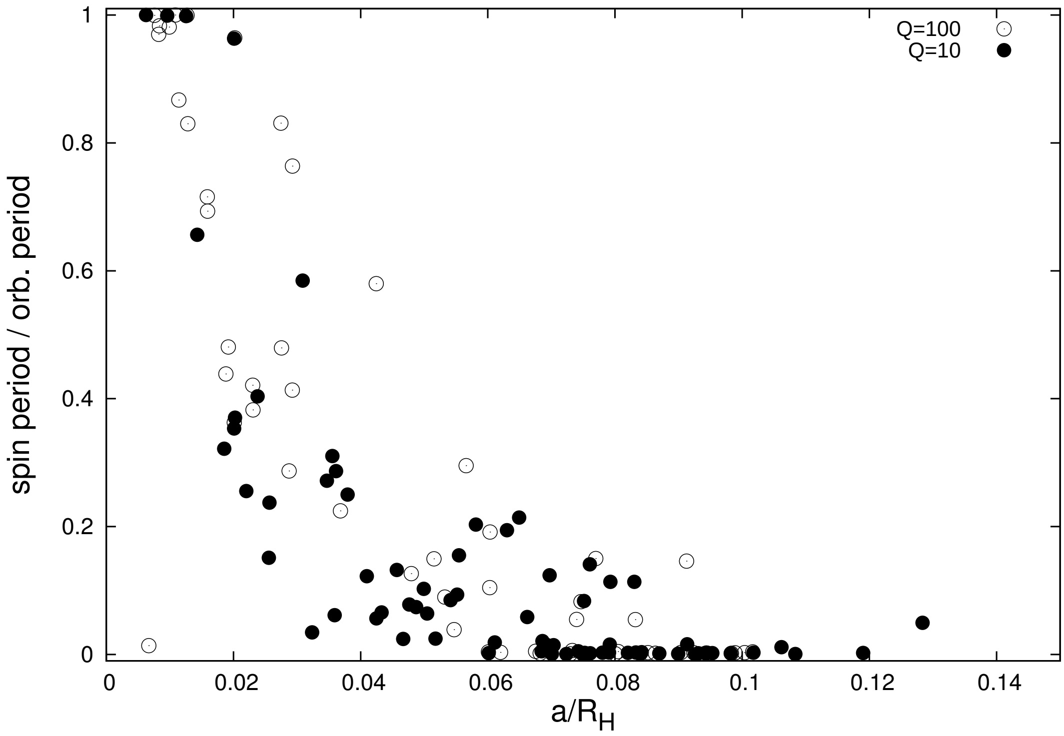

Figure 7: ***Spin periods or the surviving systems.***

We also investigate the rotational end state of the components of disrupted binary systems (those reaching the condition $a(1+e) > R_H$). Fig. 9 shows the distribution of spin periods of each component at the moment of separation. We observe a marked excess of slow rotators, with a mean spin period of ~ 100 h. Very few objects have spin periods less than 24 h, and practically none of them are fast rotators. In all cases we found that both components are rotating faster than the orbital mean motion. In this condition, the tidal bulge of each component leads the other component, and therefore the corresponding tidal torque acts to increase the orbital angular momentum, and the reaction torque on the bulge acts to decrease the spin angular momentum, slowing down the diurnal rotation rate, as it was found in Brunini (2017). The existence of slow rotators in the Neptune Trojan population would be a natural consequence of the presence of binaries. However connect this result to observations would be rather speculative. We did not include in our model important physical effects affecting the rotation rate such as the shape of the objects. Also, our rotation rate initial conditions

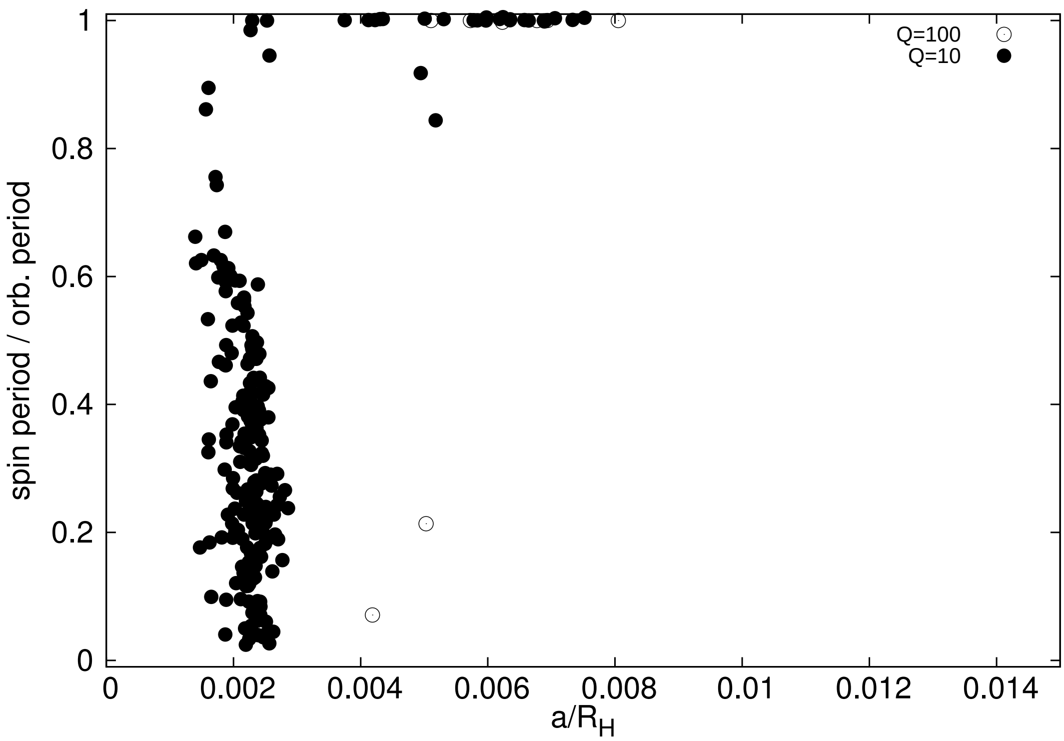

Figure 8: ***Spin periods of the circularized systems.***

are representative of the present rotation rate of the TNB sample. These facts difficult to give a more solid answer.

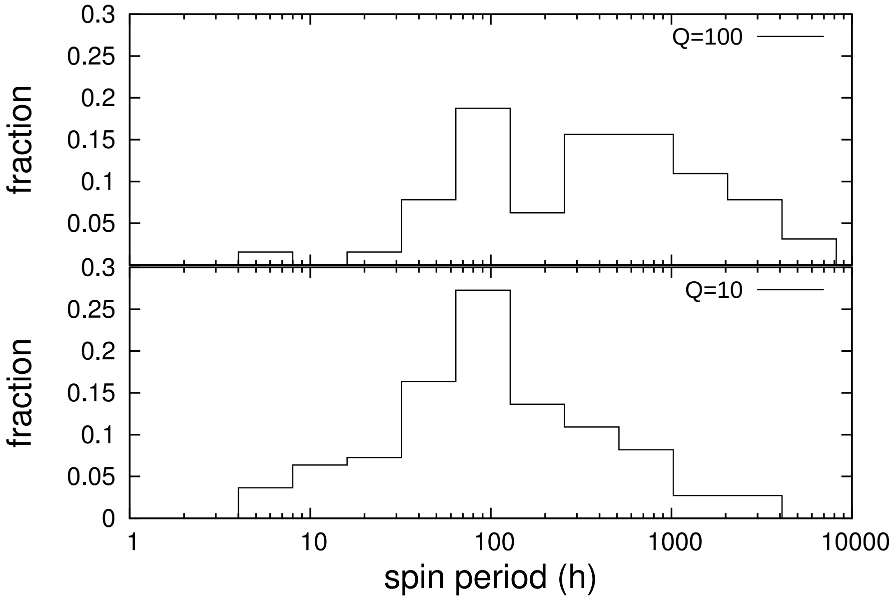

Figure 9: **Distribution of spin periods of the binary components at the moment of separation.**

4 Conclusions

We have performed a series of numerical simulations of the evolution of a synthetic population of Binary Neptune Trojans, under the influence of the solar perturbation and mutual tidal friction. Due to the symmetry in the dynamics of the L4 and L5 Lagrange stability points, we have only explored the behavior of binary objects belonging to the L4 point. In this case, in contrast to what happens for Kuiper Belt binaries, collisional evolution does not play a significant role in the dynamical sculpting of Binary Neptune Trojans.

~ 50 % of our initially tight systems end up separated as single objects, almost all of them with slow diurnal rotation rate because of the action of mutual tidal forces. A non negligible fraction of slow rotating Trojans, if they exist, could be formed through this mechanism.

The final orbital distribution of the surviving systems is statistically similar to the one found for Kuiper Belt Binaries when collisional evolution is not included in the model (Porter & Grundy 2012). Systems composed by a massive primary and a small satellite are more fragile than the ones whose components are of similar size.

If binary objects were part of the primordial population of Neptune Trojans, then ~ 50 % of them should already exist today orbiting in the L4 (and probably also in the L5) group if they are rubble pile objects. If the binary objects are composed by icy homogeneous solid bodies with $Q = 100$, then only ~ 20 % of the primordial binaries would have survive up to the present.

There are a number of different mechanism to explain the origin of Binary objects in the outer Solar System. Each one of them favors a certain primordial characteristics of binary objects. We found that, independently of their primordial nature, Binary Neptune Trojans with components of comparable size are more likely to survive for the age of the Solar System.

Due to the small number of Neptune Trojans discovered so far, the existence of binary objects in the L4 and L5 clouds could not be rejected. If they exist, their discovery could offer important clues on the formation of the outer solar system

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