Iterated Kleinian Equations *

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Abstract

A higher order field has different forms of excitation. Some of them have negative energies. The signs of the quantization rules depend on the signs of the energies. An abnormal sign implies a negative sign of the residue at the on-shell pole of the propagator, leading to a clash with unitarity.

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To change these signs we can change the identification of the creation and annihilation operators. But then the energy has no lower bound. The way out is found by adopting a symmetric vacuum state. The corresponding propagator is a half retarded and half advanced Green function. It has a zero residue at the on-shell pole. There is no associated free particle. The abnormal modes act only as mediators of interactions.

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1 Introduction

The consideration of higher order equations for possible descriptions of natural phenomena, has been present ever since the advent of differential equations in physics (See for example ref.[1]). The quantum treatment has difficulties of its own, not present in simple Klein-Gordon equations. It seems convenient to be able to overcome the technical obstacles for a better understanding of the physical implications of a given theory. In this sense it is of great help the use of lagrangian procedures for the construction of the canonical tensors. There are several expositions and one of the first was given in Courant-Hilbert's book [2]. A didactic approach can be found in ref.[3] (For a more mathematical point of view, see ref.[4]).

A Lorentz invariant equation,

$$\sum_{s=0}^{n} c_s \Box^s \psi = 0 \quad ; \quad (c_n = 1) \tag{1}$$

can be written as:

$$\prod_{r=1}^{n} \left(\Box - \lambda_r\right) \psi = 0 \tag{2}$$

where $\lambda_r (r = 1, ..., n)$ are the roots of

$$\sum_{s=0}^{n} c_s x^s = 0$$

(for negative or complex roots see respectively refs.[5] and [6]).

If all the roots are real and positive we will say that we have a "Iterated Kleinian Equation"for which we can write:

$$\prod_{r=1}^{n} \left(\Box - m_r^2\right) \psi = 0 \tag{3}$$

(We will assume that $0 < m_1 < m_2 < \cdots < m_n$).

Equation (3) implies that ψ has n different excitations or modes [7]. It can be decomposed into n "constituent fields" ϕ_r [8]. Each one obeying a normal Klein-Gordon equation:

$$\psi = \sum_{r=1}^{n} \phi_r \quad ; \quad \left(\Box - m_r^2\right) \phi_r = 0$$

$$\phi_r = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega_r}} \left(a_r e^{-ikx} + a_r^+ e^{ikx} \right) \tag{4}$$

$(k_0 = \omega_r = (\vec{k}^2 + m_r^2)^{1/2})$

The corresponding higher order lagrangian can be written:

$$\mathcal{L}_0 = \frac{1}{2} \psi \prod_{r=1}^{n} \left(\Box - m_r^2 \right) \psi \tag{5}$$

From (5) we can construct the energy-momentum tensor and in particular, the energy content of the field.

Using eq.(4) we find [2, 3]:

$$P_0 = \int \sum_{r=1}^{n} \frac{1}{2} \omega_r c_r \left(a_r a_r^+ + a_r^+ a_r \right) \tag{6}$$

where

$$c_r = \prod_{s \neq r} \left(m_s^2 - m_r^2 \right) = (-1)^{r-1} |c_r|$$

A simple redefinition:

$$a_r \to |c_r|^{-\frac{1}{2}} a_r$$

leads to:

$$P_0 = \int \sum_{r=1}^{n} (-1)^{r-1} \frac{\omega_r}{2} \left(a_r a_r^+ + a_r^+ a_r \right) \tag{7}$$

Equation (7) (or (6)) shows that the total energy of the field ψ is a superposition of the energies of the different excitations. However for r=even number the contribution of the mode is negative. This change of sign from a term to the next one, is a characteristic feature of eq.(3) [9].

The fact that P_0 is not positive definite gives rise to serious difficulties of interpretation.

Heisenberg's quantization condition (P_0 is the generator of time displacements):

$$[P_0, \psi] = i\partial_0\psi$$

implies

$$\left[a_r(k), a_r^+(k')\right] = (-1)^{r-1}\delta\left(\vec{k} - \vec{k}'\right) \tag{8}$$

From (7) and (9) we deduce the well-known relations:

$$[P_0, a_r] = -\omega_r a_r \quad ; \quad \left[P_0, a_r^+\right] = \omega_r a_r^+ \tag{9}$$

Equation (9) says that a_r^+ (resp. a_r) is a creation (resp. annihilation) operator for the energy. However, eq.(8) shows that while for r=odd number the commutation relations are the usual one, for r=even number the roles of a_r and a_r^+ are interchanged.

If we try to keep only positive energy states, we should impose for the vacuum:

$$a_r|0>= 0 \quad ; \quad (r = 1, ..., n) \tag{10}$$

Then we would find:

$$< 0|a_r^+ a_s^{'}|0 >= 0 \ \ ; \ \ < 0|a_r^{'} a_s^+|0 >= (-1)^{r-1}\delta_{rs}\delta\left(\vec{k}-\vec{k}^{'}\right) \tag{11}$$

The consequences for the propagators are important. All vacuum expectation values of products of fields operators (VEV for short) carry the signs of eqs.(11). Consequently, the propagator Δ_r for the fields ϕ_r is:

$$\Delta_r = (-1)^{r-1} F_r \tag{12}$$

where F_r is the usual Feynman propagator for the mass m_r.

Equation (12) is untenable. It implies that the residues at the on-shell poles are negative for r=even, and this circunstance is fatal for unitarity [10].

It is not difficult to change the signs of (12). We can say that actually eq.(8) shows that for r=even we should consider a_r to be a creation operator. Then we should define the vacuum by imposing:

$$a_r|0 >= 0 \quad (r = odd) \quad ; \quad a_r^+|0 >= 0 \quad (r = even) \tag{13}$$

Now we would obtain for r=even:

$$< 0|a_r^{'} a_r^+|0 >= 0 \quad ; \quad < 0|a_r^+ a_r^{'}|0 >= \delta\left(\vec{k}-\vec{k}^{'}\right) \tag{14}$$

And:

$$\Delta_r = F_r \qquad for \qquad r = 1, ..., n \tag{15}$$

So we would have no problem with the signs of the residues. However, eq.(9) says that the states created by a_r (r=even) would have negative energies.

It seems that there is no escape from these contradictions. But there is a way out if we take into account the existence of an alternative vacuum state, associated to fields that can not appear as free waves. The corresponding propagator has a null residue at the on-shell pole. As we will show in the next paragraph.

2 The symmetric vacuum

It is clear that neither (12) nor (13) will permit the elaboration of an acceptable theory. The problem lies in the r=even degrees of freedom.

We will call $\varphi(x)$ any one of the fields ϕ_r for r=even:

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega}} \left(b(k) e^{-ikx} + b^+(k) e^{ikx} \right) \tag{16}$$

The commutation relations are (cf. eq.(8)):

$$\left[b(k), b^+(k') \right] = -\delta \left(\vec{k} - \vec{k}' \right) \tag{17}$$

But now, instead of imposing (10) or (13) we will choose a "symmetric vacuum", whose implications and properties are discussed at length in reference

[11].

We define the vacuum as the “true” zero energy state. I.e.:

$$\left(bb^{+}+b^{+}b\right)|0>=0 \tag{18}$$

Equations (17) and (18) imply (compare with (11) and (14)):

$$< 0|b(k)b^{+}(k^{'})|0 >= -\frac{1}{2}\delta\left(\vec{k}-\vec{k}'\right)$$

$$< 0|b^{+}(k)b(k^{'})|0 >= \frac{1}{2}\delta\left(\vec{k}-\vec{k}'\right) \tag{19}$$

For the VEV we have:

$$< 0|\varphi(x)\varphi(0)|0 >= \frac{1}{(2\pi)^{3}}\int\frac{dk}{2\omega}\frac{1}{2}\left(e^{-i\omega t}-e^{i\omega t}\right)e^{i\vec{k}\cdot\vec{r}} =$$

$$= \frac{-i}{(2\pi)^{3}}\int dk\frac{sin\omega t}{2\omega}e^{i\vec{k}\cdot\vec{r}} \quad (= - < 0|\varphi(0)\varphi(x)|0 >)$$

And, for the chronological product:

$$< 0|T\varphi(x)\varphi(0)|0 >= \tilde{W}(x) = \frac{-isgnt}{(2\pi)^{3}}\int dk\frac{sin\omega t}{2\omega}e^{i\vec{k}\cdot\vec{r}} \tag{20}$$

We will now prove that (20) is the half advanced and half retarded Green function for the Klein-Gordon equation. In fact, a retarded Green function ($\tilde{G}_{rt}(x)$) is a Fourier transform of $(p^2+m^2)^{-1}$ where the poles at $p_0=\pm\omega$ are left below the p_0 path of integration. For $t<0$ the path can be closed on the upper half-plane and Cauchy's theorem assures a null result. For $t > 0$ the path can be closed on the lower half-plane. The sum of residues gives:

$$\tilde{G}_{rt}(x) = -\pi \frac{\Theta(t)}{(2\pi)^3} \int \frac{dk}{\omega} e^{i\vec{k}\cdot\vec{r}} \left(e^{-i\omega t} - e^{i\omega t}\right) =$$

$$= -\frac{\Theta(t)}{(2\pi)^3} 4\pi i \int dk \; e^{i\vec{k}\cdot\vec{r}} \frac{sin\omega t}{2\omega} \tag{21}$$

($\Theta(t)$ is Heaviside step function).

Similarly, for the advanced function we have:

$$\tilde{G}_{ad}(x) = \frac{\Theta(-t)}{(2\pi)^3} 4\pi i \int dk \; e^{i\vec{k}\cdot\vec{r}} \frac{sin\omega t}{2\omega} \tag{22}$$

Then:

$$\frac{1}{2}\tilde{G}_{rt}(x) + \frac{1}{2}\tilde{G}_{ad}(x) = -\frac{sgnt}{2\pi} i \int dk \; e^{i\vec{k}\cdot\vec{r}} \frac{sin\omega t}{2\omega} \tag{23}$$

A comparison with eq.(20) shows that the propagator corresponding to the symmetric vacuum can be written as:

$$\tilde{W}(x) = \frac{1}{4\pi^2} \left(\frac{1}{2}\tilde{G}_{rt}(x) + \frac{1}{2}\tilde{G}_{ad}(x)\right) \tag{24}$$

A half advanced and half retarded Green function was used by J.A.Wheeler and R.P.Feynman [12] to describe the electromagnetic interaction in a charged medium which was supposed to be a perfect absorber.

For this reason we will call (24) a "Wheeler function"(or propagator).(See also ref.[13]). Later, the same authors showed that, in spite of the advanced part it contains, the Green function (24) does not contradict causality [14].

On the real axis of the energy, $W(p)$ coincides with Cauchy's principal value, which has an on shell zero. Implying that there is no free propagation. This is the reason behind the choice of a perfect absorber in ref.[12].(No free wave can escape the system).

The Wheeler propagator has several remarkable properties (see ref.[11]). The fact that it does not contain a free component means that the corresponding field can only act virtually, as a mediator of interactions. No asociated free particle can be found. In particular, no free negative energy state can be occupied. In other words, one starts with positive energy particles and ends up with positive energy particles.

Since neither b nor b^+ annihilate the vacuum, the space of states is a two-way ladder

$$b|0>=\alpha_1|1> \quad ; \quad b|1>=\alpha_2|2>$$

$$b^+|0>=\alpha_{-1}|-1> \quad ; \quad b^+|-1>=\alpha_{-2}|-2>$$

In particular for example (cf. eq.(19)):

$$\alpha^*_{-1} < -1| - 1 > \alpha_{-1} = |\alpha_1|^2 < -1| - 1 >=< 0|b^{'} b^{+}|0 >= -\frac{1}{2}\delta\left(\vec{k} - \vec{k}^{\,\prime}\right)$$

And the norm of the state $|-1>$ is negative. (For a discussion of spaces with indefinite metric see. ref[15]).

The scalar product can be defined by means of the holomorphic representation [16]. The functional space is formed by analytic functions f(z) with the product:

$$(f, g) = \int dz\, dz^*\, e^{-zz^*} f(z)(g(z))^* \tag{25}$$

Or, in polar coordinates:

$$(f, g) = \int_0^{\infty} d\rho\, \rho\, e^{-\rho^2} \int_0^{2\pi} d\phi\, f\, g^* \tag{26}$$

The raising and lowering operators are represented by:

$$b = z \;\; ; \;\; b^* = \frac{d}{dz} \;\; ; \;\; [b, b^*] = -1 \tag{27}$$

The symmetric vacuum obeys:

$$\left(\frac{d}{dz} z + z \frac{d}{dz}\right) f_0 = \left(1 + 2z \frac{d}{dz}\right) f_0 = 0$$

Whose normalized solution is:

$$f_0 = \left(2\pi^{\frac{3}{2}}\right)^{-\frac{1}{2}} z^{-\frac{1}{2}}$$

The energy eigenfunctions are:

$$f_n = 2\pi \left| \Gamma \left(n + \frac{1}{2} \right) \right|^{-\frac{1}{2}} z^{-\frac{1}{2}} z^n \quad ; \quad (n = ..., -2, -1, 0, 1, ...) \tag{28}$$

3 Unitarity

It is evident that unitarity holds at tree level, due to the fact that when a branch is on the mass-shell the δ-function of the Feynman propagator is equivalent to the free particle appearing in external legs. On the other hand, for a Wheeler function, when a branch is on-shell we have a zero of the propagator, in correspondence with the absence of a free particle.

To study the effects of loops we are going to examine an example in which we show explicitely that the unitarity relations hold true (See also ref.[11]).

We will consider the lagrangian:

$$\mathcal{L} = \sum_{i=1}^{4} \mathcal{L}_i + \mathcal{L}' \tag{29}$$

$$\mathcal{L}_i = \frac{1}{2} \psi_i \left(\Box - m_i^2 \right) \left(\Box - M_i^2 \right) \psi_i \quad ; \quad m_i < M_i \tag{30}$$

$$\mathcal{L}' = \lambda \psi_1 \psi_2 \psi_3 \psi_4 \tag{31}$$

The equations of motion are:

$$\left(\Box - m_i^2 \right) \left(\Box - M_i^2 \right) \psi_i = -\lambda \prod_{j \neq i} \psi_j \quad (i = 1, ..., n) \tag{32}$$

According to the discussions in previous sections, we write:

$$\psi_i = \phi_i + \varphi_i \tag{33}$$

$$\phi_i = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega_i}} \left(a_i e^{-ikx} + a_i^+ e^{ikx} \right) \quad ; \quad k_0^{(i)} = \omega_i \tag{34}$$

$$\varphi_i = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\Omega_i}} \left(b_i e^{-ikx} + b_i^+ e^{ikx} \right) \quad ; \quad k_0^{(i)} = \Omega_i \tag{35}$$

$$P_0 = \int dk \sum_{i=1}^{4} \left\{ \frac{\omega_i}{2} \left(a_i a_i^+ + a_i^+ a_i \right) - \frac{\Omega_i}{2} \left(b_i b_i^+ + b_i^+ b_i \right) \right\} \tag{36}$$

(For the sake of simplicity we have taken $M_i^2 - m_i^2 = 1$).

The commutation rules are:

$$\left[a_i^{'}, a_j^+ \right] = \delta_{ij} \delta \left(\vec{k} - \vec{k}^{'} \right) \tag{37}$$

$$\left[b_i^{'}, b_j^+ \right] = -\delta_{ij} \delta \left(\vec{k} - \vec{k}^{'} \right) \tag{38}$$

The fields ϕ_i are normal. The operators a_i annihilate the vacuum state. For the φ_i modes we take (cf. eq.(18)):

$$\left(b_i b_i^+ + b_i^+ b_i \right) |0>= 0 \tag{39}$$

From the commutation rules and the vacuum relations, we deduce:

$$< 0|a_i^{'} a_i^*|0 >= -\delta \left(\vec{k} - \vec{k}^{'} \right) \quad ; \quad < 0|a_i^* a_i|0 >= 0 \tag{40}$$

$$< 0|b_i^{'} b_i^*|0 >= -\frac{1}{2} \delta \left(\vec{k} - \vec{k}^{'} \right) \; ; \; < 0|b_i^* b_i|0 >= \frac{1}{2} \delta \left(\vec{k} - \vec{k}^{'} \right) \tag{41}$$

Equations (40) imply the usual Feynman propagator for the normal modes. On the other hand the VEVs (41) imply that the propagators for φ_i are Wheeler functions.

To test unitarity we recall that:

$$SS^{+}=1 \quad ; \quad S=1-iT$$

$$-i\left(T-T^{+}\right)=TT^{+} \tag{42}$$

The perturbative development,

$$T=\sum_{s=1}^{\infty}\lambda^{s}T_{s}$$

gives:

$$-i<\alpha|T_n-T_n^{+}|\beta>=\sum_{s=1}^{n-1}\int d\sigma_{\gamma}<\alpha|T_{n-s}|\gamma><\gamma|T_s^{+}|\beta> \tag{43}$$

where we have introduced the decomposition of the unit operator:

$$\mathcal{I}=\int d\sigma_{\gamma}|\gamma><\gamma| \tag{44}$$

In particular, for n=2:

$$-i<\alpha|T_2-T_2^{+}|\beta>=\sum_{s=1}^{n-1}\int d\sigma_{\gamma}<\alpha|T_1|\gamma><\gamma|T_1^{+}|\beta> \tag{45}$$

The external legs of Feynman diagrams can only be occupied by normal ϕ-particles. We will take:

$$|\alpha >= a_1^+(p_1)a_2^+(p_2)|0> \ ; \ |\beta >= a_1^+(p_1^{'})a_2^+(p_2^{'})|0> \tag{46}$$

For the interaction we have:

$$-iT_1 = \psi_1\psi_2\psi_3\psi_4 = \prod_1^4 (\phi_i + \varphi_i) = \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\varphi_4 + \cdots +$$

$$+\varphi_1\varphi_2\varphi_3\phi_4 + \varphi_1\varphi_2\varphi_3\varphi_4$$

Due to (46), only terms containing φ_1 and φ_2 will contribute. So, we can take:

$$-iT_1 = \phi_1\phi_2\phi_3\phi_4 + \phi_1\phi_2\phi_3\varphi_4 + \phi_1\phi_2\varphi_3\phi_4 + \phi_1\phi_2\varphi_3\varphi_4 \tag{47}$$

The first term gives rise to a theory with Feynman functions and normal particles which is known to be unitary. The left-hand side (l.h.s.) of (45) contains the convolution:

$$Real\left\{\left(p^2+m_3^2-i0\right)^{-1} * \left(p^2+m_4^2-i0\right)^{-1}\right\} =$$

$$\left(p^2+m_3^2\right)_W^{-1} * \left(p^2+m_4^2\right)_W^{=1} - \pi^2\delta\left(p^2+m_3^2\right) * \delta\left(p^2+m_4^2\right) \tag{48}$$

But in the physical region the two terms in the r.h.s. coincide [11]. We have then:

$$(48) = 2\left(p^2+m_3^2\right)_W^{-1} * \left(p^2+m_4^2\right)_W^{-1} \tag{49}$$

(Where the subindex W is meant to imply that Cauchy's principal value is to be taken at the pole).

Eq.(49) shows that the l.h.s. of (48) is twice the value corresponding to the case in which one or both propagators are Wheeler functions. Let us consider now the r.h.s. of (45):

For a normal ϕ field, the decomposition (44) takes the form:

$$\mathcal{I} = |0><0| + \int dq a^{+}(q)|0><0|a(q) + \cdots \tag{50}$$

Instead, for a φ field (with indefinite metric), we have:

$$\mathcal{I} = |0><0| - \int dq\sqrt{2}b^{+}(q)|0><0|b(q)\sqrt{2}+$$

$$+\int dq\sqrt{2}b(q)|0><0|b^{+}(q)\sqrt{2} + \cdots \tag{51}$$

The sigms and the normalization factors are dictated by the VEVs (41) (take for example $\mathcal{I}b^{+}|0>$ and use (19)).

The evaluation of $<\alpha|T_1|\gamma>$ for the first term of (47) (r.h.s.) contains the matrix factor:

$$<0|\phi a^{+}|0>= \frac{1}{(2\pi)^{3/2}}\int \frac{dk^{'}}{\sqrt{2\omega^{'}}}e^{-ik^{'}x} <0|a^{'}a^{+}|0>=$$

$$\frac{1}{(2\pi)^{3/2}}\int \frac{dk^{'}}{\sqrt{2\omega^{'}}}e^{-ik^{'}x}\delta\left(\vec{k}-\vec{k}^{'}\right)$$

$$< 0|\phi a^{+}|0 >= \frac{1}{(2\pi)^{3/2}} \frac{e^{-ikx}}{\sqrt{2\omega}} \tag{52}$$

For the second term of (47) we get:

$$< 0|\varphi b^{+}|0 >= \frac{1}{(2\pi)^{3/2}} \int \frac{dk'}{\sqrt{2\Omega'}} e^{-ik'x} < 0|b' \sqrt{2} b^{+}|0 >=$$

$$\frac{1}{(2\pi)^{3/2}} \int \frac{dk'}{\sqrt{2\Omega'}} e^{-ik'x} \sqrt{2} \frac{1}{2} \delta \left(\vec{k} - \vec{k}' \right)$$

$$< 0|\varphi b^{+}|0 >= \frac{1}{(2\pi)^{3/2}} \frac{e^{-ikx}}{\sqrt{2\Omega}} \frac{1}{\sqrt{2}} \tag{53}$$

When we multiply together $< \alpha|T_1|\gamma >$ and $< \gamma|T_1^{+}|\beta >$ we find that (53) and its conjugate give a factor 1/2 as compared with (52) and its conjugate. Coinciding (resp.) with the l.h.s. for normal fields and for Wheeler excitations. Thus showing that unitarity holds true for the example discussed.

Similarly, in other cases, any proof of unitarity for normal fields, based on the decomposition (50) and the VEVs (40), can be converted into a proof of unitarity for Wheeler fields, by using (51) and (41).(See also ref.[11]).

4 Discussion

The consideration of iterated Kleinian equations, leads to fields whose energy is not positive definite. Correspondingly one has operators that create negative energy states, and the energy of the free field has no lower bound.

When we try to avoid those states by choosing an appropriately modified vacuum, we find that the propagator has a negative residue at the pole. Thus leading to a clash with unitarity. It is possible to change the unwanted sign by means of a redefinition of the vacuum. But then the negative energy states of the free field necessarily reappear

The way out of these difficulties is found by adoptind the symmetric vacuum, which leads to half advanced and half retarded propagator. This Wheeler function is equivalent, on the real axis of the energy, to Cauchy's principal value Green function. In other words, the residue at the on-shell pole is exactly zero. Consequently, there in no clash with unitarity and the physical space is the Fock space of normal free excitations bilt up with the creation operators for positive energy. The other modes are only presents as virtual states, i.e.: as mediators of interactions. They do not occupy the external legs of Feynman diagrams. Furthermore, in ref.[14] it is shown that the half advanced and half retarded Green function satisfies the requirements of a causal theory.

An example of a case in which higher order equations appear in a natural way is obtained when supersymmetry is imposed in higher dimensional spaces (Ref.[17]. Here, a connexion is established between the dimensionality of space and the order of the equations of motion. In particular, a six-dimensional Wess-Zumino model has been quantized in ref.[18].

When the possible existence of tachyons is discussed, the symmetric vacuum appears as the most reasonable starting point for the construction of the corresponding Fock-space of states (ref.[19] and ref.[20]. See also ref.[5]). In ref.[21] a family of unitary higher order equations is examined, in which an interaction with the electromagnetic field is introduced "via" the gauge covariant derivative. These interacting higher order equations can also be decomposed into second order Klein-Gordon modes.

Possible applications to string theory and to the Higgs particles, are discussed in ref.[22] and ref.[23] (resp.).

The question of unitarity when loops are present, and other interesting properties of the Wheeler propagator, has been analized in ref.[24], which has just been sent for publication.

The fact that unitarity holds true for the iterated Kleinian equations, when the Wheeler propagator is appropriately used, complements the works of references [5, 6], where it is shown that the modes corresponding to negative or complex roots of eq.(2), propagate according Wheeler functions.

We may then conclude that any Lorentz invariant higher order equation, no matter how simple it may look, can not be consistently quantized unless use is made of the half advanced and half retarded Green function for some of its modes of propagation.

References

[1] H.Bhabba: Rev. Mod. Phys. **17**, 200 (1945).

[2] R.Courant and D.Hilbert: "Methods of Mathematical Physics". Vol. II. Interscience Publ. New York (1962).

[3] C.G.Bollini and J.J.Giambiagi: Rev. Brasileira de Fisica **17**, 14 (1987).

[4] V.Aldaya and J.Azcarra: Jour. Phys. **A 13**, 2545 (1981).

[5] D.G.Barci, C.G,Bollini and M.C.Rocca: Int. J of Mod. Phys. **A 10**, 1737(1995).

[6] C.G.Bollini and L.E.Oxman: Int. J. of Mod. Phys. **A 8**, 3185(1993).

[7] H.Schnitzer and E.Sudarshan: Phys. Rev. **123**, 219 (1961).

[8] D.G.Barci, C.G.Bollini, L.E.Oxman and M.C.Rocca: Int. Jour. of Mod. Phys. **A 9**, 4169 (1994).

[9] A.Pais and G.E.Uhlenbeck: Phys. Rev. **79**, 145 (1950).

[10] S.Hawking:"Who's Afraid of (higher derivative) Ghosts". University of Cambridge preprint (1985).

[11] C.G.Bollini and M.C.Rocca:"Study of the Wheeler Propagator". La Plata preprint. UNLP (1997).

[12] J.A.Wheeler, R.P.Feynman: Rev. Mod. Phys. **A 17**, 157(1945).

[13] P.A.M.Dirac: Proc. Roy. Soc. London **A 167**, 148 (1938).

[14] J.A.Wheeler, R.P.Feynman: Rev. Mod. Phys. **A 21**, 425(1949).

[15] K.L.Nagy: Il Nuovo Cim. Suppl. **17**, 92 (1960).

[16] L.D.Faddeev and A.A.Slavnov:"Gauge Fields. Introduction to Quantum Theory". The Benjamin-Cummings Publishing Company, Inc.(1970).

[17] C.G.Bollini and J.J.Giambiagi: Phys. Rev **D 32**, 3316 (1985).

[18] D.G.Barci, C.G.Bollini, M.C.Rocca: Il Nuovo Cim. **108 A**, 797 (1995).

[19] D.G.Barci, C.G.Bollini, M.C.Rocca: Il Nuovo Cim. **106 A**, 603 (1993).

[20] D.G.Barci, C.G.Bollini, M.C.Rocca: Int. J. of Mod. Phys.**A 9**, 3497 (1994).

[21] C.G.Bollini, L.E.Oxman, M.C.Rocca: Int. J. of Mod. Phys.**A 12**, 2915 (1997).

[22] C.G.Bollini, M.C.Rocca: Il Nuovo Cim. **110 A**, 353 (1997).

[23] D.G.Barci, C.G.Bollini, M.C.Rocca: Il Nuovo Cim. **110 A**, 363 (1997).

[24] D.G.Barci, C.G.Bollini, M.C.Rocca: "The Wheeler Propagator". La Plata. UNLP (1998).