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Performance criteria based on nonlinear measures

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Abstract

To impart disturbance rejection properties is an important goal in the design of an appropriate process control structure. In a multivariable control system, the effectiveness of disturbance rejection can depend strongly on the direction of the disturbance. In this paper, we present a study of the controllability and disturbances effect for a general nonlinear plant. For this purpose, controller-independent measures are defined and computed by solving a simple optimization problem in the time-domain. Finally, several examples are considered to illustrate the proposed method.

Keywords: Disturbance Rejection; Controllability; Process Control

1. Introduction

Traditionally, the maximum obtainable profit in the chemical industry was attained by increasing the production to its admissible maximum level. However, due to the energy crisis of the early 1970s, and with the resulting sharp rise of the cost of energy, profitability became associated with decreased production costs. Therefore, the process synthesis started to involve a trade-off between the search for economically attractive designs and those that can be operated safely as well as meeting specifications. This balance involves, inevitably, the interaction of design and control.

The importance of designing processes that can be acceptably controlled, is widely recognized as a relevant topic, and has been studied by many researchers (Barhi, 1995; Fisher, Doherty, & Douglas, 1988; Hovd & Skogestad, 1992; Narraway & Perkins, 1994; Straub & Grossman, 1993). A significant consideration is whether it is possible to reduce the effect of disturbances to an acceptable level using the available manipulated variables. In this context, three relevant questions are introduced in the work by Hovd and Braatz (2000):

(i) What is the minimum output error that is obtainable for the worst possible combination of disturbances with the optimal use of the manipulated variables?
(ii) What is the minimum required magnitude for the manipulated variables to obtain an acceptable output error for the worst possible combination of disturbances?
(iii) What is the largest possible disturbance for which an acceptable output error is obtained with the available manipulated variables?

As regards processes linear representations, the mathematical formulation of each of the above questions in terms of optimization problems has already been provided (Skogestad & Wolff, 1992; Wolff, Skogestad, Hovd, & Mathisen, 1992). Moreover, an appealing discussion on the solutions of these problems has recently been presented by Hovd and Braatz (2000).

Lee, Braatz, Morari, and Packard (1995) introduced screening tools to help eliminate undesirable control structure candidates for which a robustly performing controller does not exist. They especially dealt with the problem of actuator/sensor selection. The approach is based on the Structured Singular Value Theory and uses Linear Fractional Transformation to represent the uncertain system.

During the last years, the need for developing algorithms for simultaneous solution of process and control design has

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Nomenclature

c_p	coolant heat capacity
c_v	control valve setting
C	concentration of A in CSTR
Cool^1	heat transferred between cooling jacket and reactor in CSTR #1
Cool^2	heat transferred between cooling jacket and reactor in CSTR #2
D_h	heat of reaction
E/R	activation energy
F	fuel input
k_0	Arrhenius constant
L	drum level
M	control horizon
P	drum pressure
Q	flowrate
S	steam flow
T	temperature in CSTR
T_e	feed water temperature
T_s	sample time
U	set of possible input variables
U_a	overall heat transfer coefficient
V	volume of CSTR
w_c	feed water input
W	set of possible disturbances
z_0	process profit

been tackled by many researchers. However, many of the published criteria either assume a specific design approach or a specific uncertainty description. Consequently, they are not useful as general design-independent screening tools. A contribution to this topic due to Braatz, Lee, and Morari (1996), presented screening tools for systems with general structured model uncertainty. Nonconservative estimates for the achievable performance can be attained using this screening approach. Provided the controller structure is defined, the tools can be used to select actuators, sensors, as well as appropriate variables pairings.

Kookos and Perkins (2001) introduced a decomposition algorithm for solving the combined process and control design problem. They considered that the process uncertainties can be modeled as a function of a finite number of time-invariant but uncertain parameters. The proposed calculation scheme is based on the systematic generation of lower and upper bounds to reduce effectively the size of the search space. In a more recent work, Kookos and Perkins (2003) present mixed-integer linear programming formulations for the efficient calculation of the disturbance rejection measures previously proposed by Skogestad and Wolff (1992). The formulation developed for linear systems can be solved for global optimality using the available solvers. The authors outline the equivalent formulation for nonlinear systems and they associate it with the flexibility index problem.

However, several nonlinear aspects of many real processes have not been thoroughly considered yet. For instance, the saturation feature of the manipulated variables is only considered by assuming that these variables are bounded, but sometimes this is not a realistic assumption. In this field, only some considerations related with question (iii) have been introduced by Grossman, Halamane, and Swaney (1983).

In this paper, we present a study of the controllability of a general nonlinear plant and the disturbances effects on it. Different controllability measures related to disturbances influence are proposed based on nonlinear optimization. First, a steady-state approach is formulated to solve the question (ii) above mentioned. This measure is herein used to compare several control strategies for reactors. A controllability measure of a system can be related to its operability level. Then, the system's operability can be quantified through the maximum economic profit which can be achieved (Raspanti & Figueroa, 2001). For this purpose, the problem of the maximum economic profit when the process is under disturbances, is herein dealt with as a nonlinear dynamic optimization formulation. This is an extension of the back-off problem where the operating point is moved away from the one calculated in the optimization level, in order to ensure the feasible process operation to compensate for the likely effect of the disturbances (Bandoni, Romagnoli, & Barton, 1994; Barhi, 1995). In this work, a steam-generation unit with parameter uncertainty is considered as an application example to evaluate the performance of the proposed approach.

The work is organized as follows. The steady-state measure for control effort is developed in Section 2. In Section 3, the economical profit bounds attainable by a given process are analyzed. The use of these techniques is presented via simulations in Section 4. Finally, in Section 5, the conclusions are drawn.

2. Controllability analysis

It is now a well-known fact there are some plants with better disturbance rejection capabilities than others. Usually the terms "controllability" and "dynamic resilience" are referred to as the inherent control properties of the plants. That is, if a plant has poor controllability, then the responses of that plant will be poor no matter what controller is selected to be used. In this sense, Lewin (1996) developed a graphical method to enable the diagnosis of disturbance resiliency for linear processes affected by disturbance vectors. In this way, it is possible to quantify the necessary control action for rejecting a disturbance vector. This control effort depends on the disturbance direction and frequency. The method is useful as a screening tool during the process design stage, as well as for selecting from different control structures.

In the development of disturbance effect measures, let us consider a wide set of linear transfer function models of the following form:

$$Y(s) = G(s)U(s) + G_d(s)D(s) \quad (1)$$

where U is the vector of manipulated inputs, D is the vector of disturbances, and Y is the vector of outputs (i.e. controlled variables). The objective is to keep the error $E = Y - R$ small, where R is the vector of reference signals (or set-points). G and G_d are transfer matrices that do not need to be square. Provided these considerations are held, the effect of the disturbances on the open-loop system (i.e., $U = 0$) can be stated as

$$Y_{ol}(s) = G_d(s)D(s) \quad (2)$$

Taking into account the underlying idea of using the measures for process design, it is desirable to find measures independent of the control scheme selected. Then, for most controllability measures, it is usual to consider a perfect control condition as follows:

$$U_{cl}(s) = -G(s)^{-1}G_d(s)D(s) \quad (3)$$

By considering expressions (1)–(3), some measures based on the Euclidean norm can be developed (Skogestad & Wolff, 1992). These analyses can be performed on steady state or as function of frequency (Hovd & Skogestad, 1992). However, these measures do not concern about many important considerations on process design, such as nonlinearities or operative constraints. In order to include these issues into the analysis, in this work we consider those processes described by the following general nonlinear time-domain representation:

$$\dot{x} = f(x, u, d) \quad (4)$$

$$y = h(x, u, d) \quad (5)$$

subject to a set of inequality constraints:

$$z_c = z(x, u, d) = 0 \quad (6)$$

The disturbances d are assumed to belong to a set W, i.e. $d \in W$. For instance, W can be defined as the set of all amplitude bounded step functions. On the other hand, the inputs u are assumed to belong to a set U, i.e. $u \in U$. Without loss of generality, let us assume that the steady-state solution of this system is given for $(x, u, d) = (0, 0, 0)$.

Based on the back-off concept (Figueroa, Bahri, Bandoni, & Romagnoli, 1996; Perkins & Walsh, 1994), we define the following measures.

Open-loop steady-state worst disturbance: It is the disturbance that produces the largest amplitude deviation from the desired output setpoint at steady state ($\dot{x} = 0$) and open-loop condition ($u = 0$). In order to measure that amplitude, the Euclidean norm is considered. Mathematically,

$$\begin{aligned} & z_0 = \max_{d\in W} \|y\| \\ & \text{s.t.} \\ & f(x, u, d) = 0 \\ & y - h(x, u, d) = 0 \\ & u = 0 \end{aligned} \quad (7)$$

Note that the objective function value is a measure of the steady-state process performance that can be attained, and the problem solution is obtained for the worst disturbance direction.

Open-loop dynamic worst disturbance: It is defined as the disturbance which gives rise to the largest output amplitude. To determine this measure, the open-loop time output is considered (i.e. $u = 0$). Mathematically,

$$\begin{aligned} & z_0 = \max_{d\in W} \max_{t\in[0,\infty)} \|y\| \\ & \text{s.t.} \\ & \dot{x} - f(x, u, d) = 0 \\ & y - h(x, u, d) = 0 \\ & u = 0 \end{aligned} \quad (8)$$

It must be remarked that in this case the optimization problem is also solved in the time-domain. The worst disturbance must be found, and it is the one that produces the largest output deviation for the whole time response.

Another important measure is related to the magnitude of the necessary control action to reject disturbances. This is an important measure in the case of process inputs saturation.

Steady-state control action amplitude: It is the magnitude of the control action that rejects the worst disturbance at steady-state. Mathematically,

$$\begin{aligned} & z_0 = \max_d \min_u \|u\| \\ & \text{s.t.} \\ & f(x, u, d) = 0 \\ & h(x, u, d) = 0 \end{aligned} \quad (9)$$

Note that the worst disturbance is now calculated following the criterion of the largest control action required to reject it. Sometimes, in this particular problem, it is very important to include the presence of constraints. To cope for this need, a modified version of problem (9) is stated as follows:

$$\begin{aligned} & z_0 = \max_d \min_u \|u\| \\ & \text{s.t.} \\ & f(x, u, d) = 0 \\ & h(x, u, d) = 0 \\ & z_c(x, u, d) \le 0 \end{aligned} \quad (10)$$

The solution to this problem is the answer to question (ii) for nonlinear systems described by Eqs. (4) and (5) in Section 1. A remarkable feature of the optimization problems treated in this section, is the typical existence of local minima. This fact can be overcome by introducing multiple starting points in the solution of the nonlinear programming software. In Section 4, this measure will be used to compare different schemes for reactor temperature control. In a similar mode, the problems in items (i) and (iii) can be mathematically formulated as follows.

Closed-loop dynamic minimum output error:

$$\begin{aligned} & z_0 = \max_d \min_u \max_t \|y\| \\ & \text{s.t.} \\ & \dot{x} - f(x, u, d) = 0 \\ & h(x, u, d) = 0 \\ & z_c(x, u, d) \le 0 \end{aligned} \quad (11)$$

Closed-loop dynamic worst disturbance:

$$\begin{aligned} & z_0 = \max_{d,u} \|d\| \\ & \text{s.t.} \\ & \dot{x} - f(x, u, d) = 0 \\ & h(x, u, d) = 0 \\ & z_c(x, u, d) \le 0 \end{aligned} \tag{12}$$

In the next section, the achievable performance for a particular process configuration will be studied.

3. Achievable performance

The operating point of a chemical process is usually designed to maximize (or minimize) an objective function (e.g. the profit, subject to constraints like the ones inferred from the characteristics of the plant, operating conditions, product specifications and others). This objective can be formulated as follows:

$$\begin{aligned} & z_0 = \max_{u_s} z_0(x, u_s) \\ & \text{s.t.} \\ & f(x, u_s, u_c, d^N) = 0 \\ & y - h(x, u_s, u_c, d^N) = 0 \\ & z_c(x, u_s, u_c, d^N) \le 0 \end{aligned} \tag{13}$$

where $z_0(x, u_s)$ is a performance index (typically with economic meaning), computed at steady-state free of disturbances. The vector u of control variables is divided into two vectors u_c and u_s, that represent the vector of manipulated variables and the vector of free variables, respectively. These latter variables are used to determine the optimal operating point of the process. In this expression, d^N stands for the nominal set of disturbances (perturbation-free condition). The constraints z_c define the feasible set for the possible operating points. In a second stage, a controller is designed to regulate the behavior of the plant around the desired steady-state value. The underlying idea is that the controller provides perfect control, so that the plant remains at, or at least close to, its nominal operating point against disturbances, parameter variations and uncertainties on the plant dynamics.

The effect of the disturbances at such regulation level will perturb the process causing the operating point movement away from the previously designed one. Thus, this point will be surrounded by a region within which the plant will actually operate. Under these perturbed conditions, the plant operation may become infeasible (in steady-state and/or along transient). This has led different authors (Bandoni et al., 1994; Figueroa et al., 1996; Perkins & Walsh, 1994) to include operative conditions (such as the possible presence of disturbances) at the stage of the operating point design. The mathematical formulation can be stated as follows:

$$\begin{aligned} & \max_{u_s} z_0(x, u_s) \\ & \text{s.t.} \\ & \dot{x} - f(x, u_s, u_c, d) = 0 \\ & z_c(x, u_s, u_c, d) \le 0 \\ & d \in W \end{aligned} \tag{14}$$

The main idea of this strategy is to move the operating point away from the boundary of the feasibility region to compensate for the effect the expected disturbances could have on the plant operation. This is called back-off. Through this procedure we ensure that the process will operate at its optimum, with no constraint violations. In practice, the back-off problem is usually solved by finding an operative point that guarantees the plant operation for the worst disturbances, i.e. those disturbances that provoke the largest constraint violation. It was originally calculated from the desire for evaluating and comparing control strategies on the basis of economical criteria (Figueroa et al., 1996; Narraway & Perkins, 1994).

In this paper, we propose a modification of this problem to compute the best achievable performance for the process structure independently of the particular controller selection. Taking into account that a discrete-time controller will be used to regulate the continuous-time process, we will consider that the manipulated variables can be discretized as the sequence $U_c = [u_c(0), u_c(1), \ldots, u_c(M)]$, since $u_c(t) = u_c(k)$ for $T_s k \le t < T_s(k+1)$. Notice that $k = 0, \ldots, M$, where M is the so-called control horizon and T_s is the sampling time. Therefore, the problem can be formulated as follows.

Achievable performance: It is the maximum value for the performance index z_0 attainable for the considered process structure. The index is calculated in order to guarantee that no constraints violation takes place, despite of the disturbance situation (i.e. for all $d \in W$). Mathematically,

$$\begin{aligned} & \max_{u_s, u_c(0), u_c(1), \ldots, u_c(M)} z_0(x, u_s) \\ & \text{s.t.} \\ & \dot{x} - f(x, u_s, u_c, d) = 0 \\ & z_c(x, u_s, u_c, d) \le 0 \\ & d \in W \end{aligned} \tag{15}$$

To solve this nonlinear optimization problem, it is possible to use the algorithm for the dynamic back-off computation (Figueroa et al., 1996). This is a two step algorithm. In an inner loop, a worst disturbance is computed for a given set of optimization parameters $(u_s, u_c(0), \ldots, u_c(M))$. Then, the problem stated in (15) is solved in an outer loop for a discrete set of worst perturbations. The iterative procedure is executed while no worst disturbance in the inner loop produces constraints violation (Figueroa et al., 1996).

Some remarks should be pointed out:

- The same as in the previous section, in the optimization procedure it is typical the presence of local minima. In this sense, trying multiple starting points can be useful in the solution of the nonlinear programming software.
- The dynamic optimization implicit in this problem has to be performed for a given horizon (i.e., the simulation is performed while $t < P$). This horizon should be large enough to make sure the complete transient of the process is considered.

- As in Model Predictive Control schemes, the manipulated variables are assumed constant beyond the control horizon (i.e. $u_c(t) = u_c(M)$ if $t \geq T_s M$).
- To force the process to return to the desired steady-state (i.e. complete disturbance rejection), the measured variables can be constrained to be equal to their steady-state values at the end of the simulation horizon (i.e. $|y(P) - y(0)| \leq \varepsilon$ for any small enough $\varepsilon \geq 0$).
- It could be possible that no solution is found for the set of possible disturbances. This would involve that there is not feasible operation for the analyzed process.

In the following section, the use of the above nonlinear measures is illustrated by means of two simulation examples. In Section 4.1, the application of the Steady-State Control Action Amplitude is used for evaluating different control arrays. Afterwards, in Section 4.2, the problem formulated in Eq. (15) is solved to perform the operability analysis of a steam generation unit.

4. Simulation results

4.1. Reactor temperature control

In this section, an illustrative case study is dealt with. It consists of two jacketed, cooled, continuous stirred tank reactors (CSTR) in series, with an intermediate mixer introducing a second feed (de Hennin & Perkins, 1993). The whole process is sketched in Fig. 1. The following single, irreversible, exothermic, first-order reaction takes place in both reactors:

$$\mathrm{A} \rightarrow \mathrm{B}$$

The energy and composition balances inside the reactors brings up the following differential equations:

$$V^1 \frac{\mathrm{d}C^1}{\mathrm{d}t} = -k_0 \mathrm{e}^{-(E/RT^1)} C^1 V^1 + Q_\mathrm{F}^1 (C_\mathrm{F}^1 - C^1) \qquad (16)$$

$$V^1 \frac{\mathrm{d}T^1}{\mathrm{d}t} = D_\mathrm{h} k_0 \mathrm{e}^{-(E/RT^1)} C^1 V^1 + Q_\mathrm{F}^1 (T_\mathrm{F}^1 - T^1) + \mathrm{Cool}^1 \qquad (17)$$

$$V^2 \frac{\mathrm{d}C^2}{\mathrm{d}t} = -k_0 \mathrm{e}^{-(E/RT^2)} C^2 V^2 + Q_\mathrm{F}^1 (C_\mathrm{F}^2 - C^2) \qquad (18)$$

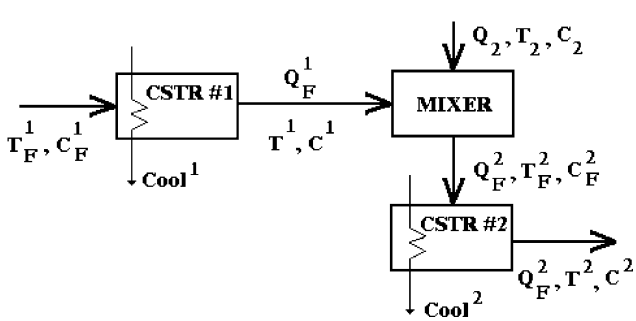

Fig. 1. Flowsheet example.

Table 1
Parameters of the CSTRs with intermediate mixer

Parameter	Value
Q_F^1 (m^3/s)	0.2062
Q_2(m^3/s)	0.3552
V^1 (m^3)	5.0
V^2 (m^3)	5.0
c_p (J/(kg K))	1.0
E/R (K)	6000
U_a (W/K)	0.35
D_h (K m^3/mol)	5.0
k_0 (s^{-1})	2.7×10^8

$$V^2 \frac{\mathrm{d}T^2}{\mathrm{d}t} = D_\mathrm{h} k_0 \mathrm{e}^{-(E/RT^2)} C^2 V^2 + Q_\mathrm{F}^2 (T_\mathrm{F}^2 - T^2) + \mathrm{Cool}^2 \qquad (19)$$

To model the mixer, the dynamics is neglected. Therefore, the balances around the mixer can be written as follows:

$$C_\mathrm{F}^2 = \frac{Q_\mathrm{F}^1 C^1 + Q_2 C_2}{Q_\mathrm{F}^2} \qquad (20)$$

$$T_\mathrm{F}^2 = \frac{Q_\mathrm{F}^1 T^1 + Q_2 T_2}{Q_\mathrm{F}^2} \qquad (21)$$

$$Q_\mathrm{F}^2 = Q_\mathrm{F}^1 + Q_2 \qquad (22)$$

The process model parameters are shown in Table 1. A detailed list is in the "Nomenclature". The temperatures and compositions of both feed streams are considered to be disturbances, and the bounds for these variables are shown in Table 2. The controlled variables are the temperatures inside the reactors (i.e. T^1 and T^2). The temperature control is performed by means of a coolant medium that flows through a jacket around the reactor. In this process, there exist many different alternatives as regards the manipulated variables selection. Traditionally, the coolant flow rate or the coolant temperature are used as control variable. The first alternative is easy to implement and it involves a minimal cost of equipment. However, it presents an operability problem due to the large nonlinearity relationship between the coolant flow rate and the removed heat. To overcome this problem, the coolant temperature can be chosen as the manipulated variable because of the linear relationship between it and the removed heat. Nevertheless, in this case, the equipment cost increases significantly due to the additional heat exchanger requirement. A third alternative has been presented by Richalet (1999). It involves the use of a combination

Table 2
Disturbances bounds

Disturbance	Lower	Nominal	Upper
C_F^1 (mol/m^3)	19.5	20	22
C_2 (mol/m^3)	19.5	20	22
T_F^1 (K)	295	300	320
T_2 (K)	295	300	320

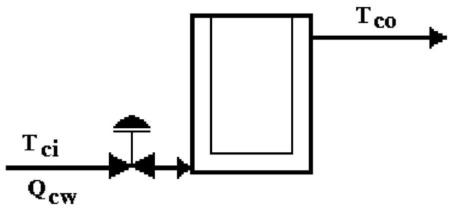

Fig. 2. Flowrate control.

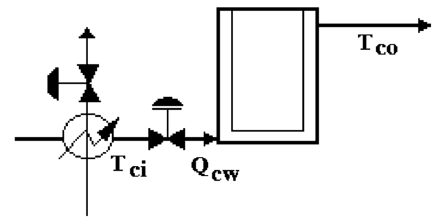

Fig. 4. Flowrate and temperature control.

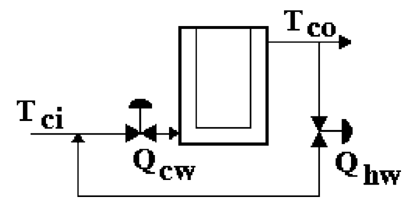

Fig. 5. Control for flowrate and temperature of coolant (without exchanger).

of the flowrate and the coolant temperature as manipulated variable.

4.1.1. Case 1: coolant flowrate control

In this case, let us consider that the internal reactor temperature is controlled by manipulating the coolant flowrate (Q_{cw}^{j}). The scheme is shown in Fig. 2. In this case, the amount of heat transferred between reactor and coolant is

$$\mathrm{Cool}^{j} = \frac{U_{a} Q_{cw}^{j} c_{p}}{U_{a} + Q_{cw}^{j} c_{p}} (T_{ci}^{j} - T^{j}) \qquad (23)$$

for $j = 1, 2$ and where the T_{ci}^{j} are constant. Note the strong nonlinearity between the transferred heat and the control variable.

4.1.2. Case 2: coolant temperature control

In this case, let us consider that the internal reactor temperature is controlled using the coolant temperature (T_{ci}^{j}) as manipulated variable. The scheme is shown in Fig. 3. In this case, the mathematical expression between the amount of heat transferred and the control variable is as in Eq. (23). But now, T_{ci}^{j} is the control variable and the Q_{cw}^{j} are constant. Note that in this form, we obtain a linear relationship between manipulated and controlled variables, but an additional equipment must be included in the process.

4.1.3. Case 3: strategy for flowrate and coolant temperature control

In this case, let us consider that the internal reactor temperature is controlled using the joint action of coolant temperature (T_{ci}^{j}) and coolant flowrate (Q_{cw}^{j}). The controlled process scheme is shown in Fig. 4.

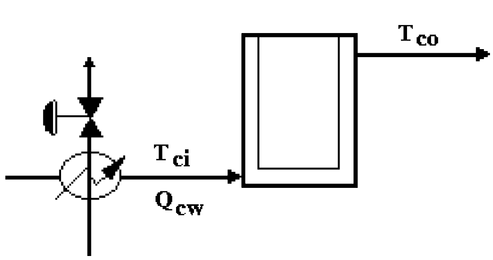

Fig. 3. Temperature control.

4.1.4. Case 4: flowrate and coolant temperature control (without heat exchanger)

In this case, as in the previous one, let us consider that the internal reactor temperature is controlled using the joint action of coolant temperature (T_{ci}^{j}) and coolant flowrate (Q_{cw}^{j}). The difference with Case 3 is that, to reduce implementation costs, the heat exchanger was excluded and a coolant recycle was added as is shown in Fig. 5. In this case, the manipulated variables are the total coolant flowrate (Q_{cw}^{j}) and the flowrate of recycle stream (Q_{hw}^{j}). The expression for the amount of heat transferred in this scheme is

$$\mathrm{Cool}^{j} = \frac{U_{a} c_{p} (Q_{cw}^{j} - Q_{hw}^{j})}{U_{a} + (Q_{cw}^{j} - Q_{hw}^{j}) c_{p}} (T^{j} - T_{ci}^{j}) \qquad (24)$$

Hence, a highly nonlinear expression is obtained.

The nominal values for the variables involved in the control schemes are shown in Table 3. Based on the Steady-State Control Action Amplitude described in Section 2, we propose a comparison between the different control schemes. For this purpose, Eq. (10) was considered and the results are shown in Table 4. In this case, the norm of the manipulated variable was normalized to its nominal value. From these results is obvious that the strategy which requires lower control efforts is the Scheme 2. It is obvious the greater availability of resources is always beneficial for the disturbance

Table 3
Values of the variables involved in the control schemes

Variables	Schemes 1–3	Scheme 4
Q_{cw}^{1} (m^3/s)	0.35	0.5
Q_{cw}^{2} (m^3/s)	0.80	0.9
T_{ci}^{1} (K)	150	150
T_{ci}^{2} (K)	150	150
Q_{hw}^{1} (m^3/s)	–	0.15
Q_{hw}^{2} (m^3/s)	–	0.10

Table 4
Results for example 1

Variables	Scheme 1	Scheme 2	Scheme 3	Scheme 4
C_F^1 (mol/m^3)	22.00	22.00	19.50	21.4646
T_F^1 (K)	320.0	315.127	320.0	320.0
C_2 (mol/m^3)	19.59	22.00	19.50	19.50
T_2 (K)	300.45	320.00	295.0	295.0
Q_{cw}^1 (m^3/s)	0.7529	–	1.20	1.20
Q_{cw}^2 (m^3/s)	0.7993	–	0.3127	1.20
T_{ci}^1 (K)	–	100.000	185.047	–
T_{ci}^2 (K)	–	129.631	100.0	–
Q_{hw}^1 (m^3/s)	–	–	–	0.50
Q_{hw}^2 (m^3/s)	–	–	–	0.496
Index	−1.325	−0.1296	−7.137	−23.196

rejection. However, this scheme involves larger cost of implementation than the others. In particular, an interesting point is that Scheme 4 presents a higher control effort than Scheme 1. It is important to remark that this is because the nominal value for the recycle flow rate is small. Then, when the manipulated control action is normalized, the division produces large values. The normalized changes show how much the system must be overdesigned to compensate for the effects of disturbances.

An efficient algorithm has been generated to solve the problems stated in Cases 1–4 using MATLAB 5.3 optimization Toolbox. A solver to deal with the algebraic equations was embedded in the optimization program used to carry out the simulations. This was performed on a 550 MHz Pentium III processor. As discussed at the end of Section 3, many starting points can be tested to avoid reaching local minima. For this purpose, 16 points were randomly generated and used to start the optimization. In order to quantify the computational time required to find the solution, an average of 20 s were needed to solve Case 1.

Table 5
Optimization variables and disturbances for the steam generating unit

Variables/parameters	Lower bound	Nominal	Upper bound
F (kg/s)	30.0	–	50.0
w_c (kg/s)	150.0	–	240.0
T_e/1000 (K)	0.28	0.29	0.32
c_v	0.7	0.8	0.9
α_1	0.9	1.0	1.05
α_2	0.9	1.0	1.05

Table 6
Critical disturbances values for the steam generating unit

Disturbance	Critical 1	Critical 2	Critical 3	Critical 4	Critical 5
T_e/1000 (K)	0.29	0.32	0.32	0.28	0.32
c_v	0.76	0.9	0.7	0.8477	0.84
α_1	1.05	0.97	1.05	1.05	0.9
α_2	0.93	1.0	0.9	1.05	0.98

4.2. Steam generation unit

In this example, the algorithm proposed in Section 3 will be used to perform the operativity analysis of a Steam Generation Unit in the presence of parameters uncertainty. The motivation for this analysis is the large operating cost involved in the operation of these units and their need to satisfy specific energy demands. The process studied in this paper consists of a 200 MW drum type boiler. The model for this unit has been developed by Ray and Majumder (1983):

$$\frac{dP}{dt} = -0.00193\alpha_1 SP^{1/8} + 0.014524F - 0.000736w_c + 0.00121L + 0.000176T_e \quad (25)$$

$$\frac{dS}{dt} = 10c_v P^{1/2} - 0.785716\alpha_2 S \quad (26)$$

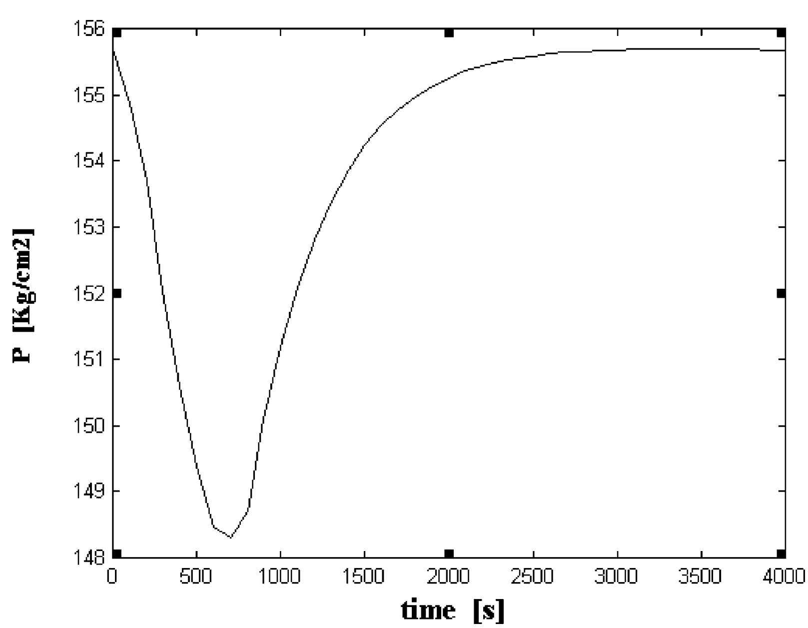

Fig. 6. Pressure inside the boiler for a critical disturbance.

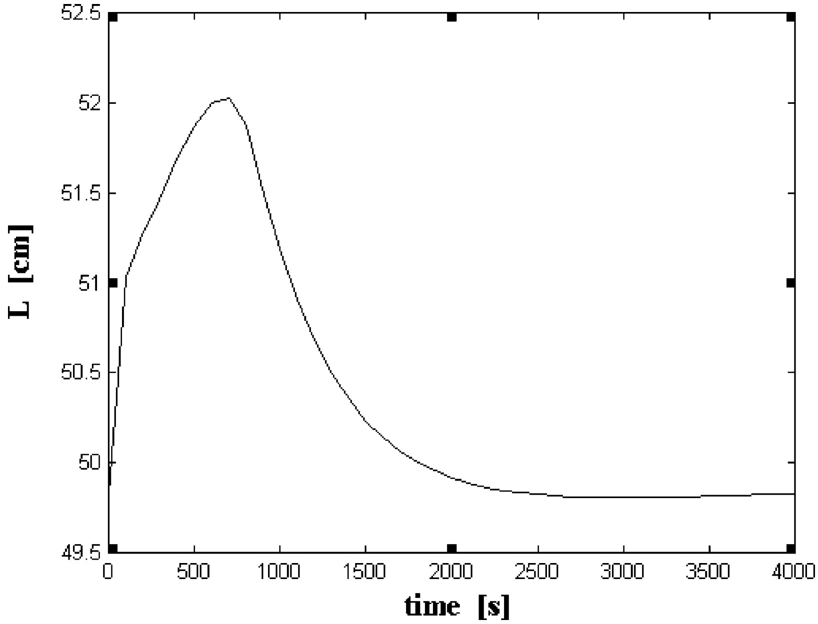

Fig. 7. Level inside the boiler for a critical disturbance.

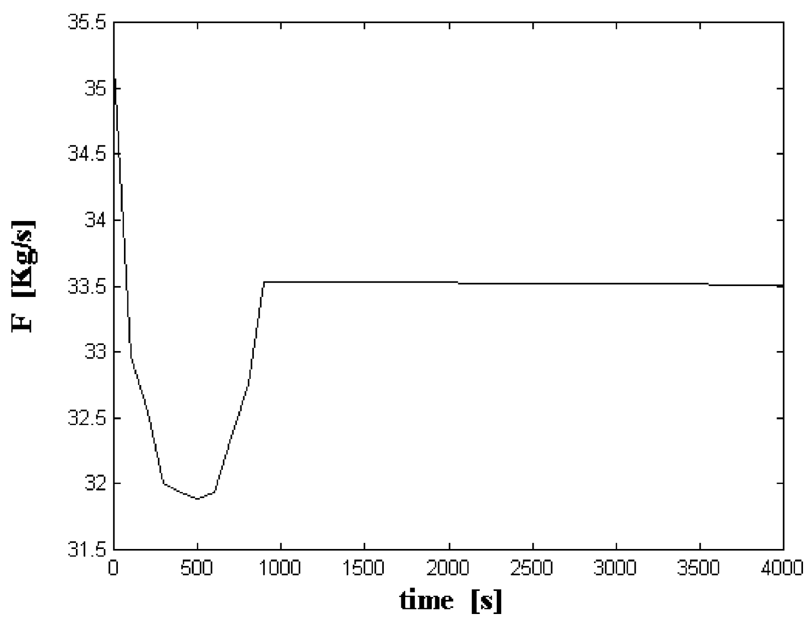

Fig. 8. Fuel flowrate to the boiler for a critical disturbance.

$$\frac{dL}{dt} = 0.00863w_c + 0.002F + 0.463c_v - 6 \times 10^{-6} P^2 - 0.00914L - 8.2 \times 10^{-5} L^2 - 0.007328S \quad (27)$$

The states of the boiler's model are the drum pressure (P), the steam flow to the H.P. turbine (S) and the drum level (L). The states P and L are the controlled variables. It has two optimization variables (they are also manipulated variables): the fuel input (F) and the feed water input (w_c), and two disturbances: the feed water temperature (T_e) and the control valve setting (c_v). The uncertain parameters included in this model are α_1 and α_2. The bounds for these variables and uncertain parameters are described in Table 5. The process constraints are $120 \leq P \leq 190$, $S \geq 90$ and $40 \leq L \leq 80$, and the objective function is $z_0 = 0.6S + 0.5P - 0.8F - 0.1w_c$. The sample time is $T_s = 100$ s, the control horizon is $M = 10$ and the simulation horizon is $40T_s$. The following restrictions were included to guarantee that both the pressure and the level return to their steady-state values: $|P(40T_s) - P(0)| \leq 10^{-3}$ and $|L(40T_s) - L(0)| \leq 10^{-3}$. As described in the previous cases, the optimization Toolbox running under MATLAB 5.3 was used to accomplish the simulation on the same hardware. The solution of the dynamic system was performed using a differential equations solver embedded in

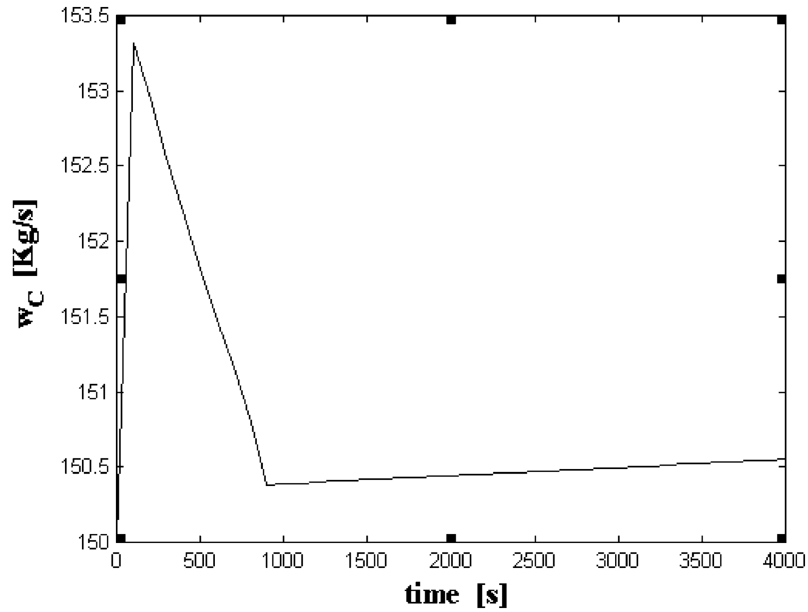

Fig. 9. Water flowrate to the boiler for a critical disturbance.

the optimization program. The average computational time consumed for solving the external loop was 1 h.

It should be noted that the developed performance criteria is controller-independent. The values of the independent process variables are calculated by the optimization of a previously defined objective function. In this way, the optimal values for the manipulated variables are obtained without making any assumption about the controller structure. Therefore, a multivariable control action is necessary to achieve the desired performance objective in this example.

The critical disturbances and uncertain parameters obtained in the solution of the problem are described in Table 6. The optimal back-off point is determined for a set of free variables. The optimum is achieved with $F = 35.1772$ kg/s and $w_c = 150.00$ kg/s which gives an objective function value of $z_0 = \$110.934$. The time responses achieved for the controlled variables P and L are shown in Figs. 6 and 7. It can be noted that both controlled variables return to their stationary steady states in spite of the presence of disturbances. The disturbances are perfectly rejected and no constraints violation is committed. The necessary movements for the manipulated variables F and w_c along the control horizon are shown in Figs. 8 and 9.

5. Conclusions

In this work, several measures for quantifying the process controllability have been introduced. They can be used as effective criteria for the purpose of assessing the performance of both an open-loop plant or a controlled one. The proposed indices are useful for measuring the ability of a certain design to operate in the presence of disturbances. Additionally, they can be used as a tool to achieve the comparison between different process design alternatives.

It must be remarked that the proposed controllability measures were defined, and are valid, for a general type of nonlinear processes. In this way, it is possible to consider a wide spectrum of processes that include some typical characteristics such as inputs saturation or complex relations.

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