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Maimon's criticism of Kant's doctrine of mathematical cognition and the possibility of metaphysics as a science

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ABSTRACT

The aim of this paper is to discuss Maimon's criticism of Kant's doctrine of mathematical cognition. In particular, we will focus on the consequences of this criticism for the problem of the possibility of metaphysics as a science. Maimon criticizes Kant's explanation of the synthetic a priori character of mathematics and develops a philosophical interpretation of differential calculus according to which mathematics and metaphysics become deeply interwoven. Maimon establishes a parallelism between two relationships: on the one hand, the mathematical relationship between the integral and the differential and on the other, the metaphysical relationship between the sensible and the supersensible. Such a parallelism will be the clue to the Maimonian solution to the Kantian problem of the possibility of metaphysics as a science.

1. Introduction

According to Kant, there are two kinds of pure rational cognition: mathematics and philosophy. Philosophical cognition is rational cognition from concepts, while mathematical cognition is rational cognition from the *construction* of concepts, i.e., from the exhibition of concepts in intuition.[1] Whereas mathematical construction allows us to "create the objects themselves" in space and time (KrV, A723 = B 751), in philosophy we cannot create the objects of cognition, but only bring the appearances given by sensibility under the concepts of the understanding, so that "we can have nothing a priori except indeterminate concepts of the synthesis of possible sensations insofar as they belong to the unity of apperception (in a possible experience)" (KrV, A723 = B 751).[2] Even though mathematics and philosophy proceed without the aid of experience, in philosophy pure reason is unable to make a priori intuitive the reality of its concepts. Thus, Kant claims that geometry and philosophy, despite their both being pure rational cognitions, "are two entirely different things, although they offer each other their hand in natural science", and therefore "the procedure of the one can never be imitated by that of the other" (KrV, A726 = B 754). In this paper we shall discuss this Kantian distinction between mathematical and philosophical cognition in light of Maimon's critical stance towards Kant's doctrine. We shall analyze how Maimon makes use of central notions and procedures of the 'hard' sciences in order to attain metaphysical knowledge of the supersensible. More precisely, we shall investigate Maimon's interpretation of differential calculus as the key to the speculative access to the noumenal realm. By means of this philosophical appropriation of methods used in pure and applied mathematics, Maimon puts forward a peculiar relationship between mathematics and metaphysics, questioning the critical limits imposed by Kant to scientific cognition in general and to metaphysical cognition in particular.

In a famous passage of the *Prolegomena*,[3] Kant criticizes Hume for not having recognized the synthetic a priori character of mathematical judgments. Kant suggests that Hume would not have grounded his metaphysical propositions in experience if he had not made that mistake, for in such case he would have had to ground mathematical axioms in experience as well, "which he was much too reasonable to do" (Prol, AA IV, 273). According to Kant, an adequate conception of mathematical knowledge would have prevented Hume from rejecting the possibility of metaphysics. We shall see that Maimon puts forward a similar criticism against Kant. For Maimon, Kant did not determine the peculiar character of mathematical cognition correctly and thus he was unable to give a satisfactory answer to the question about the possibility of metaphysics as a science. When Kant defines metaphysics as "the science of progressing by reason from knowledge of the sensible to that of the supersensible" (FM, AA XX, 260), he declares that for this transition the a priori principles of mathematics are useless, because they always refer to objects of sen-

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[1] KrV, A 713 = B 741.

[2] For an analysis of the Kantian distinction between mathematical and philosophical syntheses, see Gava (2013).

[3] Prol, AA IV, 272–273.

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sible intuition.[4] In contrast, as we will show, Maimon claims that differential calculus provides us with the tools to carry out precisely this transition from the sensible to the supersensible. According to Maimon, the possibility of metaphysical cognition of the supersensible becomes evident in differential calculus, in so far as the mathematical relationship between the integral and the differential is conceived as identical to the metaphysical one between the sensible and the supersensible. Such analogy will be the clue to the Maimonian solution to the Kantian problem of the possibility of metaphysics as a science.

The profound metaphysical consequences of Maimon's interpretation of differential calculus have not been properly discussed in the literature.[5] Even though Maimon's theory of differentials has been largely investigated in relation to the *quid juris* question,[6] Maimon's indication that with this theory he aims at the "explanation of the possibility of a metaphysics in general, through the reduction of intuitions to their elements, elements that I call ideas of the understanding" (Maimon, 2010, p. 9), has not received the same attention.[7] In discussing Maimon's theory of differentials, some scholars just refer to the problem of metaphysics, without elaborating on it.[8] Emphasis has been put on the concept of the differential as *principium individuationis*[9] or on its role in a model of cognition such that the given may be understood as a product of spontaneity, but nothing is said about the metaphysical consequences of this doctrine.[10] In what follows, we shall instead focus on Maimon's solution to the Kantian problem of metaphysics as a science. In order to accomplish this goal, we shall first analyze Maimon's criticism of Kant's doctrine of mathematical cognition. Then, we shall see how such a criticism sheds light on the relationship between mathematics and metaphysics and, in particular, how Maimon finds in differential calculus the key to the supersensible.

2. Maimon's criticism of Kant's doctrine of mathematical cognition

Let us begin by considering a classical example of a synthetic a priori judgment taken from geometry: a straight line is the shortest between two points. On this judgment, Kant maintains:

> "That the straight line between two points is the shortest is a synthetic proposition. For my concept of *straight* contains nothing of quantity, but only a quality. The concept of shortest is entirely an addition, and cannot be derived by any analysis of the concept of straight line. The aid of intuition must therefore be brought in, by means of which alone the synthesis is possible."[11]

According to Kant, the concept of "shortest" is not to be found in the concept of "straight line", for the latter does not include any note whatsoever referring to a *quantity*. Rather, the concept of straight line just contains a certain *quality* of the line, namely that of being straight. For this reason, the judgment is not analytic but synthetic. In order to connect the subject (straight line) to the predicate (shortest) of the judgment one has to go beyond the mere concept of a straight line by constructing it in intuition. Once one has acquired by construction the *intuition* of a straight line between two points, one establishes that such intuition is also the intuition of the *shortest* line. Since this construction takes place in pure intuition (more precisely, in pure space), the judgment is not only synthetic but also a priori.[12]

Against this explanation, Maimon objects that the understanding cannot use the concept of a straight line as a rule for the construction of a line in intuition, because "being straight is an intuition and consequently outside its domain" (Maimon, 2010, p. 40). In other words, we would not know how to construct a straight line unless we have already gone beyond its concept, i.e., unless we have already constructed it.[13]

But this is not the only difficulty in Kant's treatment of the problem. Maimon argues that the explanation of the possibility of synthetic a priori judgments may be understood in two different ways.[14] One may either ask for the exhibition of a certain concept in intuition, as Kant does,[15] or for a genetic explanation.[16] Provided that space and time as pure intuitions are taken as grounds for the synthetic a priori judgments of mathematics, the possibility of these judgments is understood in the first sense, i.e., one exhibits in intuition the concepts involved. However, no genetic explanation is provided herewith. For

[4] FM, AA XX, 316.

[5] Among the most recent studies on Maimon's views on metaphysics and mathematics, the following deserve special mention: Buzaglo (2002), Duffy (2014), Freudenthal (2006), Kauferstein (2006).

[6] See: Atlas (1964), pp. 109–123; Cassirer (2000), pp. 93–100; Duffy (2004); Guéroult (1929), pp. 59–86; Hoyos (2001), pp. 329–340; Kauferstein (2006), pp. 309–348; Kroner (1921), pp. 353–356.

[7] Buzaglo underlines Maimon's metaphysical inclinations, but he does not connect them to Maimon's interpretation of differential calculus: Buzaglo (2002), p. 4 and 124–128. Neither does Ehrensperger: Ehrensperger (2004), pp. XXXV-XXXIX.

[8] See: Bergman (1967), p. 270; Kuntze (1912), p. 334; Zac (1998), p. 169 and 171. According to Kuntze, Maimon's theory of differentials explains the particularity of empirical objects, once the Kantian things in themselves are rejected: Kuntze (1912), p. 331.

[9] See: Gasperoni (2012), p. 115. This is the line of interpretation also adopted by Beiser: Beiser (1987), pp. 295–300.

[10] See: Thielke (2003), p. 111. Engstler considers the relationship between the theory of differentials and the problem of metaphysics, but he overlooks the crucial analogy between mathematics and metaphysics mentioned above: Engstler (1990), pp. 182–189. Some indications regarding the Maimonian treatment of the problem of metaphysics can be found in Pringe (2016), but no discussion of Maimon's doctrine of mathematical cognition is carried out there.

[11] KrV, B16. See also: Prol, AAIV, 269.

[12] Koriako criticizes this Kantian account of mathematical knowledge and the role of *pure* intuitions in mathematical constructions: "Kants Analogie zwischen empirischen Anschauung, die zu empirischer Erkenntnis führt, und mathematischer Erkenntnis, die auf reiner Anschauung beruht, ist daher irreführend; denn die Eigenschaften des Dreiecks erkennen wir nicht durch Inspektion in der reinen Anschauung, wie wir die Eigenschaften des Mondes durch das Fernrohr in empirischen Anschauung ermitteln." Koriako (1999), p. 277. For the philosophy of mathematics, Kant's doctrine of pure intuition is "überflüssig", since pure intuition "keine Begründungsleitung bietet, die über das hinausginge, was reine mathematische Begriffe *plus* empirische Anschauungen zu leisten vermögen." Koriako (1999), p. 283.

[13] One may assume, following Wolff, that the straightness of a line consists in the fact that every part of the line has the same direction. But this presupposes the line already being constructed in intuition, since *direction* has no meaning before, or independently of, such a construction. "So," Maimon concludes, "this definition of the straight line is useless as well" (Maimon, 2010, p. 41 = GW II, 70). On this issue, Kant claims: "As for the definition of a *straight line*, this cannot be done by means of the identity of direction of all its parts because the concept of direction (as that of a straight line, through which movement is distinguished *without regard to its magnitude*) already presupposes that concept." (Br, AA XI, 53–54). Translation taken from Maimon (2010), p. 236.

[14] Maimon (2010), p. 35 = GW II, 57–58.

[15] See, e.g., Br, AA XI, 38: "All synthetic judgments of theoretical knowledge are possible only by means of the relation of the given concept to an intuition." Translation is ours.

[16] Beiser points out the rationalist roots of this genetic explanation. For example, in his *Von den Kräften des menschlichen Verstandes* Wolff claims: "Nothing more can be thought of a thing other than how it has originated or how it has become what it is. For this reason one understands the essence of a thing when one distinctly conceives how it has become what it is." Quoted by Beiser in: Beiser (1987), p. 297.

Maimon, this implies that in such a case the necessary connection between the subject and the predicate of the judgment does not receive a proper account. In this respect, Maimon asks:

> "for how is it conceivable that the understanding can establish with apodictic certainty that a relational concept (the necessary being together of the two predicates) that it thinks must be found in a given object? All that the understanding can assume with certainty in the object is what [it] itself has put into it (in so far as it has itself produced the object itself in accordance with a self-prescribed rule), and not anything that has come into the object from elsewhere."[17]

If we assume, as Kant does, that in mathematics the understanding applies its rules to the pure intuitions of space and time, then the possibility of mathematical synthetic judgments is left without a satisfactory explanation. In our example, we may learn by construction in intuition *that* the straight line is the shortest one between two points, but this does not show us *why* it is so.[18] Construction provides us with no justification of the connection between subject and predicate of the judgment. In this regard, Maimon asserts:

> *"All that cannot be constructed otherwise cannot be cognized otherwise in a construction.* Such an acclaimed principle of the possibility of a construction reduces itself to a barren identical proposition."[19]

The kind of necessity we may ascribe to this judgment is therefore not objective, but merely subjective: it might be the case that I claim that the straight line is the shortest one between two points only because I have always perceived it so.[20]

Maimon does not only challenge Kant's explanation of the synthetic character of mathematics (geometry in particular), but also that of the a priori character of this science. For this purpose, he puts forward a different definition of the determinations of a priori and *pure*:

> "The absolutely a priori [a priori *absolut betrachtet*] is, for *Kant*, a type of cognition that must be in the mind prior to any sensation. For me, on the other hand, the absolutely a priori is a type of cognition that precedes cognition of the object itself, i.e. [it is] the concept of an object in general along with everything that can be asserted about such an object, or [a type of cognition] in which the object is only determined by means of a relation, as for example the objects of pure arithmetic."[21]

According to Maimon, if we state that a mathematical judgment is established by construction of its object in intuition, then we may claim that the judgment is a priori in the Kantian sense. However, this judgment will not be a priori in Maimon's sense, because its object is first to be constructed before one can gain knowledge of it. Thus, cognition will not precede the object. This is precisely the case of the straight line:

> "Suppose I do not possess a representation of a straight line, and someone asks me, 'can a straight line be non-straight at the same time?' I will certainly not put my judgement off until I have a representation of it (assuming that I don't know what a straight line is), rather I will have my answer on hand at once: that this is impossible. By contrast if he asks me, 'is a straight line the shortest?' I will answer, 'I don't know, perhaps yes, perhaps no', until I have acquired a representation of a straight line."[22]

If a relation between objects is cognized before (i.e., independently of) the cognition of these very objects, then it is cognized a priori in the strict Maimonian sense. In contrast, should the cognition of the objects be first, even if no sensation is present, the relation is cognized a posteriori. This is the case of the axioms of mathematics (in particular geometry), as Kant understands them. Since these axioms (so conceived) do not satisfy the Maimonian apriority criterion, they do not have objective necessity and, even though they are true, they are mere assertoric judgments. Kant is unable to prove their apodicticity.[23]

But Maimon not only puts forward a new definition of the notion of a priori. He also proposes his own notion of *pure*:

> "For Kant, the *pure* is that in which nothing belonging to sensation is to be found, i.e., only a connection or a relation (as an activity [*Handlung*] of the understanding) is pure; but for me the *pure* is that in which nothing belonging to intuition is to be found (in so far as intuition is only incompletely active [*eine unvollständige Handlung ist*])."[24]

According to this Maimonian definition, pure is only that which is a product of the mere understanding, without any participation of sensibility. Thus, mathematical judgments (as understood by Kant) are not pure, because they depend on a construction in intuition, even though no sensation is thereby involved.[25]

Summing up, while Kant argues that mathematics (and in particular geometry) is apodictic, a priori and pure synthetic cognition, grounded in the construction of concepts in intuition, Maimon claims that Kant is unable to justify the necessary connection between subject and predicate in mathematical judgments. Moreover, in view of his own definitions of a priori and pure, he states that those judgments, as understood by Kant, are rather assertoric, not pure and a posteriori.

We shall now see that Maimon will explain the possibility of synthetic judgments in mathematics by showing the way in which the intuition of an object may be generated. While, for Kant, in mathemat-

[17] Maimon (2010), p. 36 = GW II, 59–60.

[18] In this sense, Maimon claims: "even if I already see the meaning of the proposition that a straight line is the shortest between two points (by constructing a straight line), I still do not know how I arrived at this proposition." (Maimon, 2010, p. 35 = GW II, 59). Analogously: "Experience (intuition) shows that a straight line is the shortest line between two points, but it does not make it the case that the straight line is the shortest." (Maimon, 2010, p. 27 = GW II, 43).

[19] GW VII, 399. Translation is ours. See also GW V, 472–473. Kant claims: "that which follows from the general conditions of the construction must also hold generally of the object of the constructed concept." (KrV A 716 = B 744).

[20] "I note, however, that even if such propositions express necessity, this does not establish that they contain (objective) necessity. For example, my judgement that a straight line is the shortest between two points can derive from my having always perceived it thus so that for me subjectively it has become necessary." Maimon (2010), p. 93 = GW II, 173.

[21] Maimon (2010), p. 91 = GW II, 168–169. See also: GW III, 187.

[22] Maimon (2010), p. 91–92 = GW II, 169–170.

[23] Maimon (2010), p. 99 = GW II, 185.

[24] Maimon (2010), p. 92 = GW II, 170. Also, Koriako recommends a different notion of *pure* in order to adequately cope with the problem of mathematical knowledge: "die `reine' Anschauung des Mathematikers erhält man nicht durch `Purifizierung' der empirischen Anschauung, sondern durch deren `Schematisierung'. `Rein' sind diese mathematischen Begriffe aber deshalb, weil sie synthetisch definierte Begriffe sind." Koriako (1999), p. 275.

[25] Since the principle of contradiction is merely a *conditio* sine *qua non* of our cognition, according to this definition we do not have any completely pure knowledge: Maimon (2010), p. 185 = GW II, 359.

ics we perform a *construction in* intuition, according to Maimon in mathematics the *arising of* intuition takes place. This can be seen if we consider the straight line once again.

Maimon puts forward a different explanation of the synthetic a priori character of the judgment that "a straight line is the shortest between two points" by taking the concept of "straight line" as the predicate rather than as the subject of the proposition. The concept of a straight line is not, as we have seen, the rule for the construction of such object, but rather the concept of the shortest line[26]:

> "As soon as the understanding prescribes the rule for drawing a line between two points (that is, that it should be the shortest), the imagination draws a straight line to satisfy this demand." [27]

According to Maimon, the concept of minimal distance is a concept of reflection that expresses a certain difference regarding magnitude. Concepts of reflection are used for establishing relations between already given objects, but in this case it can be shown that such a concept may also be the rule for the production of an object not previously given. While in other cases objects precede the concept, now it is the concept that has logical priority. The straight line, as an object, is nothing but the sensible image of the concept of minimal distance between two points.[28] Maimon acknowledges that one may learn that the straight line is the shortest by intuition, before one proves the judgment. The truth of the judgment may be anticipated in this way. But this is just a consequence of the fact that one intuits the image of the concept of minimal distance when one intuits a straight line. This sensible representation may be made clear but not distinct.[29]

However, the core of the difference between Kant's and Maimon's explanation of the synthetic character of the judgment that "a straight line is the shortest between two points" does not just concern which concept is taken to be the subject of the judgment, but also the underlying doctrine of space at play. Kant claims that space is an intuition and not a concept. For Maimon, space is an intuition but also a concept, and space as an intuition presupposes space as a concept. Space, as a concept, is the representation of the difference between things in general. Space, as intuition, is the schema of that pure concept of difference.[30] The sensible representation of the differences of given things is their being outside each other.[31] While the object of geometry is space as an *intuition*, differential calculus considers space as a *concept*[32]:

> "The object of pure arithmetic is number, whose form is pure time as a concept; on the other hand, the object of pure geometry is pure space, not as concept, but as intuition. In the differential calculus, space is considered as a concept abstracted from all quantity, but nevertheless considered [as] determined through different kinds of quality in its intuition."[33]

Maimon explains the connection between the shortest line and the straight line precisely as one may expect from the viewpoint of the calculus of variations.[34] According to this last, the proposition in question is proved by demanding that the distance between two points be minimal, thus obtaining as a result the expression of a straight line.[35] Hence, in his account, Maimon explains how a concept of difference (minimal distance) relates to its sensible schema (straight line), thereby showing in a concrete case how a certain determination of space as a concept *produces* its corresponding intuitive representation, i.e., how an intuition *arises* according to a certain conceptual rule. The judgment that "a straight line is the shortest between two points" is an example of how the two different ways in which space may be considered relate to each other. For Maimon, on the other hand, Kant unsuccessfully tries to justify the synthetic character of the judgment by appealing to a construction in the intuition of space, neglecting the conceptual origin of the latter. The Kantian doctrine of space, according to which such an intuitive representation has its origin in mere sensibility, independently of any concept, implies that we first have to construct in intuition a straight line in order to ascribe to it the property of being the shortest line between two points. In contrast, according to Maimon, it is rather the case that we impose such a property as the rule according to which the straight line can arise at all. In this case, we comprehend the way in which the straight line arises, without intuiting it as already arisen.

Precisely in view of this conception of mathematical cognition, Maimon believes that the propositions of geometry are demonstrated far more rigorously by means of differential calculus than by construction in intuition:

26 For a (quite) different explanation of Maimon's doctrine of mathematical cognition, in general, and of Maimon's views on the judgment that "a straight line is the shortest between two points", in particular, see Freudenthal (2006).

27 Maimon (2010), p. 14–15 = GW II, 19. Freudenthal overlooks this passage when he asserts that, like Kant, "Maimon has no rule of construction for the straight line and cannot construct it." Freudenthal (2010), p. 86. As a consequence, Freudenthal wrongly claims that Maimon's alternative to Kant's treatment of geometry "failed to replace the property *straight* given in intuition with a concept of understanding" and "failed to prove that the straight line is the shortest between two points." Freudenthal (2010), p. 89.

28 Maimon (2010), p. 41 = GW II, 69.

29 Maimon (2010), p. 41 = GW II, 70.

30 Maimon (2010), p. 179 = GW II, 346. See also: Maimon (2010), p. 14 = GW II, 18.

31 Maimon (2010), p. 14 = GW II, 18.

32 Bergman maintains that there is a "gap" in Maimon's system of sciences, "for if space and time are for him concept son the one hand and intuitions on the other, then there should be besides the present science of geometry that deals with intuitive space another geometry that deals with conceptual space". Bergman (1967), p. 57. But the "science" that considers space as a concept is precisely differential calculus.

33 Maimon (2010), p. 16–17 = GW II, 22–23.

34 In his analysis of Kant's example of the straight line at B16, Friedman speculates that there Kant might be "referring to the variational methods developed by Euler in 1728 for *proving* geodesicity. That is, we consider the result of integrating arc-length over all possible (neighboring) curves joining two given points, and we look for the curve that minimizes the integral." See: Friedman (1992), p. 87, footnote 54. But, as Maimon realizes, Kant does not begin with the concept of the shortest line in order to connect it to the concept of the straight line, but the concept of the shortest line is rather an addition gained by construction of the concept of the straight line. It is Maimon's and not Kant's treatment of this example that may refer to the calculus of variations. Also Freudenthal criticizes Friedman's speculation in: Freudenthal (2006), p. 8 footnote 6.

35 Kästner discusses the history of calculus of variations in: Kästner (1799[3]), pp. 857ff.

> "This is why I also hold that geometrical propositions can be demonstrated far more powerfully through the *methodus indivisibilium*, or the differential calculus, than in the usual way."[36]

For Maimon, geometrical propositions remain assertoric and a posteriori synthetic judgments if one tries to justify them by construction in intuition. In contrast, differential calculus may prove them as apodictic and a priori.[37] Thus, if we follow Kant in his explanation of the synthetic character of geometry, we are not allowed to assume geometrical judgments as *facta* in an investigation into the possibility of metaphysics as a science, as Kant does in *Prolegomena* and in the B Introduction to the first *Critique*. The reason is straightforward: "a fact that is uncertain is no fact at all" (Maimon, 2010, p. 94). In contradistinction to this situation, Maimon may take differential calculus as an Archimedean point from which to develop his investigation on metaphysics:

> "As for me, I also take a fact as ground, […] but a fact relating to a priori objects (of pure mathematics) where we connect forms (relations) with intuitions; and because this undoubted fact refers to a priori objects, it is certainly possible, and at the same time actual."[38]

Maimon's definitions of a priori and pure are not merely *alternatives* to the Kantian definitions of these concepts. Maimon's definitions capture the peculiarity of mathematical cognition in a way Kant's definitions do not. As differential calculus shows, the connection between subject and predicate in the judgment "the straight line is the shortest between two points" is not established by a *construction in* intuition, where the straight line has first to be intuited in order to be known as the shortest. On the contrary, the connection obtains because of the *arising of* intuition, i.e. by generating the intuition of the straight line in accordance with the rule of being the shortest line. Thus, the judgment is not a priori merely in the Kantian sense, but rather it is a priori in the much stronger Maimonian one.

Moreover, according to Kant, geometrical judgments are pure because no *sensation* is present in them. Conversely, Maimon claims that they are not pure, since, even in the absence of sensation, they nevertheless involve *intuition.* The Kantian definition of *pure* conceals the deep relationship between mathematical and metaphysical knowledge that Maimon seeks to bring to light. It is precisely the impurity of mathematical judgments that gives them their peculiar relevance regarding metaphysics. If these judgments are considered from the viewpoint of differential calculus, it is possible to explain how the sensible realm relates to the supersensible one and how a transition from the sensible to the intelligible world is possible.[39]

3. Metaphysics as the science of the differentials of appearances

Maimon states that he agrees with Kant on the definition of metaphysics. Metaphysics, Maimon claims, is the science of things in themselves.[40] For Kant, Maimon suggests, things in themselves are the substrata of their appearances and the former are completely heterogeneous with respect to the latter.[41] Thus, these supersensible substrata remain inexorably unknown to us, for appearances just give us cognition of the way in which we are affected but not of things as they may be beyond their being given by sensibility. Therefore, metaphysics as knowledge of the supersensible is for Kant impossible. For Maimon, in contrast, the science of things in themselves is not the science of something beyond appearances, but the science of the rules of the arising of appearances. These rules are not objects of intuition and are, in this sense, things in themselves. However, these rules may be thought of determinately precisely by means of the intuitions that arise from them. Mathematics, and in particular differential calculus, shows us that this can be done. This is the fact upon which the possibility of metaphysics as a science may be understood.

According to Maimon, there is no heterogeneity between the sensible and its intelligible ground precisely because the latter is the rule of the arising of the former. When we intuit an object, we represent it as already arisen. When we think of it, we do not represent it as arisen, but as arising[42]

> "For the understanding to think a line, it must draw it in thought, but to present a line in intuition, it must be imagined as already drawn. For the intuition of a line, only consciousness of the apprehension (of the taking together of mutually external parts) is required, whereas in order to comprehend [*begreifen*] a line, a real definition [*Sacherklärung*] is required, i.e. the explanation of the way it arises [*die Erklärung der Entstehungsart*]: in intuition the line precedes the movement of a point within it; on the other hand, in the concept it is exactly the reverse, i.e. for the concept of a line, or for the explanation of the way it arises, the movement of a point precedes the concept of the line."[43]

The rule of the arising of an object is its differential.[44] Even though the differential is not an object of intuition, it may be known

[36] Maimon (2010), p. 143 = GW II, 274. In the same sense, Maimon states: "As a result, I cannot share the opinion of *Ben David* when he asserts (*Versuch über das mathematische Unendliche* [Essay on the Mathematical Infinite]): 'That the advantage elementary geometry possesses over other sciences in regard to [its self-] evidence [Evidenz], it must also possess over higher geometry and algebra, namely that the reality of the former can be shown through construction, but that of the latter cannot'". Maimon (2010), p. 143–144 = GW II, 275. See Bendavid (1789), XXX f. Bendavid claims: "The surest touchstone for the logical correction of mathematical concepts is, as we have seen, the possibility of their construction." Bendavid (1789), XXXI. "On the other hand, no matter how one may want to explain it, the infinite is such that it cannot be constructed." Bendavid (1789), XXX. Translations are ours.

[37] As far as they are synthetic, geometrical judgments possess *outer* necessity, which Maimon distinguishes from the *inner* necessity of analytic judgments: "Necessity has two forms, inner and outer: inner necessity occurs in analytic judgements and outer necessity in synthetic judgements. A human being is an animal. In this case the necessity is inner because 'human being' cannot be thought without 'animal' as the concept of 'animal' is contained within that of 'human being'. But the judgement that a straight line is the shortest between two points expresses the relation of correspondence [Übereinstimmung] between straight and shortest; this relation of correspondence [is] not in itself, i.e. a relation of identity [Identität], but [is] rather [one of] coincidence in one and the same subject." Maimon (2010), p. 133 = GW II, 253.

[38] Maimon (2010), p. 187 = GW II, 363–364.

[39] Maimon (2010), p. 176 = GW II, 339. I thank both an anonymous reviewer and Fernando Moledo for pressing me to make this point clearer.

[40] GW III, 200.

[41] GW III, 200.

[42] Maimon (2010), p. 23 = GW II, 33.

[43] Maimon (2010), p. 23–24 = GW II, 35–36.

[44] Maimon (2010), p. 22 and 19 footnote = GW II, 33 and 28 footnote. As many scholars have already pointed out, Maimon calls "differential" not only the *rule* of the generation of the sensible, but also the *element* or smallest unit of the sensible. The inconsistency that this double characterization seems to imply (see, e.g., Engstler (1990), p. 169) may be avoided by noting that Maimon distinguishes two different perspectives from which the operations of the mind should be considered (Maimon, 2010, p. 47–48 and 193–194 = GW II, 81–82 and 376–377). What from the subjective perspective is considered an element of the sensible, is from the objective one its rule of generation. See Pringe (2016), p. 95 footnote.

by means of its sensible image or schema.[45] Consider for example a moving body in space. Its trajectory is the sensible image of its instantaneous velocity, for the rule of the arising of the trajectory is precisely the instantaneous velocity of the body. How can we determine such rule? Maimon answers:

> "Now, the velocity at each instant is a real object (a determinate intensive magnitude), a quantum of determinate quantity. But we cannot have any cognition of this determinate quantity through the velocity in itself, but only through its effect, namely through the space that a body with this velocity would traverse (if the velocity remained constant); but neither the duration of the movement nor the space traversed in this time are part of the essence of the velocity. So we must think the latter abstracted from the former, i.e. we must reduce them to an infinitely small space and an infinitely small time; but this does not make them any the less real."[46]

The instantaneous velocity cannot be known directly, but only by its effect, namely by the space that the body would traverse in a certain period of time, if it moved constantly at such velocity. As rules of the arising of trajectories, instantaneous velocities "are mere limit concepts [*Gränzbegriffe*], which we can approach nearer and nearer to, but never reach. They arise through a continuous regress or through the diminution to infinity of the consciousness of an intuition." (Maimon, 2010, p. 19 footnote). This reduction of intuition to an infinite small space and an infinite small time is what we would call, in modern terms, the limit of the average velocity when the time interval tends to zero.[47] Thus, although the instantaneous velocity is not itself an object of intuition, it may be cognized by means of its schema or sensible image: the trajectory. More precisely, the instantaneous velocity is the *derivative* of the trajectory. Thus, differential calculus teaches us how to determine the intelligible ground of a sensible object and in this way it furnishes "the explanation of the possibility of a metaphysics in general, through the reduction of intuitions to their elements, elements that I call ideas of the understanding [*Verstandsideen*]" (Maimon, 2010, p. 9). These ideas of understanding, i.e., the differentials, are the *noumena* behind the *phenomena*. Such an idea does not correspond to a transcendent reality, but rather to a *method* "for finding a passage from the representation or concept of a thing to the thing itself; it does not determines any object of intuition but still determines a real object whose schema is the object of intuition" (Maimon, 2010, p. 188). Since the differential is the rule of the arising of the sensible object and the latter is the key to the cognition of the former, one may claim that the differential is the *ratio essendi* of the sensible object, while the sensible object is the *ratio cognoscendi* of the differential.[48] In this connection, Maimon claims:

> "Our thinking essence (whatever it may be) feels itself to be a citizen in an intelligible world, and although this intelligible world is not an object of its cognition (nor indeed is this thinking essence itself), nevertheless sensible objects indicate [*hinweisen*] intelligible objects to it."[49]

Whereas differential calculus shows us how to move from the sensible to the supersensible, integral calculus teaches us how to make the inverse transition, i.e., how to go from the supersensible to the sensible. In our example, the trajectory is the integral of the instantaneous velocity.

In his account of the generation of the sensible, Maimon makes use of Newton's method of fluxions, which Kästner develops in his very influential books on mathematics and mathematical physics.[50] On the generation of mathematical quantities, Newton points out:

> "I don't here consider Mathematical Quantities as composed of Parts extremely small, but as generated by a continual motion. Lines are described, and by describing are generated, not by any apposition of Parts, but by a continual motion of Points. Surfaces are generated by the motion of Lines, Solids by the motion of Surfaces, Angles by the Rotation of their Legs, Time by a continual flux, and so in the rest. These Geneses are founded upon Nature, and are every Day seen in the motion of Bodies."[51]

The velocities of the motions that generate the quantities are called *fluxions* and the generated quantities *fluents*. Newton's fluxion corresponds to what Maimon calls the differential of an object, while the object itself, thought of as generated by its differential, is to be considered a fluent:

> "The understanding can only think objects as flowing [fliessend] (with the exception of the forms of judgement, which are not objects). The reason for this is that the business of the understanding is nothing but *thinking*, i.e. producing unity in the manifold, which means that it can only think an object by specifying [angiebt] the way it arises or the rule by which it arises [die Regel oder die Art seiner Entstehung]: this is the only way that the manifold of an object be brought under the unity of the rule, and consequently the understanding cannot think an object as having already arisen [entstanden] but only as arising [entstehend], i.e. as flowing [fliessend]."[52]

According to the Newtonian method of fluxions, the fundamental task of calculus may be stated clearly as follows: the relation of fluents being given, to find the relation of the fluxions that generate them and vice versa.[53] In Newton's own terms:

> "Prob. I. The Relation of the Flowing Quantities to one another being given, to determine the Relation of their Fluxions." [54]

> "Prob. II. An Equation being proposed, including the Fluxions of Quantities, to find the Relations of those Quantities to one another."[55]

Maimon translates the mathematical problem of the relationship between fluents and fluxions into the metaphysical one about the re-

[45] Maimon (2010), p. 188 = GW II, 366.

[46] Maimon (2010), p. 151–152 = GW II, 290–291.

[47] Neither the duration of the movement nor the space traversed belong to the essence of the *instantaneous* velocity.

[48] Maimon identifies *Entstehungsart* and *Essentia realis* in Maimon (2010), p. 213 = GW II, 415. Although Maimon later characterizes differential calculus as a method of fictions (see, e.g., GW IV, pp. 51f; GW V, pp. 263–264), he does not make this characterization in his *Essay on Transcendental Philosophy*. For a critical evaluation of the interpretations of Maimon as a fictionalist, see Engstler (1990), p. 139f. In any case, fictional, as Freudenthal rightly points out, is not to be opposed to real, but to actual, i.e. to existent *in intuition*. See Freudenthal (2010), p. 97.

[49] Maimon (2010), p. 175 = GW II, 338.

[50] See Kästner (1766) (1771) (1794) (1799). See Boyer (1949), p. 250.

[51] Newton (1964), p. 141.

[52] Maimon (2010), p. 22 = GW II, 32–33.

[53] See Boyer (1949), p. 194. On Newton's method of fluxions, see especially Guicciardini (2009), p. 169ff.

[54] Newton (1736), p. 21.

[55] Newton (1736), p. 25.

lationship between sensible objects and their supersensible ground. On the one hand, Maimon argues:

> "These differentials of objects are the so-called noumena; but the objects themselves arising from them are the phenomena. With respect to intuition = 0, the differential of any such object in itself is dx = 0, dy = 0 etc.; however, their relations are not = 0, but can rather be given determinately [bestimmt angegeben werden können] in the intuitions arising from them."[56]

The tangency problem, i.e., the problem of finding the relation of the fluxions, the fluents being given, corresponds to the problem of determining the relation of the differentials out of the intuitions that arise from them. In other words, the first problem that Newton puts forward in his *Methodus fluxionum* is understood by Maimon as the question about the determination of the noumena that ground given phenomena.

On the other hand, Maimon suggests that "out of the relations of these different differentials, which are its objects, the understanding produces the relation of the sensible objects arising from them" (Maimon, 2010, p. 21). Now it is the second main problem of calculus, i.e., that of the quadrature of curves, that receives a metaphysical reinterpretation. The determination of the fluents when the fluxions are given corresponds to the determination of the phenomena of given noumena. In short, Maimon transforms the mathematical problem of the relationship between fluents and fluxions into the metaphysical problem of the relationship between the sensible and the supersensible. Differentiation and integration are the opposite directions in which the transition between the sensible and the supersensible may take place. While differentiation takes us from the sensible to the supersensible, from the phenomena to the noumena that ground them, integration takes us the other way round, from the supersensible to the sensible, from noumena to phenomena:

> "So the result of the theory is the following. With Kant, I maintain that the objects of metaphysics are not objects of intuition and cannot be given in experience. But I depart from him in this respect: he claims that they are not objects that can be thought by the understanding as determined in any way; by contrast, I hold them to be real objects, and although they are in themselves only ideas, they can nonetheless be thought as determined by means of the intuitions that arise from them. Further, just as we are in a position to determine new relations between magnitudes themselves by reducing them to their differentials (and these in turn to their integrals), so by reducing intuitions to their elements, we are in a position to determine new relations between them, and in this way to treat metaphysics as a science."[57]

Even though the objects of metaphysical cognition are not sensible objects, the understanding may determine them by means of the intuitions that they originate. Differential calculus shows that this can be done in fact and therefore that cognition of the supersensible is possible, if only we conceive the latter not as a transcendent sphere, but as the realm where the rules of the arising of the sensible are to be found.

For Maimon, differential calculus shows, *pace* Kant, that we do have cognitive access to the intelligible ground of the sensible and that our understanding differs only in degree from an infinite one. Of course, Kant would object that we do not have "the least concept" concerning such intuitive understanding (Prol, AA IV, 316). But Maimon would promptly retort that we do have a concept of it, because we partially possess that kind of understanding. In mathematics, but especially in differential calculus, "the faculty of thought produces both the *form* and the *matter* of its thinking out of itself" (Maimon, 2010, p. 6). In this respect, "we are similar to God" (GW IV, 42).[58]

In empirical knowledge, reason produces its own objects of cognition as well. For Maimon, the forms of thought are not applied to *given* appearances by means of schemata, as Kant claims. Rather, Maimon argues "that both the forms and the objects of our cognition themselves are in us a priori".[59] Our faculty of cognition does not consist merely in recognizing given objects by means of forms of thought, but also "in producing the objects themselves by means of these forms."[60] Empirical objects are produced when the understanding thinks their differentials according to the categories.[61] Accordingly, Maimon calls his position rational dogmatism and opposes it to Kant's empirical dogmatism, which claims that the object of cognition must be given a posteriori, by experience.[62]

However, whereas we may fully determine the generating rule of a mathematical object, we can only progressively approach the rule in function of which the empirical arises. Clear and distinct consciousness of the generating rule of the empirical is possible only for an in-

[56] Maimon (2010), p. 21 = GW II, 32. Analogously Newton's method allows us to determine the relations between fluxions by considering finite lines proportional to them.

[57] Maimon (2010), p. 104 = GW II, 195–196.

[58] Translation is my own. See also GW V, p. 324, where Maimon claims that differential calculus is a "sparkle of divinity" and a "patent of nobility" that proves the lineage of human spirit from "pure intelligences". This possible response notwithstanding, Kant would still have a last resort against Maimon's position, of which he explicitly makes use in a letter to Herz: for Kant, if we assumed, as Maimon does, that our understanding is of the same kind as a divine one, though merely limited, then the antinomies of pure reason could not be resolved. (Br, AA XI, 54). In order to avoid these antinomies, Kant argues, we must accept that the matter of cognition (even in mathematics) is given to the understanding and not spontaneously produced by the latter. A reconstruction of the Kantian antinomies from a Maimonian viewpoint, aimed at determining whether they may nevertheless get solved or even whether they would first arise, goes beyond the scope of this paper. However, we may indicate that Maimon, far from trying to escape this problem, doubles the Kantian challenge and claims that not only in metaphysics do we face antinomies, but also in physics and mathematics (Maimon (2010), p. 119 = GW II, 227). As a matter of fact, Maimon acknowledges that it is the very structure of our limited faculty of cognition that leads our reason to an antinomy. The Maimonian antinomy consists in the following theses: 1) Without given matter we cannot achieve consciousness of the form of thought. 2) For the thought of an object to be complete, it is required that nothing in it should be given (GW III, p. 186). The way out of this antinomy does not rely on the Kantian distinction between a spontaneity and a receptivity irreducible thereto. Rather, the matter of cognition, which according to Kant is received by sensibility, is to be guided back to the intellectual form that grounds it. As we have seen, the sensible matter of cognition is the schema of the intellectual form. Since the former is the *ratio cognoscendi* of the latter, the first demand—according to which matter is a condition of our consciousness of the form—is satisfied. And since the form is the *ratio essendi* of the matter, the second demand—according to which, thought should progressively be shown as the ground of the given—may be fulfilled as well. Therefore, the Maimonian antinomy of thought may be resolved precisely by the reduction of sensible matter to intellectual form carried out by differential calculus. On the Maimonian antinomy of thought, see Bransen (1989).

[59] Maimon (2010), p. 222 = GW II, 432.

[60] Maimon (2010), p. 222 = GW II, 432.

[61] "If one thus judges that fire melts wax, then this judgement does not relate to fire and wax as objects of intuition, but to their elements, which are thought by the understanding in the relation of cause and effect to one another." Maimon (2010), p. 183–184 = GW II, 356. Since there is no heterogeneity between the category and the object upon which it is applied, for both are representations of the understanding, the *quid juris?* question does not arise. On this issue, see Pringe (2016).

[62] Maimon (2010), p. 222 = GW II, 432.

finite understanding.[63] In the case of a limited understanding like our own, there is just the possibility of a progressive approximation to the consciousness of that rule.[64] This approximation is possible because the rule (i.e., the differential) is a representation of a spontaneity that differs from ours only quantitatively (and not qualitatively). But precisely because of this difference, the approximation never ends. In view of this infinite character of the path connecting the empirical and its supersensible ground, Maimon calls his position empirical skepticism, in opposition to Kant's rational skepticism, according to which there is *no* such a path *at all*.[65] Maimon's metaphysics is this infinite quest for the supersensible, towards which our limited intellect may draw ever closer in a never-ending progression.[66]

4. A brief remark on Maimon's criticism of Kant

At this point we could raise some concerns about Maimon's reconstruction of Kant's position regarding geometrical cognition. As a matter of fact, references to the method of fluxions can be found not only in Maimon's but also in Kant's work.[67] In a passage of the "Anticipations of Perception", Kant claims:

> "Magnitudes of this sort can also be called flowing, since the synthesis (of the productive imagination) in their generation is a progress in time, the continuity of which is customarily designated by the expression "flowing" ("elapsing")."[68]

Kant follows Newton in claiming that spatial quantities are not composed of points but generated by motion. This motion is not empirical, but pure:

> "Motion of an *object* in space does not belong in a pure science, thus also not in geometry; for that something is movable cannot be cognized a priori but only through experience. But motion, as *description* of a space, is a pure act of the successive synthesis of the manifold in outer intuition in general through productive imagination, and belongs not only to geometry but even to transcendental philosophy."[69]

When explaining the relation of irrational numbers to intuition, Kant explicitly appeals to the generation of a line through its fluxion.[70] This successive generation of a line is stressed in the "Axioms of Intuition" as well:

> "I cannot represent to myself any line, no matter how small it may be, without drawing it in thought, i.e., successively generating all its parts from one point, and thereby first sketching this intuition. […] On this successive synthesis of the productive imagination, in the generation of shapes, is grounded the mathematics of extension (geometry) with its axioms, which express the conditions of sensible intuition a priori, under which alone the schema of a pure concept of outer appearance can come about."[71]

These references to the Newtonian method of fluxions have not gone unnoticed.[72] In this connection, Friedman goes as far as to claim that:

> "It is extremely likely that some such understanding of the calculus (the method of fluxions) underlies Kant's insistence on the kinematic character of construction in pure intuition. When he speaks of the 'productive synthesis' involved in the 'mathematics of extension' Kant is referring to what we would now call calculus in a Euclidean space; he is not simply thinking of Euclidean geometry proper."[73]

Thus, one might argue that the crucial Maimonian distinction between the exhibition of a concept in intuition by means of Kantian construction and Maimon's own genetic explanation of the arising of a geometrical object from its differential is not as clear as it should be. According to this possible objection, the exhibition of a geometrical concept in intuition would amount, for Kant, precisely to its genetic explanation, appropriately described by means of calculus in Euclidean space. Maimon's criticism of Kant's notion of construction would therefore have missed its target.

However, for Kant, space, as a mere form of outer sense, is irreducible to the spontaneity of the subject. Its origin is sensibility, not understanding. Therefore, even if the exhibition of a geometrical concept were understood as a generation according to the method of fluxions, for Kant that generation would occur *in* space, i.e., it would take the spatial manifold as its pure *given* matter:

> "Thus the mere form of outer sensible intuition, space, is not yet cognition at all; it only gives the manifold of intuition a priori for a possible cognition. But in order to cognize something in space, e.g., a line, I must *draw* it, and thus synthetically bring about a determinate combination of the given manifold, so that the unity of this action is at the same time the unity of consciousness (in the concept of a line), and thereby is an object (a determinate space) first cognized."[74]

In contrast, according to Maimon, the genetic process mathematically expressed by the method of fluxions does not take place *in* space, but rather *produces* space in the first place.[75] Not only the *unity* of space has an intellectual origin (something that a Kantian would still accept) but also the spatial *manifold*: "Space as the matter

[63] Maimon's theory of differentials enables us to understand *how* this infinite understanding operates: "God creates the objects of nature in the same manner that we create the objects of mathematics: by real thought, i.e. by construction." GW IV, 58.

[64] As Bergman puts it, "God, as it were, thinks in differentials and we in integrals." Bergman (1967), p. 63.

[65] Maimon (2010), p. 222 = GW II, 432. "To discover a passage from the sensible to the intelligible world", Maimon claims, whatever the politicians may say, "is certainly more important than the discovery of a route to the East Indies." Maimon (2010), p. 176 = GW II, 339.

[66] In the same sense, Bergman claims that a "metaphysics constructed on the basis of a consistent idealism in the manner of Leibniz is therefore *possible* (in contradistinction to Kant) since our intuitions keep approaching ideas just as in mathematics a series keeps approaching its limit." Bergman (1967), p. 67.

[67] Kant, just like Maimon, most probably received this Newtonian influence through the work of Abraham Kästner. See Friedman (1992), p. 75.

[68] KrV, A169-170/B211-212.

[69] KrV, B155 footnote.

[70] HN, AA 14, 53. See also: HN, AA 18, 167. On this issue, see: Friedman (1992), p. 76 footnote 29.

[71] KrV, A162-163/B203-204.

[72] See, especially: Büchel (1987), pp. 221–299. It is worth pointing out that in his deep and extensive investigation, Koriako does not take this issue into account. See Koriako (1999).

[73] Friedman (1992), p. 76.

[74] B137-138. For a discussion of the pre-critical roots of Kant's critical notion of mathematical knowledge as the representation of the universal *in concreto*, see De Jong (1995).

[75] Even if we "consider only the act whereby we construct the concept" of a geometrical figure, as Kant demands (KrV, A714/B742), this construction presupposes space as pure intuition.

of these objects [of the objects of mathematics] is produced by the very faculty of cognition" (GW IV, 629).[76]

Space, as an extensive magnitude, is nothing but the sensible image of the relations between ideas of understanding, i.e., differentials. These ideas may be indirectly perceived by means of their extensive schemata:

> "The extensive magnitude is, so to speak, the schema of the intensive magnitude because intensive magnitude, along with its relations, cannot be perceived directly and in itself but only by means of extensive magnitude. […] With quanta, intensive magnitude is the differential of the extensive, and the extensive is, in turn, the integral of the intensive."[77]

In other words, Maimon would reject a Kantian explanation of the genesis of mathematical objects *in* space, by pointing out that differential calculus rather shows the genesis *of* space itself. For Maimon, this is a crucial distinction, because if space is assumed as given, or in other words, if space is taken to be an a priori intuition in the Kantian sense, then the question regarding the possibility of synthetic propositions in mathematics remains[78]:

> "So, assuming that time and space are a priori intuitions, they are still only *intuitions* and not a priori *concepts*; they make only the terms of the relation intuitive for us, and by this means the relation itself, but not the truth and legitimacy of its use. So the question remains: how are synthetic propositions possible in mathematics? or: how do we arrive at their evident nature [Evidenz]?"[79]

In order for this question to be answered one has to consider space as being grounded in concepts or, more precisely, in ideas of understanding, just as differential calculus does.

[76] Translation is ours. See also GW VII, 91f.

[77] Maimon (2010), p. 69 = GW II, 121–122. Kant characterizes the intensive magnitude as containing the ground of the extensive one in: HN, AA 14, 496. Such an intensive magnitude is a differential magnitude. See: Cohen (1918), p. 538ff; in particular, p. 544. For a discussion of the relationship between Maimon and Cohen, see: Bergman (1939).

[78] I thank an anonymous reviewer for pressing me to make this point clearer. Moreover, it would be interesting to contrast Maimon's position not only with Kant's but also with that of German idealism. Although this task goes beyond the scope of this paper, we may here quote the following suggestion made by Franks concerning this problem: "Kant also maintains that all construction is mathematical and that all mathematical construction is in pure sensible intuition. Here Maimon and the German idealists part ways with Kant and with each other. All agree that science requires construction of a priori concepts, in which form immediately constitutes its object. However, whereas Maimon wants to free mathematical construction from its dependence on intuition, the German idealists want to free construction in intuition from its restriction to mathematics." Franks (2005), p. 191.

[79] Maimon (2010), p. 36 = GW II, 60. The genetic character of the Kantian geometrical construction is underlined by Wolff-Metternich in Wolff-Metternich (1995), pp. 60ff. Against Eberhard's objection, according to which the mathematician, "ohne seinem Begriffe eine *genau* correspondirende Anschauung in der Einbildungskraft zu geben, den Gegenstand desselben durch den Verstand gar wohl mit verschiedenen Prädicaten belegen und ihn also auch ohne jede Bedingung erkennen könne" (Wolff-Metternich, 1995, p. 45), Wolff-Metternich argues "daβ die Konstruierbarkeit von Begriffen nach Kant *nicht* von der *Faßbarkeit in einem Bild* abhängig ist." Wolff-Metternich (1995), p. 56. However, despite its productive character, construction still presupposes space as a priori *given*. Thus, Maimon's criticism remains unanswered. A reference to Maimon may have been useful to complement Koriako's thorough analysis of the discussion between "Kantianier und Leibnizianer" on the foundations of mathematics in Eberhard's *Philosophisches Magazin*. Unfortunately, Koriako does not consider Maimon's position. See Koriako (1999), p. 320ff.

5. Conclusions

As we have indicated in our introduction, in the *Prolegomena* Kant claims that Hume's rejection of the possibility of metaphysics as a science was conditioned by an erroneous understanding of the character of mathematical cognition. If Hume had realized that mathematical judgments were synthetic, he would have had either to ground them in experience or to look for an explanation of the possibility of a synthesis a priori. Since the first option would have been unsatisfactory (for the apodictic character of mathematics could not have been explained), Hume would have had to tackle the critical question concerning the possibility of synthetic a priori judgments, on which the possibility of metaphysics as a science relies. We have seen that Maimon puts forward a similar criticism against Kant. According to Maimon, if Kant had had a correct understanding of the synthesis present in mathematical judgments, in particular in differential calculus, he would not have rejected the possibility of theoretical cognition of the supersensible.[80] More precisely, he would have had to accept that the supersensible is not merely a realm beyond the sensible one, but it is rather the realm of the rules of the arising of the sensible. These rules, as differential calculus shows, can be cognized precisely through the sensible objects that arise in accordance with them. Therefore, Maimon puts forward a very peculiar connection between mathematics and metaphysics. He claims that an analogy holds between these sciences, in so far as two relationships are identical: the mathematical one between the integral and the differential and the metaphysical one between the sensible and the supersensible. Against Kant, Maimon argues that this analogy grounds the possibility of metaphysics, understood as the science of the supersensible rules of the arising of the sensible. For Maimon, metaphysics may thereby finally start along the secure (albeit infinite) path of a science.[81]

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[80] Kant develops a *practical-dogmatic* metaphysics of the supersensible in Kant (2002a,bThe reference is Kant (2002b)). On this issue, see Caimi (1989).

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