



# A note on 4D heterotic string vacua, FI-terms and the swampland

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## ABSTRACT

We present a conjecture for the massless sector of perturbative 4D  $N = 1$  heterotic  $(0, 2)$  string vacua, including  $U(1)^n$  gauge symmetries, one of them possibly anomalous (like in standard heterotic compactifications). Mathematically it states that the positive hull generated by the charges of the massless chiral multiplets spans a sublattice of the full charge lattice. We have tested this conjecture in many heterotic  $N = 1$  compactifications in 4D. Our motivation for this conjecture is that it allows to understand a very old puzzle in  $(0, 2)$   $N = 1$  heterotic compactification with an anomalous  $U(1)$ . The conjecture guarantees that there is always a D-flat direction cancelling the FI-term and restoring  $N = 1$  SUSY in a nearby vacuum. This is something that has been verified in the past in a large number of cases, but whose origin has remained obscure for decades. We argue that the existence of a lattice generated by massless states guarantees the instability of heterotic non-BPS extremal blackholes, as required by Weak Gravity Conjecture arguments. Thus the pervasive existence of these nearby FI-cancelling vacua would be connected with WGC arguments.

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## 1. Introduction

Four dimensional string vacua often have a number of gauged  $U(1)$  symmetries. Some of them are sometimes anomalous with anomalies cancelled by the 4D version of the GS mechanism. In heterotic vacua obtained from CY with non-Abelian bundles or standard  $(0, 2)$  Abelian orbifolds at most one anomalous  $U(1)_X$  is allowed, whose anomaly is cancelled by the shift transformation of the axionic partner of the axi-dilaton  $\text{Im}S$ . Supersymmetry then tells us that there is an associated FI-coupling [1] such that one has D-term of the form

$$V_X = \frac{1}{S + S^*} \left[ \xi_X + \sum_i q_{X,i} |\Phi_i|^2 \right]^2, \quad (1.1)$$

with

$$\xi_X = \frac{\text{Tr} Q_X}{48(2\pi)^2 \kappa_4^2} \frac{1}{S + S^*}. \quad (1.2)$$

Here the sum runs over all scalars in the theory charged under the anomalous  $U(1)$ , and  $\text{Tr} Q_X$  is the trace over all massless charged

chiral multiplets in the theory. Perturbatively the potential of the dilaton is flat and, for non-vanishing values a field-dependent FI-term,  $\xi_X \neq 0$  appears to break SUSY. From the very early days of heterotic compactification it was found that, in any such  $(0, 2)$  4D heterotic vacua, there is always a nearby SUSY vacuum in which some scalars  $\Phi_i$  with the correct charge get appropriate vevs to get a vanishing D-term in a new SUSY vacuum. However, the reason why this is true was never fully understood in the literature. In the present paper we come back to this puzzle and take advantage of recent efforts [2–5] to sort out the set of theories which may be embedded into a consistent theory of quantum gravity from those which cannot and belong to the *swampland* [6–8] (see [9] for a review). We argue that putative theories in which the FI-term is not cancelled would be inconsistent or belong to the *swampland*.

It is well known that a description in terms of holomorphic scalar operators [10] provides a useful way to look for D-flat directions for an arbitrary gauge group in  $N = 1$  SUSY. Having a flat direction cancelling all the D-terms corresponds to the existence of an operator involving the scalar components of massless chiral scalar fields  $\phi_i$  (but not the conjugates)

$$I = (\phi_i \phi_j \dots \phi_k) \quad (1.3)$$

such that it has  $U(1)_X$  charge opposite to  $\text{Tr} Q_X$  and zero charge with respect to any other charge or gauge interaction. Setting the vevs fulfilling

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$$\frac{\partial I}{\partial \phi_i} = c \phi_i^*, \quad (1.4)$$

all D-terms vanish ( $c$  is a constant). Note that the operators  $I$  are just a book-keeping device to see what fields may have simultaneously vevs with a vanishing D-term, they do not need to be present in the effective action. What experience tells us is that such operators with the required sign of the  $Q_X$  always exist in any (0, 2) heterotic model so far analyzed. This suggests that there could be some deeper principle why this is always the case. It could well be, in particular, that consistency with quantum gravity forces this to happen, for some reason still to be understood.

It is reasonable to ask whether the existence of scalars with the correct charge, opposite to the FI-term, is a direct consequence of anomaly cancellation and  $N = 1$  SUSY. After all, the cancellation of the mixed  $U(1)$ -gravitational anomalies seems to require the presence of fermions with opposite charges. However this is not so when the  $U(1)$  is anomalous. In the class of theories under consideration the coefficients of the cube and the mixed-gravitation anomalies of  $U(1)_X$  are related by Green–Schwarz anomaly cancellation constraints (see e.g. [11] and references therein)

$$\frac{1}{3k_X} \sum_i q_{X,i}^3 = \frac{1}{24} \text{Tr} Q_X, \quad (1.5)$$

where  $k_X$  is the normalization of the  $U(1)_X$  coupling. In principle this may be fulfilled with all scalars having the same sign, so that the FI-term would never cancel. For example one may have a model with two chiral multiplets with charge  $q_1 = 1, q_2 = 1/2$ . The reader may check that for  $k_X = 6$  Eq. (1.5) is fulfilled, and anomalies cancel through the GS mechanism. We intend to put forth that this cannot happen in  $N = 1$  heterotic vacua and such model would be in the swampland. It seems that anomaly cancellation is not strong enough to guarantee FI-term cancellation.

In Section 2 we formulate a conjecture concerning massless sector of  $N = 1$  heterotic vacua and provide some examples. In Section 3 we discuss our conjecture and its possible connection to the Weak Gravity Conjecture (WGC) [7]. We conclude with some comments in Section 4

## 2. The positive cone conjecture

Consider a  $N = 1$   $D = 4$  heterotic vacuum with gauge group  $G = H \times U(1)^N$ , where  $H$  is some semisimple group and where a combination  $U(1)_X$  of the  $N$   $U(1)$ 's may be anomalous. There are massless chiral multiplets with complex scalar components with a vector of charges

$$\phi_i = (R_i; q_1^i, \dots, q_N^i), \quad (2.1)$$

where  $i$  runs over all the massless chiral spectrum. Here  $R_i$  is some representation of the non-Abelian semisimple group  $H$ . By holomorphically multiplying these scalars it is possible to obtain operators  $\Phi^a$  which are singlets under the non-Abelian group

$$\Phi^a = (\phi_1 \dots \phi_k)^a \quad (2.2)$$

each one with a vector of charges

$$q_a = (q_1^a, \dots, q_N^a). \quad (2.3)$$

Consider now all the vectors of charges generated by:

$$\Lambda_0 = \left\{ \sum_a M_a q_a, M_a \in \mathbf{Z}^+ \right\} \quad (2.4)$$

where  $a$  runs over all possible  $\Phi^a$  chiral operators.  $\Lambda_0$  is the positive hull generated by the charges of all massless chiral fields. The conjecture is then:

*The positive hull  $\Lambda_0$  generated by the charges of all massless chiral fields is a sublattice of the full charge lattice  $\Lambda$ .*

The important point here is that  $\Lambda_0$ , being a sublattice, contains a given vector *and* its opposite. Note that the statement is non-trivial, in principle it could have happened that  $\Lambda_0$  was *not* a lattice, but just a set of vectors, not including the opposite of each vector. In fact, the positive hull of the example given above with two charged particles is not a lattice.

If the conjecture was true, there should always be a flat direction at which the FI-term would be cancelled. Indeed, to each member of the sublattice  $\Lambda_0$  (choice of integers  $M_a$ ) corresponds an operator  $(\phi_1 \dots \phi_N)$ . In particular,  $\Lambda_0$  should contain an operator  $I_X$  which is only charged under  $U(1)_X$  and another operator  $I_{-X}$  with opposite charge. Either one or the other will be able to cancel the FI-term by assigning vevs as in eq. (1.4). So the existence of these sublattices would guarantee the existence of a D-flat direction cancelling the FI-term and preserving  $N = 1$ .

We now discuss examples of heterotic compactifications, showing how sublattices always arise in the massless sector of the theory.

The reader uninterested in the details of these models may safely jump to Section 3.

### 2.1. Examples

We have tested this conjecture in many Abelian (0, 2)  $Z_N$  orbifolds of the heterotic  $E_8 \times E_8$  and  $SO(32)$  strings leading to chiral  $D = 4, N = 1$  theories with or without an anomalous  $U(1)$ . Here we show a couple of representative examples (see e.g. [11] for a review on heterotic orbifold constructions) and present further ones in the Appendix.

#### 2.1.1. $Z_3$ orbifold $E_8 \times E_8$ examples

A simple example with a single anomalous  $U(1)$  is the  $Z_3$  orbifold with shift  $V = 1/3(11112000) \times (20000000)$  acting on the  $E_8 \times E_8$  gauge lattice. This model has gauge group  $SU(9) \times SO(14) \times U(1)_X$  and charged massless chiral spectrum given by

$$U: 3[(\mathbf{84}, \mathbf{1})_0 + (\mathbf{1}, \mathbf{14})_{-1} + (\mathbf{1}, \mathbf{64})_{1/2}]; \quad (2.5)$$

$$T: 27[(\bar{\mathbf{9}}, \mathbf{1})_{2/3}], \quad (2.6)$$

where  $U$  and  $T$  denote untwisted and twisted spectrum and the subindex is the charge under the  $U(1)$  generator  $Q_X = (1, 0, \dots, 0)$  in the second  $E_8$ . In this simple case the sublattice is generated by

$$\Lambda_0 = (M(\pm 2), M \in \mathbf{Z}). \quad (2.7)$$

The minimum charge for this lattice comes from operators like  $(\mathbf{1}, \mathbf{14})_{-2}^2, (\mathbf{1}, \mathbf{64})_2^4, [(\bar{\mathbf{9}}, \mathbf{1})^3(\mathbf{84}, \mathbf{1})_2]$ , etc. Here  $\text{Tr} Q_X = 24 \times 9$  and the FI could be cancelled with vevs corresponding to the operator  $(14)_{-2}^2$ , and  $SO(14)$  is broken to  $SO(12)$ .

A simple model with two  $U(1)$ 's is provided by the  $Z_3$  orbifold with embedding  $V = 1/3(110..0) \times (200..0)$  yielding gauge group  $E_7 \times U(1)_X \times SO(14) \times U(1)$ . The chiral spectrum is given by

$$U: 3[(\mathbf{56}, \mathbf{1})_{1,0} + (\mathbf{1}, \mathbf{1})_{-2,0}] + 3[(\mathbf{1}, \mathbf{14})_{0,-1} + (\mathbf{1}, \mathbf{64})_{0,1/2}] \quad (2.8)$$

$$T: 27[(\mathbf{1}, \mathbf{14})_{2/3,-1/3} + (\mathbf{1}, \mathbf{1})_{2/3,2/3} + (\mathbf{1}, \mathbf{1})_{-4/3,2/3}], \quad (2.9)$$

and the first  $U(1)_X$  is anomalous. A sublattice is given by:

$$\Lambda_0 = \{M \times (4/3, -2/3) + N \times (2/3, 2/3), M, N \in \mathbf{Z}\}. \quad (2.10)$$

In this case it is generated by single twisted fields but there is a smaller lattice generated from the untwisted fields with vector charges  $(\pm 2, 0)$ ,  $(0, \pm 2)$ . coming from the operators  $[(\mathbf{56}, \mathbf{1})_{2,0}^2, (\mathbf{1}, \mathbf{1})_{-2,0}$  and  $[(\mathbf{1}, \mathbf{64})_{0,2}^4, [(\mathbf{1}, \mathbf{14})_{0,-2}^2]$  respectively. In this example  $\text{Tr} Q_X = 18 \times 24$  so that the FI may be simply cancelled by the untwisted singlet  $(\mathbf{1}, \mathbf{1})_{-2,0}$  which is already in the massless sector. However cancellation can be also achieved by e.g. the operator  $[(\mathbf{1}, \mathbf{14})_{0,-1}^2 (\mathbf{1}, \mathbf{1})_{-4/3, 2/3}^3]_{-2,0}^3$  which involves twisted states. The normalizations are  $k_X = 4$ ,  $k' = 2$  from  $Q_X = (1, 1, 0 \dots 0)$ ,  $Q' = (1, 0, 0 \dots 0)$ .

In the above examples the FI-term could be cancelled by using operators/fields making use only of untwisted fields. However there are plenty of examples in which the untwisted subsector does not generate a sublattice by itself, and a full sublattice only arises from the complete untwisted and twisted spectrum. The following example has this property.

### 2.1.2. An $SO(32)$ , $Z_7$ example

Consider  $v = \frac{1}{7}(1, 2, -3)$  and a gauge shift with embedding  $V = \frac{1}{7}(3, \dots, 3, 0, 0)$  leading to gauge group  $SU(14) \times U(1) \times SO(4)$  with  $Q_X = (1, 1, \dots, 1; 0, 0)$ . The untwisted sector massless field content is generated by left handed lattice momenta (underlining means all possible permutations)  $P = (-1, -1, 0, 0, \dots; 0, 0)$  with  $PV \equiv \frac{1}{7}$  and  $(-1, 0, 0, \dots; \pm 1, 0)$  with  $PV \equiv -\frac{3}{7}$  and reads

$$U : (\overline{\mathbf{91}}, \mathbf{1})_{-2} + (\mathbf{14}, \overline{\mathbf{4}})_{(-1)}$$

with no massless states in the  $\frac{2}{7}$  sector. The  $m$  twisted sector ( $m = 1, 2, 4$ ) massless states can be read from the  $P$  states satisfying  $\frac{(P+mV)}{2} + E_0 + N_L - 1 = 0$  with  $E_0 = \frac{2}{7}$  and  $N_L = 0, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}$  the left oscillator number with associated multiplicities  $1, 1, 2, 3, 5$  respectively. Then one finds the twisted chiral fields

$$T : 7[3(\mathbf{1}, \mathbf{2})_{(-1)} + (\mathbf{14}, \mathbf{1})_{(0)} + 5(\mathbf{1}, \mathbf{1})_{(-2)} + (\mathbf{1}, \mathbf{2})_{(3)} + (\mathbf{1}, \mathbf{1})_{(-4)}] \quad (2.11)$$

Notice that  $\frac{1}{24} \text{Tr} Q_X = -14 = \frac{1}{28.3} \text{Tr} Q_X^3$ . The singlets

$$[(\mathbf{1}, \mathbf{2})_{(3)}^2 (\mathbf{1}, \mathbf{1})_{(-4)}]_{(2)}; (\mathbf{1}, \mathbf{1})_{(-2)} \quad (2.12)$$

generate the sublattice  $\{2, -2\}$ . The singlets  $[(\mathbf{1}, \mathbf{2})_{(3)}]_{(6)}^2$  or  $[(\mathbf{1}, \mathbf{2})_{(3)} (\mathbf{1}, \mathbf{1})_{(-4)}]_{(2)}$  constructed up from the only positive charge massless field  $(\mathbf{1}, \mathbf{2})_{(3)}$  could be used to cancel FI term. In this example only the twisted sector had fields with positive charge and hence the untwisted fields cannot generate a sublattice by themselves. Further examples are presented in the Appendix.

## 3. Blackholes, the WGC and FI-terms

We see that the existence of the above sublattice guarantees D-flat directions in which the FI-term in the potential is cancelled by the vev of opposite charge scalars. We will now argue that the WGC could be at the origin of the existence of this sublattice. We do not have any formal proof of this statement but we want to present in this section some circumstantial evidence in this direction.

In general terms, the Weak Gravity Conjecture [2,3,7] (see [9] for a review) states that gravity is the weakest force. In the context of a  $U(1)$  gauge theory, it states that any (non-BPS) extremal charged blackhole should be able to decay into a superextremal particle with mass  $m < Q$  in Planck units. This is required if we want to avoid a tower of remnant stable extremal blackholes which are problematic from different points of view. Interestingly, the

toroidal compactifications of the heterotic strings down to 4D provide the prototypical example in which indeed the appropriate superextremal particles exist with the appropriate characteristics. A. Sen first described in [12] the structure of extremal blackholes in heterotic toroidal compactifications. There are extremal BPS blackholes with masses  $m^2 = P_R^2/2$  in Planck units and extremal non-BPS blackholes with masses  $m^2 = P_L^2/2$ . On the other hand the spectrum of masses of the heterotic string states is given by the expression

$$\alpha' M^2 = \alpha' 2M_L^2 = 4 \left( \frac{P_L^2}{2} + N_L - 1 \right) = 4 \left( \frac{P_R^2}{2} + N_R \right). \quad (3.1)$$

Here  $P_L$  and  $P_R$  are the left- and right-handed momenta. They span lattices with signature  $(22, 6)$ , with  $P_L$  including the  $E_8 \times E_8$  or  $Spin(32)$  gauge degrees of freedom. For  $P_R^2 > P_L^2$  one finds BPS states for  $N_R = 0$  and  $N_L = \frac{1}{2}(P_R^2 - P_L^2) + 1$ . They have mass  $M^2 = P_R^2/2$ . In addition, for  $P_L^2 > P_R^2$  and  $N_L = 0$ ,  $N_R = (P_L^2 - P_R^2)/2 - 1$  one has non-BPS states with mass  $M^2 = P_L^2/2 - 1$ . The masses of these non-BPS states tends to  $M^2 = P_L^2/2$  for large charges. This nicely fits with the spectrum of blackholes found in [12]. It also shows an explicit realization of the WGC bounds. Indeed, as we go to smaller values of the charge we find string states obeying the WGC with the inequality saturated for the BPS states with  $P_R^2 > P_L^2$ . For  $P_R^2 < P_L^2$  however the extremal blackholes have  $m^2 = P_L^2/2$  whereas there is always a lighter string state with  $M^2 = P_L^2/2 - 1$ . This canonical example of heterotic realization of the WGC was first presented in [7].

This example has  $N = 4$  whereas the theories that we are studying have  $N = 1$  and are chiral. However, in the case of toroidal orbifold compactifications we might expect that, at least in the untwisted sector of the theory, towers of non-BPS extremal blackholes with masses  $M^2/4 = P_L^2/2$  still remain in the spectrum. Interestingly, in some cases the wanted instability requires the existence of massless chiral fields in the spectrum, and these required massless fields have the correct charge to cancel a FI-term in the potential, giving a connection between BH instability and FI-term cancellation.

An example in which this happens is the simple  $Z_3$  orbifold with gauge group  $E_7 \times U(1)_X \times SO(14) \times U(1)$  discussed above. Consider the  $E_8$  lattice vector  $P_L = (-1, -1, 0, 0, 0, 0, 0, 0)$ . Associated to this there would be an extremal non-BPS blackhole with mass  $m^2/4 = P_L^2/2$  and charges  $(-2, 0)$  with respect to the  $U(1)$ 's. But precisely for this lattice vector there is a massless chiral field with the same charges  $\mathbf{1}_{-2,0}$ , verifying WGC bounds. On the other hand, as explained in the previous section, this singlet can cancel the FI-term associated to the anomalous  $U(1)$ . So this is an example in which the massless singlet plays a double roll of insuring WGC constraints and FI-term cancellation. This is just an example, and there are many others. In many of them however there are no appropriate single field states in the untwisted massless sector that could play this role. In general multi-particle states both from the untwisted and twisted massless field sectors are needed to build the required sublattice. But at least these simplest examples show the possible connection between the need for states to verify the WGC and the presence of the required massless fields to cancel the FI term.

Another important point to take into account is the corrections to mass/charge ratio in non-BPS extremal blackholes. It has been shown that corrections involving 4-derivative interactions drive  $M^2 < Q^2$  [13]. In fact those authors find that for  $D = 4$  the corrected mass of extremal back holes in the heterotic string is

$$\frac{M^2}{M_p^2} = Q^2 - \frac{3h\Omega_2^2}{20} \frac{M_p^2}{M_s^2} \quad (3.2)$$

where  $\Omega_2 = 2\pi^{3/2}/\Gamma(3/2)$  and  $h$  is the dilaton. The correction is always negative and may become numerically important as the compact volume increases. Analogous corrections are expected to arise for the case here considered of  $N = 1$  compactifications. If this was the case the risk of extremal blackholes becoming lighter than their prospective string states into which they could decay appears, rendering them stable. A cure to this possible disease would be that the massless chiral sector of the theory is sufficiently rich so that all extremal blackholes can decay always at least to sets of massless (typically multiparticle) states. The sublattice structure of the states spanned by the massless sector would provide for the appropriate decay products. Note that in addition to the towers of extremal blackholes associated to the untwisted sector one would also expect extremal blackholes with a charge lattice generated by the shifted lattice  $(P + n_i V^i)$ . These typically correspond to fractional charges. For these additional blackholes not to be stable (typically multiparticle) states constructed using twisted massless chiral fields would then exist. Summarizing, we conjecture that for any node in the sublattice generated by the massless chiral fields an extremal blackhole with the same charge should exist. The existence of the sublattice would then guarantee both extremal BH instability and cancellation of the FI-term in D-flat directions.

#### 4. Comments

The above conjecture for the existence of a lattice  $\Lambda_0$  is only a *sufficient condition* for the FI-term cancellation. In order for the new shifted vacuum to be supersymmetric the corresponding D-flat direction should also be F-flat. It would be interesting to prove that the presence of the appropriate (multiparticle) decay channels of the blackholes would, in addition, force the cancellation of F-terms.

The sublattice of states discussed in this note is reminiscent of the sublattice of  $U(1)^N$  charges discussed in [2]. In the third paper in there it was conjectured that in a theory of quantum gravity with multiple  $U(1)$ 's a sublattice of the charge lattice with a superextremal particle at every site must exist. As made clear in [14], this can only be true in more than 4D because there are plenty of examples in 4D in which only massless particles may be superextremal. But then we would have an infinite number of massless particles. In our case this is not what happens. There are no infinite particles but rather a charge lattice generated by a finite number of massless fields. At each node there is a multiparticle state to which potential non-BPS extremal blackholes could decay into.

A natural question is whether the conjecture of the existence of the positive cone sublattice should apply to all  $N = 1$  string vacua. It seems that the answer is no, and indeed it is easy to find e.g. Type I or Type IIA orientifolds with  $Dp$ -branes in which the conjecture does not work. There are a number of reasons for this to be the case. Consider for example the Type I duals of the  $Spin(32)$  heterotic models.<sup>1</sup> Unlike the heterotic case, in the perturbative Type I duals there are no towers of non-BPS blackholes and there are no spinorial states either. The duals of the  $Spin(32)$  lattice and the spinorials appear only at the non-perturbative level from the dynamics of D1-branes, which decouple in the perturbative regime. So an argument for a sublattice based on the stability of extremal

blackholes does not hold. This is also consistent with the different structure of anomalous  $U(1)$ 's in Type I orbifold vacua. Indeed in the latter class of models there can be more than one anomalous  $U(1)$  and the multiple FI-terms associated to those are related to the twisted blowing-up modes rather than to the overall dilaton [16]. These blowing up modes can be put to zero without generating a decoupling of the anomalous  $U(1)$  couplings whatsoever. The same is expected to happen in Heterotic compactifications with  $U(N)$  bundles (see [17] and references therein).

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#### Appendix A. (2, 2) compactifications of $SO(32)$ heterotic on a CY

In any such compactification the gauge group is generically  $SO(26) \times U(1)_X$ , with a massless spectrum given by

$$b_{11}(\mathbf{26}_1 + \mathbf{1}_{-2}) + b_{12}(\mathbf{26}_{-1} + \mathbf{1}_2), \quad (A.1)$$

where the subindex is the  $U(1)_X$  charge in some integer normalization. We see that  $Tr Q_X = 24(b_{11} - b_{12})$ , which is in general non vanishing. But note that for any values of the Betti numbers, the sublattice generated is generated by

$$\Lambda_0 = (2, -2). \quad (A.2)$$

In particular e.g. even if  $b_{12} = 0$ ,  $\Lambda_0$  contains not only  $(-2)$ , but also  $(+2)$  from the  $SO(26)$  singlet operator  $(\mathbf{26}_1)^2$ . In this case the sublattice has index 2 with respect to the full charge lattice  $\Lambda$  generated by  $(\pm 1)$ . In fact the index is larger since in the massive spectrum there will be spinorial states which will have semiinteger charges. This case is relatively trivial since there is always the required singlet scalar  $\mathbf{1}_{\pm 2}$  already in the massless sector of the theory, one does not need several scalars to cancel the FI. This is in general not the case, as the following examples show.

#### Appendix B. (2, 2) compactification of heterotic on $Z_3$ orbifold

In this case there is a gauge shift embedding:

$$V = \frac{1}{3}(11200\dots 0), \quad (B.1)$$

and the gauge group is  $SO(26) \times SU(3) \times U(1)_X$ . The chiral spectrum contains from untwisted and twisted sectors:

$$U : P = (100; \dots \pm 1 \dots) \text{ etc.} \rightarrow 3[(\mathbf{26}, \mathbf{3})_1 + (\mathbf{1}, \mathbf{3})_{-2}] \quad (B.2)$$

$$T : (P + V) = (1/3, 1/3, -1/3, \dots \pm 1 \dots) \rightarrow 27(\mathbf{26}, \mathbf{1})_{1/3} \quad (B.3)$$

$$T_{osc} : (P + V) = (1/3, 1/3, 2/3, 000\dots 0) \rightarrow 3 \times 27(\mathbf{1}, \bar{\mathbf{3}})_{4/3} \quad (B.4)$$

Here the anomalous  $U(1)_X$  is generated by the charge vector  $Q_X = (1, 1, 1, 0, \dots, 0)$ . Charges of fields are given by the scalar products e.g.  $Q_X \cdot (P + V)$ . In this case we have the sublattice:

$$\Lambda_0 = (M(\pm 2/3), M \in \mathbf{Z}). \quad (B.5)$$

The operators associated to the shortest charges in the lattice would be in this case  $[(\mathbf{26}, \mathbf{1})]_{2/3}^2$  and  $[(\mathbf{1}, \mathbf{3})(\mathbf{1}, \bar{\mathbf{3}})]_{-2/3}$ . There is a

<sup>1</sup> See as an example the heterotic  $Z_3$ ,  $U(4)^4$  model in [15] and its Type I dual. The massless twisted states in the heterotic side do generate a sublattice. In the Type I side the massless chiral fields do not.

variety of choices which can cancel the FI. One has  $Tr Q_X = 24 \times 36$  so one could cancel the FI with e.g.  $(1, \mathbf{3})_{-2}^3$  or  $[(1, \mathbf{3})(1, \overline{\mathbf{3}})]_{-2/3}$ . Along these directions  $SU(3)$  is also broken. Note in this examples several scalar are required to cancel the FI, there is no singlet in the massless sector to do the job.

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