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Discussion

## Free vibrations of Bernoulli–Euler beams with intermediate elastic support: a concise thematic recension

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The problem of a beam elastically restrained at intermediate points is often encountered in the design of structural members of buildings, aircrafts, ships, pipes, robotic applications and an extensive variety of other structures.

In Ref. [1], Albarracín et al. presented some exact and numerical considerations of the problem of a uniform beam with intermediate constraints and ends elastically restrained against rotation and translation.

Their results deserve credit and with regard to previous information they state that “In contrast..., there is only a limited amount of information for beams elastically restrained at intermediate points”. However, 50 years ago Lee and Saibel [2] obtained a general expression from which the frequency equation for the vibration of a constrained beam with any combination of intermediate elastic or rigid supports can be found readily.

In his book published in 1975 Gorman [3] provided a set of simultaneous equations for mode shapes which can be used in conjunction with frequency tables for the problem of free lateral

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vibrations of clamped–clamped uniform Bernoulli–Euler beams with arbitrarily located intermediate elastic support.

In 1987 Maurizi and Bambill [4] presented an  $8 \times 8$  matrix frequency equation of transverse vibrations of clamped beams with an intermediate translational constraint. Results for the first eigenfrequency are presented in graphical fashion as a function of the stiffness and the location of the spring attachment. The same analytical procedure is applicable in the case of additional complexities: elastic restraints at the beam ends, elastic rotational constraint at an intermediate point, etc.

Almost coincidentally Kameswara Rao [5] furnished explicit and exact frequency and mode-shape expressions, and extensive design data for the first five frequencies of vibration for various values of non-dimensional position and stiffness parameters of the intermediate elastic support. The numerical results presented in this paper demonstrate that the natural frequencies and mode shapes are sensitive to the position and stiffness of the intermediate elastic support. It can be also seen that the effect of finite stiffness of the elastic support on the first few modes is considerable and becomes negligible on higher mode frequencies.

De Rosa et al. [6] examined a stepped beam with rotational and translational flexibilities at the ends, with an additional constraint at the step, which is supposed to be flexible against rotations and translations. In this way, the most general stepped system is investigated, and all the previous results can be obtained as limiting cases.

Very recently Wang [7] presents the optimum location and the minimum stiffness of internal support for beams with various end conditions. For accuracy, the author uses the exact characteristic equation to compute the eigenfrequencies.

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