LFIs and methods of classical recapture

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Abstract

In this paper, I will argue that Logics of Formal Inconsistency (*LFIs*) can be used as very sophisticated and powerful methods of classical recapture. I will compare *LFIs* with the well-known non-monotonic logics by Batens (2001, *Log. Anal.*, 45–68) and Priest (2006, *In Contradiction*, Oxford University) and the 'shrieking' rules of Beall (2013, *Rev. Symb. Log.*, 6, 755–764). I will show that these proposals can be represented in *LFIs* and that *LFIs* give room to more complex and varied recapturing strategies.

Keywords: Paraconsistency, LFIs, classical recapture, shrieking, adaptative logics.

1 Introduction

In the past decades, paraconsistency emerged as a reasonable theory for dealing with inconsistency. As many authors observed, the main problem with paraconsistency is its weakness. Departing from the same axioms of a classical theory, a paraconsistent system will typically provide a much weaker theory. Pointing at this problem, many authors have developed methods for 'recapturing' classicality inside a paraconsistent theory.

There is no common definition of 'classical recapture', so I will clarify what I mean by the expression in this context. As I see it, a process of classical recapture does not need to be absolute, i.e. one may not want to recover the classicality of a whole theory. In this way, one may want to see what happens if only one part of a theory is assumed to be consistent. Moreover, one could take for granted that either one part is consistent or another one is consistent. There are many possible consistency assumptions and therefore many ways of recapturing classicality.

In this paper I will mention four recapturing methods: Batens' adaptive logic, Priest's minimally inconsistent Logic of Paradox (*miLP*), Beall's shrieking rules and finally *LFIs*. *LFIs* are more complex and linguistically richer than the other theories. I will show that the other theories can be framed in *LFIs* and that a simple *LFI* approach can make many useful and sophisticated distinctions that the other proposals cannot make.

2 Four theories

As most readers already know, Logic of Paradox (*LP*) is a three-valued logic with values 1, i and 0. The logical matrixes are as follows:

$\vee$	1	i	0
1	1	1	1
i	1	i	i
0	1	i	0

$\wedge$	1	i	0
1	1	i	0
i	i	i	0
0	0	0	0

A	$\neg A$
1	0
i	i
0	1

The set of designated values is $\{1, i\}$. *LP* is paraconsistent, i.e. there are formulas A and B such that $A, \neg A \not\models B$. The conditional $A \to B$ can be defined as $\neg A \vee B$.

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In what follows, I will describe four strategies of recapturing, three of which are *LP*-based and the last one is based on a stronger system.

2.1 Shrieking rules

In a recent paper, Beall [4] introduced a method of recapturing classicality in paraconsistent theories. The general idea is to express by meta-theoretical statements ('rules') that a particular sentence is consistent. The proposition 'A is consistent' (or more literally, 'A does not have value i in any model') can be expressed in the following way:

$A \wedge \neg A \vdash \bot$

or in Beall's terminology (where $A!$ means $A \wedge \neg A$)

$A! \vdash \bot$.

Beall shows that some classical theories can be recovered by shrieking the axioms, such as Peano Arithmetic (*PA*) ([4], Theorem 2). This does not apply to every theory. There is, anyway, a general method for recovering classicality. If the premises Γ involve the predicates $P_1, \ldots P_n$, you may introduce the following shrieking rules for each predicate P_i of arity n:

$\exists x_1 ... \exists x_n (Px_1 ... x_n \wedge \neg Px_1 ... x_n) \vdash \bot$.

Once the rules are added to the theory, Γ implies A in *LP* whenever Γ implies A in classical logic. Propositional theories can be recaptured by taking propositional letters as 0-adic predicates, i.e. the shrieking rules will be $p, \neg p \vdash \bot$, for every p in the axioms.

2.2 Priest's miLP

A logic is monotonic whenever adding premises to a valid argument does not make it invalid. Priest's [14] logic *miLP* is non-monotonic. It assumes that the premises are consistent as far as possible. Each *LP* model M has an inconsistency set $|M|$, which is the set of propositional letters it makes inconsistent (i.e. gives value i). We say that a model M of Γ is minimally inconsistent whenever, for every M' such that $|M| \subset |M|$, M' is not a model of Γ. Now the notion of validity can be defined as follows:

$\Gamma \vDash_{miLP} A$ whenever every minimally inconsistent model of Γ is a model of A.

For example, $p, p \supset q \vDash_{miLP} q$, for every minimally inconsistent model of the premises is classical. But $p, p \supset q, \neg p \nvDash_{miLP} q$, for there are minimally inconsistent models of the premises where p has value i and q has value 0.

Clearly, if the premises are classically consistent, they have the same conclusions in *miLP* as in classical logic, since their minimally inconsistent models are just their classical models.

2.3 Batens' adaptive logic

Batens [2] developed a very complex approach to classical recapture which is non-monotonic. Unlike Priest's, Batens' approach is mainly proof theoretical. The idea of Batens is that we should argue classically as far as the premises behave classically. When doing this, we should take note of which formulas are assumed to be consistent. If an inconsistency appears, we should revise the sentences that have been obtained. The only safe formulas are those which can be obtained in the lower limit logic, which for simplicity we will take it to be *LP* (Batens uses *CLUNs*[1]). For example, suppose that we have the following proof:

[1] *CLUNs* is certainly closer to an *LFI* than to *LP*, for it has a detachable conditional. As Carnielli and Coniglio [8, p. 179] observe, the propositional *CLUNs* is functionally equivalent to $J3$ and therefore also to $LP_\circ$. However, this is not particularly important for Batens' recapturing strategy, for in every adaptive logic the premises are assumed consistent as far as possible. In other words, even though *CLUNs* is an *LFI*, Batens has not used *CLUNs* in the way we are proposing to use the *LFIs*.

1.	$\neg p$	premise	
2.	$p \vee q$	premise	
3.	$p \wedge r$	premise	
4.	~~q~~	DS, 1, 2	p is consistent marked at 6
5.	p	$\wedge$ E, 3	
6.	$p \wedge \neg p$	$\wedge$ I, 1,5.	

At step 6, p is declared inconsistent. Therefore, the step 4 (which assumed the consistency of p) is cancelled.

There are two different strategies in adaptive logic. We can exemplify with the following proof:

1.	$\neg p$	premise	
2.	$\neg q$	premise	
3.	$p \vee r$	premise	
4.	$q \vee s$	premise	
5.	$p \vee q$	premise	
6.	~~r~~	DS, 1,3	p is consistent marked at 8
7.	~~s~~	DS, 2,4	q is consistent marked at 8
8.	$p! \vee q!$	*LP*, 1,2,5.	

At step 8, we know that either p or q is inconsistent. Can we still infer r in step 6 and s in step 7? Both strategies consider that this is not legitimate. According to the *reliable* approach, steps 6 and 7 should be cancelled for p and q are now ‘unreliable’ (i.e. they might be inconsistent). The reliable approach claims that a disjunction between inconsistency statements has the same ‘adaptive’ effect as a conjunction; we cannot assume the consistency of any of the possibly inconsistent propositional letters.

On the other hand, the *minimal* approach also claims that r and s should be cancelled because neither r nor s is true in every minimally inconsistent model of $p! \vee q!$ and the premises. For example, there is a minimally inconsistent model of the premises where p is inconsistent, q and r are just false and s is just true; and another minimally inconsistent model where q is inconsistent, p and s are just false and r is just true.

The difference between both strategies becomes clear with the inference $r \vee s$. According to the *reliable* strategy, $r \vee s$ cannot be inferred since p and q are both unreliable; i.e. we should take them as inconsistent:

9.	$r \vee s$	$\vee$I,6	p is consistent marked at 8
10.	$r \vee s$	$\vee$I,7	q is consistent marked at 8.

While according to the *minimal* strategy, $r \vee s$ can be obtained, since $r \vee s$ is true in every minimally inconsistent model of the premises:

9.	$r \vee s$	$\vee$I,6	p is consistent marked at 8
10.	$r \vee s$	$\vee$I,7	q is consistent marked at 8.

The semantical approach to adaptive logics is more familiar and easier to understand. The minimal adaptive logic (according to our *LP*-based theory) is simply *miLP*. The reliable approach needs the notion of a minimal *Dab* (‘disjunctive abnormality’) consequence. For every set Γ, A is a *Dab* consequence whenever it is a consequence of Γ and it has the form $(p_1! \vee ... \vee p_n!)$, whereas A is a *minimal Dab* consequence of Γ whenever it is a *Dab* consequence and there is no shorter *Dab* consequence included in A. For example, $\{\neg p, \neg q, p \vee q\}$ has $p! \vee q!$ as a minimal *Dab* consequence, but not $p! \vee q! \vee r!$.

Let $U(\Gamma)$ contain the letters of every minimal *Dab* consequence of Γ. Let $Ab(M)$ be the set of propositional letters which get value i under M. A model M of Γ is *reliable* whenever $Ab(M) \subseteq U(\Gamma)$, i.e. every abnormal letter is unreliable. A sentence A is a reliable consequence of Γ whenever it is true in every reliable model of Γ. It is almost trivial to prove that if the premises Γ are classically consistent the reliable or minimal consequences will be identical to the consequences in classical logic, for the set of minimally (or reliably) inconsistent models of Γ is just the set of classical models of Γ.

2.4 The LFI approach

LFIs are a family of paraconsistent logics [7], which are able to express consistency.[2] In particular, they can define a consistency operator $\circ$, such that for all formulas A and B, it holds that $A, \neg A, \circ A \vDash B$, whereas $\{A, \circ A\}$ and $\{\neg A, \circ A\}$ are not necessarily trivial. There is some discussion about how to read this consistency operator; Carnielli and Rodrigues [9] e.g. have an epistemic reading of the connective, where $\circ A$ means that the truth value of A has been conclusively established. In the rest of the paper, I will only care about the technical aspects of the *LFIs*, and I will not take into account the more sophisticated epistemic readings of its semantics. It is important to observe that the consistency operator cannot be defined in *LP*, but it can be introduced as an additional connective:

A	$\circ A$
1	1
i	0
0	1

In the rest of the paper, I will focus on *LP* and $LP_\circ$, which is *LP* with the consistency operator.[3]

In the literature regarding *LFIs*, the concept of *derivability adjustment theorem* (*DAT*) can be seen as playing the role of a classical recapture (see [12]). In general, the idea is that once the normality of a set of sentences Σ has been established, the premises Γ imply A in a non-classical system *NC* whenever Γ implies A in classical logic *CL*. Formally speaking,

$\forall \Gamma \forall \alpha (\Gamma \vDash_{CL} \alpha \Leftrightarrow \exists \Sigma (*\Sigma, \Gamma \vDash_{NC} \alpha))$.

DATs can be proved for many non-classical theories. In the contexts of *LFIs*, in general, the operation * corresponds to the consistency operator $\circ$. In particular, for the *LFI* system *mbC* it can be proved that [7, p. 46]

$\forall \Gamma \forall \alpha (\Gamma \vDash_{CL+} \alpha \Leftrightarrow \exists \Sigma (\circ\Sigma, \Gamma \vDash_{mbC} \alpha))$.

In this case, $CL+$ is classical logic with the symbol $\circ$ and the axiom $\circ A$. For an exhaustive treatment of *DATs*, see [10].

A *DAT* is a recapture result, since it shows that, with some additional assumtions, the classical reasoning can be captured inside a non-classical theory. However, not every recapturing method has this structure. The non-monotonic approaches e.g. take for granted that the premises are normal, as far as possible; but they do not need (at least explicitly) new premises to carry out the classical reasoning. As for the shrieking method, it does not establish normality using new sentences but new logical rules.

[2] *LFIs* have a long history, and they were developed under different names during the past decades. A complete reference of the history and the different *LFIs* can be found in [8].

[3] $LP_\circ$ is not a particularly special or original *LFI* system. The operator $\circ$ can be defined in Ottaviano and DaCosta's $J3$ using their 'possibility' operator ∇: $\circ A = \neg\nabla A \vee \neg\nabla\neg A$. On the other hand, the operator ∇ can be defined in $LP_\circ$ as $\neg\circ A \vee (A \wedge \circ A)$. This means that $LP_\circ$ is functionally equivalent to $J3$, so it is also functionally equivalent to $LFI1$, $LPT1$, MPT, $L3$ and *CLUNs* (see [8, Section 4.4]).

On the other hand, I will argue that the possibility of proving a *DAT* is not the only reason to use *LFIs* to recapture classicality. Indeed, the kind of recapture that can be obtained with *DATs* can also be obtained with other methods. I will claim that *LFIs* give room to more sophisticated consistency claims and therefore to more interesting and complex recapturing strategies.

In short, the *LFI* approach to recapture does not need to be specific. The proposals of Beall, Priest and Batens intend to express the notion of consistency without introducing a consistency operator. According to the *LFI* approach, a consistency operator can provide simpler, more expressive and more elegant recapturing strategies. In the following section, I will show how $LP_\circ$ can recapture the proposals of Beall, Priest and Batens and go beyond them.

3 Capturing recapturing strategies with *LFIs*

In this section, we will see how $LP_\circ$ can represent the different recapturing strategies that I presented before.

3.1 Shrieking rules

As we mentioned, Beall's idea was to introduce meta-theoretical statements claiming that some sentences are consistent. In some cases (such as *PA*), by shrieking the axioms of a theory, one recovers its classicality. It is almost trivial that this can be done in $LP_\circ$. The only thing we need to do is to add, for every axiom γ of Γ, the consistency axiom $\circ\gamma$.

However, as we claimed before, this is not necessarily the case in every theory. A general way of recovering every theory was to shriek the predicates. This can also be done in a first-order version of $LP_\circ$, by adding new axioms of this form for every predicate P in the premises/axioms:

$\forall x_1 ... \forall x_n \circ Px_1 ... x_n.$

Again, the propositional theories can be captured in the same way, by taking propositional letters as 0-adic predicates. The additional axioms will have the form $\circ p$, for every p in the original axioms. This new theory will be conservative over the classical theory Γ. I give the proof for propositional logic.[4]

THEOREM 3.1
The $LP_\circ$ theory $Cn_{LP_\circ}(\Gamma \cup \circ\Gamma)$, where $\circ\Gamma$ includes $\circ p$ for every p in the formulas of Γ, is conservative over the classical theory $Cn_{CL}\Gamma$.

PROOF. Let $\Gamma, A \in \mathcal{L}_{CL}$. It is easy to see that if $\Gamma \cup \circ\Gamma \vDash_{LP_\circ} A$, then $\Gamma \vDash_{CL} A$. For suppose Γ does not imply A in classical logic. Clearly, the classical models which give value 1 to Γ and 0 to A can be extended to $LP_\circ$ models which also give 1 to $\circ\Gamma$.

For the other side, suppose that if $\Gamma \vDash_{CL} A$ but $\Gamma \cup \circ\Gamma \nvDash_{LP_\circ} A$. If this is the case, then in every classical model where Γ is true, A is true. As ([5], Lemma 2) established, if an LP model is classical for a set X, then there is a classical model which agrees on the truth values of every formula in X. Now let M be an $LP_\circ$ model of $\Gamma \cup \circ\Gamma$. M will be classical for every formula in Γ (it is well known that a complex formula will be classical if the components are classical). Now suppose that $M \nvDash A$, i.e. it gives A the value 0 and $\neg A$ the value 1. Given Beall's lemma, there will be a classical model for $\Gamma \cup \neg A$. But this is impossible. □

[4] This result can be seen as a DAT.

Beall [4] recognizes the similarity of his proposal with the *LFI* theories, but he claims that his theory does similar things as $LP_\circ$ without introducing new terminology.[5] This is partly true; some $LP_\circ$ sentences can be expressed with shrieking rules. For example,

$\circ A$ can be translated as $A! \vdash \bot$

$\circ A \wedge \circ B$ can be translated as $A! \vdash \bot$ and $B! \vdash \bot$

$\circ A \vee \circ B$ can be translated $A! \wedge B! \vdash \bot$.

However, $LP_\circ$ is more expressive than *LP* with shrieking. In particular, $LP_\circ$ can make mixed consistency statements, such as $\circ p \vee q$ (i.e. either p is consistent or q is true). The most usual recapturing strategies intend to recover consistency as much as possible, but it should also be possible to have a more complex view of consistency. For example, it should be possible to say 'if the part Γ of the theory is consistent, the sentence A is true' ($\circ \Gamma \rightarrow A$),[6] or 'if the part Γ of the theory is inconsistent, the sentence A is true' ($\circ \Gamma \vee A$ or equivalently $\neg \circ \Gamma \rightarrow A$). There is no reason to see recapturing as an all-or-nothing process; once we are able to *express* consistency, we can obtain a more flexible and dynamic view of consistency assumptions.

It is easy to see why shrieking cannot express $\circ p \vee q$.

THEOREM 3.2
$\circ p \vee q$ cannot be expressed by shrieking rules.

PROOF. Suppose that $\circ p \vee q$ can be expressed by shrieking rules and *LP*.

Case A. It can be expressed with shrieking rules alone. This is impossible. A shrieking rule $A! \vdash \bot$ establishes the consistency of an *LP* sentence A. There are no models which give to the sentence A the value i. However, $\circ p \vee q$ is true under the trivial model, where every letter p (and therefore every *LP* formula) gets the value i.

Case B. It can be expressed in *LP*. This is impossible but not so trivial. If the function $\circ A \vee B$ was expressible in *LP*, then the function $(A \rightarrow B) \wedge (\circ A \vee B)$ would also be expressible in *LP*. But this last function is equivalent to a detachable conditional $A \Rightarrow B$, such that $A \Rightarrow B$ does not imply B, but $A, A \Rightarrow B$ implies B. This is impossible as it was shown in [6].

Case C. It can be expressed in *LP* and some shrieking rules. It has the same problem as CASE A.[7] □

Therefore, $LP_\circ$ has some advantages over shrieking rules. First, it can provide the same recapturing results. Second, this can be done without appealing to meta-theoretical 'rules'. Finally, $LP_\circ$ can not only recapture classicality but also express many mixed consistency statements that cannot be expressed by shrieking rules.

3.2 miLP

The theory *miLP* is non-monotonic, so it can only be represented by a non-monotonic *LFI*. The well-known theory of default assumption consequence [11, p. 30] is a supra-classical theory where, apart

[5] This is particularly relevant for truth theories. Some *LFIs* such as $LP_\circ$ are incompatible with a transparent truth predicate, unlike weaker paraconsistent theories such as *LP*. For example, in an *LFI*, the liar sentence $\lambda = \circ T(\ulcorner \lambda \urcorner) \wedge \neg T(\ulcorner \lambda \urcorner)$ cannot get a stable truth value. This result can be avoided if the diagonalization instances contain a weak and non-detachable *LP* conditional (see [1]). In this paper we are mostly concerned with classical recapture strategies, so we are not focusing on the prospects of $LP_\circ$ as a background logic for a theory of truth.

[6] In these formulas, the expression $\circ \Gamma$ is used as an abbreviation of $\bigwedge_{A_i \in \Gamma} \circ A_i$.

[7] A similar proof may be applied to $\circ p \rightarrow q$. This sentence is *just true* in the trivial *LP* model, so it cannot be expressed by shrieking rules (which establish the consistency of some *LP* sentences), and it also cannot be expressed in *LP* (whose formulas have all value i in the trivial model).

from premises Γ, there are default assumptions Δ in the notion of logical consequence. When the assumptions do not conflict with the premises, they can all be taken as additional premises. However, some assumptions may be inconsistent with the premises. In these cases, we consider the subsets of Δ that are maximally consistent with the premises. To put it in clear formal terms,

we say that $\Gamma \vDash_{\Delta} A$ whenever $\Gamma \cup \Delta' \vDash A$ for every set $\Delta' \subseteq \Delta$ maximally consistent with Γ.

For example, in a classical default assumption consequence,

$p \vee q \vDash_{\{\neg p\}} q$.

However,

$p \vee q, p \nvDash_{\{\neg p\}} q$.

In order to capture *miLP*, we just need to take as default assumptions all the sentences $\circ p$, where p is a propositional letter. Let Δ be the set of these sentences $\circ p$. The theory default $LP_\circ$ ($DLP_\circ$) will be a default logic with $LP_\circ$ as the background logic, where Δ is taken as the default set of assumptions. Of course, given that the premises may be inconsistent, the notion of 'maximal consistency' will now be taken as maximal non-triviality.[8] In other words,

we say that $\Gamma \vDash_{DLP_\circ} A$ whenever $\Gamma \cup \Delta' \vDash_{LP_\circ} A$ for every set $\Delta' \subseteq \Delta$ maximally non-trivial with Γ (where Δ is the set of sentences $\circ p$ for every propositional letter p).

Therefore,

$p, p \supset q \vDash_{DLP_\circ} q$.

But $p, p \supset q, \neg p \nvDash_{DLP_\circ} q$, since $\circ p$ has been ruled out.

THEOREM 3.3

$DLP_\circ$ is conservative over *miLP*.

PROOF. Let $\Gamma, A \in \mathcal{L}_{LP}$. First suppose that Γ implies A in *miLP*, i.e. the minimally inconsistent models of Γ are models of A. Now suppose for contradiction that $\Gamma \nvDash_{DLP_\circ} A$. So there are sets Δ' maximally non-trivial with Γ such that $\Gamma \cup \Delta' \nvDash_{LP_\circ} A$. Now there is an $LP_\circ$ model which satisfies $\Gamma \cup \Delta'$ but dissatisfies A. M cannot be minimally inconsistent (for it would contradict the assumption). This means that there is a less inconsistent model M' of Γ. Therefore, there is a propositional letter p such that M' gives a classical value to p, and M gives a non-classical value. But if this is the case, Δ' should include $\circ p$ (and it does not). Therefore Δ' is not maximally non-trivial with Γ. This is a contradiction.

Now suppose that Γ implies A in $DLP_\circ$ but not in *miLP*. Therefore, there is a minimally inconsistent model M of *LP* which makes Γ true and A simply false. Now take the letters that M makes consistent. The set of default assumptions $\circ p$ for every consistent p in M, let us call it Δ', is non-trivial with Γ. It is also maximally non-trivial; if another letter q can be assumed as consistent, then it could be taken as consistent in M and so M would not be minimally inconsistent. Now there is an $LP_\circ$ model which extends M and makes $\Gamma \cup \Delta'$ true but A just false. Then Γ does not imply A in $DLP_\circ$. This is a contradiction. □

As it happened in Beall's approach, $DLP_\circ$ is a much more powerful theory than *miLP*, for it can not only explain *miLP* (by assuming the consistency of every propositional letter), but it can also depart from more complex consistency default assumptions. A different non-monotonic logic can be obtained if the set Δ includes not every sentence $\circ p$, or if it includes more sophisticated assumptions such as 'either p is consistent or q is consistent', etc.

[8] As an anonymous referee suggested, it should be stressed that this theory cannot be expressed in *LP*, where no set of sentences is trivial.

An anonymous referee observes that the system $DLP_\circ$ validates some apparently problematic inferences. For example, consider the following set of premises: $\{p, \neg p, p \to \neg \circ q\}$. In $DLP_\circ$ this implies $\circ q$, for in every minimally inconsistent model of the premises, the letter q is consistent. This can be seen as undesirable, since according to the conditional statement in the premises, if p is true then q should be inconsistent.

But this is not really something to worry about. The underlying problem is the conditional of the system. $LP_\circ$ has a non-detachable conditional, which is too weak for many purposes. In the context of $LP_\circ$, the fact that the antecedent is contradictory makes the entire conditional irrelevant and trivially true. A way of solving this problem would be to use a different conditional which validates modus ponens; such a conditional is definable in $LP_\circ$ as $(A \to B) \wedge \circ(A \to B)$.

3.3 Adaptive logic

The adaptive logics can also be obtained with $LP_\circ$. The minimal strategy is actually equivalent to *miLP*; indeed, *miLP* can be seen as an adaptive logic.[9] As we saw above, in order to obtain this system with $LP_\circ$, you just need to use a default assumption consequence, taking as a default assumption the consistency of every propositional letter, and considering the consequences of the maximally non-trivial subsets of default assumptions. The reliable strategy is more difficult to be captured.

For every set of propositional letters $p_1, ..., p_n$, there is a consistency set $\{\circ p_1, \dots, \circ p_n\}$. We say that a consistency set X is *minimally incompatible* with Γ whenever $\Gamma \cup X$ is $LP_\circ$ trivial, and for every set $X' \subset X$, $\Gamma \cup X'$ is not $LP_\circ$ trivial.

Now you start by assuming the consistency of every propositional letter p. But when there is a consistency set X minimally incompatible with the premises Γ, you need to drop $\circ p$ for every $\circ p$ in X, out from the set of consistency assumptions. Once you make this process with every minimally incompatible consistency set, the resulting set is Δ, the set of reliable consistency assumptions. More precisely, let $C(\Gamma)$ be the set of consistency sets which are minimally incompatible with Γ. Then $\Delta = \{\circ p | \circ p \in \mathcal{L} - \bigcup C(\Gamma)\}$.

The logic reliable $LP_\circ$ ($RLP_\circ$) can be formally defined in the following way:

$\Gamma \vDash_{RLP_\circ} A$ iff $\Gamma \cup \Delta \vDash_{LP_\circ} A$, where Δ is the set of reliable consistency assumptions.

For example, $\{\neg p, \neg q, p \vee r, q \vee s, p \vee q\}$ implies $r \vee s$ in $DLP_\circ$; there is a maximally non-trivial subset $\{\circ p, \circ r, \circ s, ...\}$ and another maximally non-trivial subset $\{\circ q, \circ r, \circ s, ...\}$. In both cases, the premises together with the subsets do imply $r \vee s$.

This is different in the $RLP_\circ$. Now we should consider the consistency sets

$\{\circ p\}, \{\circ q\}, \{\circ r\}, \{\circ s\}$,
$\{\circ p, \circ q\}, \{\circ p, \circ r\}, \{\circ p, \circ s\}, \{\circ q, \circ r\}, \{\circ q, \circ s\}, \{\circ r, \circ s\}$,
$\{\circ p, \circ q, \circ r\}, \{\circ p, \circ q, \circ s\}, \{\circ p, \circ r, \circ s\}, \{\circ q, \circ r, \circ s\}$,
$\{\circ p, \circ q, \circ r, \circ s\}$.

The consistency set $\{\circ p, \circ q\}$ is minimally incompatible with $\{\neg p, \neg q, p \vee r, q \vee s, p \vee q\}$, given that it is incompatible, and both $\{\circ p\}$ and $\{\circ q\}$ are compatible with the premises (the supersets of $\{\circ p, \circ q\}$ are also incompatible with the premises but not minimally). So we drop the statements $\circ p$ and $\circ q$ from Δ. Now, from the premises and the reliable consistency assumptions $\{\circ r, \circ s, ...\}$ there is no way of obtaining $r \vee s$.

It is worth observing that whenever the premises Γ belong to $\mathcal{L}_{LP}$, the non-reliable letters will just be the propositional letters in the minimal *Dab* consequences of Γ, i.e. the additional premises Δ will

[9] I would like to thank an anonymous referee for stressing this point. It is also worth remarking that the result involves a particular *instance* of the minimal strategy, where *LP* is the lower logic and *CL* is the upper logic.

include $\circ p$ for every p not in Dab_Γ (see Section 2.3 for the notion of 'minimal *Dab* consequence'). For whenever $p_1! \vee ... \vee p_n!$ is a minimal *Dab* consequence of Γ, the set $\{\circ p_1, ..., \circ p_n\}$ is minimally incompatible with Γ. It is clearly incompatible; besides, it is minimally incompatible because, if $\{\circ p_1, ..., \circ p_{n-1}\}$ was also incompatible with Γ, the sentence $p_1! \vee ... \vee p_n!$ would not be a *minimal Dab* consequence. With this result, it is easy to prove that $RLP_\circ$ is conservative over reliable adaptive logic.

THEOREM 3.4
$RLP_\circ$ is conservative with respect to the reliable strategy of adaptive logic.

PROOF. Let $\Gamma, A \in \mathcal{L}_{LP}$. Let Dab_Γ be the set of *Dab* formulas that can be derived from Γ in *LP*.

Suppose that Γ implies A in the reliable strategy of adaptive logic. This means that every *LP* model of Γ which gives a classical value to every p outside Dab_Γ is also a model of A. Now suppose for contradiction that Γ does not imply A in $RLP_\circ$. Then, when Δ is the set of sentences $\circ p$ for every p outside Dab_Γ, it holds that $\Gamma \cup \Delta \nvDash_{LP_\circ} A$. Now let M be a model of $LP_\circ$ which satisfies $\Gamma \cup \Delta$ but not A. Then, following the fact that Γ implies A in reliable adaptive logic, there is some $p \notin Dab_\Gamma$ such that M gives i to p. But this is impossible, since Δ includes all the p which are not in Dab_Γ.

For the other side, suppose that Γ implies A in $RLP_\circ$. This means that $\Gamma \cup \Delta' \vDash_{LP_\circ} A$, where Δ' is the set of sentences $\circ p$ for every p outside Dab_Γ. Now suppose that Γ does not imply A in the reliable strategy of adaptive logic. This means that there is an *LP* model M which makes every letter outside Dab_Γ classical, makes Γ true and also makes A simply false. Now, an $LP_\circ$ extension of M would make $\Gamma \cup \Delta'$ true and A simply false. And this is impossible. □

In a similar way as the previous paragraph, it might be observed that *LFIs* can be used in a more sophisticated way than in $DLP_\circ$ or $RLP_\circ$. In particular, there is no need to assume the consistency of every propositional letter. This method makes it possible to assume the consistency of some letters but not of the whole language. Moreover, in *LFI* one can make more complex claims, such as consistency disjunctions, conditionals or mixed statements such as $\circ p \vee q$.

4 Conclusion

In this paper we have argued that using *LFIs* is a promising way of recapturing classicality. The non-monotonic proposals by Priest and Batens can be expressed using default assumption consequence (for *miLP* and the minimal adaptive logic) and a slightly different default-like theory (for the reliable adaptive logic) over $LP_\circ$. This general approach makes it possible not to assume the consistency of the entire language, but just the consistency of some sentences or propositional letters. This is impossible in Batens' and Priest's approaches.

These local consistency assumptions can actually be expressed by Beall's shrieking rules. However, *LFIs* can very easily capture shrieking rules, and their expressive power is much richer. For example, *LFIs* can express mixed consistency assumptions such as $\circ p \vee q$, which cannot be expressed using shrieking rules.

Does this mean that *LFIs* are always preferable over other approaches to represent theories with inconsistent parts? Well, not necessarily. *LFIs* involve an additional symbol with respect to *LP*, and this is usually seen as undesirable.[10] However, the paper shows that *LFIs* are much more

[10] As an anonymous referee suggested, the problem with $\circ$ is mostly related to truth theories. In other areas of application, the introduction of the notion of consistency can be really useful. For example, in paraconsistent belief revision (see [13]) the formula $\circ p$ could express that there is a special kind of information regarding p.

sophisticated tools than the most popular methods of recapture (shrieking and non-monotonicity). Therefore, *LFIs* could be used in the same cases and also in more complex cases where more distinctions should be made. In this way, *LFIs* as methods for classical recapture are a promising way of dealing with cases where we have more complex information regarding the consistency of the target theory.

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