Granular flow through an aperture: Pressure and flow rate are independent

María Alejandra Aguirre,[1,2] Juan Gabriel Grande,[1,2] Adriana Calvo,[1,2,*] Luis A. Pugnaloni,[3] and Jean-Christophe Géminard[4]
[1]*Grupo de Medios Porosos, Fac. de Ingeniería, Universidad de Buenos Aires, Paseo Colón 850, (C1063ACV) Buenos Aires, Argentina*
[2]*LIA PMF-FMF (Franco-Argentinian International Associated Laboratory in the Physics and Mechanics of Fluids), Argentina - France*
[3]*Instituto de Física de Líquidos y Sistemas Biológicos (UNLP, CONICET La Plata), Casilla de Correo 565, 1900 La Plata, Argentina*
[4]*Université de Lyon, Laboratoire de Physique, Ecole Normale Supérieure de Lyon, CNRS, 46 Allée d'Italie, F-69364 Lyon Cedex 07, France*
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We simultaneously measure the flow rate and the normal force on the base, near the outlet, during the discharge through an orifice of a dense packing of monosized disks driven by a conveyor belt. We find that the normal force on the base decreases even when a constant flow rate is measured. In addition, we show, by changing the mass of the disks, that pressure can be changed while the flow rate remains constant. Conversely, we are able, by changing the belt velocity, to set different flow rates for the same pressure. The experiment confirms that, contrary to what has been implicitly assumed in numerous works, the flow rate through an aperture is not controlled by the pressure in the outlet region.

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I. INTRODUCTION AND BACKGROUND

Granular flows through an aperture are of great practical importance to several industries (e.g., pharmaceutical, mining, and agriculture) and disciplines (e.g., chemistry, physics, and engineering) and therefore have been intensely studied [1–11]. In general, the discharge of a silo through an orifice can present three regimes. Depending on the size of the outlet relative to the size of the grains, one observes the following: a continuous flow, an intermittent flow, or a complete blockage of the system due to arching [12–14].

In the continuous-flow regime, generally observed for large outlets, the *mass flow rate*, W, is generally well described by the so-called Beverloo law [1,15]: $W = C\rho\sqrt{g}(A - kD)^{5/2}$, where A is the diameter of the opening, ρ the bulk density of the granular sample, g the acceleration of gravity, and D the diameter of the grains. The parameters k and C are empirical dimensionless constants. The constant k accounts for the boundary effects at the aperture edges which lead to two boundary layers having a thickness of the order of the size of the grains (the so-called *empty annulus* [16]). Hence, one considers an effective aperture, $A_{\rm eff} \equiv A - kD$, instead of A. Therefore, the Beverloo law predicts a value $A_c \equiv kD$ of the aperture at which the flow rate is expected to vanish. The value of k has been found to be independent of the size of the particle and to generally take values between 1 and 2, depending on the particle and container properties [17]. However, there are some exceptions and, notably, for sand $k = 2.9$. It should be noted that Zhang and Rudolph claim that the only plausible value is $k = 1$ [18]. Mankoc *et al.* introduced an exponential correction to Beverloo scaling to accommodate the empirical law to the experimental observations [12]. In a two-dimensional (2D) setup—or similarly for slit-shaped apertures—one expects Beverloo law to read $W = C\rho\sqrt{g}(A - kD)^{3/2}$, where ρ then denotes the density in 2D and A the width of the aperture [19].

In the jamming regime, the jamming probability has been shown to be mainly controlled by the ratio A/D of the aperture size to the grain diameter [13,14,20–23].

The study of the discharge of particulate systems not driven by gravity is also of interest. Indeed, in many industrial applications, the grains are horizontally transported at constant velocity, as on a conveyor belt [24] or floating on the surface of a flowing liquid [25]. Recently, we reported on the discharge of disks driven through an aperture by a conveyor belt [26]. We measured the flow rate, Q, defined to be the number of disks that escape the system per unit time, as a function of the belt velocity V and aperture width A. We measured that, for large apertures ($A/D \geqslant 6$), the flow is continuous throughout the discharge and that, for given values of V, D, and A, the flow-rate Q is constant and obeys:

$$Q = \mathcal{C}\frac{V}{D}(A/D - k). \quad (1)$$

Note that the scaling law (1) is equivalent to a 2D Beverloo law in which the typical output velocity $\sqrt{g A_{\rm eff}}$ is considered to be the belt velocity V. The dimensionless flow rate $Q^* \equiv QD/V$ is expected to be independent of V and to depend linearly on the ratio A/D. From the experimental data, we indeed obtained that the proportionality constant $\mathcal{C}$ depends on the system packing fraction Φ according to $\mathcal{C} = \frac{4}{\pi}\Phi$ and that the empty annulus is accounted for by $k \approx 2$. Interestingly, the later empirical law was observed to be valid even for smaller apertures ($4.5 < A/D < 6$), when the system is likely to jam. Measurements for ratios $A/D < 4.5$ are difficult because the system systematically jams and there is no evidence that the linear relation (1) still holds for very small apertures. Note that, for very small apertures, a marked deviation from the 5/2 Beverloo scaling has been observed in three-dimensional configurations [12].

In accordance with the experimental observations, the flow rate given by the Beverloo law does not depend on the height, h, of material in the container above the outlet. This behavior differs qualitatively from the one observed for the discharge of a container filled with a viscous liquid during which the mass flow rate depends linearly on the height of the liquid above the outlet. The explanation most frequently used for

[*]Sadly, A. Calvo has passed away during the preparation stages of this article.

this independence is based on the so-called Janssen effect: friction redirects the weight of the material onto the silo walls, which leads to a saturation of the pressure at the bottom of the container. Therefore, since the pressure at the outlet does not depend on h, the flow rate is also independent of h [27]. On the one hand, it should be noted that the Janssen effect has been proven to be at work only for static situations, the condition of applicability being obviously not fulfilled during the discharge process. On the other hand, the pressure is implicitly assumed to govern the flow rate as it does in fluids, which has not been directly tested experimentally. In contrast, in the horizontal configuration, it was experimentally suggested that the above argument is improper [26]. Indeed, different flow rates were achieved even while the bottom pressure was expected to be the same and the flow rate was observed to be constant even while the pressure near the outlet was likely to decrease during the discharge. However, in [26], the pressure at the outlet was not measured.

Concerning pressure inside a silo, some works report measurements at the bottom of closed containers (i.e., static condition) [28–32]. However, much less has been reported about stress variations during the discharge process (i.e., dynamic condition). Few groups have experimentally or numerically [33,34] studied flow rates and stress variations in vertical containers. In [33], the authors study the discharge of a silo while its walls are shaken at different frequencies and amplitudes, therefore obtaining different stress conditions inside the container. They measure a constant flow rate and, failing to find any other reason, they account for the observation by assuming that upon discharge a screening effect is recovered (i.e., a dynamical Janssen effect). In addition, in [34], the authors also study the flow rate and the pressure at the bottom of a hopper. Different granular column heights in the tube yield different values of normal stress (pressure) at the aperture. They find three regimes: unchocked, transitional, and chocked regimes. In the chocked regime they observe that, independent of the normal stress (column height), the flow rate remains constant and follows Beverloo law. Nevertheless, they state later that, as in the case of compressible flows, the granular discharge rate is expected to be a function of the pressure. It is clear that these works also hold onto the erroneous belief [27] that the flow rate is influenced by the pressure, which leads them to overlook the flow rate independence from the pressure as an explanation for the experimental observations.

Although there are some experimental data for vertical systems, none exist for horizontal configurations. Therefore, in the present article, we extend our former experimental work to simultaneous measurements of the flow rate and of the normal force exerted by the material on the outlet plane. We show that the local stress in the outlet region and the flow rate can be changed independently and, thus, that the flow rate is not governed by the pressure in the outlet region.

II. EXPERIMENTAL SETUP AND PROCEDURE

The experimental setup (Fig. 1), is similar to the one used in [26]. It consists of a conveyor belt (width 40 cm, length 1 m) above which a confining Plexiglas frame (width 26 cm, length 54 cm) is maintained at a fixed position in the frame of the laboratory. A motor drives the belt at a constant velocity V which ranges from 0.4 cm/s to 4 cm/s.

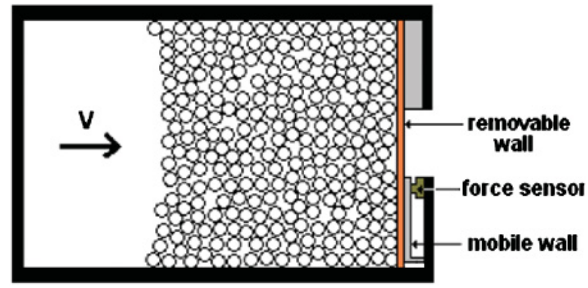

FIG. 1. (Color online) Sketch of the experimental setup. The granular material (disks) lies on a conveyor belt driven at constant velocity V. The disks, confined in a frame which is fixed in the frame of the laboratory, are pushed against a downstream wall. Once the confining wall is removed, the disks start flowing through the aperture. On one side, the mobile wall is equipped with a force sensor which makes it possible to assess the total force F exerted by the granular material on the downstream wall in the direction of the flow.

In most experiments, the granular material consists of $N_0 =$ 600 Plexiglas disks of thickness $e = (10.25 \pm 0.25)$ mm, diameter $D = (1.00 \pm 0.01)$ cm, and mass $m = (0.98 \pm 0.02)$ g. Nevertheless, in some experiments smaller disks are used in order to vary the force range (friction force). In this case, $e = (3.1 \pm 0.1)$ mm, diameter $D = (1.04 \pm 0.01)$ cm, and mass $m = (0.306 \pm 0.002)$ g.

Downstream, the Plexiglas frame exhibits, at the center, a sharp aperture of total width A. In practice, A can range from 0 to 10 cm but, in order to work in the continuous flow regime ($A/D \geqslant 6$, [26]), we report results obtained for $A =$ (8.1 ± 0.1) cm ($A/D \approx 8$). The sides of the aperture consist of two 9-cm-long bars that complete the base of the frame: one thick bar is firmly attached to the frame; the second bar (mobile wall) is thinner and L shaped. It is in frictional contact with the belt and dragged against the downstream wall of the frame. It leans punctually on the frame at its two ends. The normal force exerted by the bar onto the frame, in the direction of the flow, is assessed by measuring the force exerted in the direction of the flow at one end, near the outlet, thanks to a force sensor (Honeywell FSG15N1A). In this configuration, we do not directly measure the total force, F_T, exerted by the grains onto the downstream wall. However, assuming that the mobile wall is free to rotate at one end and that the stress σ in the direction of the flow (normal stress) is constant in the outlet plane, we estimate that the force measured by the sensor is, the constant contribution of the bar being already suppressed, $F \simeq L\sigma/2$, where $L = 9$ cm stands for the length of one bar. In the same conditions, the total force exerted by the granular material onto the downstream wall would be $F_T = 2L\sigma$, such that $F_T \simeq 4F$. It should be noted that, by construction, in the case of an inhomogeneous stress in the outlet plane, the measured force F would be more sensitive to the stress near the outlet. Even if we are aware that we do not measure F_T directly, we assume that measurements of F give a reliable image of the behavior of the total force. During the discharge, F is measured at 1 Hz.

The initial state of the system is achieved by depositing, in a disordered manner, N_0 disks on the conveyor belt, inside the confining frame. The aperture is closed ($A = 0$) using a thin (2-mm) removable wall (Fig. 1). The belt is then moved at a small, constant velocity until all grains are packed against the downstream wall and, then, stopped. Subsequently, the flow is initiated, first by removing the downstream wall, which opens the aperture to the desired width $A = 8.1$ cm, and second by moving the belt at the selected constant velocity V. The belt is stopped when the disks stop flowing through the aperture. We point out that, even if the disks are not confined by a top plate (and thus in frictional contact only with the belt and their neighbors), the granular layer is not subjected to any buckling instability and remains in contact with the belt during the whole discharge.

A video camera (Pixelink, PL-A741) is used to image the system from above. We assess the dynamics of the granular material during the discharge process by acquiring, at 1 Hz, images of the disks that remain inside the confining frame. An intensity threshold is used to convert the pictures into binary images. The number $N_{\rm in}(t)$ of disks that remain inside the confining frame at time t is calculated from the number of white pixels in the images, considering that the surface area of one disk is $S_0 = (733 \pm 8)$ pixels. We then obtain the number of disks that flowed out the system at time t, $N(t) \equiv N_0 - N_{\rm in}(t)$. The instantaneous flow rate [i.e., the number of disks that flowed out the system per unit time (averaged over 1 s because of the acquisition frequency)] is defined by $Q \equiv \frac{dN}{dt}$.

III. EXPERIMENTAL RESULTS

A. One typical run

As an example, we first report on the discharge process of the heavier disks (Fig. 2). The flow rate exhibits two distinct behaviors, similar to those observed in vertical silos [15].

First, a constant flow rate Q, characteristic of continuous flows, is observed: the number N of disks displaced out of the frame increases linearly with time. In this case, the area fraction occupied by the disks inside the frame, Φ, does not noticeably vary during the discharge and remains almost equal to that of the initial configuration $\Phi = (0.85 \pm 0.02)$. As expected, Q is given by Eq. (1) and, from the experimental value of Φ, we get $\mathcal{C} = (1.08 \pm 0.02)$. Note that, in this regime, the dimensionless flow rate $Q^* \equiv QD/V$ has been proven to be independent of V and to depend linearly on the aperture width A [26].

Later, the flow rate starts decreasing and then vanishes. We observe in the images that the transition occurs when a depletion of the disks occurs along the longitudinal axis of the container (Fig. 3; $t = 55$ s). In this regime, the aperture starts to receive only disks from each wedge formed on the side bars and not from the bulk. The aperture is not completely filled with disks and, effectively, Φ decreases. From simple geometrical arguments, we deduce that this regime takes place when the height of grains above the outlet is below $W \tan(\alpha)/2$, where W stands for width of the container and α for the angle of the pile (i.e., the angle of avalanche). In our experimental conditions, the transition between the two regimes thus occurs when about 70% of the grains have flowed out the container.

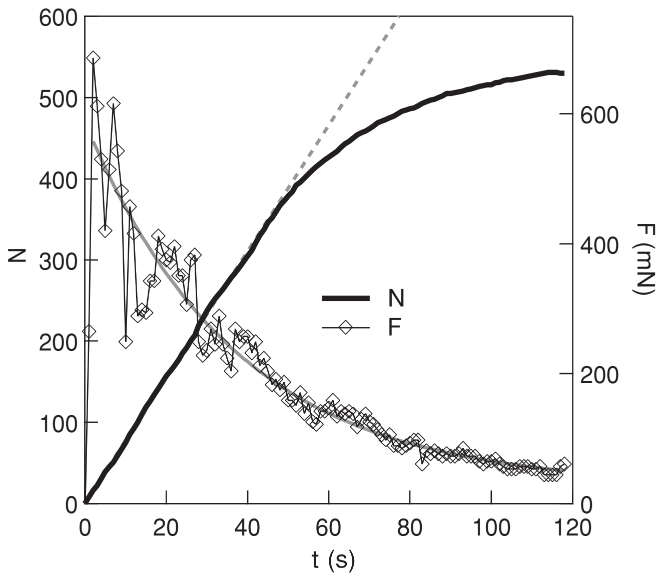

FIG. 2. Number N and force F versus time t. (Left) In the constant flow regime, N increases linearly with t and we measure $Q = (7.65 \pm 0.02)$ s^{-1} (dashed line). After about 50 s, the flow rate Q starts decreasing. At the same time, one observes a depletion of the disks on the axis of the container (Fig. 3). (Right) After a short transitory regime which is due to the preparation of the initial state and lasts less than 2 s, we observe that, in spite of large fluctuations, the force F continuously decreases throughout the discharge even when Q is constant (The gray line is only a guide for the eye; $V = 1.1$ cm/s, $m = 0.98$ g, $A = 8.1$ cm, and $A/D \approx 8$).

The temporal evolution of the force F during the discharge is reported in Fig. 2. Due to the preparation of the initial state, the grains do not touch the downstream wall at $t = 0$ and the force $F(0) = 0$. When the belt is put in motion at the velocity V, the grains rapidly enter in contact with the wall around the outlet and F reaches a maximum value $F_{\rm max} \simeq 0.65$ N in less than 2 s.

From the maximum value $F_{\rm max}$, we can estimate that the force applied by the disks on the container is almost entirely applied to the downstream wall, the force supported by the lateral walls in the direction of the flow being negligible (negligible Janssen effect). Indeed, we estimate from $F_{\rm max}$ that the total force on the downstream wall reaches a maximum of about $F_{\rm T} \simeq 4F_{\rm max} \simeq 2.6\,N$. In addition, we get the estimate $\mathcal{F} \equiv \mu_{\rm d} m g N_0 \simeq 3.0 \pm 0.3$ N of the total force

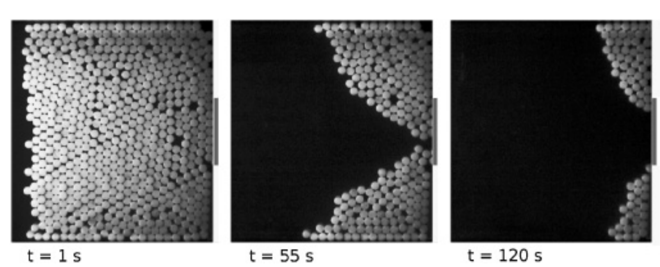

FIG. 3. Images of the disks in the container. The position of the aperture is marked with the gray bar on the right side. At $t = 55$ s, one observes a depletion of disks on the axis of the container (72% of the disks flowed out the container). At $t = 120$ s, Q almost vanishes ($V = 1.1$ cm/s, $m = 0.98$ g, $A = 8.1$ cm, and $A/D \approx 8$).

exerted by the disks on the container from the total weight $mgN_0 = (5.8 \pm 0.1)$ N of the disks and from the disk-belt dynamical frictional coefficient $\mu_d = 0.51 \pm 0.04$ (measured independently by pushing directly a column of disks against the force sensor). Therefore about 87% of $\mathcal{F}$ is applied to the downstream wall indicating that the Janssen effect is negligible in our experimental conditions.

After having quickly reached the maximum value F_{max}, the force F, in spite of the large oscillations, continuously decreases throughout the discharge even when a constant flow rate is measured. At the end of the discharge, F tends to a finite value, $F_\infty = (20 \pm 2)$ mN, which is due to the two piles that remain on the sides (Fig. 3; $t = 120$ s). During the discharge, the large fluctuations in F are likely to be due to the dynamics of stress chains in the packing. A similar behavior has been reported in other works where force is measured in vertical silos [28,32].

B. General results

In order to assess the characteristic behavior of the force F during the discharge at constant flow rate Q, we consider results obtained after F has reached its maximum (i.e., after the disks have packed against the downstream wall) and before Q starts decreasing (i.e., until about 50% of the initial amount of disk flowed out the container). The results are averaged over 10 experimental runs similar to that reported in Sec. III A.

In Fig. 4, we report F as a function of the belt displacement $V\,t$ at time t for two different belt velocities V. First, at a given velocity V, even if the data set is limited to a constant flow rate Q, we observe that the force decreases, at least, by a factor 2 in the considered range. In addition, the results are observed to be independent of V even if the flow rate Q, which is proportional to V, is more than doubled ($V = 1.1$ and $V = 2.2$ cm/s). The results directly evidence that the flow rate and the pressure (i.e., by the force) in the outlet region are independent one from another.

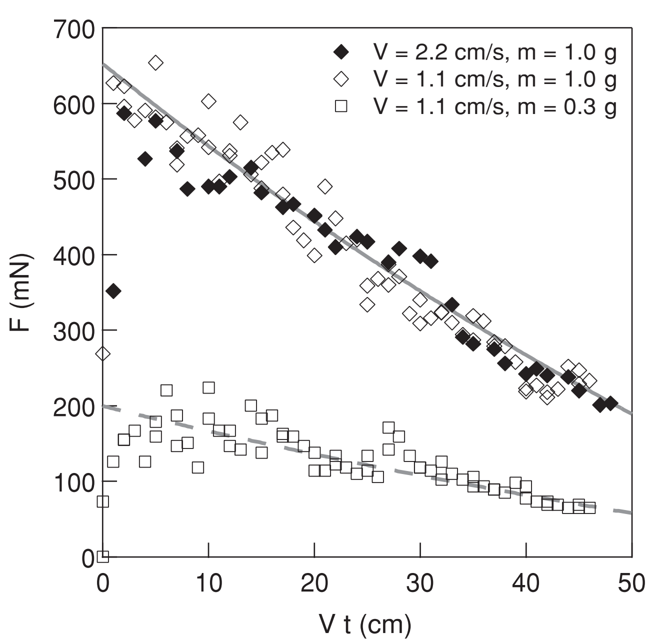

FIG. 4. Force F versus belt displacement $V\,t$. For given velocity V and mass m, we observe a significant decrease of F during the discharge even if the flow rate Q is constant. We observe that, for a given mass m (open and solid diamonds), F takes almost the same value for a given displacement $V\,t$ even if the velocity V and, thus, the flow rate Q is doubled. In addition, when the disks are almost three times lighter, for the same velocity V (open symbols), the value of F, measured for the same displacement $V\,t$, is almost divided by three even if the flow rate Q remains unchanged. (The gray lines are only guides for the eye; $A = 8.1$ cm and $A/D \approx 8$.)

One can get an additional proof of the independence of Q from F by changing the mass m of the disks. Indeed, the force F is due to the frictional contact between the disks and the belt and, thus, expected to be proportional to m. In Fig. 4, we report results obtained in the same experimental conditions ($V = 1.1$ cm/s and $A = 8.1$ cm) with disks made of the same material, having the same diameter $D = 1$ cm but a different mass, $m = 0.31$ g. We observe that the flow rate is the same for the two types of disks (not shown) but that the normal force is typically three times smaller for the lighter disks. Therefore, the same flow rate can be achieved for two different ranges of the force in the outlet region.

IV. DISCUSSION AND CONCLUSIONS

In most previous works [27], the independence of the flow rate from the granular column height h is explained by means of an experimental fact plus a postulate. The experimental fact is as follows: if the height is larger than twice the diameter of the silo, the pressure at the bottom in a static condition is constant (Janssen effect). The postulate is as follows: if during the discharge, the pressure at the bottom is constant, the flow rate is, as a consequence, also constant.

In our horizontal experimental system, the pressure decreases during the discharge whereas the flow rate remains constant. In addition, for the same pressure in the outlet region, the flow rate varies with the belt velocity [Eq. (1)]. Besides, the same flow rate can be obtained for different values of the pressure in the outlet region, for instance, by simply changing the weight of the grains.

In conclusion, using an experimental setup which makes it possible to control the escape velocity independently from other parameters of the flow, we have shown that the granular flow rate through an orifice is not controlled by the local pressure in the outlet region and that invoking the Janssen effect is not pertinent to explaining the constant flow rate measured during a silo discharge.

The flow rate is rather controlled by the mechanism that drives the grains out of the container as shown by the linear dependence of Q on V. In our experiment, this mechanism is the displacement of the conveyor belt at constant velocity which advects the grains by friction. In vertical silos, the mechanism is the acceleration due to the gravity. The independence of the flow rate from the pressure is a robust result. In a separated paper, we show experimental evidences that, in gravity-driven granular flows of polydisperse glass beads, the flow rate is also independent from the pressure in the outlet region [35].

It is worth noting that Eq. (1) only holds for low belt velocities and that the material cannot be driven at any desired velocity using a conveyor belt. Inside the horizontal silo, disks slide on the conveyor belt until they set free from the packing and move away—at rest on the belt— through the opening. In order to stop on the conveyor belt, a period of time Δt is required for each disk. The disk accelerates at $a = \mu_d g$. Therefore, to change its velocity in the laboratory reference system from zero to the belt velocity V, the disk needs $\Delta t = V/(\mu_d g)$. Over this time the disk travels $\Delta x = 0.5a\Delta t^2$. If the acceleration occurs over a very short distance, the disk will move away from the pack and reach the outlet at constant velocity V from nearly the beginning (low-velocity regime). However, if the acceleration phase takes some time, the disk will reach the outlet still with acceleration a (high-velocity regime). In the low-velocity regime $Q \propto A_{\rm eff} V$, however, in the high-velocity regime $Q \propto A_{\rm eff}\sqrt{aA_{\rm eff}} = \sqrt{a}A_{\rm eff}^{3/2}$ since $\sqrt{aA_{\rm eff}}$ is a characteristic velocity just as $\sqrt{gA_{\rm eff}}$ is used in gravity driven setups. In the high-velocity regime, we get back to a discharge of accelerated grains. Assuming that the distance from the point of detachment of the grains from the rest of the packing to the plane of the orifice is about $A/2$, the critical belt velocity around which a change of regime is expected can be estimated as $V_c \approx \sqrt{g\mu_d A}$. Notice that this is independent of the disk size. In this work we focused only on the low-velocity regime. However, our main conclusion on the independence of flow rate and pressure are also applicable to the high-velocity regime.

It is important to emphasize that valuable industrial applications can be derived from our findings. We are taught that the pressure in the silo does not need to be increased to achieve higher throughput. Rather, an appropriate mechanism to drive particles close to the outlet would suffice. In some applications, for example, air pressure is increased inside the silo to force powders trough a funnel [36]. However, a more local intervention could be used by application of electric or magnetic fields near the outlet to increase the velocities of the grains that break free from the bulk and approach the outlet.

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