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STRATEGIC EQUILIBRIA IN A MODEL OF ECONOMIC GROWTH WITH INPUT INTERDEPENDENCE

Juan M.C. Larrosa
CONICET - Departamento de Economía, Universidad Nacional del Sur, Argentina

ABSTRACT

Two countries face a strategic interdependence in producing intermediate goods. Producing these intermediate goods requires both domestic capital and another imported intermediate good. Individually, both economies determine a balanced growth path by taking into account this interdependence in different grades of awareness. By allowing for strategic interactions in the analysis, we adapted a two-agent dynamic setting and find an interior Markov perfect equilibrium as well as an open-loop equilibrium reflecting these different degrees of reaction. We find that main results resemble each other but growth rates will be higher when strategies are dynamically updated.

Keywords: difference game, economic growth, intermediate goods
JEL classification numbers: C73, F15

I. INTRODUCTION

Interdependence among economies has increased due to globalization (Ventura, 1997; Jones, 2000). There exist many production and complementation agreements among multinational firms conjointly to trade liberalization processes among countries. For instance, a considerable part of the car production in any given economy requires importing spare parts and non-locally produced components. This way, the manufacturing in diverse economic sectors is internationally complemented through local production and the importing of raw and intermediate inputs from abroad. This has been already noted by Sanyal and Jones (1982), who conclude that the majority of international trade is mainly composed of intermediate and raw goods, which require local processing before reaching the final consumer. Trade agreements have fostered this process and helped firms in diverse economies to specify their production plans based on free tariff inputs. In fact, local produce and foreign inputs usually compete in the domestic market (Chen *et al.*, 2004). In these highly integrated economies, the production of intermediate inputs in one economy becomes crucial for production of final or intermediate goods in the associated economy. What is the optimal growth policy in this context of interdependence?

Correspondence: CONICET - Departamento de Economía, Universidad Nacional del Sur, San Juan y 12 de Octubre, 8000 Bahía Blanca, Argentina. Tel: +54 291 459 5101 x2735; Fax: +54 291 459 5139; Email: jlarrosa@uns.edu.ar. The author thanks two anonymous referees for helpful comments.

We study a simple model of optimal economic growth in which two countries maintain an interdependence relation in intermediate goods production. Each economy produces a final good and an intermediate good that exports to its trading partner. Producing one unit of its own capital requires a non-domestically produced input (intermediate good) that is imported from another economy. Both economies have liberated their middle product trade and intermediate goods and services are available to both economies free of any commercial restriction. The reasons for this trade liberalization can be associated to comparative advantages in production or extraction cost or maybe the result of a political trade agreement previously signed. We assume that both economies acknowledge this mutual interdependence and plan their long-run growth paths assuming this interdependence.[1]

We work with a two-economy model, where each economy is represented by an agent that maximizes consumption by producing two types of goods: productive capital and an intermediate good. The production of intermediate goods has strategic considerations: each economy requires intermediate goods from another economy to produce its own intermediate goods. The model is strongly based on Dockner and Nishimura (2004)'s analytical framework. It also draws upon game-theoretic differential game literature (Zeeuw and van der Ploeg, 1991; Fischer and Mirman, 1996), and on literature on intermediate good trade (Ventura, 1997; Sanyal and Jones, 1982; Chen *et al.*, 2004). We model two types of equilibria: open-loop and Markovian strategy equilibria. The first type of equilibrium is associated with policy makers in one economy believing that the policy makers of the other economy will not react to a change the former makes in the intensity of intermediate good production. In the second type of equilibrium, policy makers of one economy react directly to changes the former economy makes in the production of intermediate goods.

Empirically, the first equilibrium can be related to international trade arrangements based on grounds not strictly focused on obtaining economic profits where the main reason for the link is to maintain an open link with less care going towards the efficiency of the economic outcome of the exchange. One example of this kind of relationship was, at one point, the USSR and its trade relations with the COMECON countries.[2] Under these agreements, international trade was supposed to fulfill certain production goals with less focus on conjunctural variations of the availability and price of intermediate goods required in the production process. The second type of equilibrium, on the other hand, can be associated with the actual globalized trade agreements where the main focus of the link is to improve production efficiency by relocating production processes or accessing to cheaper inputs in the world market.

The main findings of the model are that equilibria exist under the open-loop and Markov strategies. When the internalization of the effect is updated period to period, economies grow faster. By internalizing the dependence of foreign intermediate products, agents obtain greater rates of growth. For this to be observed, each economy must have its own technology of intermediate goods production with strictly decreasing returns to scale.

The paper is organized as follows. Section II describes the model and definitions. Section III presents the results from the strategic growth model and its main findings. Section IV ends the paper with conclusions.

[1] Another model of optimal growth and trade is Ventura (1997). However, he relies on a Ramsey model with trade that includes no strategic interactions and he focus on the international interdependence in the middle products sector and its influence in the growth process. Peng *et al.* (2006) also model a two-economy (or two-region) model focusing on the population agglomeration effects and differential wage incidence on growth and trade.

[2] Tsokhas (1980) presents the relationship between Cuba and the USSR, a liaison mainly focused on sugar trade, where the main interest of both countries was to keep the economic exchange open for political reasons. Evanson (1985) widens the contributions for other similar cases in Latin America.

II. THE MODEL

Consider two economies, each of which is represented by a single agent indexed by $i \neq j$, which accumulates productive capital and produces intermediate goods through capital investment and input imports and derives utility from consumption.[3] Each country tries to maximize economic growth observing this mutual interdependence. Output of one country's middle products not only depends on its capital stock and labour but also on the use of intermediate goods from the other country. Thus, each country faces an externality in its production depending of the adopted behaviour of the other economy.

The structure of the model is as follows. Let K_t^i, Z_t^i, Y_t^i and C_t^i be the productive capital stock, intermediate-goods stock, output and consumption of agent i in period t, respectively. For simplicity, we assume that total labour used in each country is constant and given by L^i. The production function of agent i is $Y_t^i = \pi_i(\theta K_t^i)^{\alpha_i}(Z_t^i)^{\beta_i}(L^i)^{1-\alpha_i-\beta_i}$, where $\pi_i > 0$ is a total factor productivity index and θ is the proportion of capital allocated in producing the final good.[4] We assume that the level of $Z_t^i = f(K_t^i, Z_t^j)$, with $j \neq i$, in a formulation of the type

$$Z_t^i = \left(\frac{(1-\theta)K_t^i}{L^i}\right)^{\frac{\varsigma_i}{\beta_i}} \left(\frac{Z_t^j}{L^j}\right)^{\frac{\varphi_j}{\beta_i}} L^i \tag{1}$$

where $\varsigma_i + \varphi_j = \beta_i < 1$. This function shows that intermediate goods Z_t^i are produced as a combination of the stock of productive capital of economy i, K_t^i and intermediate inputs from the economy j, Z_t^j.

Assuming a constant rate of depreciation for the stock of capital, δ, and applying the income identity, that is, output in the current period is used for consumption and investment, results in the following accumulation equation:

$$K_{t+1}^i = \pi_i \left(\theta K_t^i\right)^{\alpha_i} \left(Z_t^i\right)^{\beta_i} (L^i)^{1-\alpha_i-\beta_i} - C_t^i + (1-\delta)K_t^i$$

If we redefine variables in terms of units per labour employed, that is, $k_t^i \equiv K_t^i/L^i$, $z_t^i \equiv Z_t^i/L^i$, $c_t^i \equiv C_t^i/L^i$, and assume that there is full depreciation,[5] δ and χ, we obtain the following:

$$k_{t+1}^i = \pi_i \left(\theta k_t^i\right)^{\alpha_i} \left(z_t^i\right)^{\beta_i} - c_t^i \tag{2}$$

The initial state dynamics are represented by (2). But for (1) we have that $z_t^i \equiv Z^i/L^i \equiv ((1-\theta)k_t^i)^{\frac{\varsigma_i}{\beta_i}}(z_t^j)^{\frac{\varphi_j}{\beta_i}}$, from which we obtain $k_{t+1}^i = \pi_i(\theta k_t^i)^{\alpha_i}((1-\theta)k_t^i)^{\varsigma_i}(z_t^j)^{\varphi_i} - c_t^i$ this way, in case the interaction is fully internalized by the two agents the state dynamics becomes

$$k_{t+1}^i = \pi_i \left(\xi k_t^i\right)^{\alpha_i+\varsigma_i} \left(z_t^j\right)^{\varphi_j} - c_t^i \tag{3}$$

where $\xi = \theta^{\alpha_i}(1-\theta)^{\varsigma_i}$. The accumulation equation (3) gives rise to a model of strategic growth in which agents recognize that their respective productions do influence each others so this is internalized and emerges as a strategic externality resulting in a model that can be either formulated as an open-loop (precommitment) or a Markov game without commitment.[6] In the case where agents play a game in which they have access to Markov strategies they design their current actions as decision rules that depend on the current stocks of both countries, whereas in

[3] The model could also be interpreted as a two-region one, with different agents in each region.
[4] Thanks go to an anonymous referee for this suggestion.
[5] While unrealistic this assumption is made for the sake of simplicity.
[6] Readers interested in this concept of solutions should consult Dockner *et al.* (2000) for the differential games case and Zeeuw and van der Ploeg (1991) for the difference games case.

the case of an open-loop game agents choose their strategies as simple time paths that do not depend on the current level of the capital stocks.

We assume that the production function, $y_t^i = \pi_i(\xi k_t^i)^{\alpha_i+\varsigma_i}(z_t^j)^{\varphi_j}$, satisfies the following parameter restrictions: π_i is strictly positive and constant and $0 < \alpha_i, \varsigma_i, \varphi_j < 1$ and $\alpha_i + \varsigma_i + \varphi_j \leqslant 1$. Each agent derives utility in period t only from current consumption and the utility function, $u^i(c_t^i)$ is logarithmic, that is, $u^i(c_i^t) = \ln c_t^i$. Each agent lives forever, that is, $T = \infty$, and maximizes the discounted stream of utility with the discount rate given by $\rho = (1/(1+r))$, where r is the rate of interest satisfying $r > 0$. We consider only domestic consumption in this model. Intermediate importing and exporting is fully liberalized between countries. Finally, we assume that trade balance is always in equilibrium and does not require any capital flows compensation.

Each agent maximizes the discounted stream of utility given by

$$J^i = \sum_{i=0}^{\infty} \rho^i \ln c_t^i,$$

subject to

$$k_{t+1}^i = \pi_i \left(\xi k_t^i\right)^{\alpha_i+\varsigma_i} \left(z_t^j\right)^{\varphi_j} - c_t^i$$

and given initial conditions (k_0^i, z_0^j).

Specifically, this strategic growth game in reduced form looks as follows:

$$\max_{\{k_{t+1}^i\}_{t=0}^{\infty}} \left\{ J^i = \sum_{t=0}^{\infty} \rho^t \ln \left(\pi_i \left(\xi k_t^i\right)^{\alpha_i+\varsigma_i} \left(z_t^j\right)^{\varphi_j} - k_{t+1}^i\right) \right\} \tag{4}$$

subject to the initial conditions (k_0^i, z_0^j).

We follow with the derivation of equilibria of the two postulated models.

III. STRATEGIC GROWTH GAME EQUILIBRIA

We mentioned earlier that we have two possible equilibrium strategies for the agents. The first one that we are going to analyse is the open-loop solution that requires that agents commit to some initial strategy of capital augmentation during the complete period of analysis. In the second, agents used Markov strategies that reconsider in each period of time their strategy during the complete period of analysis.

In both open-loop and Markov perfect equilibria, countries understand that a change in their actions (e.g., a deviation from the equilibrium production value) causes a change in the state variable. In open-loop equilibrium, countries believe that interdependent countries will not respond to such changes. In Markov perfect equilibrium (MPE), on the contrary, countries realize that interdependent countries will respond to changes in the state variable. That way, MPE assumption attributes greater rationality to policy makers in charge of conducting these production agreements.

III.1 The open-loop game

In open-loop games, agents choose their consumption as simple time paths and commit themselves to stick to these time profiles during the entire game. It is clear that such a game cannot capture all the strategic interactions present in the dynamic game. It resembles many features of a one shot game and the equilibrium optimum could be analysed as the Nash equilibrium of this game. This leads to a static view of the interaction analysis.

The best-response function in static game determines, in our case, how a country would want to respond to an arbitrary action by its interdependent country if it were able to respond at all. However, the assumption of simultaneous moves prevents this actual response from occurring. In a static setting what a best-response function shows is the direction of the desired contemporaneous response to the linked country's actions.

To characterize the open-loop equilibrium we apply the first-order conditions, which are also sufficient given strict concavity, and get

$$\frac{-1}{\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}-k_{t+1}^i}+\rho\frac{\pi_i(\alpha_i+\varsigma_i)\left(\xi k_{t+1}^i\right)^{\alpha_i+\varsigma_i-1}\left(z_{t+1}^j\right)^{\varphi_j}}{\pi_i\left(\xi k_{t+1}^i\right)^{\alpha_i+\varsigma_i}\left(z_{t+1}^j\right)^{\varphi_j}-k_{t+2}^i}=0 \tag{5}$$

Rearranging terms and dividing (5) by $(\xi k_{t+1}^i)^{\alpha_i+\varsigma_i}(z_{t+1}^j)^{\varphi_j}$ we obtain

$$\pi_i+\rho\alpha_i\pi_i-\rho\alpha_i\pi_i^2\frac{\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}}{k_{t+1}}-\frac{k_{t+2}^i}{\left(\xi k_{t+1}^i\right)^{\alpha_i+\varsigma_i}\left(z_{t+1}^j\right)^{\varphi_j}}=0$$

Now we create a variable $x_{t+1}^i=\frac{k_{t+1}^i}{(\xi k_t^i)^{\alpha_i+\varsigma_i}(z_t^j)^{\varphi_j}}$ which we replace in the first-order condition, to get the dynamic equation

$$\pi_i+\rho\alpha_i\pi_i-\rho\alpha_i\pi_i^2\frac{1}{x_{t+1}}-x_{t+2}^i=0 \tag{6}$$

Any growth process is governed by the dynamical system (6).

Lemma 1. The followings results hold: There exists an open-loop equilibrium of the strategic growth model that results in consumption rules given by

$$c^i\left(k_t^i,z_t^j\right)=(1-\alpha_i\rho)\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{7}$$

Departing from initial stocks (k_0^i,z_0^j), equilibrium dynamics are governed by

$$k_{t+1}^i=\omega_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{8}$$

$$k_{t+1}^j=\omega_j\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i} \tag{9}$$

where $\omega_i=(\alpha_i+\varsigma_i)\rho\pi_i$ and $\omega_j=(\alpha_j+\varsigma_j)\rho\pi_j$.

Proof. See Appendix.

The system develops governed by (8) and (9). For characterizing the system we propose the following theorem.

Lemma 2. In case there are decreasing returns to scale, that is, $\alpha_i+\varsigma_i+\varphi_j<1$, equilibrium dynamics admit a unique and stable steady state given by

$$\left(\bar{k}_0^i,\bar{z}_0^j\right)=\left([(\omega_i)]^{\frac{1-\alpha_j-\varsigma_j}{\Phi}}[\omega_j]^{\frac{\varphi_j}{\Phi}},[\omega_i]^{\frac{1-\alpha_i-\varsigma_i}{\Phi}}[\omega_j]^{\frac{\varphi_j}{\Phi}}\right) \tag{10}$$

where $\Phi=(1-\alpha_i-\varsigma_i)(1-\alpha_j-\varsigma_j)-\varphi_i\varphi_j$

Proof. See Appendix.

The case of decreasing returns of scale leads to results that are directly interpretable. The case of constant returns to scale follows as another interesting case for analysis.

Lemma 3. Assume that the technologies exhibit constant returns to scale, that is, $\alpha_i + \varsigma_i + \varphi_j = 1$ *and that* π_i *is chosen in a way that* $\pi_i(\alpha_i + \varsigma_i)\rho > 1$ *and* $\pi_j(\alpha_j + \varsigma_j)\rho > 1$ *is verified. This then verifies that there exists a balanced growth path factor given by*

$$\iota = [\omega_i]^{\frac{\varphi_j}{\varphi_i+\varphi_j}} [\omega_j]^{\frac{\varphi_i}{\varphi_i+\varphi_j}} \tag{11}$$

where $\omega_i = \pi_i(\alpha_i + \varsigma_i)\rho, \forall i, j$.

Proof. See Appendix.

Notice that these results also resemble the AK-model. For instance, if both countries begin with the same initial capital stocks, then the equilibrium dynamics are represented by

$$k_{t+1} = \omega(\xi k_t)^{\alpha+\varsigma+\varphi}$$

but if the available technology shows constant returns to scale we derive that the dynamics of balanced-growth path is represented by

$$k_{t+1} = \omega\xi k_t = Ak_t$$

known as a type of AK-growth model.

III.2 *The Markov perfect game*

We follow with the derivation of a Markov Perfect Equilibrium of the strategic growth model. An MPE is a Markov Nash Equilibrium that is at the same time a subgame perfect equilibrium (SPE). The issue of existence of MPE for difference games is not an easy test. The approach resembles many features of a repeated game where agents can revise their strategies and the equilibrium optimum could be analysed as the SPE of this game. This leads to a dynamic setting of the interaction. When countries use Markov strategies in a dynamic setting they are able to respond to their rivals' actions, although they respond after they observe rivals' movements. In a dynamic setting, a best-response function shows the actual future response to a interdependent country's actions.

We begin with the following lemma.

Lemma 4. The following results hold: An MPE equilibrium exists in the strategic growth game that results in consumption rules given by

$$c^i\left(k_t^i, z_t^j\right) = \left(1 - \rho\left((\alpha_i + \varsigma_i) + \frac{\rho\varphi_i\varphi_j}{1-(\alpha_i+\varsigma_i)\rho}\right)\right)\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{12}$$

Departing from initial stocks (k_0^i, z_0^j)*, equilibrium dynamics are governed by*

$$k_{t+1}^i = \psi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{13}$$

$$k_{t+1}^j = \psi_j\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i} \tag{14}$$

for $i \neq j$*, where*

$$\psi_i = \pi_i\left((\alpha_i + \varsigma_i)\rho + \rho^2\frac{\varphi_i\varphi_j}{1-(\alpha_i+\varsigma_i)\rho}\right)$$

and $\psi_j = \pi_j((\alpha_j + \varsigma_j)\rho + \rho^2\frac{\varphi_i\varphi_j}{1-(\alpha_j+\varsigma_j)\rho})$

If there are decreasing returns to scale, that is, $\alpha_i + \varsigma_i + \varphi_j < 1$, equilibrium dynamics admit a unique steady state given by

$$\left(\bar{k}_0^i, \bar{z}_0^j\right) = \left([\psi_i]^{\frac{1-\alpha_j-\varsigma_j}{\Phi}} [\psi_j]^{\frac{\varphi_i}{\Phi}}, [\psi_j]^{\frac{1-\alpha_i-\varsigma_i}{\Phi}} [\psi_i]^{\frac{\varphi_j}{\Phi}}\right) \tag{15}$$

where $\Phi = (1 - \alpha_i - \varsigma_i)(1 - \alpha_j - \varsigma_j) - \varphi_i\varphi_j$

If there are constant returns to scale, that is, $\alpha_i + \beta_i + \nu_j = 1$, there exists a balanced growth path with the growth factor given by

$$\vartheta = \psi_i^{\frac{\varphi_j}{\varphi_i+\varphi_j}} \psi_j^{\frac{\varphi_i}{\varphi_i+\varphi_j}} \tag{16}$$

Proof. See Appendix.

Next section is dedicated to analyse the results of both games.

III.3 Analysis of results

The MPE is closer to an expected rational behaviour in equilibrium for the policy maker. That is to say, it shows the actions taken by one economy in response to the actions taken by the other economy in the recent past. In this respect, the MPE shows the equilibrium at each step of the time. In our case, MPE steady state rate of growth is higher than the open-loop case. This is because $\psi > \omega$ for i, j given that $\rho^2 \frac{\varphi_i\varphi_j}{1-(\alpha_j+\varsigma_j)\rho} > 0$. The difference is directed explained by the intensity in the use of intermediate good production in each economy. This additional part in the MPE explains how each economy is reacting to changes in intermediate goods production. Whenever both economies increase the allocation of capital in producing both intermediate goods it would result in an increase in the rate of growth.

In open-loop strategies, policy makers estimate an initial optimal allocation in intermediate input production and they believe that any changes they may make would not generate any response in partner behaviour. It is a plan that has to be followed in a rigid way. So, the rate of growth using open-loop strategies makes no use of the information in the composition of capital allocated in the middle products. On the other hand, in Markovian strategies, economies consider that if they make changes in the intensity of capital use in the middle products market, they will have a response in the counterpart.

In the empirics of international trade, the static game outcome resembles some aspects of the commercial agreements signed with political interest rather than economic interest at heart. For instance, the trade arrangements signed by many countries of the COMECON with the former Soviet Union were rather rigid deals where the absence of conjunctural adjustments or at least with highly bureaucratized procedures for adapting the production process were common practice. International trade under this framework was focused on meeting the planned objectives for a 5-year master plan (Pelzman, 1978). The model's open-loop equilibrium can be understood as an equilibrium where production decisions are taken without analysing all the relevant economic information available to the economy at every step of the time but focusing instead on a long run objective that has to be accomplished. That objective may include political or development justifications.[7]

In terms of policy implications, the result expresses that agreements of inter-industrial cooperation require constant updating for a continuous and smooth adaptation to changes

[7] Another reason can be national defence objectives. Independently of the political system, several international defence programmes are continued and concluded even if intermediate goods prices rise sensitively.

in the production process. By doing this it is granted a more efficient use of capital in both economies and that ensures joint production efficiency providing greater growth rates.

It is interesting to note that a typical current account can have international transactions that reflect both types of equilibria. A part of the international trade could reflect programmes that allow for small or null adaption to shocks whereas another part of the commerce could reflect open exchange fully adaptable to conjunctural variations. It is foreseeable that the greater the proportion of the international trade that is based on the first scenario, the effect on growth would be in the direction of lower growth rates.

IV. CONCLUSIONS

The model represents two economies that strategically interact in the process of growth. This interaction, when taken into account dynamically, enhances the growth rate of both economies compared to an open-loop equilibrium. By applying Markovian strategies, strategically connected economies recognize the role of increasing capital stock in favouring increasing rates of growth. This equilibrium presupposes that agents are fully rational in terms of economic behaviour. It seems that not only by internalizing but for dynamically reviewing the optimality of the investment decision at each time step requires a higher rate of economic growth. The open-loop equilibrium, on the other hand, implies agents that are not focused in pursuing only economic goals in the exchange.

According to this scenario if both countries are hoping for greater rates of growth they should enhance trade between them in such a way that intermediate input provision never affects the production process. Again, a simple model like this strongly suggests that trade coordination policies should be an essential part of the international exchange.

Inputs and domestic capital produce the exportable intermediate good under decreasing scale returns. Although this is consistent with the structure of the model, empirical evidence shows that international intermediate goods usually embody increasing scale returns associated with technological improvements (Ciccone, 2002). This is an issue that deserves further investigation.

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APPENDIX

The following demonstrations are adapted from Dockner and Nishimura (2004).

Proof of Lemma 1. The dynamical system (6) has two negative steady states given by

$$\hat{x}^i = (\alpha_i + \varsigma_i)\rho\pi_i$$
$$\tilde{x}^i = \pi_i$$

Given that $\tilde{x}^i = \pi_i$ implies zero consumption it is not an interesting equilibrium, because consumption becomes null. Even more, it is easy to show that $\hat{x}^i = (\alpha_i + \varsigma_i)\rho\pi_i$, is unstable. Now, we rely on the transversality condition to derive an equilibrium

$$\lim_{t\to\infty} \rho^{t+1} \frac{\partial \ln\left(\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - k_{t+1}^i\right)}{\partial k_{t+1}^i} k_{t+1}^i = 0$$

where we follow that

$$\rho^{t+1} \frac{\partial \ln\left(\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - k_{t+1}^i\right)}{\partial k_{t+1}^i} k_{t+1}^i = \frac{-\rho^{t+1}k_{t+1}^i}{\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - k_{t+1}^i}$$

and the transversality condition becomes

$$\lim_{t\to\infty} \rho^{t+1} \frac{x_{t+1}^i}{\pi_i - x_{t+1}^i} = 0$$

and it is satisfied if x_{t+1}^i is bounded away from π_i. However, because $\hat{x}^i$ is unstable, the path which does not start from $\hat{x}^i$ converges to zero or to π_i. Since this last result makes consumption go to zero, we look forward for the path converging to zero. From (6) any path $\{x_t^i\}$ converging to π_i is monotone and satisfies

$$\frac{x_{t+1}^i}{\pi_i - x_{t+1}^i} = \frac{x_{t+1}^i}{\rho(\alpha_i + \varsigma_i)\pi_i}\left(\frac{x_t^i}{\pi_i - x_t^i}\right)$$

Iterating we obtain

$$\rho^{t+1} \frac{x_{t+1}^i}{\pi_i - x_{t+1}^i} = \frac{x_{t+1}^i x_t^i \cdots x_1^i}{((\alpha_i + \varsigma_i)\pi_i)^{t+1}}\left(\frac{x_0^i}{\pi_i - x_0^i}\right)$$

As $x_{t+1}^i \to \pi_i$ the right hand side goes to infinity. Therefore, a path violates the transversality condition. The other possible solution, the steady state $\hat{x}^i = (\alpha_i - \varsigma_i)\rho\pi_i$ does not violate the transversality condition and hence it corresponds to an equilibrium path. This steady state results in the following optimum dynamics

$$k_{t+1}^{i} = (\alpha_i + \varsigma_i)\rho\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}$$

which implies, by (2), a consumption function equal to

$$c^i\left(k_t^i, z_t^j\right) = \pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - (\alpha_i+\varsigma_i)\rho\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}$$
$$c^i\left(k_t^i, z_t^j\right) = (1-(\alpha_i+\varsigma_i)\rho)\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \qquad \blacksquare$$

Proof of Lemma 2. We begin by defining the Jacobian function of the dynamical system for analysing the characteristic equation. This would be the first step for dealing with the existence of a steady state in this system.

$$\begin{bmatrix} \alpha_i+\varsigma_i & \varphi_i\pi_i(\alpha_i+\varsigma_i)\rho^{\frac{1-\alpha_i-\varsigma_i-\varphi_i}{\Phi}}\pi_j(\alpha_j+\varsigma_j)\rho^{\frac{\alpha_i+\varsigma_i+\varphi_i-1}{\Phi}} \\ \varphi_j\pi_i(\alpha_i+\varsigma_i)\rho^{\frac{\alpha_i+\varsigma_i+\varphi_i}{\Phi}}\pi_j(\alpha_j+\varsigma_j)\rho^{\frac{1-\alpha_i-\varsigma_i-\varphi_i}{\Phi}} & \alpha_j+\varsigma_j \end{bmatrix}$$

where $\Phi = (1-\alpha_i-\varsigma_i)(1-\alpha_j-\varsigma_j) - \varphi_i\varphi_j$.

By getting the characteristic equation of this system we obtain

$$f(\lambda) = \lambda^2 - (\alpha_i+\varsigma_i+\alpha_j+\varsigma_j)\lambda + ((\alpha_i+\varsigma_i)(\alpha_j+\varsigma_j)) - \varphi_i\varphi_j$$

Roots are real and we have that

$$f(1) = (1-\alpha_i-\varsigma_i)(1-\alpha_j-\varsigma_j) - \varphi_i\varphi_j > \varphi_i\varphi_j - \varphi_i\varphi_j = 0$$

and

$$f(-1) = (1-\alpha_i-\varsigma_i)(1-\alpha_j-\varsigma_j) - \varphi_i\varphi_j > 0$$

The function is strictly convex because $f'' = 2 > 0$ and its minimum is at $(\alpha_i+\varsigma_i+\alpha_j+\varsigma_j)/2 < 1$ with two roots, λ_i, $i = 1, 2$, such that $|\lambda_i| < 1$. Both roots are positive if $(\alpha_i+\varsigma_i)(\alpha_j+\varsigma_j) > \varphi_i\varphi_j$; one is positive and the other is negative if $(\alpha_i+\varsigma_i)(\alpha_j+\varsigma_j) < \varphi_i\varphi_j$. $\blacksquare$

Proof of Lemma 3. Assume that the technologies exhibit constant returns to scale, that is, $\alpha_i + \varsigma_i + \varphi_j = 1$ and that π_i is chosen in a way that $\pi_i(\alpha_i+\varsigma_i)\rho > 1$ and $\pi_j(\alpha_j+\varsigma_j)\rho > 1$ is verified.

We get the following result:

The rate of growth for the economy in the open-loop case is giving by

$$\iota = [(\pi_i(\alpha_i+\varsigma_i)\rho)]^{\frac{\varphi_j}{\varphi_i+\varphi_j}}[\pi_j((\alpha_j+\varsigma_j)\rho)]^{\frac{\varphi_i}{\varphi_i+\varphi_j}}$$

Proof. Assuming that

$$k_t^i = \iota^t k_0^i \quad \text{and} \quad k_t^j = \iota^t k_0^j \tag{A.1}$$

is verified. Then we get that $k_t^j/k_t^i = k_t^i/k_t^j$. Substituting (A.1) into (8) we get that

$$\iota^{t+1}k_0^i = ((\alpha_i+\varsigma_i)\rho\pi_i)\iota^{t\varphi_i}\iota^{t(\alpha_i+\varsigma_i)}\left(k_0^i\right)^{\alpha_i+\varsigma_i}\left(k_0^j\right)^{\varphi_i}$$
$$\iota^{t+1}k_0^i = ((\alpha_i+\varsigma_i)\rho\pi_i)\left(k_0^i\right)^{\alpha_i+\varsigma_i-1}\left(k_0^j\right)^{\varphi_i}$$
$$\iota = ((\alpha_i+\varsigma_i)\rho\pi_i)\left(\frac{k_0^j}{k_0^i}\right)^{\varphi_i}$$

An identical argument states that

$$\iota = ((\alpha_j + \varsigma_j)\rho\pi_j)\left(\frac{k_0^i}{k_0^j}\right)^{\varphi_j}$$

By joining both results we obtain

$$\iota = \left[\pi_i^{((\alpha_i+\varsigma_i)\rho\pi_i)}\right]^{\frac{\varphi_j}{\varphi_i+\varphi_j}}\left[\pi_j^{((\alpha_j+\varsigma_j)\rho\pi_j)}\right]^{\frac{\varphi_i}{\varphi_i+\varphi_j}}$$

■

Proof of Lemma 4. Employing Markovian decision rules means that representative agents choose strategies of the type $c_t^i = c^i(k_t^i, z_t^j)$ when designing their actions. To derive an MPE, we therefore make use of dynamic programming techniques. Let us define the value function for agent *i* as

$$V^i\left(k_t^i, z_t^j\right) \equiv \max \sum_{s=t}^{\infty} \rho^s \ln c_s^i \tag{A.2}$$

These value functions must satisfy the Bellman equations

$$V^i\left(k_t^i, z_t^j\right) \equiv \max_{c^i}\left\{\ln c^i + \rho V^i\left(\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - c^i, \pi_j\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i} - c^j\right)\right\} \tag{A.3}$$

By following Dockner and Nishimura (2004) and Fischer and Mirman (1996), we guess a candidate solution for the value function of the type

$$V^i\left(k_t^i, z_t^j\right) \equiv A_{ii}\ln\left(\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}\right) + B_{ij}\ln\left(\pi_j\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i}\right) + D_i \tag{A.4}$$

where A_{ii}, B_{ij} and D_i are appropriately chosen constants. Based on these value functions we can guess a policy function for the consumption strategy of agent *i* by

$$c^i\left(k_t^i, z_t^j\right) = a_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{A.5}$$

where a_i, in the same way, needs to be determined from the equilibrium conditions. Taking under consideration these specifications of the policy functions the corresponding dynamical system of the capital stock becomes

$$k_{t+1}^i = (\pi_i - a_i)\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} \tag{A.6}$$

If we now substitute the policy functions (A.5) and the proposed value functions (A.4) into the Bellman equation (A.3) we get

$$\begin{aligned} &A_{ii}\ln\left(\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}\right) + B_{ij}\ln\left(\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i}\right) + D_i \\ &\quad= \ln\left(a_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}\right) \\ &\qquad+ \rho\left[A_{ii}\ln\left(\pi_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j} - a_i\left(\xi k_t^i\right)^{\alpha_i+\varsigma_i}\left(z_t^j\right)^{\varphi_j}\right)\right] \\ &\qquad+ \rho\left[B_{ij}\ln\left(\pi_j\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i} - a_i\left(\xi k_t^j\right)^{\alpha_j+\varsigma_j}\left(z_t^i\right)^{\varphi_i}\right) + D_i\right] \end{aligned}$$

where we see that the constants A_{ii} and B_{ij} satisfy the equations

$$A_{ii} = 1 + \rho[(\alpha_i + \varsigma_i)A_{ii} + \varphi_j B_{ij}] \tag{A.7}$$

$$B_{ij} = \rho[\varphi_i A_{ii} + (\alpha_j + \varsigma_j)B_{ij}] \tag{A.8}$$

Then (A.4) and (A.5) solve (A.3). A solution to the system (A.7) and (A.8) is easily found and given by

$$A_{ii} = \frac{1 - (\alpha_j + \varsigma_j)\rho}{(1 - (\alpha_i + \varsigma_i)\rho)(1 - \rho) - \rho^2 \varphi_j \varphi_i} \tag{A.9}$$

$$B_{ij} = \frac{\varphi_j \rho}{(1 - (\alpha_i + \varsigma_i)\rho)(1 - \rho) - \rho^2 \varphi_i \varphi_j} \tag{A.10}$$

for $i \neq j$

Proving that the policy function (A.5) is in fact an equilibrium we need to show the maximum of the right hand side of (A.3) given the specification (A.4).

The maximization gives:

$$\frac{1}{c^i} = \frac{\rho((\alpha_i + \varsigma_i)A_{ii} + \varphi_i B_{ij})}{\pi_i \left(\xi k_t^j\right)^{\alpha_j + \varsigma_j} \left(z_t^i\right)^{\varphi_i} - c^i}$$

Solving for the optimal policy rule shows that the functional form given by (A.1) is achieved by using A_{ii} and B_{ij} as obtained in (A.9) and (A.10), that is,

$$c^i \left(k^i, z_t^j\right) = \frac{\pi_i \left(\xi k_t^j\right)^{\alpha_j + \varsigma_j} \left(z_t^i\right)^{\varphi_i}}{1 + \rho((\alpha_i + \varsigma_i)A_{ii} + \varphi_i B_j)} = a_i \left(\xi k_t^j\right)^{\alpha_j + \varsigma_j} \left(z_t^i\right)^{\varphi_i}$$

where $a_i = \pi_i / 1 + \rho((\alpha_i + \varsigma_i)A_{ii} + \varphi_i B_j)$.

Hence, the equilibrium dynamics becomes

$$k_{t+1}^i = \psi_i \left(\xi k_t^i\right)^{\alpha_i + \varsigma_i} \left(z_t^j\right)^{\varphi_j} \tag{A.11}$$

$$k_{t+1}^j = \psi_j \left(\xi k_t^j\right)^{\alpha_j + \varsigma_j} \left(z_t^i\right)^{\varphi_i} \tag{A.12}$$

for $i \neq j$, where

$$\psi_i = \pi_i \left((\alpha_i + \varsigma_i)\rho + \rho^2 \frac{\varphi_i \varphi_j}{1 - (\alpha_i + \varsigma_i)\rho}\right) \text{ and } \psi_j = \pi_j \left((\alpha_j + \varsigma_j)\rho + \rho^2 \frac{\varphi_i \varphi_j}{1 - (\alpha_j + \varsigma_j)\rho}\right)$$

In the case of constant returns to scale, the balanced growth path with the growth factor is given by

$$\vartheta = \psi_i^{\frac{\varphi_i}{\varphi_i + \varphi_j}} \psi_j^{\frac{\varphi_j}{\varphi_i + \varphi_j}} \qquad \blacksquare$$