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Research Article

Self-Dual Effective Compact and True Compacton Configurations in Generalized Abelian Higgs Models

Rodolfo Casana[1], **Guillermo Lazar**[2], **and Lucas Sourrouille**[3,4]

[1]*Departamento de Física, Universidade Federal do Maranhão, Campus Universitário do Bacanga, 65080-805 São Luís, MA, Brazil*
[2]*Coordenação do Curso de Licenciatura em Ciências Naturais, Universidade Federal do Maranhão, Campus III, 65700-000 Bacabal, MA, Brazil*
[3]*IMDEA Nanociencia, Calle de Faraday, 9, Cantoblanco, 28049 Madrid, Spain*
[4]*IFISUR, Departamento de Física (UNS-CONICET), Avenida Alem 1253, Bahía Blanca, Buenos Aires, Argentina*

Correspondence should be addressed to Rodolfo Casana; rodolfo.casana@gmail.com

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We have studied the existence of self-dual effective compact and true compacton configurations in Abelian Higgs models with generalized dynamics. We have named an effective compact solution the one whose profile behavior is very similar to the one of a compacton structure but still preserves a tail in its asymptotic decay. In particular, we have investigated the electrically neutral configurations of the Maxwell-Higgs and Born-Infeld-Higgs models and the electrically charged ones of the Chern-Simons-Higgs and Maxwell-Chern-Simons-Higgs models. The generalization of the kinetic terms is performed by means of dielectric functions in gauge and Higgs sectors. The implementation of the BPS formalism without the need to use a specific *Ansatz* has led us to the explicit determination for the dielectric function associated with the Higgs sector to be proportional to $\lambda|\phi|^{2\lambda-2}$, $\lambda > 1$. Consequently, the followed procedure allows us to determine explicitly new families of self-dual potential for every model. We have also observed that, for sufficiently large values of λ, every model supports effective compact vortices. The true compacton solutions arising for $\lambda = \infty$ are analytical. Therefore, these new self-dual structures enhance the space of BPS solutions of the Abelian Higgs models and they probably will imply interesting applications in physics and mathematics.

1. Introduction

Topological defects produced by field theories including generalized kinematic terms have been an issue of great interest in the latest years. Usually these models include higher derivatives dynamic terms, but sometimes the generalizations are caused by the introduction of some generalized parameter or function in the kinetic terms. These modified theories named as k-theories arose initially as effective cosmological models for inflationary evolution [1]. Later the k-theories were permeating other issues of field theory and cosmology such as dark matter [2], strong gravitational waves [3], the tachyon matter problem [4], and ghost condensates [5]. It is worth emphasizing the possibility that these theories can arise naturally within the context of string theory. Several studies concerning the topological structure of the k-theories have shown that they also engender topological solitons [6–22]; in general, they can present some new characteristics when compared those of the usual ones [23].

Compactons were defined as solitons with finite wavelength in the pioneer work [1] and so far they have been the subject of several studies, since models containing topological defects have been used to represent particles and cosmological objects such as cosmic strings [24]. A particular arrangement of particles can be represented by a group of compactons and in this case we will not have the problem of the superposition of particles (or defects) due to the fact that compactons do not carry a "tail" in its asymptotic decay. We also point out that compact vortices and skyrmions are intrinsically connected with recent advances in the miniaturization of magnetic materials at the nanometric scale for spintronic applications [25–27]. Compact topological defects have gained greater attention as effective low-energy models for QCD concerning skyrmions, where nonperturbative results at the classic level have been reached [28]. Compact solutions were also successfully employed in the description of boson stars [29] and in baby Skyrme models [30, 31].

At the classical level there is a widely employed mechanism to achieve field equations, namely, the Bogomol'nyi-Prasad-Sommerfield (BPS) formalism [32, 33]. The BPS method consists of building a set of first-order differential equations which solve as well the second-order Euler-Lagrange equations. One interesting aspect of this mechanism is that all the equations are built up for static field configurations. As a consequence, the first-order equations of motion coming out from the BPS formalism describe field configurations minimizing the total system energy. The static characteristic of the fields in the BPS limit has been applied to investigate topological defects in several frameworks. For example, in the context of planar gauge theories, vortices structures arise from the BPS equations, specially, magnetic vortices which were found in Maxwell-Higgs electrodynamics [24]. Also we can mention the Chern-Simons-Higgs electrodynamics [34, 35] and the Maxwell-Chern-Simons-Higgs model [36] both describing electrically charged magnetic vortices. Other interesting frameworks involving first-order BPS solutions are the nonlinear sigma models (NLσM) [37] in the presence of a gauge field. These theories have been widely applied in the study of field theory and condensed matter physics [38, 39]. We can mention, in this sense, the topological defects in a $O(3)$ nonlinear sigma model with the Maxwell term, as it is shown in [40–45]. Concerning the Chern-Simons term, topological and nontopological defects were analyzed in [46, 47]. Also, the gauged $O(3)$ sigma model with both Maxwell and the Chern-Simons terms was studied in [48, 49].

The existence of vortex solutions with compact-like profiles in k-generalized Abelian Maxwell-Higgs model was studied in [50, 51] and in k-generalized Born-Infeld model [52]; however, only in [51] were the first-order vortices with compact-like profiles found. Therefore, the aim of this manuscript is to study the topological vortices engendered by the self-dual configurations obtained from the generalization of the following Abelian Higgs models: Maxwell-Higgs (MH), Born-Infeld-Higgs (BIH), Chern-Simons-Higgs (CSH), and Maxwell-Chern-Simons-Higgs (MCSH). Firstly, we have performed a consistent implementation of the Bogomol'nyi method for every model and obtained the respective generalized self-dual or BPS equations. The development of the BPS formalism has allowed fixing the form of the function $\omega(|\phi|)$, composing the generalized term $\omega(|\phi|)|D_\mu\phi|^2$, which only can be proportional to $\lambda|\phi|^{2\lambda-2}$ for $\lambda > 1$ (a similar result was obtained in [53]). Secondly, we use the usual vortex *Ansatz* to obtain the self-dual equations describing axially symmetric configurations. We have observed that independently of the model an effective compact behavior of the vortices arises for a sufficiently large value of the parameter λ and only in the limit $\lambda \to \infty$ are the true compacton structures achieved. Finally, we give our remarks and conclusions.

2. The Maxwell-Higgs Case

The Maxwell-Higgs model is a classical field theory where the gauge field dynamics are controlled by the Maxwell term and the matter field is represented by the complex scalar Higgs field. The model presents vortex solutions when it is endowed with a fourth-order self-interacting potential promoting a spontaneous symmetry breaking. Although the model seems very simple it presents characteristics very similar to the phenomenological Ginzburg-Landau model for superconductivity [54] or superfluidity in He^4. The applications of the Maxwell-Higgs model extend from the condensed matter to inflationary cosmology [55] or as an effective field theory for cosmic strings [24].

The generalized Maxwell-Higgs model [11] is described by the following Lagrangian density:

$$\mathscr{L} = -\frac{G(|\phi|)}{4}F_{\mu\nu}F^{\mu\nu} + \omega(|\phi|)\left|D_\mu\phi\right|^2 - V(|\phi|). \quad (1)$$

The nonstandard dynamics are introduced by two nonnegative functions $G(|\phi|)$ and $\omega(|\phi|)$ depending of the Higgs field. The Greek index runs from 0 to 2. The vector A_μ is the electromagnetic field, the Maxwell strength tensor is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $D_\mu\phi$ defines the covariant derivative of the Higgs field ϕ;

$$D_\mu\phi = \partial_\mu\phi - ieA_\mu\phi. \quad (2)$$

The function $V(|\phi|)$ is a self-interacting scalar potential.

From (1), the gauge field equation reads

$$\partial_\nu\left(\mathrm{G}F^{\nu\mu}\right) = e\omega J^\mu, \quad (3)$$

where ωJ^μ is the conserved current density, that is, $\partial_\mu(\omega J^\mu) = 0$, and J^μ is the usual current density

$$J^\mu = i\left[\phi\left(D^\mu\phi\right)^* - \phi^*\left(D^\mu\phi\right)\right]. \quad (4)$$

Throughout the remainder of the section, we are interested in time-independent soliton solutions that ensure the finiteness of the action engendered by (1). Then, from (3), we read the static Gauss law

$$\partial_k\left(G\partial_k A_0\right) = 2e^2\omega A_0\left|\phi\right|^2 \quad (5)$$

and the respective Ampère law

$$\epsilon_{kj}\partial_j(GB) = e\omega J_k. \quad (6)$$

It is clear from the Gauss law that $e\omega J_0$ stands for the electric charge density, so that the total electric charge of the configurations is

$$Q = 2e^2\int d^2x\,\omega A_0\left|\phi\right|^2 \quad (7)$$

which is shown to be null ($Q = 0$) by integration of the Gauss law under suitable boundary conditions for the fields at infinity; that is, $A_0 \to 0$, $\phi \to cte$, and $G(|\phi|)$ a well behaved function. Therefore, the field configurations will be electrically neutral, like it happens in the usual Maxwell-Higgs model.

The fact that the configurations are electrically neutral is compatible with the gauge condition, $A_0 = 0$, which satisfies identically the Gauss law (5). With the choice $A_0 = 0$, the static and electrically neutral configurations are described by the Ampère law (6) and the reduced equation for the Higgs field

$$D_k(\omega D_k\phi) - \frac{1}{2}B^2\frac{\partial G}{\partial\phi^*} - \frac{\partial\omega}{\partial\phi^*}\left|D_k\phi\right|^2 - \frac{\partial V}{\partial\phi^*} = 0. \quad (8)$$

To implement the BPS formalism, we first establish the energy for the static field configuration in the gauge $A_0 = 0$, so it reads

$$E = \int d^2x\left[\frac{G}{2}B^2 + \omega\left|D_k\phi\right|^2 + V\right]. \quad (9)$$

To proceed, we need the fundamental identity

$$\left|D_i\phi\right|^2 = \left|D_\pm\phi\right|^2 \pm eB\left|\phi\right|^2 \pm \frac{1}{2}\epsilon_{ik}\partial_i J_k, \quad (10)$$

where $D_\pm\phi = D_1\phi \pm iD_2\phi$. With it, the energy (9) is written as being

$$E = \int d^2x\left[\frac{G}{2}B^2 + V(|\phi|) + \omega\left|D_\pm\phi\right|^2 \pm e\omega B\left|\phi\right|^2 \pm \frac{1}{2}\omega\epsilon_{ik}\partial_i J_k\right]. \quad (11)$$

We observe that the term $\omega\epsilon_{ik}\partial_i J_k$ precludes the implementation of the BPS procedure, that is, expressing the integrand as a sum of squared terms plus a total derivative plus a term proportional to the magnetic field. This inconvenience already was observed in [56]; such a problem was circumvented by analyzing only axially symmetric solutions in polar coordinates.

The key question about the functional form of $\omega(|\phi|)$ allowing a well-defined implementation of the BPS formalism was solved in [53]. In the following we reproduce some details on looking for the function $\omega(|\phi|)$. The starting point is the following expression:

$$\omega\epsilon_{ik}\partial_i J_k = \epsilon_{ik}\partial_i(\omega J_k) - \epsilon_{ik}(\partial_i\omega)J_k. \quad (12)$$

By considering ω to be an explicit function of $|\phi|^2$, after some algebraic manipulations, the last term $\epsilon_{ik}(\partial_i\omega)J_k$ becomes expressed as

$$\epsilon_{ik}(\partial_i\omega)J_k = |\phi|^2\frac{\partial\omega}{\partial|\phi|^2}\epsilon_{ik}\partial_i J_k + 2eB|\phi|^4\frac{\partial\omega}{\partial|\phi|^2}, \quad (13)$$

which, after being substituted in (12), allows obtaining

$$\left(\omega + |\phi|^2\frac{\partial\omega}{\partial|\phi|^2}\right)\epsilon_{ik}\partial_i J_k = \epsilon_{ik}\partial_i(\omega J_k) - 2eB|\phi|^4\frac{\partial\omega}{\partial|\phi|^2}. \quad (14)$$

At this point, we establish the function ω to satisfy the following condition:

$$\omega + |\phi|^2\frac{\partial\omega}{\partial|\phi|^2} = \lambda\omega, \quad \lambda > 0, \quad (15)$$

whose solutions provide the explicit functional form of $\omega(|\phi|)$ to be

$$\omega(|\phi|) = \lambda\frac{|\phi|^{2\lambda-2}}{v^{2\lambda-2}}, \quad (16)$$

guaranteeing that the vacuum expectation value of the Higgs field is $|\phi| = v$.

With the key condition (15), (14) allows expressing the term $\omega\epsilon_{ik}\partial_i J_k$ in the following way:

$$\omega\epsilon_{ik}\partial_i J_k = \frac{1}{\lambda}\epsilon_{ik}\partial_i(\omega J_k) - 2ev^2(\lambda - 1)\frac{|\phi|^{2\lambda}}{v^{2\lambda}}B. \quad (17)$$

By putting the expression (17) in the energy (11), it becomes

$$E = \int d^2x\left[\frac{G}{2}B^2 + V(|\phi|) + \omega\left|D_\pm\phi\right|^2 \pm ev^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}}B \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i(\omega J_k)\right]. \quad (18)$$

We now manipulate the two first terms in such a form that the energy can be written as

$$E = \int d^2x\left[\frac{G}{2}\left(B \mp \sqrt{\frac{2V}{G}}\right)^2 + \omega\left|D_\pm\phi\right|^2 \pm B\left(\sqrt{2GV} + ev^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right) \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i(\omega J_k)\right]. \quad (19)$$

With the objective being that the integrand has a term proportional to the magnetic field, we impose that the factor multiplying it must be a constant; that is,

$$\sqrt{2GV} + ev^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}} = ev^2. \quad (20)$$

Consequently, the self-dual potential $V(|\phi|)$ becomes

$$V(|\phi|) = \frac{1}{G}U^{(\lambda)}(|\phi|), \quad (21)$$

where we have defined the potential $U^{(\lambda)}(|\phi|)$ given by

$$U^{(\lambda)}(|\phi|) = \frac{e^2v^4}{2}\left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right)^2. \quad (22)$$

We can note that, for $\lambda = 1$, it becomes the self-dual potential of the Maxwell-Higgs model.

Hence, the energy (77) reads as

$$E = \int d^2x \left\{ \pm ev^2 B + \omega \left| D_\pm \phi \right|^2 + \frac{G}{2}\left(B \mp \frac{\sqrt{2U^{(\lambda)}}}{G} \right)^2 \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i\left(\omega J_k\right) \right\}. \tag{23}$$

Now by imposing appropriate boundary conditions, the contribution to the total energy of the total derivative is null and the energy has a lower bound proportional to the magnitude of the magnetic flux,

$$E \geq \pm ev^2 \int d^2x B = ev^2 \left|\Phi\right|, \tag{24}$$

where for positive flux we choose the upper signal and for negative flux we choose the lower signal.

The lower bound is saturated by fields satisfying the first-order Bogomol'nyi or self-dual equations [32, 33]

$$D_\pm \phi = 0, \qquad B = \pm\frac{ev^2}{G}\left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right). \tag{25}$$

The function $G(|\phi|)$ must be a function providing a finite magnetic field such that $B(|\vec{x}| \rightarrow \infty) \rightarrow 0$ is sufficiently rapid to provide a finite total magnetic flux.

In the BPS limit the energy (9) provides the energy density of the self-dual configurations

$$\varepsilon_{\mathrm{BPS}} = \frac{2U^{(\lambda)}}{G} + \lambda\frac{|\phi|^{2\lambda-2}}{v^{2\lambda-2}}\left|D_k\phi\right|^2; \tag{26}$$

it will be finite and positive-definite for $\lambda > 0$. We here also require the function $G(|\phi|)$ yielding a finite BPS energy density such that $\varepsilon_{\mathrm{BPS}}(|\vec{x}| \rightarrow \infty) \rightarrow 0$ is sufficiently rapid to provide a finite total energy.

2.1. Maxwell-Higgs Effective Compact Vortices for λ Finite.

In the following, with loss of generality, we have chosen $G(|\phi|) = 1$ (for this one and for all other models analyzed throughout the manuscript) with the aim of studying the influence of the generalized dynamic in Higgs sector in the formation of effective compact vortices and true compactons. Such a generalization provided by the function $\omega(|\phi|)$ defined in (16) and its effects in the formation of effective compact vortices apparently remains unexplored in the literature.

Thus, we seek axially symmetric solutions according to the usual vortex *Ansatz* [24]

$$\phi(r,\theta) = vg(r)e^{in\theta}, \qquad A_\theta(r) = -\frac{a(r) - n}{er}, \tag{27}$$

with $n = \pm 1, \pm 2, \pm 3, \ldots$ standing for the winding number of the vortex solutions.

The profiles $g(r)$ and $a(r)$ are regular functions describing solutions possessing finite energy and obeying the boundary conditions,

$$g(0) = 0, \quad a(0) = n, \quad g(\infty) = 1, \quad a(\infty) = 0. \tag{28}$$

Under *Ansatz* (27), the magnetic field reads as

$$B(r) = -\frac{1}{er}\frac{da}{dr}. \tag{29}$$

The correspondent quantized magnetic flux is given by

$$\Phi = \int d^2x B = \frac{2\pi}{e}n, \tag{30}$$

as expected.

The BPS equations (25) are written as

$$g' = \pm\frac{ag}{r}, \qquad -\frac{a'}{r} = \pm e^2v^2\left(1 - g^{2\lambda}\right). \tag{31}$$

The upper (lower) signal corresponds to the vortex (antivortex) solution with winding number $n > 0$ ($n < 0$).

The self-dual energy density (26) is expressed by

$$\varepsilon_{\mathrm{BPS}} = e^2v^4\left(1 - g^{2\lambda}\right)^2 + 2\lambda v^2 g^{2\lambda-2}\left(\frac{ag}{r}\right)^2. \tag{32}$$

It will be finite and positive-definite for $\lambda \geq 1$.

The total energy of the self-dual solutions is given by the lower bound (24),

$$E_{\mathrm{BPS}} = \pm ev^2\Phi_B = \pm 2\pi v^2 n; \tag{33}$$

it is proportional to the winding number of the vortex solution, as expected.

The behavior of $g(r)$ and $a(r)$ near the boundaries can be easily determined by solving the self-dual equations (31) around the boundary conditions (28). Then, for $r \rightarrow 0$, the profiles behave as

$$g(r) \approx C_n r^n + \cdots, \qquad a(r) \approx n - \frac{e^2v^2}{2}r^2 + \cdots, \tag{34}$$

where the constant $C_n > 0$ is computed numerically.

By considering the profile $0 \leq g(r) < 1$, in the limit $\lambda \rightarrow \infty$, the potential (22) acquires the following form:

$$U^{(\infty)}(g) = \frac{e^2v^4}{2}\Theta(1 - g), \tag{37}$$

where $\Theta(1 - g)$ is the Heaviside function.

The BPS equations (31), in the limit $\lambda \to \infty$, are written as

$$g' = \pm \frac{ag}{r}, \qquad -\frac{a'}{r} = \pm e^2 v^2 \Theta(1-g). \tag{38}$$

The boundary conditions for compacton solutions are

$$g(0) = 0, \quad a(0) = n, \quad g(r) = 1, \quad a(r) = 0 \quad r_c \le r < \infty. \tag{39}$$

The radial distance $r_c < \infty$ is the value where the profile $g(r)$ reaches the vacuum value and the gauge field profile $a(r)$ becomes null.

The solutions (for $n > 0$) of the compacton BPS equations (38) provide analytical profiles for the Higgs and gauge field,

$$g^{(\infty)}(r) = \left(\frac{r}{r_c}\right)^n \exp\left[\frac{n}{2}\left(1-\frac{r^2}{r_c^2}\right)\right]\Theta(r_c - r) + \Theta(r - r_c), \qquad a^{(\infty)}(r) = n\left(1-\frac{r^2}{r_c^2}\right)\Theta(r_c - r), \tag{40}$$

where the radial distance r_c is given by

$$r_c = \frac{\sqrt{2n}}{|ev|}. \tag{41}$$

The magnetic field and BPS energy density of the Maxwell-Higgs compacton are

$$B^{(\infty)}(r) = ev^2\Theta(r_c - r), \qquad \varepsilon_{\mathrm{BPS}}^{(\infty)}(r) = e^2 v^4 \Theta(r_c - r). \tag{42}$$

The numerical solutions (for all model analyzed in the manuscript) were performed using the routines for boundary value problems of the software Maple 2015. We have chosen the upper signals in BPS equations (31). We have fixed $e = v = 1$, the winding number $n = 1$, and calculated the numerical solutions for some finite values of λ. The profiles for the Higgs and gauge fields are given in Figure 1 and the correspondent ones for the magnetic field and the self-dual energy density are depicted in Figure 2.

On the other hand, when $r \to \infty$, they behave as

$$g(r) \approx 1 - \frac{C_\infty}{\sqrt{r}} e^{-mr}, \qquad a(r) \approx mC_\infty \sqrt{r} e^{-mr}; \tag{35}$$

the constant C_∞ is determined numerically and m, the self-dual mass, is given by

$$m = ev\sqrt{2\lambda}, \tag{36}$$

reminding us that $ev\sqrt{2}$ is the mass scale of the usual Maxwell-Higgs model. The influence of the generalization in the mass scale explains the changes in the vortex-core size for large values of λ observed in Figures 1 and 2.

2.2. Maxwell-Higgs Compactons for $\lambda = \infty$. The numerical analysis shows that for sufficiently large but finite values of λ, the profiles are very similar to compacton solution ones; we have named them Maxwell-Higgs effective compact vortices. The true Maxwell-Higgs compacton is formed when $\lambda = \infty$ (see black line profiles in Figures 1 and 2).

3. The Born-Infeld-Higgs Case

The Born-Infeld theory [57, 58] is on nonlinear electrodynamics that was introduced to remove the divergence of the electron self-energy. It is the only completely exceptional nonlinear electrodynamics because to the absence of shock waves and birefringence in its propagation properties [59, 60]. Concerning topological defects in the Born-Infeld-Higgs model, vortex solutions were found in [61]. One generalization of BIH model was firstly done in [52] but no self-dual solutions were found. On the other hand, the self-dual or BPS topological vortex solutions were found in a generalized Born-Infeld-Higgs model introduced in [62].

The Lagrangian density of our $(2+1)$-dimensional theory is written as

$$\mathcal{L} = \beta^2(1-\mathcal{R}) + \omega(|\phi|)\left|D_\mu\phi\right|^2 - W(|\phi|), \tag{43}$$

where we go to consider the function $\omega(|\phi|)$ given by (16). We have also defined the following functions:

$$\mathcal{R} = \sqrt{1 + \frac{G(|\phi|)}{2\beta^2} F_{\mu\nu}F^{\mu\nu}}, \qquad W(|\phi|) = \beta^2\left[1 - V(|\phi|)\right]. \tag{44}$$

The generalized potential $W(|\phi|)$, a nonnegative function, inherits its structure from the function $V(|\phi|)$, which is restricted by the condition $0 < V(|\phi|) \le 1$, so $W(\phi) > 0$. The Born-Infeld parameter β provides modified dynamics for both scalar and gauge fields further enriching the family of possible models.

From the action (43) the gauge field equation of motion reads as

$$\partial_\nu\left(\frac{G}{\mathcal{R}}F^{\nu\mu}\right) = e\omega J^\mu. \tag{45}$$

We are interested in stationary solutions, so the Gauss law becomes

$$\partial_j\left(\frac{G}{\mathcal{R}}\partial_j A_0\right) = 2\omega e^2 A_0 |\phi|^2. \tag{46}$$

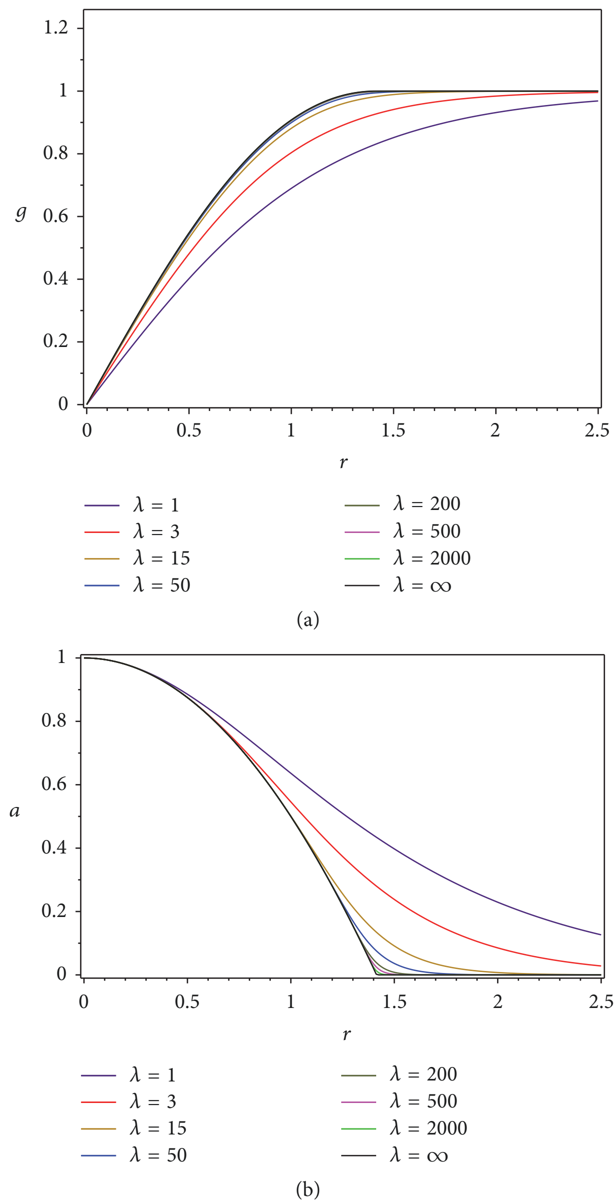


FIGURE 1: The profiles $g(r)$ (a) and $a(r)$ (b) come from generalized Maxwell-Higgs model (1) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual MH model and the true compacton solution is given by $\lambda = \infty$ (black lines).

Similarly to what happens in Maxwell-Higgs model, the field configurations are electrically neutral; therefore we go to work in the gauge $A_0 = 0$.

Consequently, at static regime, in the gauge $A_0 = 0$, the Ampère law is given by

$$\epsilon_{kj}\partial_j\left(\frac{G}{\mathscr{R}}B\right) - e\omega J_k = 0 \tag{47}$$

and the Higgs field equation reads as

$$\begin{aligned} 0 = {} & \omega\left(D_j D_j \phi\right) + \left(\partial_j w\right) D_j\phi - \frac{\partial\omega}{\partial\phi^*}\left|D_j\phi\right|^2 \\ & - \frac{B^2}{2\mathscr{R}}\frac{\partial G}{\partial\phi^*} - \frac{\partial W}{\partial\phi^*}. \end{aligned} \tag{48}$$

In the last two equations $\mathscr{R}$ reads as

$$\mathscr{R} = \left(1 + \frac{G}{\beta^2}B^2\right)^{1/2}. \tag{49}$$

The energy of the system, in static regime and in the gauge $A_0 = 0$, is given by

$$E = \int d^2x \left[\beta^2\left(\mathscr{R} - V\right) + \omega\left|D_k\phi\right|^2\right] \tag{50}$$

and will be nonnegative whenever the condition $\mathscr{R} \geq V$ is satisfied.

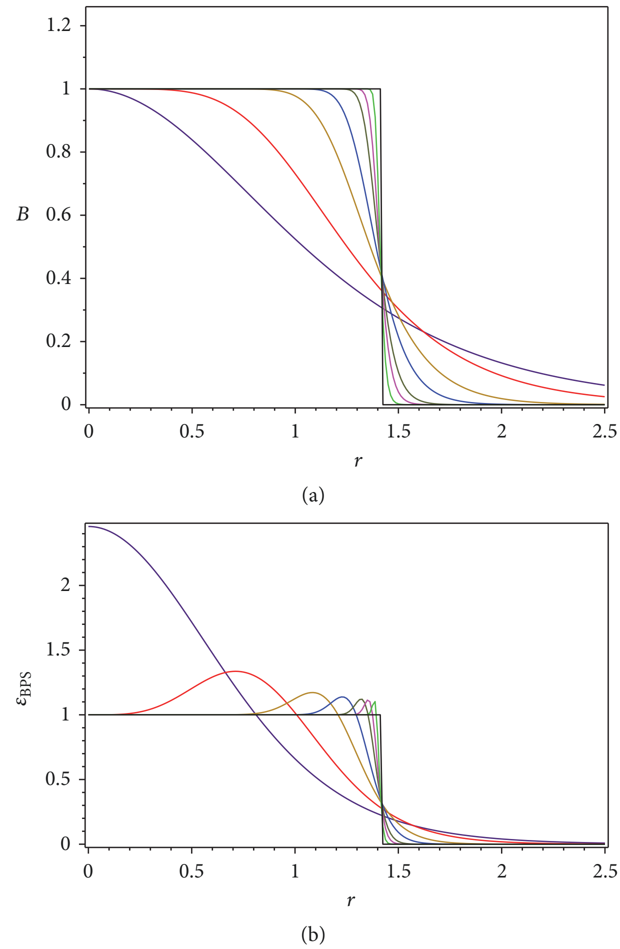


Figure 2: The magnetic field $B(r)$ (a) and the BPS energy density $\varepsilon_{\mathrm{BPS}}(r)$ (b) come from generalized Maxwell-Higgs model (1) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual MH model and the true compacton solution is given by $\lambda = \infty$ (black lines). The convention for the color of the lines is the same given in Figure 1.

To proceed with the BPS formalism, we use the identities (10) and (17) such that (50) becomes

$$E = 2\pi \int d^2x \left[\pm ev^2 B + \omega \left|D_{\pm}\phi\right|^2 \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i \left(\omega J_k\right) + \frac{\mathcal{R}}{2G}\left(\frac{G}{\mathcal{R}}B \mp \sqrt{2U^{(\lambda)}}\right)^2 + \beta^2\left(\mathcal{R} - V\right) - \frac{1}{2}\frac{GB^2}{\mathcal{R}} - \frac{\mathcal{R}}{G}U^{(\lambda)} \right]. \tag{51}$$

We have introduced the potential $U^{(\lambda)}(|\phi|)$ given by (22) with the aim of obtaining the term proportional to the magnetic field ev^2B.

The Bogomol'nyi procedure would be complete if we require the third row in (51) to be null, so we obtain

$$V + \frac{\mathcal{R}U^{(\lambda)}}{\beta^2 G} = \frac{1}{2}\mathcal{R} + \frac{1}{2\mathcal{R}}. \tag{52}$$

This provides a relation between the functions V, G, and $\mathcal{R}$. We here clarify that (52) is not arbitrary because, as we will observe later, in the BPS limit it becomes equivalent to the condition on the diagonal components of the energy-momentum tensor $T_{\mu\nu}$: $T_{11} + T_{22} = 0$, proposed by Schaposnik and Vega [63] to obtain self-dual configurations.

Then, the condition (52) allows writing the energy (50) in the Bogomol'nyi form,

$$E = \int d^2x \left\{ \omega \left|D_{\pm}\phi\right|^2 + \frac{\mathcal{R}}{2G}\left(\frac{G}{\mathcal{R}}B \mp \sqrt{2U^{(\lambda)}}\right)^2 \pm ev^2 B \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i\left(\omega J_k\right)\right\}. \tag{53}$$

Under suitable boundary conditions, the integration of the total derivative in (53) gives null contribution to the energy. Hence, it becomes clear that the energy possesses a lower bound

$$E \geq ev^2 \left|\Phi\right|, \tag{54}$$

with Φ being the total magnetic flux. Such a lower bound is saturated when the fields satisfy the BPS or self-dual equations

$$D_{\pm}\phi = 0, \tag{55}$$

$$\frac{G}{\mathscr{R}}B = \pm ev^2\left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right). \tag{56}$$

By using the BPS equations in (52) we compute the self-dual potential $V(|\phi|)$,

$$V = \frac{1}{\mathscr{R}} = \sqrt{1 - \frac{2U^{(\lambda)}}{\beta^2 G}}. \tag{57}$$

This way the second BPS equation (56) becomes

$$B = \pm\frac{ev^2}{GV}\left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right). \tag{58}$$

By using the BPS equations in (50) we find that the BPS energy density is given by

$$\varepsilon_{\mathrm{BPS}} = \frac{2U^{(\lambda)}}{GV} + \lambda\frac{|\phi|^{2\lambda-2}}{v^{2\lambda-2}}\left|D_k\phi\right|^2; \tag{59}$$

it will be positive-definite $\lambda > 0$.

3.1. Born-Infeld-Higgs Effective Compact Vortices for λ Finite. In [52] the existence of effective compact vortex solutions was explored but the self-dual ones were not found. In this section we show the existence of such self-dual effective compact solutions in BIH model. Without loss of generality, we perform the study by considering $G(|\phi|) = 1$ in the Lagrangian density (43).

The searching for vortex solutions is made by means of the vortex *Ansatz* introduced in (27). Thus, the BPS equations (55) and (56) read as

$$g' = \pm\frac{ag}{r},$$
$$-\frac{a'}{r} = \pm\frac{e^2v^2\left(1 - g^{2\lambda}\right)}{\sqrt{1 - e^2v^4\left(1 - g^{2\lambda}\right)^2/\beta^2}}. \tag{60}$$

The behavior of the profiles $g(r)$ and $a(r)$ when $r \to 0$ is determined by solving the self-dual equations (60), so we have

$$g(r) \approx C_n r^n + \cdots,$$
$$a(r) \approx n - \frac{e^2v^2\beta}{2\sqrt{\beta^2 - e^2v^4}}r^2 + \cdots. \tag{61}$$

Similarly, the behavior of the profiles for $r \to \infty$ is

$$g(r) \approx 1 - \frac{C_\infty}{\sqrt{r}}e^{-mr},$$
$$a(r) \approx mC_\infty\sqrt{r}e^{-mr}, \tag{62}$$

where m, the self-dual mass, is given by

$$m = ev\sqrt{2\lambda}; \tag{63}$$

it is exactly the same obtained for the generalized MH model analyzed in the previous section.

The BPS energy density for the self-dual vortices reads as

$$\varepsilon_{\mathrm{BPS}} = \frac{e^2v^4\left(1 - g^{2\lambda}\right)^2}{\sqrt{1 - e^2v^4\left(1 - g^{2\lambda}\right)^2/\beta^2}} + 2\lambda v^2 g^{2\lambda-2}\left(\frac{ag}{r}\right)^2; \tag{64}$$

it will be positive-definite and finite for $\lambda \geq 1$.

3.2. Born-Infeld-Higgs Compactons for $\lambda = \infty$. In the limit $\lambda \to \infty$, the BPS equations (60) read as

$$g' = \pm\frac{ag}{r},$$
$$-\frac{a'}{r} = \pm\frac{e^2v^2\Theta\left(1 - g\right)}{\sqrt{1 - e^2v^4/\beta^2}}, \tag{65}$$

with the profiles $g(r)$ and $a(r)$ satisfying the boundary conditions (39).

By solving the BPS compacton equations for the BIH model, we obtain also analytical solutions

$$g^{(\infty)}(r) = \left(\frac{r}{r_c}\right)^n \exp\left[\frac{n}{2}\left(1 - \frac{r^2}{r_c^2}\right)\right]\Theta\left(r_c - r\right) + \Theta\left(r - r_c\right),$$
$$a^{(\infty)}(r) = n\left(1 - \frac{r^2}{r_c^2}\right)\Theta\left(r_c - r\right), \tag{66}$$

where the radial distance r_c now is given by

$$r_c = \frac{\sqrt{2n}}{|ev|}\left(1 - \frac{e^2v^4}{\beta^2}\right)^{1/4}. \tag{67}$$

The magnetic field and BPS energy density profiles of the Born-Infeld-Higgs compacton are

$$B^{(\infty)}(r) = ev^2\left(1 - \frac{e^2v^4}{\beta^2}\right)^{-1/2}\Theta\left(r_c - r\right),$$
$$\varepsilon^{(\infty)}_{\mathrm{BPS}}(r) = e^2v^4\left(1 - \frac{e^2v^4}{\beta^2}\right)^{-1/2}\Theta\left(r_c - r\right). \tag{68}$$

In order to compute the numerical solutions we choose the upper signs in (60), $e = 1$, $v = 1$, $\beta = 3/\sqrt{5}$, and winding number $n = 1$. Similarly to the MH model, the effective compacton behavior appears for sufficiently large values of λ; see Figures 3 and 4. The true Born-Infeld-Higgs compactons arising for $\lambda = \infty$ also are depicted (see black line profiles) in Figures 3 and 4.

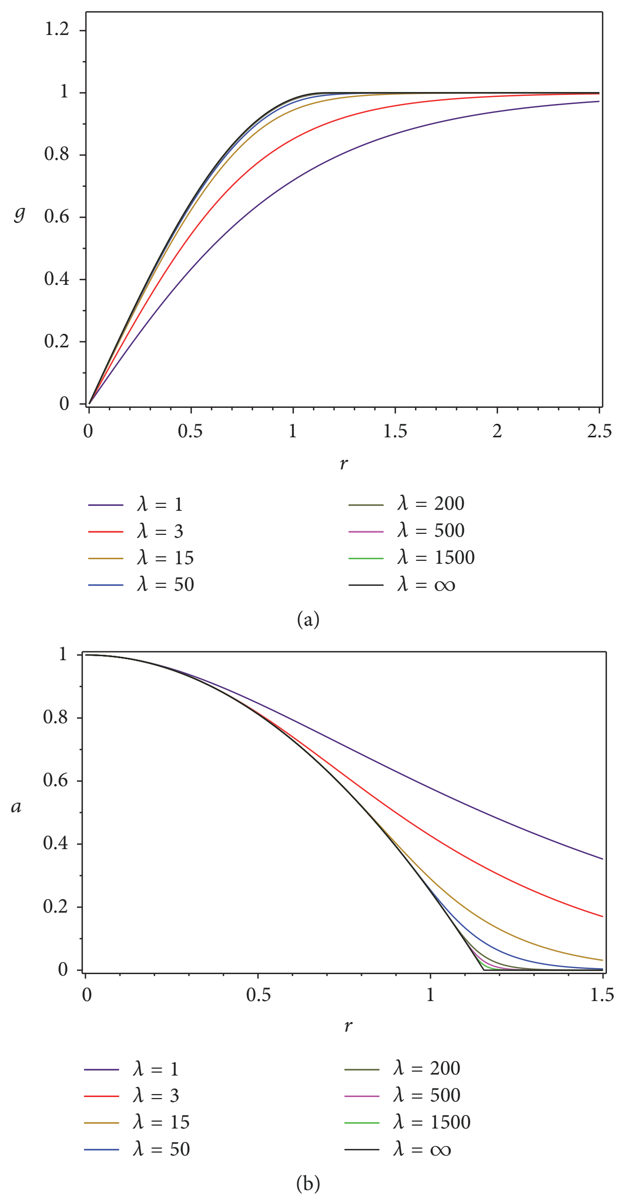


Figure 3: The profiles $g(r)$ (a) and $a(r)$ (b) come from the generalized Born-Infeld-Higgs model (43) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual BIH model and the true compacton solution is given by $\lambda = \infty$ (black lines).

4. The Chern-Simons-Higgs Case

In this section we apply the same formalism to construct self-dual solutions in the generalized Abelian Chern-Simons-Higgs model. Physics in two spatial dimensions is closely linked to CS theory, which contains theoretical novelties besides practical application in various phenomena of condensed matter, such as the physics of Anyons, and it is related to the fractional quantum Hall effect [64]. It can be found in extensive literature about CS theory that some of the pioneer papers concerning topological and nontopological solutions as well as relativistic and nonrelativistic models can be found in [65–74]. Also a close connection between CS theory and supersymmetry exists. This connection was firstly demonstrated in [75], where from $N = 2$ supersymmetric extension of CS model the specific potential for the Bogomol'nyi equations was found, which arise naturally.

The generalized Chern-Simons-Higgs model is described by the following Lagrangian density:

$$\mathscr{L} = \frac{\kappa}{4}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + \omega\left(|\phi|\right)\left|D_\mu\phi\right|^2 - V\left(|\phi|\right), \tag{69}$$

with the function $\omega(|\phi|)$ given by (16). The gauge field equation is to be

$$\frac{\kappa}{2}\epsilon^{\mu\alpha\beta}F_{\alpha\beta} - e\omega J^\mu = 0, \tag{70}$$

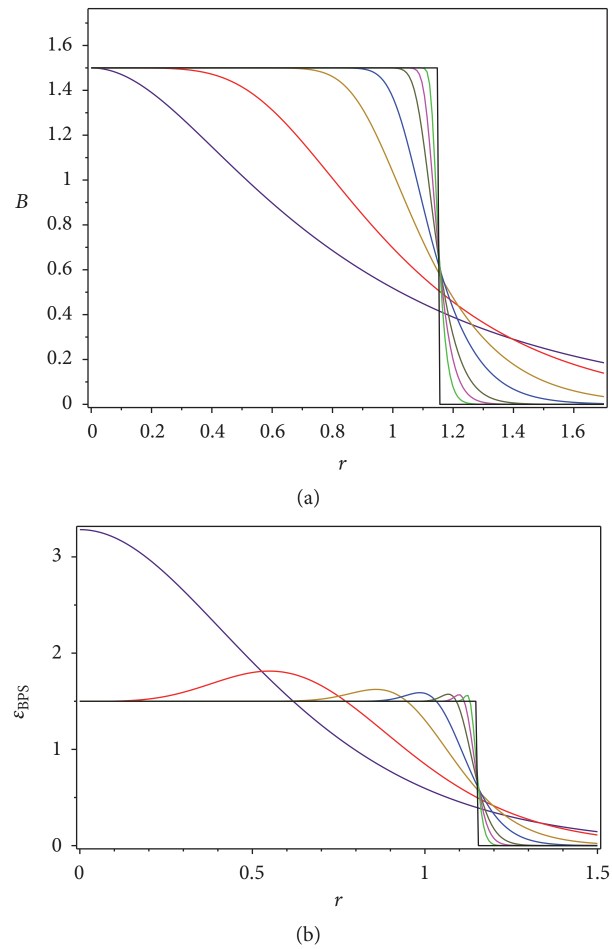


FIGURE 4: The magnetic field $B(r)$ (a) and the BPS energy density $\varepsilon_{\text{BPS}}(r)$ (b) come from the generalized Born-Infeld-Higgs model (43) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual BIH model and the true compacton solution is given by $\lambda = \infty$ (black lines). The convention for the color of the lines is the same given in Figure 3.

and the Gauss law reads as

$$\kappa B = e\omega J_0. \tag{71}$$

It is clear that the electric charge density is $e\omega J_0$ whose integration performed via the Gauss law gives

$$Q = \int d^2 x e\omega J_0 = \kappa \int d^2 x B = \kappa\Phi. \tag{72}$$

As such it happens in usual CSH model, the electric charge is nonnull and proportional to the magnetic flux, so the field configurations always will be electrically charged.

These are the stationary points of the energy which for the static field configuration reads as

$$E = \int d^2 x \cdot \left[-\kappa A_0 B - e^2 \omega A_0^2 |\phi|^2 + \omega |D_i\phi|^2 + V(|\phi|)\right]. \tag{73}$$

From the static Gauss law, we obtain the relation

$$A_0 = -\frac{\kappa}{2e^2}\frac{B}{\omega|\phi|^2}, \tag{74}$$

which substituted in (73) leads to the following expression for the energy:

$$E = \int d^2 x \left[\frac{\kappa^2}{4e^2}\frac{B^2}{\omega|\phi|^2} + \omega |D_i\phi|^2 + V(|\phi|)\right]. \tag{75}$$

We now use the identities (10) and (17) in (73) such that the energy becomes

$$E = \int d^2 x \left[\frac{\kappa^2}{4e^2}\frac{B^2}{\omega|\phi|^2} + V(|\phi|) + \omega |D_\pm\phi|^2 \pm ev^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}} B \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i(\omega J_k)\right]. \tag{76}$$

After some manipulation, the energy can be expressed almost in the Bogomol'nyi form

$$E = \int d^2x \left[\frac{\kappa^2}{4e^2} \frac{1}{|\phi|^2 \omega} \left(B \mp \frac{2e|\phi|}{\kappa} \sqrt{\omega V} \right)^2 + \omega |D_\pm \phi|^2 \pm \frac{1}{2\lambda} \epsilon_{ik} \partial_i (\omega J_k) \pm B \left(\frac{\kappa}{e|\phi|} \sqrt{\frac{V}{\omega}} + ev^2 \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \right) \right]. \quad (77)$$

We observe that the Bogomol'nyi procedure will be complete if the term multiplying the magnetic field is a constant; that is,

$$\frac{\kappa}{e|\phi|} \sqrt{\frac{V}{\omega}} + ev^2 \frac{|\phi|^{2\lambda}}{v^{2\lambda}} = ev^2, \quad (78)$$

such that a condition allows determining the self-dual potential $V(|\phi|)$ to be

$$V(|\phi|) = \lambda \frac{e^4 v^6}{\kappa^2} \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \right)^2. \quad (79)$$

We can see that, for $\lambda = 1$, the $|\phi|^6$-potential of the usual Chern-Simon-Higgs model is recovered.

Hence, the energy (77) reads as

$$E = \int d^2x \left\{ \pm ev^2 B \pm \frac{1}{2\lambda} \epsilon_{ik} \partial_i (\omega J_k) + \omega |D_\pm \phi|^2 + \frac{\kappa^2}{4e^2 |\phi|^2 \omega} \left[B \mp \frac{2e^2 v^2}{\kappa^2} \lambda \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \sqrt{2U^{(\lambda)}} \right]^2 \right\}. \quad (80)$$

We see that under appropriated boundary conditions the total derivative gives null contribution to the energy. Then, the energy is bounded below by a multiple of the magnetic flux magnitude

$$E \geq \pm ev^2 \int d^2x B = ev^2 |\Phi|. \quad (81)$$

This bound is saturated by fields satisfying the first-order Bogomol'nyi or self-dual equations [32, 33]

$$D_\pm \phi = 0, \qquad B = \pm \frac{2e^3 v^4}{\kappa^2} \lambda \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \right). \quad (82)$$

In order for the magnetic field to be nonsingular at origin, it is required that $\lambda > 0$.

By using the BPS equation in (75) we find that the energy density is given by

$$\varepsilon_{\text{BPS}} = 2\lambda \frac{e^4 v^6}{\kappa^2} \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \left(1 - \frac{|\phi|^{2\lambda}}{v^{2\lambda}} \right)^2 + \lambda \frac{|\phi|^{2\lambda-2}}{v^{2\lambda-2}} |D_k \phi|^2; \quad (83)$$

it will be positive-definite $\lambda > 0$.

4.1. Chern-Simons-Higgs Effective Compact Vortices for λ Finite. The BPS equations (82) read as

$$g' = \pm \frac{ag}{r}, \qquad -\frac{a'}{r} = \pm \frac{2e^4 v^4}{\kappa^2} \lambda g^{2\lambda} \left(1 - g^{2\lambda} \right). \quad (84)$$

The behavior of $g(r)$ and $a(r)$ near the boundaries can be easily determined by solving the self-dual equations (84) around the boundary values (28). Thus, for $r \to 0$, the profile functions behave as

$$g(r) \approx C_n r^n + \cdots, \qquad a(r) \approx n \mp \frac{\lambda e^4 v^4}{(n\lambda + 1)\kappa^2} (C_n)^{2\lambda} r^{2n\lambda+2} + \cdots, \quad (85)$$

where the constant $C_n > 0$ is determined only numerically.

On the other hand, when $r \to \infty$ they behave as

$$g(r) \approx 1 - \frac{C_\infty}{\sqrt{r}} e^{-mr}, \qquad a(r) \approx C_\infty m \sqrt{r} e^{-mr}, \quad (86)$$

with the constant C_∞ computed numerically and m being the self-dual mass

$$m = \frac{2\lambda e^2 v^2}{\kappa}. \quad (87)$$

We observe that for $\lambda = 1$, the mass becomes the same one of the usual self-dual Chern-Simons-Higgs bosons.

The BPS energy density for the self-dual vortices is given by

$$\varepsilon_{\text{BPS}} = 2\lambda \frac{e^4 v^6}{\kappa^2} g^{2\lambda} \left(1 - g^{2\lambda} \right)^2 + 2\lambda v^2 g^{2\lambda-2} \left(\frac{ag}{r} \right)^2, \quad (88)$$

and it will be positive-definite and finite for $\lambda \geq 1$.

4.2. Chern-Simons-Higgs Compactons for $\lambda = \infty$. From (84) we obtain the BPS equations for the Chern-Simons-Higgs compactons

$$g' = \pm \frac{ag}{r}, \qquad -\frac{a'}{r} = \frac{2e^4 v^4}{\kappa^2} \delta(1 - g), \quad (89)$$

with the boundary condition given in (39).

By solving the BPS compacton equations for $n > 0$, we obtain the analytic profiles

$$g^{(\infty)}(r) = \left(\frac{r}{r_c} \right)^n \Theta(r_c - r) + \Theta(r - r_c), \qquad a^{(\infty)}(r) = n\Theta(r_c - r). \quad (90)$$

The radial distance r_c is calculated to be

$$r_c = \frac{n|\kappa|}{e^2 v^2}. \quad (91)$$

The magnetic field and BPS energy density of the Chern-Simons-Higgs compacton are

$$B^{(\infty)}(r) = \frac{n}{e r_c}\delta(r_c - r), \qquad \varepsilon_{\mathrm{BPS}}^{(\infty)}(r) = \frac{n v^2}{r_c}\delta(r_c - r). \quad (92)$$

In order to compute the numerical solutions we choose the upper signs in (84), $e = 1$, $v = 1$, $\kappa = 1$, and winding number $n = 1$. From the numerical analysis, we can see the appearance of the effective compacton behavior for not very large values of λ such that it is explicitly shown in Figures 5 and 6. An interesting feature of the Chern-Simons-Higgs effective compact vortices is the enhancement of the ring shape (inclusive for $n = 1$), for increasing values of λ in the profiles for the magnetic field and the BPS energy density (see Figure 6). The analytic CSH compacton structures appearing in $\lambda = \infty$ are represented by the black line profiles in Figures 5 and 6.

5. The Maxwell-Chern-Simons-Higgs Case

Electrically charged vortices were first found in the Abelian Higgs model by Paul and Khare [76], where the Chern-Simons term was included in the usual Maxwell-Higgs action. This was an ingenious manner to avoid the temporal gauge, $A_0 \neq 0$, coupling the electric charge density to the magnetic field. This model has also been generalized by multiplying a dielectric (scalar) function in the Maxwell kinetic term [77, 78] yielding topological and nontopological solutions satisfying a Bogomol'nyi bound.

The generalized Maxwell-Chern-Simons-Higgs model is described by the following Lagrangian density:

$$\mathscr{L} = -\frac{G(|\phi|)}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} + \omega(|\phi|)\left|D_\mu\phi\right|^2 + \frac{G(|\phi|)}{2}\partial_\mu N\partial^\mu N - e^2\omega(|\phi|)N^2|\phi|^2 - V(|\phi|), \quad (93)$$

where the function $\omega(|\phi|)$ is given by (16). The gauge field equation reads as

$$\partial_\nu\left(\mathrm{G}F^{\nu\mu}\right) + \frac{\kappa}{2}\epsilon^{\mu\alpha\beta}F_{\alpha\beta} - e\omega J^\mu = 0; \quad (94)$$

the static Gauss law is

$$\partial_k\left(G\partial_k A_0\right) - \kappa B = 2e^2\omega A_0|\phi|^2. \quad (95)$$

Similarly, the equation of motion of the Higgs field is

$$0 = D_\mu\left(\omega D^\mu\phi\right) + \left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu N\partial^\mu N\right)\frac{\partial G}{\partial\phi^*} - \frac{\partial\omega}{\partial\phi^*}\left|D_\mu\phi\right|^2 + 2e^2\omega N|\phi|^2 + \frac{\partial V}{\partial\phi^*}. \quad (96)$$

In the same way of the previous models, we are interested in time-independent soliton solutions that ensure the finiteness of the action (69). These are the stationary points of the energy which for the static field configuration reads as

$$E = \int d^2x\left[\frac{G}{2}B^2 + \frac{G}{2}\left(\partial_j A_0\right)^2 + e^2\omega\left(A_0\right)^2|\phi|^2 + \omega\left|D_j\phi\right|^2 + \frac{G}{2}\left(\partial_j N\right)^2 + e^2\omega N^2|\phi|^2 + V(|\phi|)\right]. \quad (97)$$

To proceed, we use the identities (10) and (17) such that the energy becomes

$$E = \int d^2x\left[\frac{G}{2}B^2 + V(|\phi|) + \frac{G}{2}\left(\partial_j A_0\right)^2 + \frac{G}{2}\left(\partial_j N\right)^2 + e^2\omega\left(A_0\right)^2|\phi|^2 + e^2\omega N^2|\phi|^2 + \omega\left|D_\pm\phi\right|^2 \pm e v^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}}B \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i\left(\omega J_k\right)\right]. \quad (98)$$

After some algebraic manipulations, it can be expressed by

$$E = \int d^2x\left[\omega\left|D_\pm\phi\right|^2 + \frac{G}{2}\left(B \mp \sqrt{\frac{2V}{G}}\right)^2 + \frac{G}{2}\left(\partial_j A_0 \pm \partial_j N\right)^2 + e^2\omega|\phi|^2\left(A_0 \pm N\right)^2 \pm B\left(\sqrt{2GV} + e v^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}} + \kappa N\right) \mp \partial_j\left(NG\partial_j A_0\right) \pm \frac{1}{2\lambda}\epsilon_{ik}\partial_i\left(\omega J_k\right)\right]. \quad (99)$$

At this point, with the purpose for the total energy to have a lower bound proportional to the magnetic field, we chose the potential $V(|\phi|)$ to be

$$V(|\phi|) = \frac{1}{2G}\left(e v^2 - e v^2\frac{|\phi|^{2\lambda}}{v^{2\lambda}} - \kappa N\right)^2, \quad (100)$$

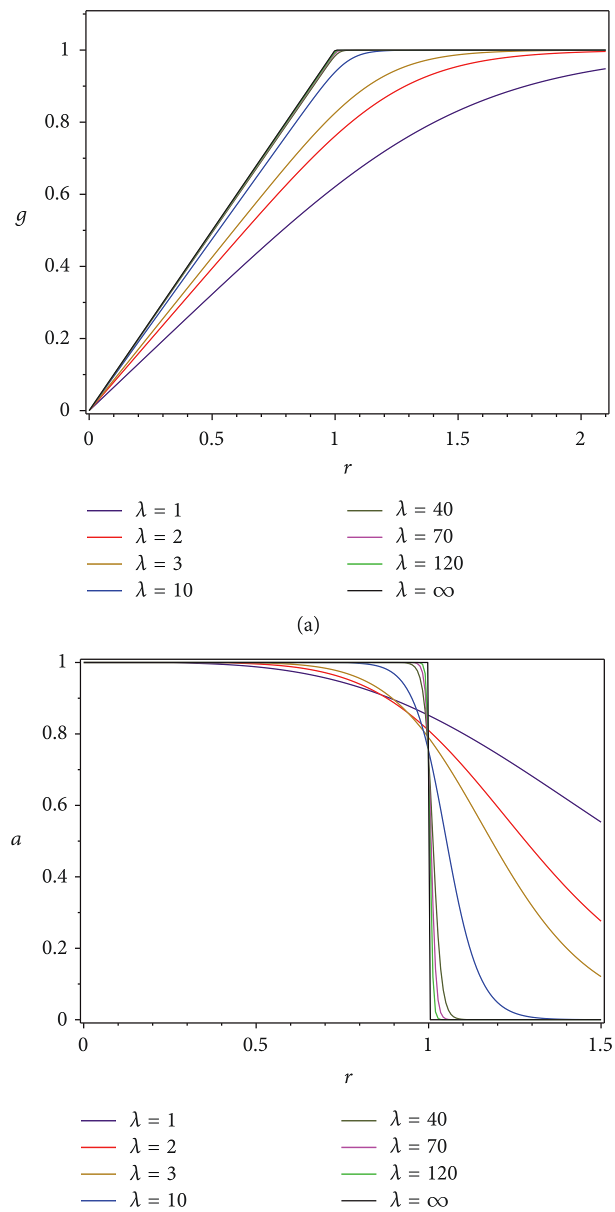


Figure 5: The profiles $g(r)$ (a) and $a(r)$ (b) come from the generalized Chern-Simons-Higgs model (69) with $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual CSH model and the true compacton solution is given by $\lambda = \infty$ (vertical black lines representing the δ-Dirac function).

where $C = \lambda v^{2-2\lambda}$ in order for the vacuum expectation value of the Higgs field to be $|\phi| = v$. Hence, the energy (99) reads as

$$E = \int d^2x \left\{ \pm e v^2 B + \omega \left| D_\pm \phi \right|^2 + \frac{G}{2} \left[B \mp \frac{1}{G} \left(e v^2 - e v^2 \frac{|\phi|^{2\lambda}}{v^{2\lambda}} - \kappa N \right) \right]^2 + \frac{G}{2} \left(\partial_j A_0 \pm \partial_j N \right)^2 + e^2 \omega |\phi|^2 \left(A_0 \pm N \right)^2 \mp \partial_j \left(N G \partial_j A_0 \right) \pm \frac{1}{2\lambda} \epsilon_{ik} \partial_i \left(\omega J_k \right) \right\}. \tag{101}$$

Under appropriated boundary conditions on the fields, the integration of the total derivatives becomes null; then the total energy is bounded below by a multiple of the magnetic flux magnitude

$$E \geq \pm e v^2 \int d^2x B = e v^2 |\Phi|. \tag{102}$$

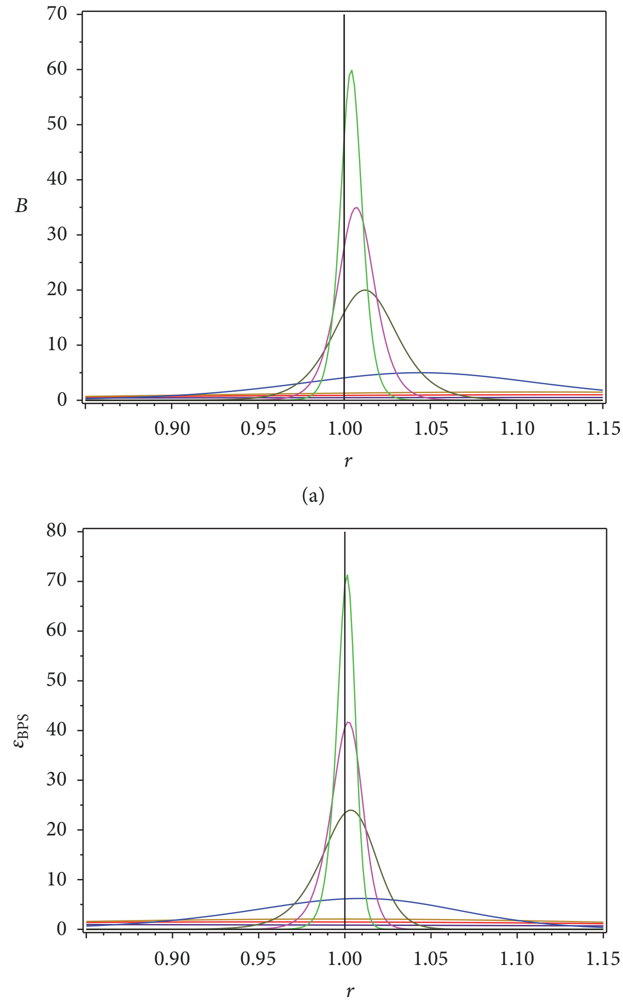


Figure 6: The magnetic field $B(r)$ (a) and the BPS energy density $\varepsilon_{\rm BPS}(r)$ (b) come from the generalized Chern-Simons-Higgs model (69) with $\omega(g)=\lambda g^{2\lambda-2}$. Observe that $\lambda=1$ (indigo lines) represents the usual CSH model and the true compacton solution is given by $\lambda=\infty$ (vertical black lines representing the δ-Dirac function). The convention for the color of the lines is the same given in Figure 5.

The lower bound (102) is saturated by fields satisfying the first-order Bogomol'nyi or self-dual equations [32, 33]

$$D_\pm\phi=0, \tag{103}$$

$$B=\pm\frac{ev^2}{G}\left(1-\frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right)\mp\kappa\frac{N}{G}, \tag{104}$$

$$\partial_j A_0\pm\partial_j N=0, \tag{105}$$

$$A_0\pm N=0. \tag{106}$$

The condition $N=\mp A_0$ saturates the two last equations (105) and (106) so the self-dual solutions are obtained by solving the self-dual equations

$$D_\pm\phi=0,$$

$$B=\pm\frac{ev^2}{G}\left(1-\frac{|\phi|^{2\lambda}}{v^{2\lambda}}\right)+\kappa\frac{A_0}{G}, \tag{107}$$

and the Gauss law

$$\partial_k\left(G\partial_k A_0\right)-\kappa B=2e^2v^2\lambda\frac{|\phi|^{2\lambda}}{v^{2\lambda}}A_0. \tag{108}$$

The BPS energy density is

$$\varepsilon_{\rm BPS}=GB^2+G\left(\partial_j A_0\right)^2+\lambda\frac{|\phi|^{2\lambda-2}}{v^{2\lambda-2}}\left|D_j\phi\right|^2 + 2e^2v^2\lambda\frac{|\phi|^{2\lambda}}{v^{2\lambda}}\left(A_0\right)^2. \tag{109}$$

5.1. Maxwell-Chern-Simons-Higgs Effective Compact Vortices for λ Finite. By considering that $G(|\phi|) = 1$ and using *Ansatz* (27), the BPS equations (107) read as

$$g' = \pm \frac{ag}{r},$$
$$-\frac{a'}{r} = \pm e^2 v^2 \left(1 - g^{2\lambda}\right) + e\kappa A_0 \tag{110}$$

and the Gauss law (108) becomes

$$\frac{1}{r}\left(rA_0'\right)' - \kappa B = 2\lambda e^2 v^2 g^{2\lambda} A_0. \tag{111}$$

We analyze the behavior of the profiles $g(r)$ and $a(r)$ and $A_0(r)$ at boundaries. This way, for $r \to 0$, the profiles behave as

$$g(r) \approx C_n r^n + \cdots,$$
$$a(r) \approx n - \frac{e\left[ev^2 + \kappa A_0(0)\right]}{2} r^2 + \cdots, \tag{112}$$
$$A_0(r) \approx A_0(0) + \frac{\kappa\left[ev^2 + \kappa A_0(0)\right]}{4} r^2 + \cdots,$$

with the constants $C_n > 0$ and $A_0(0)$ being determined numerically for every n.

The behavior at the origin for $A_0(r)$ provides the boundary condition

$$A_0'(0) = 0. \tag{113}$$

On the other hand, for large values of r ($r \to \infty$), they have the Abrikosov-Nielsen-Olesen behavior,

$$g(r) \approx 1 - \frac{C_\infty}{\sqrt{r}} e^{-mr}, \tag{114}$$

$$a(r) \approx C_\infty m \sqrt{r} e^{-mr}, \tag{115}$$

$$A_0(r) \approx -\frac{|\kappa|}{\kappa}\frac{m}{e}\frac{C_\infty}{\sqrt{r}} e^{-mr}, \tag{116}$$

with the constant C_∞ being computed numerically and m being the self-dual mass

$$m = \frac{1}{2}\sqrt{\kappa^2 + 8\lambda e^2 v^2} - \frac{|\kappa|}{2}; \tag{117}$$

for $\lambda = 1$, we recover self-dual mass of the usual Maxwell-Chern-Simons-Higgs bosons.

In this way we obtain from (116) the boundary condition for $A_0(r)$ when $r \to \infty$:

$$A_0(\infty) = 0. \tag{118}$$

The BPS energy density of the self-dual vortices reads as

$$\varepsilon_{\text{BPS}} = B^2 + \left(A_0'\right)^2 + 2v^2\lambda g^{2\lambda-2}\left(\frac{ag}{r}\right)^2 + 2e^2v^2\lambda g^{2\lambda}\left(A_0\right)^2, \tag{119}$$

being positive-definite and finite for $\lambda \geq 1$.

5.2. Maxwell-Chern-Simons-Higgs Compactons for $\lambda = \infty$. From (110), the limit $\lambda \to \infty$ provides the BPS equation for the compacton configurations

$$g' = \pm \frac{ag}{r},$$
$$-\frac{a'}{r} = \pm e^2 v^2 \Theta\left(1 - g\right) + e\kappa A_0. \tag{120}$$

The compacton Gauss law obtained from (111) becomes

$$A_0' + \frac{\kappa}{e}\frac{(a - n)}{r} = 0. \tag{121}$$

The compacton boundary conditions satisfied by the profiles $g(r)$, $a(r)$, and $A_0(r)$ are

$$\begin{aligned} g(0) &= 0, \\ a(0) &= n, \\ A_0'(0) &= 0, \\ g(r) &= 1, \\ a(r) &= 0, \\ A_0(r) &= 0, \\ r_c \leq r &< \infty. \end{aligned} \tag{122}$$

The radial distance $r_c < \infty$ is the value where the profile $g(r)$ reaches the vacuum value and the gauge field profile $a(r)$ and scalar potential $A_0(r)$ become null.

The system is solved analytically to be

$$\begin{aligned} g^{(\infty)}(r) &= \left(\frac{r}{r_c}\right)^n \exp\left[\frac{e^2v^2}{\kappa^2}\left(1 - \frac{I(0,\kappa r)}{I(0,\kappa r_c)}\right)\right]\Theta\left(r_c - r\right) + \Theta\left(r - r_c\right), \\ a^{(\infty)}(r) &= n\left(1 - \frac{rI(1,\kappa r)}{r_c I(1,\kappa r_c)}\right)\Theta\left(r_c - r\right), \\ A_0^{(\infty)}(r) &= \frac{ev^2}{\kappa}\left(-1 + \frac{I(0,\kappa r)}{I(0,\kappa r_c)}\right)\Theta\left(r_c - r\right). \end{aligned} \tag{123}$$

The radial distance r_c is computed from

$$I\left(0,\kappa r_c\right) = r_c \frac{e^2v^2}{n\kappa} I\left(1,\kappa r_c\right), \tag{124}$$

where the function $I(\nu, x)$ is the modified Bessel function of the first kind and order ν.

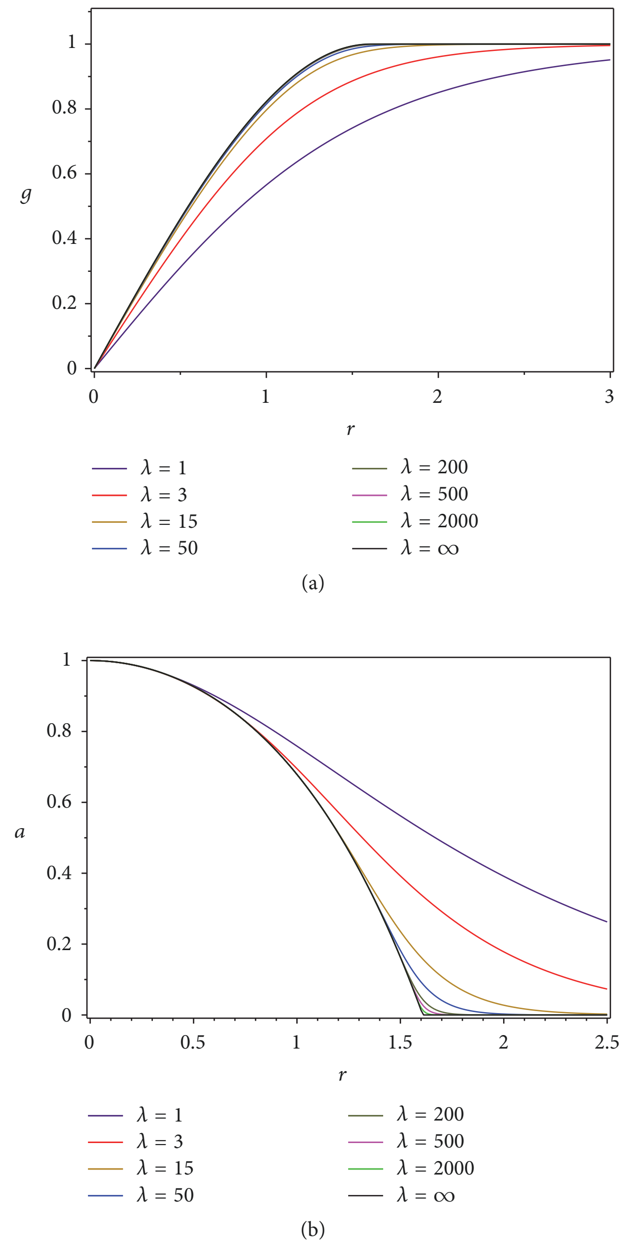


Figure 7: The profiles $g(r)$ (a) and $a(r)$ (b) come from generalized Maxwell-Chern-Simons-Higgs model (93) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual MCSH model and the true compacton solution is given by $\lambda = \infty$ (black lines).

The magnetic field and BPS energy density of the Maxwell-Chern-Simons-Higgs compacton are

$$B^{(\infty)}(r) = ev^2 \frac{I(0,\kappa r)}{I(0,\kappa r_c)} \Theta(r_c - r),$$

$$\varepsilon_{\text{BPS}}^{(\infty)}(r) = e^2 v^4 \left(\frac{I(0,\kappa r)}{I(0,\kappa r_c)}\right)^2 \Theta(r_c - r) + e^2 v^4 \left(\frac{I(1,\kappa r)}{I(0,\kappa r_c)}\right)^2 \Theta(r_c - r). \quad (125)$$

In order to compute the numerical solutions we choose the upper signs in (110), $e = 1$, $v = 1$, $\kappa = -1$, and winding number $n = 1$. The profiles for the Higgs and gauge fields are given in Figure 7; the correspondent ones for the scalar potential and for the electric field are depicted in Figure 8. We can note again that an effective compact topological defect is formed for large values of λ. This feature can be seen from the magnetic field and BPS energy density profiles in Figure 9. Similarly to the previous studied models, the analytic MCSH compactons are formed for $\lambda = \infty$; they are represented by the solid black lines in Figures 7, 8, and 9.

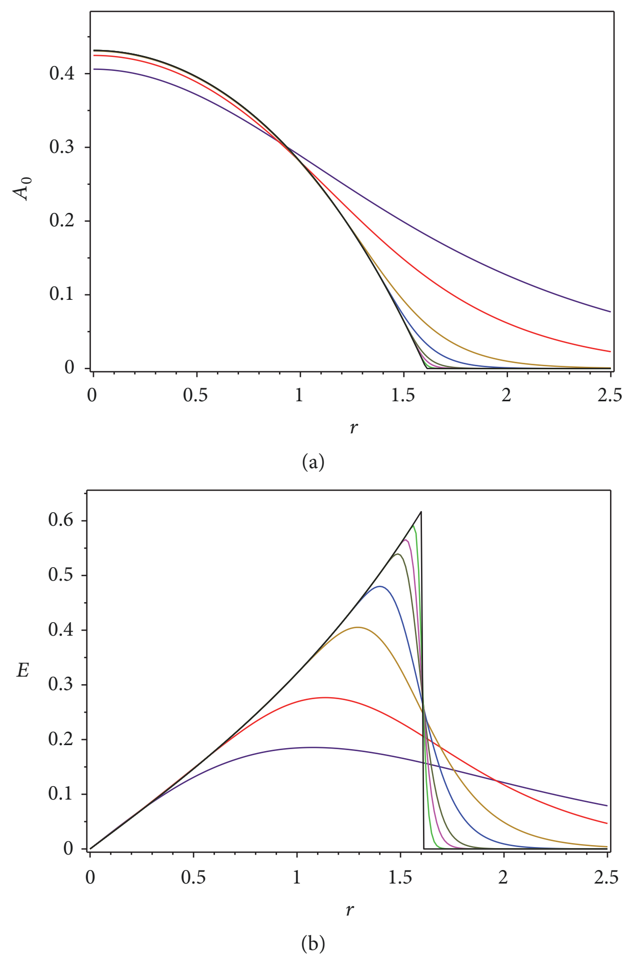


FIGURE 8: The scalar potential $A_0(r)$ (a) and the electric field $E(r)$ (b) come from generalized Maxwell-Chern-Simons-Higgs model (93) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual MCSH model and the true compacton solution is given by $\lambda = \infty$ (black lines). The convention for the color of the lines is the same given in Figure 7.

6. Remarks and Conclusions

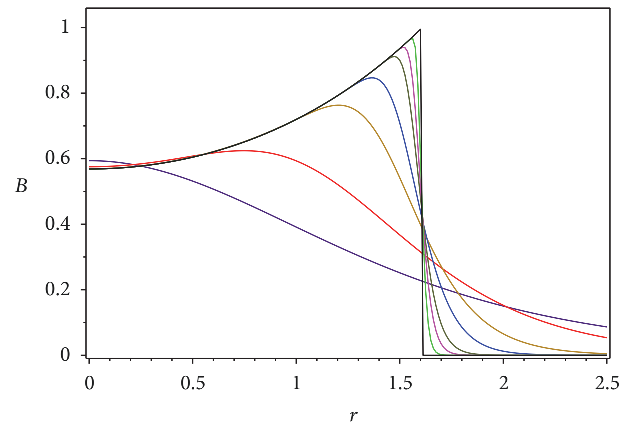
We have found some generalized Abelian Higgs models whose BPS or self-dual equations give origin to both effective compact solutions and true compacton configurations. Our goal was obtained by means of a consistent implementation of the BPS formalism which besides providing the self-dual or BPS equations has also allowed finding the explicit form of the generalizing function $\omega(|\phi|)$ (see (16)) which is parameterized by the positive parameter λ. Such a parameter determines explicitly new families of self-dual potential for every model and consequently characterizes their self-dual configurations. We draw attention to the importance to obtain self-dual effective compact and analyze true compacton configurations in Abelian Higgs. These models enhance the space of self-dual solutions which probably will imply interesting applications in physics and mathematics, for example, the construction of the respective supersymmetric extensions [79–82].

For every model, we have studied the vortex solutions arising from the respective self-dual equations. The numerical analysis has shown that, for sufficiently large values of λ, the profiles (of the Higgs field, gauge field, magnetic field, and BPS energy density) are very similar to the ones of an effective compacton solution but still preserve a tail in their asymptotic decay. For every model, we have also analyzed the limit $\lambda \rightarrow \infty$ for arbitrary winding number n. Our analysis has shown that, for $\lambda = \infty$, the analytical compacton structures arise in all models (see black line profiles in all figures in the manuscript).

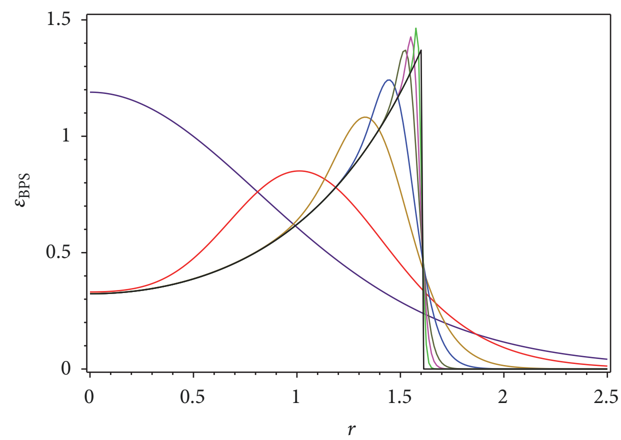
Finally, we are considering the interesting challenge of looking for effective compact structures in gauge field models which engender monopoles or skyrmions, for example. Advances in this direction will be reported elsewhere.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.



(a)



(b)

FIGURE 9: The magnetic field $B(r)$ (a) and the BPS energy density $\varepsilon_{\mathrm{BPS}}(r)$ (b) come from generalized Maxwell-Chern-Simons-Higgs model (93) with $G(g) = 1$ and $\omega(g) = \lambda g^{2\lambda-2}$. Observe that $\lambda = 1$ (indigo lines) represents the usual MCSH model and the true compacton solution is given by $\lambda = \infty$ (black lines). The convention for the color of the lines is the same given in Figure 7.

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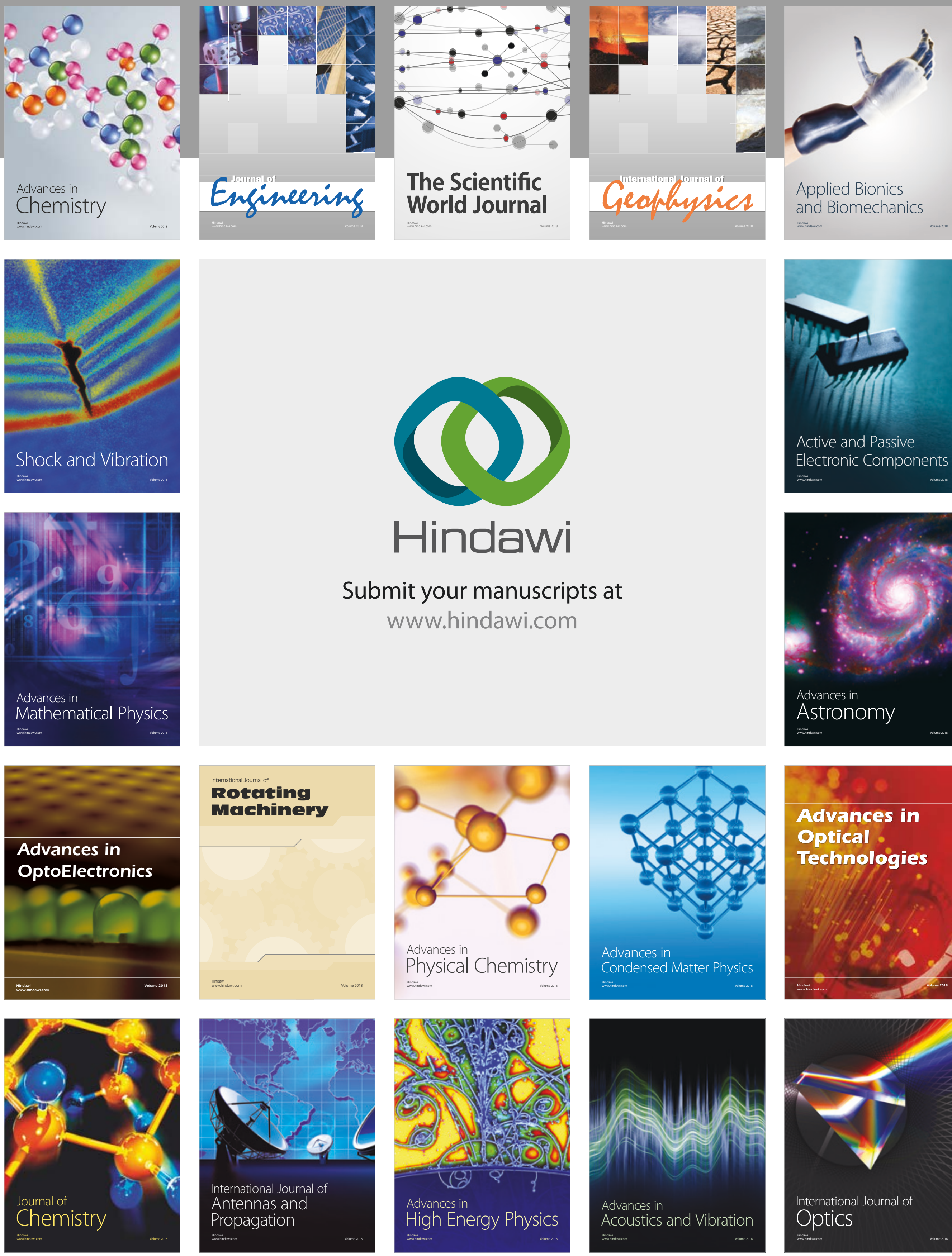
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