Very efficient correlator for loosely synchronised codes

M.C. Pérez, J. Ureña, C. De Marziani, A. Hernández, J.J. García and F.J. Álvarez

A new approach for the efficient correlation of loosely synchronised (LS) codes, when they are generated from Golay pairs, is presented. By employing this new approach, the total number of operations to be performed in the correlation is notably decreased, if compared with existing proposals based on classical efficient Golay correlators or straightforward methods. The proposed correlator can be easily implemented on configurable hardware devices to achieve real-time operation even with long codes.

Introduction: Loosely synchronised (LS) codes have been successfully used in quasi-synchronised CDMA applications to reduce multiple access interference (MAI) and inter-symbol interference (ISI) [1]. This reduction in interferences is due to the zero correlation zone (ZCZ) that LS codes exhibit in their aperiodic autocorrelation and cross-correlation functions.

LS codes can be obtained from Golay pairs and then detected by means of a correlator based on the well-known efficient Golay correlator (EGC) [2, 3]. Such a correlator, called efficient LS correlator (ELSC), has been presented in [4] and reduces the number of multiplications and additions to be carried out, compared with those required by standard straightforward correlators. In this Letter, a modification is introduced in the ELSC to minimise the total number of operations and memory elements needed in the pulse compression. Thus, very long LS code pulse compression can be easily achieved.

LS codes: A set of K LS codes with length $L\{G = g_k[l]; 0 \le k \le K-1; 0 \le l \le L-1\}$ is a set of codes composed of three types of elements $g_k[l] \in \{0, +1, -1\}$, which have the property that their aperiodic correlation functions equal zero in a zone W_0 around the origin. They can be generated from two orthogonal Golay pairs (c_0,s_0) and (c_1,s_1) of length N and elements $\{+1, -1\}$ as follows:

$$G_k[z] = \sum_{i=0}^{K/2-1} h_{k,i}\, z^{-i \cdot N}\left[C_{\pi_i}[z] + z^{-(K/2N+W_0)} S_{\pi_i}[z]\right] \tag{1}$$

where $h_{k,i} \in \{+1, -1\}$ are the elements of a $(\frac{K}{2} \times \frac{K}{2})$ Hadamard matrix; the vector $\Pi = [\pi_0, \pi_1, \cdots, \pi_{K/2-1}] \in \{0, 1\}$ denotes a binary expansion of an arbitrary integer n, $0 \le n \le 2^{K/2}$, and $W_0 \le N-1$. Equation (2) shows that the first $K/2$ codes of a LS family basically consists of two mutually orthogonal (MO) Golay pairs linked in the order established by Π, and with the polarity indicated by the coefficients $h_{k,i}$ of the Hadamard matrix. Note that a set of W_0 zeros appears in the centre of the LS code (separating sequences c_i from sequences s_i). The codes $g_{K/2}, \ldots, g_{K-1}$ are obtained when Π is replaced by its complement $\Pi^* = [\pi_0^*, \pi_1^* \cdots, \pi_{K/2-1}^*]$; $\pi_k^* = \pi_k + 1(mod\ 2); 0 \le k \le K/2-1$. In this way the total length of the LS code is given by $L = K^*N + W_0$.

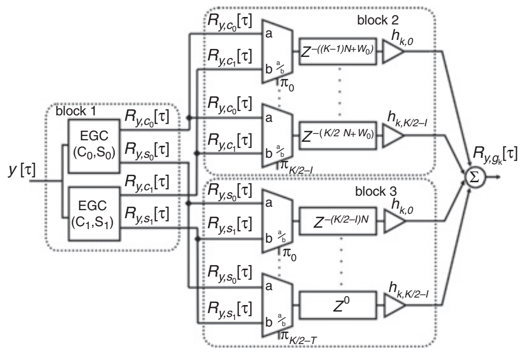

Fig. 1 *Block diagram of efficient correlator (ELSC)*

Efficient LS correlator (ELSC): The efficient correlator proposed in [4] starts with two EGCs, as is shown in Fig. 1. One carries out the correlation of the input signal $y[\tau]$ with the sequences (c_0, s_0) of the first Golay pair, providing the outputs $R_{y,c_0}[\tau]$ and $R_{y,s_0}[\tau]$. The other EGC performs the correlation with the sequences of the orthogonal pair (c_1, s_1), and gives the outputs $R_{y,c_1}[\tau]$ and $R_{y,s_1}[\tau]$. Fig. 2*a* summarises the operations carried out in each EGC. A set of multiplexers governed by Π, or Π^*, determines the delays to be applied to the previous correlation outputs. The $K/2$ multiplexers in block 2 apply the delays $(K-i-1)\,N + W_0$, $(0 \le i \le K/2-1)$ to $R_{y,c_{\pi_i}}[\tau]$; whereas the $K/2$ multiplexers in block 3 apply the delays $(K/2-i-1)\,N$ to $R_{y,s_{\pi_i}}[\tau]$. The delayed outputs are multiplied by the corresponding element $h_{k,i}$ of the Hadamard matrix, and finally they are added according to (2) to obtain the correlation result:

$$\begin{aligned} R_{Y,G_k}[z] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K/2-i-1)\,N} \\ &\quad \times [z^{-(K/2N+W_0)} \times R_{Y,C_{\pi_i}}[z] + R_{Y,S_{\pi_i}}[z]] \\ R_{Y,G_{k+K/2}}[z] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K/2-i-1)\,N} \\ &\quad \times [z^{-(K/2N+W_0)} \times R_{Y,C_{\pi_i}}[z] + R_{Y,S_{\pi_i}}[z]] \end{aligned} \tag{2}$$

Very efficient LS correlator (VELSC): The proposed algorithm takes advantage of the orthogonality of the Golay pairs from which LS codes are derived. In [4], two EGCs are used to obtain the correlation of the input signal with each Golay pair. Since (c_0, s_0) and (c_1 s_1) are MO, both EGCs only differ in the coefficient w_1 (see Fig. 2*a*), where $\{w_1, w_2, \ldots, w_T\} \in \{+1, -1\}$, $T \in Z^+$ and $N = 2^T$. The architecture of the EGC can be modified as is indicated in Fig. 2*b* [5]. Note that the delays $D_i\{D_1, D_2, \ldots, D_T = 2^{T-1}, 2^{T-2}, \ldots, 2^0\}$ and the coefficients w_i appear reverted. An adder is included in the last stage of the filter to obtain the output $a[\tau]$ which is the sum of correlation functions with c_0 and s_0 (corresponding to coefficient w_1). Also, a subtracter is included in the last stage of the filter to obtain the output $b[\tau]$ which is the sum of correlation functions with c_1 and s_1 (corresponding to coefficient $-w_1$). When the inputs of the systems are $x_0[\tau] = c_0[\tau] + c_1[\tau]$ and $y_0[\tau] = s_0[\tau] + s_1[\tau]$, the outputs are $a[\tau] = R_{c_0,c_0}[\tau] + R_{c_1,c_0}[\tau] + R_{s_0,s_0}[\tau] + R_{s_1,s_0}[\tau] = R_{c_0,c_0}[\tau] + R_{s_0,s_0}[\tau]$ and $b[\tau] = R_{c_0,c_1}[\tau] + R_{c_1,c_1}[\tau] + R_{s_0,s_1}[\tau] + R_{s_1,s_1}[\tau] = R_{c_1,c_1}[\tau] + R_{s_1,s_1}[\tau]$, as ($c_0$, s_0) and (c_1, s_1) are MO. Considering that, and restricting the values of Π and Π^* to $\Pi = [0,\ 0,\ \ldots,\ 0]$ and $\Pi^* = [1,\ 1,\ \ldots,\ 1]$, it is possible to obtain a very efficient architecture to correlate LS codes as follows (see Fig. 3):

The input signal $y[\tau]$ is added or subtracted with different shifts in block 1 and block 2. Note that the delays applied to the input signal in these blocks correspond to those applied to $R_{y,c_{\pi_i}}[\tau]$ and $R_{y,s_{\pi_i}}[\tau]$, respectively, in the ELSC depicted in Fig. 1. Then, the delayed versions of $y[\tau]$ are multiplied by the corresponding element $h_{k,i}$, used in the generation of the LS code g_k and the resulting values are added, obtaining

$$\begin{aligned} x_0[\tau] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K-i-1)N+W_0} \times y[\tau] \\ y_0[\tau] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K/2-i-1)N} \times y[\tau] \end{aligned} \tag{3}$$

In block 3 the resulting signals x_0 and y_0 are processed with the aid of the filter described in Fig. 2*b*. The final correlation result $R_{y,g_k}[\tau]$, see (4), is obtained after block 4. The operation carried out in that block 4 will be an addition ($m = 1$) if $\Pi = [0,\ 0, \ldots,\ 0]$ has been used in the generation of the LS code g_k $(0 \le k \le K/2-1)$, or a subtraction ($m = -1$) if $\Pi^* = [1,\ 1,\ \ldots,\ 1]$ was used for the generation of the code g_k $(K/2 \le k \le K-1)$.

$$\begin{aligned} R_{R,G_k}[z] = R_{X_0,C_0}[z] + R_{Y_0,S_0}[z] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K/2-i-1)\,N} \\ &\quad \times [z^{-((K/2)^*N+W_0)} \times R_{Y,C_0}[z] + R_{Y,S_0}[z]] \\ R_{R,G_{k,K/2}}[z] = R_{X_0,C_1}[z] + R_{Y_0,S_1}[z] &= \sum_{i=0}^{K/2-1} h_{k,i} \times z^{-(K/2-i-1)\,N} \\ &\quad \times [z^{-((K/2)^*N+W_0)} \times R_{Y,C_1}[z] + R_{Y,S_1}[z]] \end{aligned} \tag{4}$$

Restricting the values of Π and Π^* affects neither to the number K of codes in the LS family nor to the size of the ZCZ, and significantly reduces the number of operations and memory required for the detection of a LS code. For the sake of clarity, in Fig. 3 the different delays in block 1 and 2 appear separated, but in a real implementation only the bigger delay memory $z^{-((K-1)N+W_0)}$ is needed, as the input signal is always the same. A comparison among the number of multiplications, additions and memory positions required by a straightforward implementation, the ELSC and the proposed VELSC, can be found in Table 1. In practice, all the implementations can be achieved without any multiplication, since the coefficients w_i and $h_{k,i}$ are $\{+1, -1\}$.

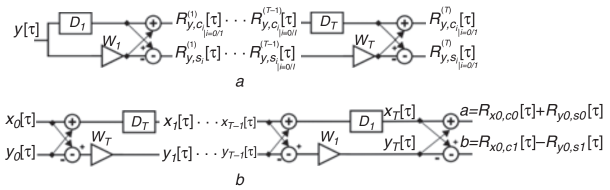

Fig. 2 *Efficient correlator structures for Golay pairs*

a Efficient Golay correlator
b Simultaneous correlation of two MO Golay pairs using reverted version of EGC

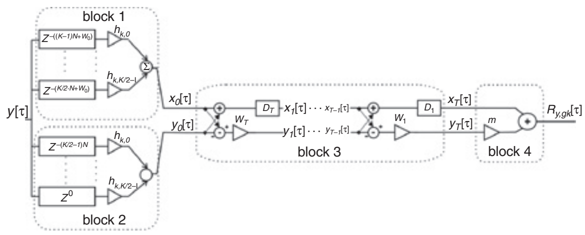

Fig. 3 *Block diagram of proposed very efficient correlator (VELSC)*

Table 1: Resources to obtain correlation function of LS codes from different methods, against length N of initial Golay pairs, family size K, and length of ZCZ $(2W_0 + 1)$

	Multipliers	Adders	Memory positions
Straightforward	$K{\cdot}N$	$K{\cdot}N - 1$	$K{\cdot}N + W_0$
ELSC	$2{\cdot}log_2(N) + K$	$4{\cdot}log_2(N) + K - 1$	$\frac{K}{2} \begin{matrix} 2\cdot(N-1)+ \\ (K\cdot N - N + W_0) \end{matrix}$
VELSC	$log_2(N) + K$	$2{\cdot}log_2(N) + K - 1$	$K{\cdot}N + W_0 - 1$

As an example, in the case of a family of $K = 4$ LS codes, $L = 1279$ and $W_0 = 63$ (and then generated from Golay pairs of length $N = 256$), the multipliers, adders and memory positions demanded by the different implementations are, respectively: straightforward (1024, 1023, 1279); ELSC (20, 35, 2556); and VELSC (12, 19, 1278).

Conclusions: A new architecture for the correlation of LS codes generated from Golay pairs is presented. This new architecture demands less computations than previous works based on straightforward implementations or efficient Golay correlators. Hence, the use of long codes and real-time operation can be most feasible. Furthermore, this optimised correlator can be easily implemented in configurable devices to be used in quasi-synchronised CDMA applications.

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