Optimized Correlator for LS Codes-Based CDMA Systems

Mª Carmen Pérez, *Student Member, IEEE,* Jesús Ureña, *Member, IEEE,* Álvaro Hernández, *Member, IEEE,* Carlos De Marziani, J. Jesús García, *Member, IEEE,* and Ana Jiménez

***Abstract*—Loosely Synchronized codes, which have gained considerable interest in CDMA-based wireless communications and sensory systems, can be constructed from a set of mutually orthogonal complementary sets of sequences. In this paper a new architecture is provided for the correlation of such codes. It demands less memory and number of operations than previous ones based on efficient correlators for complementary set of sequences, and can be easily implemented in programmable devices due to its regular structure.**

***Index Terms*—Code division multi-access, correlators, pulse compression methods, sequences.**

I. Introduction

THE aperiodic correlation functions of Loosely Synchronized (LS) codes exhibit zero values around the in-phase shift. Thus, the inter-symbol and multiple-access interferences can be significantly mitigated, provided that the relative delays between the receptions do not exceed the size of this zero correlation zone (ZCZ). These codes can be obtained from Complementary Sets of Sequences (CSSs) as is indicated in [1], and then be correlated by using efficient methods that reduce the number of operations to be carried out in comparison with straightforward implementations. Hence, real-time operation can be more feasible for quasi-synchronous CDMA applications, such as space communications or local positioning systems, that demand long sequence pulse compression to mitigate the noise or the multiple-access interference [2]. A previous technique, called Efficient LS Correlator (ELSC), was proposed in [3] and it achieves this reduction in the number of operations at the expense of a significant increase of the memory requirements. In this paper, the architecture of the ELSC is modified to obtain a new filter that minimizes the number of multiplications and additions without penalizing the memory requirements (the memory needs of both the straightforward and the proposed implementation grow with N^{M+2}, whereas in the ELSC grow with N^{M+4}, where N is the number of orthogonal CSSs of length $N^M, M \in \mathbb{Z}^+$, used to construct the LS codes).

Manuscript received October 14, 2010. The associate editor coordinating the review of this letter and approving it for publication was C. Sacchi.

This work has been possible thanks to the the support of the Spanish Ministry of Science and Technology (TIN2009-14114-C04-01 and FOM P13/08).

M. C. Pérez, J. Ureña, A. Hernández, J. J. García, and A. Jiménez are with the Department of Electronics, University of Alcalá, Escuela Politécnica, Campus Universitario s/n, 28805, Alcalá de Henares, Madrid, Spain (e-mail: {carmen, urena, alvaro, jesus, ajimenez}@depeca.uah.es).

C. De Marziani is with the Department of Electronics Engineering, National University of Patagonia San Juan Bosco (Argentina), Ciudad Universitaria, Ruta Provincial nº1 s/n, Km 4, 9000 Comodoro Rivadavia (Chubut), Argentina (e-mail: marziani@unpata.edu.ar).

Digital Object Identifier 10.1109/LCOMM.2010.121310.101952

II. LS codes

A set of K LS codes with length L $\{G = g_k[l]; 0 \leq k \leq K-1; 0 \leq l \leq L-1\}$ is a set of ternary sequences which have the property that their aperiodic correlation functions equal zero in a zone W around the origin and whose elements $g_k[l] \in \{-1, 0, +1\}$. In [1] a general method for constructing LS codes from N orthogonal CSSs is given. This method is more general than previous ones [4] based on Golay pairs, and can be expressed as follows:

$$G_{n,m}(z) = \sum_{i=0}^{N-1} h_{m,i} \cdot z^{-iL_0} \cdot \left[\sum_{j=0}^{N-1} z^{-j(NL_0+W)} S_{\pi_{n,i},j}(z) \right] \tag{1}$$

where $G_{n,m}(z)$ represent the Z transforms of $\{g_{n,m}(\tau);\ 0 \leq n, m \leq N-1\}$; $S_{i,j}(z)$ are the Z transforms of the N sequences $\{s_{i,j}[l]; 0 \leq j \leq N-1; 0 \leq l \leq L_0-1\}$ of length $L_0 = N^M$ that compose each set of N orthogonal CSSs $\{S_i; 0 \leq i \leq N-1\}$, where $s_{i,j} \in \{-1, +1\}$, $N = 2^P$ and P, $M \in \mathbb{Z}^+$; $h_{m,i} \in \{+1, -1\}$ are the elements of a $N \times N$ Hadamard matrix; $\pi_{n,i} = (n+i) \bmod N$; and $W \leq L_0 - 1$. From equation (1) it can be seen that every code of a LS family with size $K = N^2$ and length $L = N^2 L_0 + (N-1)W$ is composed by $s_{i,j}$ complementary sequences arranged according to $\pi_{n,i}$, with the polarity indicated by the coefficients $h_{m,i}$ of a Hadamard matrix and with a set of W zeros between each change of j.

III. Optimized Efficient LS Correlator (O-ELSC)

The ELSC proposed in [3] is based on the Efficient Complementary Set of Sequences Correlator (ESSC) described in [5]. It uses N ESSCs to obtain the correlation of the input signal $r(\tau)$ with every sequence $s_{i,j}$, $0 \leq j \leq N-1$, of the N orthogonal CSSs S_i, $0 \leq i \leq N-1$ (see Step 1 in Fig. 1.a). Afterwards, in agreement with (1), the outputs of each ESSC are added with different time shifts (Step 2, Fig. 1.a). Then, the results obtained are delayed according to $\pi_{n,i}$ and added with the polarity specified by the components $h_{m,i}$ of the Hadamard matrix (Steps 3 and 4, respectively, of Fig 2). Although the ELSC requires less operations than the straightforward implementation, it approximately demands $\frac{N^2}{2}$ times the memory needed by the straightforward one. The optimization proposed in this letter requires less operations than the former ELSC and, at most, less than twice the memory used by the straightforward implementation. It has been called Optimized Efficient LS Correlator (O-ELSC) and differs from the ELSC in the Steps 1 and 2 as follows (see Fig. 1.b):

Step 1: In the proposed filter the delays $z^{-(N-1-j)\cdot(NL_0+W)}$, $0 \leq j \leq N-1$ that appear in

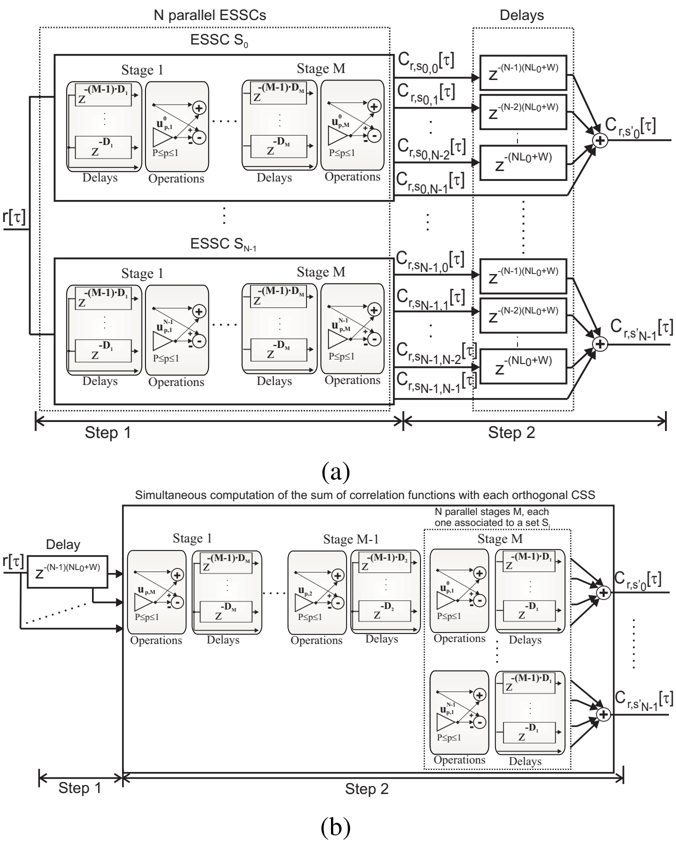

Fig. 1. (a) Block diagram of the first two steps of the Efficient LS code Correlator (ELSC); (b) block diagram of the first two steps of the proposed Optimized Efficient LS Correlator (O-ELSC).

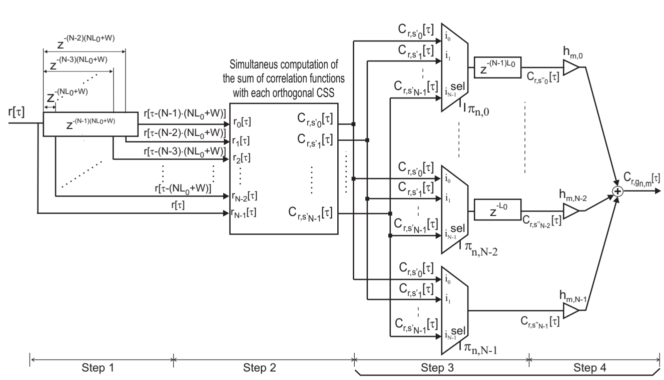

Fig. 2. Block diagram of proposed Optimized Efficient LS Correlator (O-ELSC).

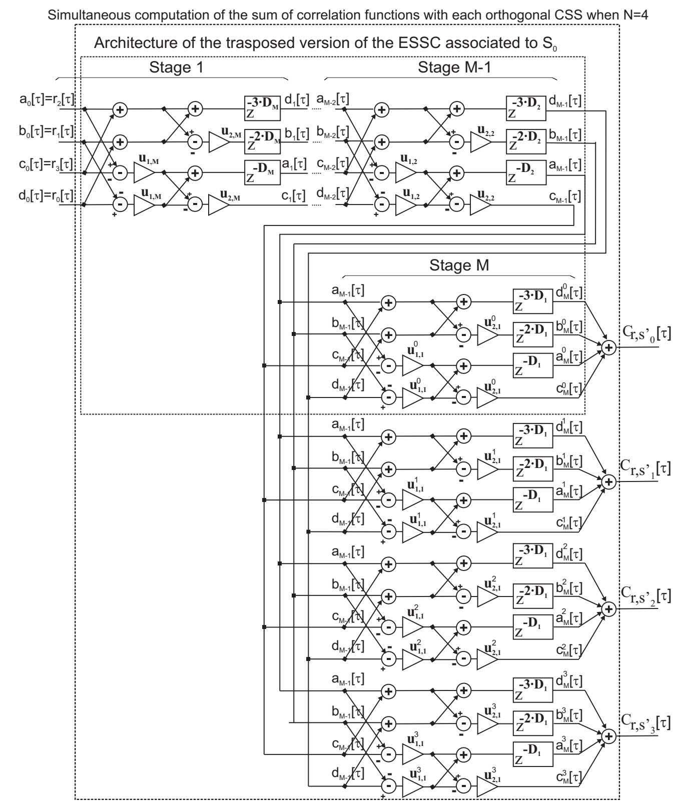

Fig. 3. Example of Step 2 in the O-ELSC when $N = 4$.

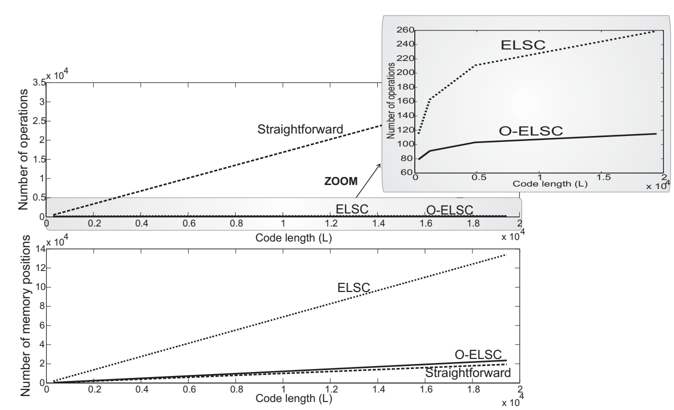

Fig. 4. Number of operations and memory resources required by correlators straightforward, ELSC and O-ELSC, when $N = 4$.

Step 2 of the ELSC are applied to the input signal $r(\tau)$ (see Step 1 in Fig. 1.b), thus obtaining N different outputs $R_j(z) = z^{-(N-1-j)\cdot(NL_0+W)}R(z)$, $0 \leq j \leq N-1$. Since all the delays share the input of the system $r(\tau)$, it is enough to implement the largest delay $z^{-(N-1)\cdot(NL_0+W)}$ and allow the access to intermediate positions. This modification means a save of $(N-1)(NL_0+W)(\frac{N^2}{2}-1)$ memory positions with regard to the existing ELSC.

Step 2: The filter proposed here takes advantage of the orthogonality of the CSSs from which LS codes are derived and of the regular structure of the ESSC. Since the N sets $S_i, 0 \leq i \leq N-1$, are mutually orthogonal, the ESSC associated to each S_i only differ from the others in the coefficients $(u^i_{P,1}, u^i_{P-1,1}, \cdots, u^i_{1,1})$ of the generation seed at the first stage (see [5] for further details regarding the ESSC). Then, the architecture of each ESSC can be transposed as is indicated in Fig. 1.b, where the delays $\{D_1, D_2, \cdots, D_M\} = \{N^0, N^1, \cdots, N^{M-1}\}$ and the coefficients $(u^i_{P,1}, u^i_{P-1,1}, \cdots, u^i_{1,1}, u^i_{P,2}, \cdots, u^i_{1,2}, \cdots, u^i_{P,M}, \cdots, u^i_{1,M})$ appear reverted. The sum $C_{r,s'_i}(\tau)$ of correlation functions with each sequence of the set S_i is obtained by adding the N outputs of each reverted ESSC. The N transposed ESSCs can share the first $M-1$ stages, since they only differ in the coefficients $(u^i_{P,1}, u^i_{P-1,1}, \cdots, u^i_{1,1})$ of the last one (stage M). Thus, in Fig. 1.b the superscript i, which indicates the CSS S_i to be detected, only appears in stage M. This new algorithm

TABLE I

NUMBER OF OPERATIONS AND MEMORY REQUIREMENTS IN THE STRAIGHTFORWARD CORRELATOR, THE ELSC AND THE PROPOSED O-ELSC, AS A FUNCTION OF N, M AND W (N^M IS THE LENGTH OF THE INITIAL CSS; $W \leq L_0 - 1$ IS THE LENGTH OF THE SET OF ZEROS THAT APPEAR IN THE LS CODE; AND $L = N^{M+2} + (N-1)W$ IS THE LENGTH OF THE LS CODE).

	Straightforward	ELSC		OELSC	
Products	$N^{(M+2)}$	Step 1	$\frac{M \cdot N^2}{2} log_2(N)$	Step 2	$\frac{N}{2} \cdot (N+M-1) \cdot log_2(N)$
		Step 4	N	Step 4	N
		TOTAL	$N(1 + \frac{M \cdot N}{2} \cdot log_2(N))$	TOTAL	$N \cdot (1 + \frac{N+M-1}{2} \cdot log_2(N))$
Additions	$N^{(M+2)} - 1$	Step 1	$M \cdot N^2 \cdot log_2(N)$	Step 2	$N \cdot (N+M-1) \cdot log_2(N) + N^2 - N$
		Step 2	$N(N-1)$		
		Step 4	$N-1$	Step 4	$N-1$
		TOTAL	$M \cdot N^2 \cdot log_2(N) + N^2 - 1$	TOTAL	$N \cdot (N+M-1) \cdot log_2(N) + N^2 - 1$
Memory Positions	$N^{M+2} + (N-1)W$	Step 1	$\frac{3N^{M+1} - 2N^M - N^2}{2}$	Step 1	$(N-1) \cdot (N^{M+1} + W)$
		Step 2	$\frac{(N^{M+1}+W)(N^3-N^2)}{2}$	Step 2	$\frac{N^2-N}{2}\left(\frac{N^M+N^2-2N}{N-1}\right)$
		Step 3	$\frac{N^M(N^2-N)}{2}$	Step 3	$\frac{N^M(N^2-N)}{2}$
		TOTAL	$\frac{N^{M+4} - N^{M+3} + N^{M+2} + 2N^{M+1} - 2N^M + WN^3 - N^2(1+W)}{2}$	TOTAL	$\frac{3N^{M+2} - 2N^{M+1} + N^3 - 2N^2 + 2NW - 2W}{2}$

allows to directly obtain the N sum of correlation functions $C_{R,S'_i(z)} = \sum_{j=0}^{N-1} z^{-(N-1-j)(NL_0+W)} C_{R,S_{i,j}}(z), 0 \leq i \leq N-1$, with the N sequences of each set S_i, where $C_{R,S_{i,j}}$ is the Z transform of the aperiodic correlation between $r(\tau)$ and the sequence $s_{i,j}(\tau)$. This modification means a significant save of memory and operations in comparison with the use of N ESSCs, as the $M-1$ first stages are shared. For a better understanding, Fig. 3 represents an example of the modification proposed to perform the simultaneous correlation with each orthogonal S_i when $N = 4$.

Later, as can be seen in Fig. 2, a set of N multiplexers governed by $\pi_{n,i}$ determines the delays to be applied to every partial correlation result $C_{R,S'_i(z)}$ (Step 3). The obtained outputs are multiplied in Step 4 by the corresponding element $h_{m,i}$ of the Hadamard matrix and then added to get the final correlation result (2).

$$\begin{aligned} C_{R,G_{n,m}}(z) \quad = \quad & \sum_{i=0}^{N-1} h_{m,i} \cdot z^{-(N-1-i)L_0} \cdot \\ & \cdot [\sum_{j=0}^{N-1} z^{-(N-1-j)(NL_0+W)} C_{R,S_{\pi_{n,i},j}}(z)] \end{aligned} \tag{2}$$

The total number of operations and memory positions' requirements of the O-ELSC, the ELSC and the straightforward implementation can be observed in Table I. If only binary values are employed in the complementary sequences used to construct the LS codes, the three implementations can be performed without any multiplication. Anyway, multiplications have been included in the table to consider a general case where multilevel or polyphase LS codes could be involved. For an easy comparison, Fig. 4 shows the computational requirements for the correlation of LS codes of different lengths when $N = 4$. As can be observed in Table I and Fig. 4, the O-ELSC demands less operations than the previous proposals and its memory requirements are comparable with those of the straightforward correlator.

IV. CONCLUSION

A fast algorithm for the correlation of LS codes derived from CSSs is presented. The proposed method is much more efficient than previous ones based on straightforward implementations or efficient CSS correlators. It minimizes the number of operations needed and do not penalize the memory requirements. The proposed correlator is suitable in all those CDMA systems that work under a low signal-to-noise ratio and demand a real-time detection of very long LS codes. Additionally, it can be modified to detect other codes derived from orthogonal CSSs, as some types of ZCZ codes.

REFERENCES

[1] C. Zhang, X. Lin, S. Yamada, and M. Hatori, "General method to construct LS codes by complete complementary sequences," *IEICE Trans. Commun.*, vol. E88-B, no. 8, pp. 3484–3487, 2005.

[2] P. Fan, "Spreading sequence design and theoretical limits for quasisynchronous CDMA systems," *EURASIP J. Wireless Commun. and Networking*, vol. 2004, no. 1, pp. 19–31, 2004.

[3] M. C. Pérez, J. Ureña, A. Hernández, F. J. Álvarez, and A. Jiménez, "Efficient correlator for LS codes generated from orthogonal CSS," *IEEE Commun. Lett.*, vol. 2, no. 10, pp. 764–766, 2008.

[4] H. Wei, L. L. Yang, and L. Hanzo, "On the uplink performance of LAS-CDMA," *IEEE Trans. Wireless Commun.*, vol. 5, no. 5, pp. 1187–1196, 2006.

[5] C. Marziani, J. Ureña, A. Hernández, M. Mazo, F. Álvarez, J. García, and P. Donato, "Modular architecture for efficient generation and correlation of complementary sets of sequences," *IEEE Trans. Signal Process.*, vol. 55, no. 5, pp. 2323–2337, 2007.