



# Estimation of high cycle fatigue behaviour using a threshold curve concept

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## ABSTRACT

Estimation of short crack propagation rates has proven to be a key factor in design and maintenance of metallic components subjected to loading conditions related to high cycle fatigue (HCF). Extrapolation of long crack fracture mechanics approaches fails to correlate short crack behaviour, since crack length dependence of the short crack propagation threshold is not accounted for. In this paper, a fracture mechanics approach for prediction of HCF behaviour involving short cracks is analysed. The method is defined through application of the resistance curve concept and a short crack propagation threshold prediction model. Different threshold estimation methods are reviewed and compared. Application of the proposed model is exemplified and results of predicted crack propagation rates and estimated fatigue lives are presented and discussed.

## 1. Introduction

In high cycle fatigue of metallic components, a relatively large fraction of the total life is required to initiate a crack which size can be detected by inspection. In these cases, as much as 80% of the total fatigue life is needed to create a 1 mm long crack [1,2]. Thus, it is very important to rely on an accurate estimation model to account for the short crack propagation behaviour and the number of fatigue load cycles that are necessary to create such a crack.

In the analysis of cracks longer than 1 mm, several modifications of the Paris law have been proposed by different researchers in order to consider the fatigue crack propagation threshold,  $\Delta K_{thr}$ . Two of the simplest expressions for crack propagation rates,  $da/dN$ , are:

$$da/dN = C(\Delta K - \Delta K_{thr})^m \quad (1)$$

$$da/dN = C^* (\Delta K^{m^*} - \Delta K_{thr}^{m^*}) \quad (2)$$

where  $\Delta K$  is the applied stress intensity factor range and  $C$ ,  $C^*$ ,  $m$  and  $m^*$  are constants that depend on material, environment and load ratio. Expression (1) was proposed by Zheng and Hirt [3], and expression (2) by Klesnil and Lukáš [4]. Fatigue limit can be defined by the minimum nominal applied stress range for which the resultant applied  $\Delta K$  is equal or greater than the threshold for fatigue crack propagation,  $\Delta K_{th}$ , for any crack length. Hence, both expressions give the same result when the fatigue limit or endurance of different configurations is estimated. In expression (1) the difference between the total applied  $\Delta K$  and  $\Delta K_{thr}$  can be considered as an effective driving force concept, so it can be assumed to be a physically relevant propagation driving force

parameter. In expression (2), the difference between  $\Delta K$  and  $\Delta K_{thr}$  is initially modified by the  $m^*$  exponent, reducing the equation to a Paris law expression that is shifted a constant value. At the same time, it provides better fitting of experimental results, mainly for applied  $\Delta K$  values in the near-threshold region. However, expression (2) has some problems when trying to account for the fatigue crack propagation rate of short cracks, as we will discuss later.

Generally speaking, cracks shorter than 1 mm do not follow the crack growth laws derived for long cracks, for which the threshold for fatigue crack propagation,  $\Delta K_{thr}$ , is constant for a given load ratio. In the case of short cracks,  $\Delta K_{th}$  is a function of crack length, and increases from a minimum value that is associated to the fatigue limit of the material [1–8], to a maximum value given by  $\Delta K_{thr}$ .

Section 2 of this paper presents an overview of the most widely used short crack threshold estimation models. Additionally, a comparison of these models with the Chapetti model [7] is made to emphasize the differences between them in order to clarify some cases of misuse, given several misunderstandings of the fundamental hypotheses behind it. Section 3 presents the development of a methodology, based on a fracture mechanics approach, that is able to predict the high cycle fatigue behaviour of short cracks, applying the resistance curve concept. The method is fully defined for cracks larger than the characteristic microstructural dimension of the studied material, which can usually be found to be the grain size, pearlite colony size, bainite sheaf length, etc. The equations and parameters needed to make the fatigue crack growth estimations are also presented and estimations are compared with experimental results taken from the bibliography. Finally, some recommendations on future work are given.

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Nomenclature			
$a$	crack length	$\Delta K_{th}$	crack length dependent threshold
$C, C^*$	environmentally sensitive material constant	$\Delta K_{thR}$	long crack threshold
$d$	position from the surface of the strongest microstructural barrier to fatigue crack propagation	$\Delta K_{eff,th}$	long crack effective threshold
$da/dN$	crack growth rate	$K_{op}$	crack opening stress intensity
$k$	exponential factor	$K_{op,max}$	maximum crack opening stress intensity
$m, m^*$	environmentally sensitive material exponent	$L_0$	critical distance
$\Delta K$	applied stress intensity factor range	$L_{0eff}$	effective critical distance
$\Delta K_{dr}$	microstructural threshold	$\Delta \sigma$	applied stress range
$\Delta K_c$	extrinsic component of $\Delta K_{th}$	$\Delta \sigma_{er}$	plain fatigue limit
$\Delta K_{cR}$	extrinsic component of $\Delta K_{thR}$	$\Delta \sigma_{th}$	threshold stress range
		$R$	load ratio
		$Y$	geometrical factor

## 2. Overview and comparison of short fatigue crack growth threshold estimation models

As we have mentioned before, it is now well understood that the complex behaviour of short cracks is related to the threshold for short fatigue crack propagation, which is a function of crack length. Many efforts have been made to study small fatigue crack growth [1–17] and various kinds of models have been proposed to try to correlate this behaviour (see for instance Refs. [5–8,13]). Common fatigue analyses usually contemplate the use of the resistance curve concept, expressed by Eqs. (1) or (2). However if the anomalous crack propagation behaviour usually shown by short cracks is to be included, estimation of short fatigue crack growth rates must consider a crack length dependent propagation threshold,  $\Delta K_{th}$ , as follows:

$$da/dN = C (\Delta K - \Delta K_{th})^m \quad (3)$$

$$da/dN = C^* (\Delta K^{m^*} - \Delta K_{th}^{m^*}) \quad (4)$$

The total applied stress intensity factor range,  $\Delta K$ , can be properly evaluated for any configuration, and since experimental measurements of crack length dependent threshold values are difficult to obtain, they are usually estimated using available prediction models. Three of these models are introduced next, plotted schematically in Figs. 1 and 2. Fig. 1 shows the estimated threshold stress range as a function of crack length (Kitagawa and Takahashi diagram [18]), and Fig. 2 plots the estimated threshold in terms of the stress intensity factor range as a function of the square root of crack length.

### 2.1. El Haddad model

El Haddad model proposes a mathematical transition between the plain fatigue limit,  $\Delta \sigma_{er}$ , and the crack propagation threshold for long cracks,  $\Delta K_{thR}$ , and defines a critical distance,  $L_0$ , using these two parameters. It is worth noting that this  $L_0$  parameter does not hold any relation to an actual crack size or physical dimension (see Figs. 1 and 2). El Haddad and co-workers proposed the following expression to estimate the threshold as a function of an “effective” length ( $a + L_0$ ) [6]:

$$\Delta K_{th} = \Delta K_{thR} \sqrt{\frac{a}{a + L_0}} \quad (5)$$

Definition of  $L_0$  is given by:

$$\Delta K_{thR} = \Delta \sigma_{er} \sqrt{\pi L_0} \Rightarrow L_0 = \frac{1}{\pi} \left( \frac{\Delta K_{thR}}{\Delta \sigma_{er}} \right)^2 \quad (6)$$

From expressions (5) and (6), the threshold stress range can be expressed as:

$$\Delta \sigma_{th} = \Delta \sigma_{er} \sqrt{\frac{L_0}{a + L_0}} \quad (7)$$

Several modifications of this basic model have been proposed, adding the parameter  $Y$  in expression (5) to account for different geometrical and loading configurations, or modifying  $L_0$  to account for the plastic zone size and other aspects related to short crack propagation [19–21].

It is important to emphasize that in the El Haddad model,  $\Delta K_{th} = 0$  for  $a = 0$ , (see Figs. 1 and 2) since the model does not allow a definition of a minimum  $\Delta K_{th}$  value, associated to the fatigue limit (microstructural threshold). This means that, according to this model, there is not minimum threshold for fatigue short crack propagation.

### 2.2. Crack closure based models

Some authors attribute the short crack propagation behaviour to the initial stages of crack closure development. Propagation models based on this theory postulate that the total threshold is obtained as the sum of the effective threshold stress intensity range,  $\Delta K_{eff,th}$ , and the opening stress intensity factor  $K_{op}$ . In these models,  $K_{op}$  is the crack length dependent parameter, while the  $\Delta K_{eff,th}$  value is constant, irrespective of crack length. Effective threshold values are obtained from standard fatigue tests of specimens with long cracks, in which it is necessary to measure the crack closure component with a certain precision. The threshold for fatigue crack propagation as a function of crack length is then given by:

$$\Delta K_{th} = \Delta K_{eff,th} + K_{op} \quad (8)$$

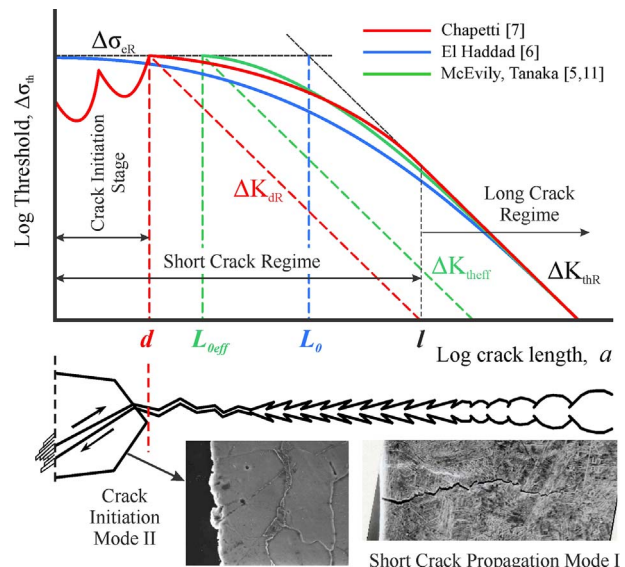


Fig. 1. Schematic representation of the analysed models in a Kitagawa-Takahashi type diagram showing the threshold between propagation and non-propagating cracks.

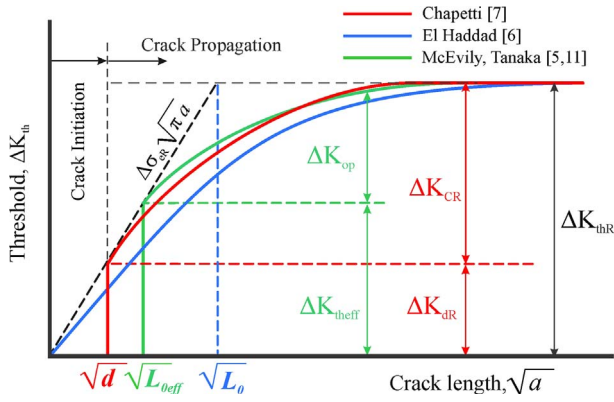


Fig. 2. Schematic representation of the analysed models in a Modified Kitagawa-Takahashi type diagram showing threshold curves in terms of  $\Delta K_{th}$  vs  $a^{1/2}$ .

In the model of McEvily and Minakawa the development of the opening stress intensity factor,  $K_{op}$ , (due to the development of the crack closure effect), which should be zero for zero crack length and maximum for long cracks, is estimated by the following exponential expression [5]:

$$\Delta K_{op} = (1 - e^{-ka}) K_{op,max} \quad (9)$$

where  $K_{op,max}$  is the maximum value of the closure component and  $k$  is a parameter of units  $\text{mm}^{-1}$  that must be experimentally measured for a given material and load ratio. According to the authors,  $k$  varies from 2 to  $10 \text{ mm}^{-1}$  for steels, depending on the tensile resistance of the alloy [22,23].

Tanaka and Akiniwa [11] proposed a different expression to estimate the development of  $K_{op}$ , given by:

$$\Delta K_{op} = \left( \frac{a - c_1}{c_2 - c_1} \right) K_{op,max} \quad (10)$$

where  $c_1$  is taken as the length of the stage I crack, and  $c_2$  is the crack length above which  $K_{op} = K_{op,max}$  (limit between the short crack and long crack regimes). The parameter  $c_1$  could be associated to the microstructural dimension (grain size  $d$ , for instance), but the parameter  $c_2$  must be experimentally measured for each material and loading ratio.

These methods have another source of uncertainties associated with the value of  $K_{op,max}$ , which is measured using standardised tests that present several implementation difficulties, and give results that are highly dependent on experimental procedures and techniques [24].

### 2.3. Chapetti model

As it was described by Miller [16], fatigue resistance is characterized by the existence of a microstructural and a mechanical threshold. The microstructural threshold is strictly related to the intrinsic microstructural properties of the material. It can be defined as the stress level needed for a microstructurally short crack (MSC) to overcome the strongest microstructural barrier, usually found to be a characteristic microstructural dimension (grain size, pearlite colony size, bainite sheaf length, etc.). Previous work by Chapetti et al. [7,25] provided evidence that this intrinsic threshold stress level matches the material's plain fatigue limit. Taking this into consideration, the Chapetti model [7] defines the microstructural or intrinsic threshold as:

$$\Delta K_{dR} = Y \Delta \sigma_{eR} \sqrt{\pi d} \quad (11)$$

where  $\Delta \sigma_{eR}$  is the material's plain fatigue limit,  $d$  is the position of the strongest microstructural barrier (e.g. grain size), and  $Y$  is the geometric correction factor. As in most cases MSC nucleated at surfaces are considered semi-circular [26],  $Y$  is taken as 0.65. The 'R' subscript indicates that as  $\Delta \sigma_{eR}$  is R-ratio dependent,  $\Delta K_{dR}$  also is.

The Chapetti model proposes that, in addition to the microstructural threshold, the cracks propagation threshold is also composed by an "extrinsic" component,  $\Delta K_C$ , which is dependent on crack length. Once this component has fully developed, it reaches a maximum value (for long cracks),  $\Delta K_{CR}$ , which is constant for a given material and load ratio. This maximum extrinsic component is defined as the difference between the long crack propagation threshold,  $\Delta K_{thR}$ , and the microstructural threshold,  $\Delta K_{dR}$ , given by:

$$\Delta K_{CR} = \Delta K_{thR} - \Delta K_{dR} \quad (12)$$

The development of  $\Delta K_C$  as a function of crack length can be modelled with an exponential function of the form:

$$\Delta K_C = \Delta K_{CR} (1 - e^{-ka}) \quad (13)$$

where  $a$  is the crack length, and  $k$  is a material and R-ratio dependent constant that defines the shape of the  $\Delta K_C$  curve. The material threshold as a function of crack length is then defined as:

$$\Delta K_{th} = \Delta K_{dR} + \Delta K_C = Y \Delta \sigma_{eR} \sqrt{\pi a} \quad a \geq d \quad (14)$$

From expressions (12) and (13) we finally get:

$$\Delta K_{th} = \Delta K_{dR} + (\Delta K_{thR} - \Delta K_{dR}) [1 - e^{-k(a-d)}] \quad a \geq d \quad (15)$$

It is now important to define the value of the constant  $k$ , responsible for the behaviour of  $\Delta K_C$ , the extrinsic component of  $\Delta K_{th}$ . The first hypothesis the model considers is that the plain fatigue limit,  $\Delta \sigma_{eR}$ , is defined by the strongest microstructural barrier located at position  $d$ . A second hypothesis of this model postulates that, for cracks longer than  $d$ , the threshold stress level must be equal or lower than the plain fatigue limit. Graphically, we can exemplify this condition in Fig. 1, where from  $a = d$ , the slope of the  $\Delta \sigma_{th}$  vs.  $a$  curve must be equal (tangent) or lower than the horizontal line defined by the fatigue limit,  $\Delta \sigma_{eR}$ . Considering this restriction, we can obtain an upper limiting value for  $k$ . When comparing this upper value with two sets of experimental data for steel, it was found that better correspondence is obtained when using one half of the given upper limit. This gave rise to the final form of the expression used to estimate the parameter  $k$ , without the need of any further calibration or additional fitting procedures [7]:

$$k = \frac{1}{4d} \frac{\Delta K_{dR}}{(\Delta K_{thR} - \Delta K_{dR})} = \frac{1}{4d} \frac{\Delta K_{dR}}{\Delta K_{CR}} \quad (16)$$

This expression was later proved to work very well with experimental data for eight different materials. See Ref. [7] for more details.

It can be seen that the Chapetti model is fully defined once  $\Delta \sigma_{eR}$ ,  $\Delta K_{thR}$  and  $d$  are known, all being parameters that can be easily obtained from common standardised fatigue tests and metallographic analysis.

### 2.4. Comparison of the models

Figs. 1 and 2 help us analyse the differences between the three models: the El Haddad model, which is a mathematical transition between the fatigue limit,  $\Delta \sigma_{eR}$ , and the fatigue propagation threshold for long cracks, is plotted in blue colour; the representation of a propagation model based on the development of the crack closure component with crack length (in its different versions, proposed by McEvily et al. [5] or Tanaka et al. [11]), plotted in green colour; and the Chapetti model, plotted in red colour. Even though the Kitagawa diagram (Fig. 1) is the most popular way of representing threshold values, plotting the driving force as a function of the crack length in terms of the stress intensity factor range,  $\Delta K$ , seems to be more practical when using expressions like (5), (8) or (15). Additionally, using the square root of the crack length yields a clarified analysis of the differences between the models (Fig. 2). A straight line given by the following expression can be plotted in order to relate the basic concepts of the models:

$$\Delta K = Y \Delta \sigma_{eR} \sqrt{\pi a} \quad (17)$$

It can be seen that the El Haddad model does not allow a definition of the transition between the fatigue crack initiation and propagation stages. In the case of the models that use the crack closure concepts, an intrinsic crack length  $L_{0eff}$  is defined, associated to the fatigue limit by using the effective fatigue crack propagation threshold for long cracks ( $\Delta K_{eff,th}$ , the threshold for long-crack propagation without the crack closure component). This length seems to be related with the extension of the mode II crack propagation stage (crack initiation), but there is not clear experimental evidence to prove this. On the other hand, the Chapetti model allows a clear definition of the crack initiation stage, and a fatigue crack initiation life, as the number of cycles necessary to nucleate a micro crack with length equal to  $d$ .

Application of the crack closure models requires estimation of the  $k$  or  $c_1$  parameters in expressions (9) and (10) respectively. These values are obtained by fitting of experimental results of short crack propagation, and by measurement of the effective threshold for long cracks (that is to say, to measure the crack closure component at threshold levels). Technical difficulties and large scatter associated with these experimental requirements, reduces significantly the practical application of this models. Consequently, the El Haddad and Chapetti models are usually preferred for fatigue life estimations [12,27,28], requiring only measurement of the long crack propagation threshold,  $\Delta K_{thR}$ , and the fatigue limit or endurance,  $\Delta\sigma_{eR}$ , (plus  $d$  in the case of the Chapetti model). In those cases where it is necessary to deal with a minimum  $\Delta K_{th}$  for fatigue crack propagation, the Chapetti model is preferred, since that minimum value is defined by the microstructural threshold,  $\Delta K_{dR}$ . El Haddad model usually gives lower threshold values than  $\Delta K_{dR}$  for  $a = d$  (see Figs. 1 and 2). This driving force difference can add up to a considerably high number of cycles when dealing with high cycle fatigue, resulting in over-conservative fatigue life estimations.

Different analyses of the Chapetti model can be found in the literature and some of them consider that the  $\Delta K_C$  term refers to the closure component of the threshold  $\Delta K_{th}$  (see expression (13)), and so, that it is similar to the model proposed by McEvily (see expression (9)). For instance, Santus and Taylor [12] express textually that “ $\Delta K_C$  is the closure term” of the threshold. Other analyses of the model in the same publication are based on this misunderstanding. The only similarity between the Chapetti and McEvily models is the mathematical expression used for the estimation of the development of  $\Delta K_{th}$ , an exponential function that requires only one constant ( $k$ ) to describe the behaviour of the curve. In the McEvily model,  $\Delta K_{op}$  is defined as the crack length dependent closure component, and it is intrinsically related to the conceptual basis of the model. In the Chapetti model,  $\Delta K_C$  is defined as the difference between the total threshold,  $\Delta K_{th}$ , and the microstructural threshold,  $\Delta K_{dR}$ , which is obtained through the fatigue limit and the position of the strongest microstructural barrier,  $d$ . The microstructural threshold associated to the plain fatigue limit is not only defined by the absence of the crack closure effect in a micro crack with length equal to  $d$ , but also influenced by the surface strain concentration, change in crack propagation mode and other differences with the fatigue behaviour and nature of long cracks. In the case of MSC propagation, the experimental evidence behind the fatigue limit being a microstructural threshold related to the strongest microstructural barrier [16], makes the definition of  $\Delta K_{dR}$  much more realistic than the extrapolation of the effective threshold for long crack propagation,  $\Delta K_{eff,th}$ . Another important difference between the models is that the parameter  $k$  used to describe the development of  $\Delta K_{th}$  in the closure models (see expression (9)), must be obtained by fitting experimental data of short crack thresholds. In the Chapetti model, the parameter  $k$  is estimated with  $\Delta K_{thR}$ ,  $\Delta\sigma_{eR}$  and  $d$  in expression (16), which is deduced from conditions associated to the definition of the microstructural threshold and a final calibration against a few sets of experimental results. Additionally, expression (15) can be applied for any alloy making the model a practical and useful estimation tool.

Several other misunderstandings of the Chapetti model can be found in the literature [29]. A clear one is the recent publication of Wang

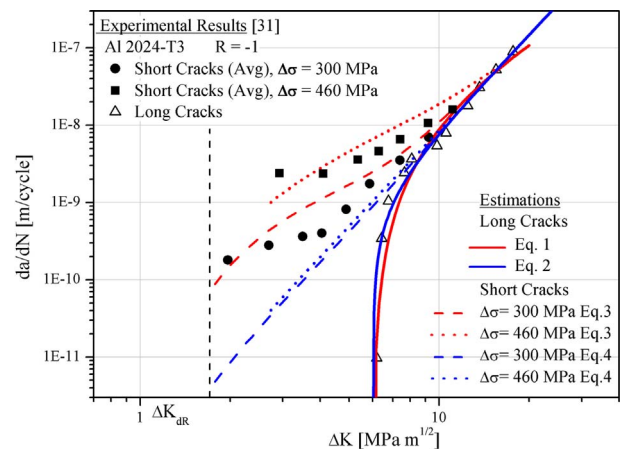


Fig. 3. Comparison of estimated short and long crack propagation rates with Eqs. (3) and (4) for Al 2324-T3 [31], using the Chapetti model for the short crack propagation threshold prediction.

et al. [30] that misunderstood not only the model itself, but also the hypothesis that were used for the proposal.

In the following section an application to estimate short crack propagation behaviour and high cycle fatigue lives is presented. The model proposed by Chapetti [7] to estimate the short fatigue crack propagation threshold as a function of crack length is used for the analysis.

### 3. Application, results and discussion

#### 3.1. Fatigue propagation rates estimation for short cracks

Fig. 3 shows experimental results for long and short crack propagation published by Akiniwa and Tanaka [31] for Al 2024-T3 and load ratio  $R = -1$ . Open symbols represent long crack data while solid symbols represent average short crack growth rates. Solid lines represent long crack behaviour described by Eq. (1) in red and Eq. (2) in blue. Parameters  $C$  and  $m$  used in Eq. (1) are optimized to fit the experimental values at the near threshold region so that the high cycle fatigue behaviour is properly accounted for, while parameters  $C^*$  and  $m^*$  in Eq. (2) are the standard Paris coefficients. Values used for the curve fitting are:  $C = 5.5 \cdot 10^{-10}$ ,  $C^* = 6.95 \cdot 10^{-13}$  (when  $\Delta K$  values are expressed in units of  $\text{MPa}(\text{m})^{1/2}$  and  $da/dN$  in units of  $\text{m}/\text{cycle}$ ),  $m = 2$  and  $m^* = 4.1$ . It can be seen that Eq. (2) produces a better fit of the threshold behaviour of long cracks. However, when the dependence of the propagation threshold with crack length is included in the calculation, expression (2) does not show a proper fitting of the short crack propagation data. Discontinuous lines (dashed and dotted) in Fig. 3, shows prediction results using the Chapetti model for the estimation of the propagation threshold as a function of crack length by means of Eq. (15). These estimations are plotted against short crack propagation data (average) [31] for two different applied stress ranges,  $\Delta\sigma$ , of 300 MPa and 460 MPa drawn as dashed and dotted lines respectively. Red lines were obtained with Eq. (3) while blue lines were obtained through Eq. (4). The parameters used in the model were extracted from Refs. [31] and [32], being:  $\Delta K_{thR} = 6 \text{ MPa}(\text{m})^{1/2}$ ,  $\Delta\sigma_{eR} = 275 \text{ MPa}$  and  $d = 19 \mu\text{m}$ . It is clear that Eq. (4) gives is a mathematical limitation to the crack propagation rate values, and that it does not adequately represent the short crack propagation behaviour. However, introduction of the crack length dependent threshold given by the Chapetti model in Eq. (3) allows accounting for the higher crack propagation rates shown by short cracks as the nominal applied stress is increased for the same applied  $\Delta K$ . Fig. 4 shows only the results from expression (3) adding the experimental propagation rates of a main crack reported by Akiniwa et al. for each nominal stress range. It can be seen that good agreement is



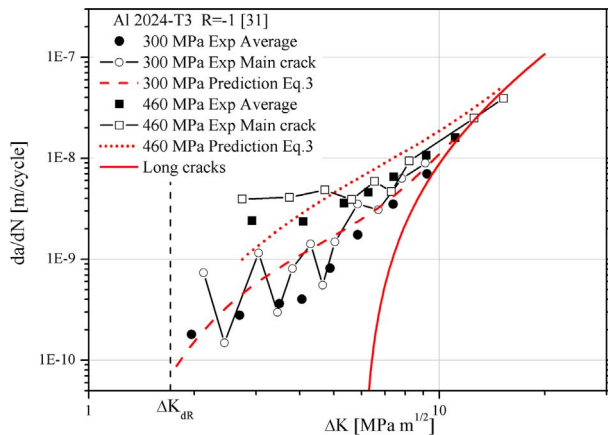


Fig. 4. Short crack propagation rate estimation using Eq. (3) and the Chapetti model against experimental results for average and main short crack propagation rates in Al 2024-T3 [31].

found with the tendencies of experimental data. These experimental values show the usual saw-tooth profile associated with the interaction between the micro-crack front and the first 2, 3 or 4 microstructural barriers. After that, threshold values can be averaged as the crack grows and the propagating front crosses more than 5 or 10 microstructural identities. This is a well-accepted assumption for the most widely used threshold prediction models (El Haddad, Tanaka, McEvily, Chapetti, etc).

### 3.2. High cycle fatigue life estimation

The previous analysis of Fig. 4 is complemented with the results presented in Fig. 5, which shows the predicted crack length as a function of number of cycles by integrating expression (3), for the same Al alloy analysed by Akiniwa and Tanaka [31]. Integration of experimental results through linear interpolation is also plotted for the average short crack propagation data and for main cracks. It can be seen that for the 460 MPa stress level, predicted propagation behaviour yields very good agreement with experimental data. For the 300 MPa nominal stress level the fatigue life is underestimated, however, given the large scatter usually encountered with short crack propagation at stress levels near the fatigue limit, it can be concluded that the prediction presents an adequate, conservative result.

In a recent publication, Santus and Taylor [12] proposed that the physically short crack propagation is similarly modelled by means of a driving force equation, but independent from the long crack propagation. They concluded that, with the use of this equation, a better description of the short crack behaviour is provided. They obtained physically short crack propagation model parameters by fitting expression (3) to experimental data taken from the literature, for two aluminium alloys and a titanium alloy at two different heat treatment conditions and load ratios. The proposed model offers a phenomenological tool to describe the higher short crack propagation rates than those expected for long cracks at similar applied  $\Delta K$ . However, it cannot be used for prediction purposes due to the need of experimental results to fit and obtain the parameters needed for the model.

In the same publication they also presented the possibility to estimate the initiation period by subtracting the predicted propagation cycles from the total fatigue life obtained from smooth samples. Fig. 6 shows an S-N plot with experimental results for a Ti-6Al-4V alloy [12], together with the propagation estimation presented by Santus and Taylor (dashed line), obtained using their proposal and the El Haddad model for the estimation of the short crack propagation threshold. The propagation lives obtained by their procedure results very small when compared to total lives of the fatigue specimens. Given this fact, Santus et al. concluded that most of the fatigue life is spent at the initiation

period, characterizing the propagation stage as almost negligible.

Fig. 6 also shows the prediction of the propagation stage by integration of expression (3) and using the Chapetti model for the short crack propagation threshold prediction. Data used in the calculation are  $\Delta\sigma_{eR} = 470$  MPa (extracted from lowest value at  $10^7$  cycles),  $\Delta K_{thR} = 4$  MPa(m) $^{1/2}$  and  $d = 20$   $\mu$ m [33]. It can be seen that there is a large difference between both predictions, up to two orders of magnitude at stress levels near the fatigue limit, contrary to the conclusion of similarity between both El Haddad and Chapetti models as it has been reported [12]. Additionally, the Santus model does not seem to be able to predict the fatigue limit of the given configuration, as the Chapetti model does. Considering the experimental scatter, the curve integrated from the Chapetti model falls close to the experimental results while keeping a conservative result. The authors believe that the result obtained by Santus and Taylor is related to the choice of over-conservative values of  $C$  and  $m$  parameters used in the propagation equation, whereas a proper expression has been used within the Chapetti model to integrate  $da/dN$  values. Furthermore, using the El Haddad model for estimation of the short crack propagation threshold, gives a relatively high fatigue crack propagation rate for nominal stress level near the fatigue limit. As it was mentioned before, for  $a = d$ , the fatigue threshold estimated by the El Haddad model is smaller than  $\Delta K_{dR}$ , the minimum threshold for fatigue crack propagation at the fatigue limit,  $\Delta\sigma_{eR}$  (see Figs. 1 and 2).

## 4. Conclusion

An integrated fracture mechanics approach to estimate the high cycle fatigue behaviour of metallic components is presented. Definition of the intrinsic transition crack length given by the parameter  $d$ , which separates the crack initiation and crack growth periods, allows the proposed method to include the short crack regime analysis, where the threshold for fatigue crack propagation is a function of crack length. Application of the model to both long and short crack propagation was carried out using a single  $da/dN$  and results were in good agreement with experimental data for different values of nominal applied stress level.

Given the lack of need for additional fitting parameters or difficult experimental procedures and the simplicity of application, authors believe that the proposed model can be utilized in different types of analyses based on fracture mechanics principles. Simplification of the saw-tooth profiles usually found in short crack propagation rates (Fig. 4) into a continuum equation yields good results, and adequate and conservative crack propagation rates are easily estimated with the proposed methodology. These results can vary significantly with misunderstanding and misuse of the Chapetti model (see Fig. 6).

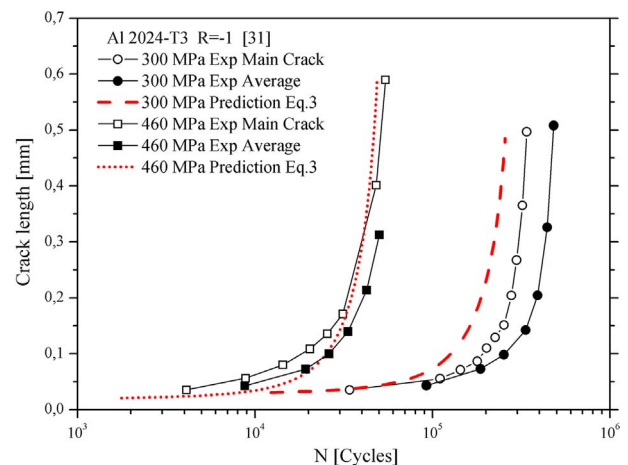


Fig. 5. Prediction of crack propagation behaviour against integration of experimental results.

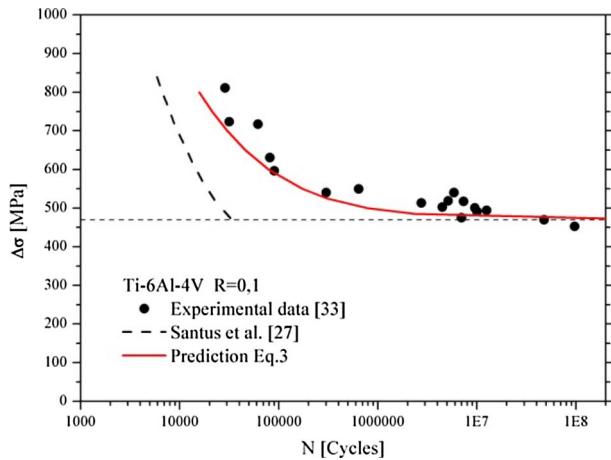


Fig. 6. S-N curve showing experimental data for Ti-6Al-4V, and total life predictions given by the Chapetti model and the model proposed by Santus et al. [12].

Thus, the method can be used as a simple and practical estimation tool, and aid in the study of interesting fatigue phenomena related with small fatigue cracks and high cycle fatigue. This includes the study of the mechanism of non-propagating crack development, notch size effect and fatigue notch sensitivity. Further analysis and applications should be carried out in order to demonstrate its ability and the reliability of the estimations.

## 5. Future work

As the proposed method encompasses the propagation of both short and long cracks, future work would have to focus on expanding the predictive capabilities of the procedure into the field of variable amplitude loading. This would require reliable experimental data, for short and long cracks for different stress ratios, in order to correlate mean stress effects. Additionally, the load interaction phenomena should be included in the form of a modified driving force, accounting for the influence of residual stress fields and the occurrence of crack closure effects.

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