Modelling and Solving the Perfect Edge Domination Problem

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Abstract

A formulation is proposed for the Perfect Edge Domination Problem and some exact algorithms based on it are designed and tested. So far, perfect edge domination has been investigated mostly in computational complexity terms. Indeed, we could find no previous explicit mathematical formulation or exact algorithm for the problem. Furthermore, testing our algorithms also represented a challenge. Standard randomly generated graphs tend to contain a single perfect edge dominating solution, i.e., the trivial one, containing all edges in the graph. Accordingly, some quite elaborated procedures had to be devised to have access to more challenging instances. A total of 736 graphs were thus generated, all of them containing feasible solutions other than the trivial ones. Every graph giving rise to a weighted and a non weighted instance, all instances solved to proven optimality by two of the algorithms tested.

Keywords: Perfect Edge Domination, Exact Algorithms, Instance Generation, Computational Results.

1 Introduction

Let $G = (V, E)$ be a simple undirected graph with a set of vertices V and a set of edges E. Denote by $\delta(i) \subseteq E$ the set of edges incident to $i \in V$ and respectively by $N[e] = \delta(i) \cup \delta(j)$ and $N(e) = N[e] \setminus \{e\}$ the *closed* and the *open edge neighborhoods* of $e = \{i, j\} \in E$. A set $D \subseteq E$ is said to *dominate* all the edges of $N[D]$ and is called an *Edge Dominating Set* (EDS) if $D \cap N[e] \neq \emptyset$ holds for every $e \in E$, i.e., if D dominates all the edges of G. The Minimum Edge Dominating Set Problem (MEDSP) is to find an EDS of G with cardinality as small as possible.

Edge domination has also been investigated in the literature under more restricted forms than the one described above. Indeed, *perfect* and *efficient* edge domination have probably attracted even more interest than the general problem. Given an EDS D, perfect domination applies if $|N(e) \cap D| = 1$ holds for every $e \in E \setminus D$, i.e., if $e \in E \setminus D$ is dominated by a single edge of D. Domination is efficient if, in addition, every edge of D is dominated just by itself. The Perfect Edge Domination Problem (PEDP) is to find a Perfect EDS (PEDS) of G with cardinality as small as possible. The Efficient Edge Domination Problem (EEDP) is defined along similar lines.

MEDSP is known to be NP-complete for some time. Moreover, it still remains NP-complete for graphs of maximum degree at most 3 that are either bipartite or planar [27]. In [14] the hardness has been further proved for planar bipartite graphs, planar cubic graphs, line graphs and total graphs. Additionally, MEDSP was also shown to be NP-complete for regular bipartite graphs [11]. On the other hand, polynomial-time algorithms are known for trees [27], block graphs [10], series-parallel graphs [23], bipartite-permutation graphs [24], and co-triangulated graphs [24]. As for exact algorithms for general graphs, references [2, 25] investigate Integer Programming (IP) based approaches for finding Minimum Maximal Matchings (MMMs), where a MMM is a restricted type of EDS (see [2, 25] for details).

The problem of deciding whether a given graph admits an efficient EDS (EEDS) is known to be NP-complete [12]. Furthermore, it remains so even for planar bipartite graphs of maximum degree 3 [4], k-regular graphs with $k \geq 3$ [8] and, as indicated in [9], for subcubic $(C_3, \ldots, C_k, H_1, \ldots, H_k)$-free bipartite graphs for any fixed k (where H_i is the graph obtained by subdividing $i-1$ times the middle edge of an H graph). On the other hand, polynomial time algorithms do exist for some graph classes, such as chordal graphs [21], generalized series-parallel graphs [21] (for both weighted and non weighted variants), claw-free graphs [9], weighted claw-free graphs [18], long claw-free graphs [13], graphs with bounded clique-width [9], hole-free graphs [4], convex graphs [16], dually-chordal graphs [5], P_7-free graphs [6], P_8-free graphs [7], bipartite permutation graphs [22], AT-free graphs [5], interval-filament graphs [5], and weakly chordal graphs [5]. Additionally, various exact algorithms do exist for general graphs [20, 26]. In particular, the one proposed in [20] also allows the counting of the number of EEDSs. So far, no exact IP based algorithm has yet been proposed for EEDP.

Less is known about PEDSs. It is NP-complete to determine if a given graph contains a PEDS of a given size [21] and that result also holds true for claw-free graphs of degree at most 3, bipartite graphs [21], k-regular graphs with $k \geq 3$, and bounded-degree graphs of large girth or bounded-degree F-free graphs, except when F is a set of disjoint paths (see [17], for all of these particular cases). Polynomial time algorithms are known for chordal graphs [21], circular-arc graphs [19], P_5-free graphs [17], cubic claw-free graphs and bounded-degree F-free graphs [17], where F is a set of disjoint paths. No exact algorithm, combinatorial or IP based, appears to exist for PEDP. Indeed, we could not even find in the literature any explicit mathematical formulation for the problem.

Suggesting a formulation for PEDP and designing and testing some accompanying exact algorithm for it are two of the contributions of this paper. An additional one, as we shall see, is to generate challenging graphs $G = (V, E)$ that, apart from the trivial solution, E, also contain additional feasible solutions for the problem.

Closing this brief overview on edge domination, it should be mentioned that it finds practical applications in the design and analysis of communication networks, network routing and coding theory [27, 12, 26].

Finally, it should also be stressed that we distinguish between the weighted and the non weighted variants of the problem. The former is called here Weighted PEDP (WPEDP) while the latter is simply called PEDP.

This paper contains four additional sections. Section 2 introduces our PEDSP formulation and describes some exact algorithms for it. Section 3 then follows with some quite elaborated procedures to generate graphs that contain PEDSs other than trivial ones. Section 4 describes the computational experiments we carried out for our algorithms. Finally, Section 5 closes the paper with some conclusions and suggestions for future work.

2 Problem Formulation

Our WPEDP formulation associates variables $\mathbf{x} \in \mathbb{R}_+^{|E|}$ with the edges of $G = (V, E)$ and enforces that $x_e = 1$ holds if $e \in E$ is selected as a dominating edge, with $x_e = 0$ applying otherwise. It is given by

$$\min \{\sum_{e \in E} c_e x_e : \mathbf{x} \in \mathcal{R}_0 \cap \mathbb{Z}^{|E|}\}, \tag{1}$$

where $\mathcal{R}_0$ is the polyhedral region defined by the intersection of inequalities

$$\sum_{f \in N[e]} x_f \geq 1, \ \forall e \in E \tag{2}$$

$$\sum_{e \in N(a)} x_e + (1 - N(a)) x_a \leq 1, \ a \in E, \ |N(a)| \geq 2 \tag{3}$$

$$0 \leq x_e \leq 1, \ e \in E. \tag{4}$$

The formulation takes edge weights $\{c_e \in \mathbb{R} : e \in E\}$ and specializes to PEDP when $\{c_e = 1 : e \in E\}$ applies.

Inequalities (2) ensure that at least one edge is selected for every neighborhood $N[e]$, $e \in E$, thus guaranteeing that (1) returns an EDS of G. In turn, as required for perfect edge domination, inequalities (3) enforce that any non dominating edge, i.e., any $a \in E$ with $x_a = 0$, must be dominated by no more than one of its neighbor vertices. These inequalities thus become redundant when $x_a = 1$ applies. However, when $x_a = 0$ holds, they impose the required dominance condition, in combination with (2), written for a. As previously remarked, formulation (1) appears to be the only one so far proposed for PEDP.

A Linear Programming (LP) relaxation for the formulation is defined as

$$\min \{\sum_{e \in E} c_e x_e : \mathbf{x} \in \mathcal{R}_0\} \tag{5}$$

and may be reinforced with inequalities

$$\sum_{e \in F} x_e + (1 - |F|) x_a \leq 1, \ a \in E, \ F \subseteq N(a), |F| \geq 2, \tag{6}$$

that contain (3) and are valid for PEDP. It should be pointed out that inequalities (6) subsume

$$x_b + (1 - x_a) + x_c \leq 2, \ a = \{i, j\} \in E, \ b \in \delta(i) \setminus \{a\}, \ c \in \delta(j) \setminus \{a\} \tag{7}$$

and

$$\sum_{e \in F \setminus \{a\}} x_e + (2 - |F|) x_a \leq 1, \ i \in V, \ F \subseteq \delta(i), \ a \in F, \ |F| \geq 3, \tag{8}$$

and that we have two basic reasons to highlight that fact. The first is that we came across inequalities (7) and (8) prior to reaching (6). The second and most important one is that, out of our computational experiments, the best performing BC algorithm we obtained is one that reinforces LP relaxation (5) with inequalities in (7) and (8), and not with the additional inequalities in (6). Although stronger LP relaxation bounds are attained by restricting oneself to using only the inequalities in (6), the resulting algorithm did not pay off in CPU time and RAM memory terms. Quoting specific figures for that, the BC algorithm based on the latter bounds took, on average, 71% more CPU time than its counterpart, in spite of exploring, on average, 10% less enumeration tree nodes. Furthermore, under the CPU time limit imposed, it failed to solve 6 instances of our test set. That compares with only one for its counterpart. For that reason and also due to space limitation constraints, computational results in Section 4 are restricted to those obtained by IP solver CPLEX, version 12.6 [15], used as a stand alone algorithm, and the BC algorithm based on the reinforced formulation

$$\min \{\sum_{e \in E} c_e x_e : \mathbf{x} \in \mathcal{R} \cap \mathbb{Z}^{|E|}\}, \tag{9}$$

where $\mathcal{R}$ is defined as the intersection of (2)-(4), (7), and (8).

2.0.1 Reinforcing the LP Relaxation with Cutting Planes

Out of the inequalities that define $\mathcal{R}$, the very first LP relaxation for our BC algorithm only uses (2)-(4) and (7). The number of inequalities in either (2) or (3) is precisely $|E|$. On the other hand, $O(|E|.|V|^2)$ inequalities (7) do exist for general graphs. These, however, would reduce to as low as $O(|E|)$, as it applies to the 3-regular graphs we will consider in Section 4. Finally, the exponentially many inequalities in (8) are separated *on the fly* by the algorithm and are only appended to LP relaxations when violated.

Assume that $\overline{\mathbf{x}}$ is an optimal solution to the LP relaxation problem in hand and that $\overline{G} = (\overline{V}, \overline{E})$ is its corresponding support graph, i.e., the subgraph of G defined by the edges $e \in E$ with $\overline{x}_e > 0$. We use notation $\overline{\delta}(i)$ to identify support graph edges incident to $i \in \overline{V}$. Additionally, inequalities (8) are rewritten as

$$\sum_{e \in F \setminus \{a\}} (x_e - x_a) \leq (1 - x_a),\ i \in V,\ F \subseteq \delta(i),\ a \in F,\ |F| \geq 3 \tag{10}$$

and for every $i \in \overline{V}$ with $|\overline{\delta}(i)| \geq 3$ and for every $a \in \overline{\delta}(i)$, a particular set F is defined. Namely, a set formed by a and all edges $e \in \overline{\delta}(i) \setminus \{a\}$ with non negative $(\overline{x}_e - \overline{x}_a)$ values. If $|F| \geq 3$ and $\sum_{e \in F \setminus \{a\}}(\overline{x}_e - \overline{x}_a) > (1 - \overline{x}_a)$ holds, inequality (10) corresponding to the triplet i, a, F is violated and is used to reinforce the relaxation. Conversely, if $|F| < 3$ applies, one then checks, in decreasing order of $(\overline{x}_e - \overline{x}_a)$ values, if there exists a subset $F_c \subset \overline{\delta}(i) \setminus F$ with $|F| + |F_c| = 3$ and $\sum_{e \in (F \cup F_c) \setminus \{a\}}(\overline{x}_e - \overline{x}_a) > (1 - \overline{x}_a)$. In case it does, $\sum_{e \in F \cup F_c \setminus \{a\}}(x_e - x_a) \leq (1 - x_a)$ is then violated by $\overline{x}$ and the inequality is used to reinforce the relaxation.

3 Test Instances

We initially tested our algorithms over general graphs $G = (V, E)$, randomly generated so as to enforce that a given pre-defined percentage density is attained. Graph connectivity was ensured by initializing the scheme with a randomly generated Hamiltonian path for G. Additional edges would then follow, until the desired graph density was reached. However, as pointed out before, after experimenting with literally hundreds of these graphs, they all contained no PEDS apart from the trivial one, E. That outcome prompted us to look for alternative generation schemes that would escape that pattern and produce more challenging instances.

PEDP is known to be NP-hard for some specific graph classes. Among them, bipartite graphs [21] and K_3-free 3-regular graphs [17], where K_3 stands for a complete graph on three vertices. We have thus devised generation schemes that produce particular graphs belonging to these two classes. Additionally, we also consider graphs from an additional class we call *Efficient Edge Domination* (EED) graphs. The reasoning behind that denomination is that these graphs, as we shall see, always contain a feasible EEDP solution. As such they also contain a feasible solution for PEDP, given that efficient edge domination implies perfect edge domination. Finally, as it will be indicated, that solution is a non trivial PEDP one.

EED graphs and the generation scheme we implemented for them will be described first, followed by our particular types of connected bipartite and K_3-free 3-regular graphs. Every graph we generate gives rise to a PEDP and a WPEDP instance, the latter instances with edge weights randomly drawn from the uniform distribution in the range $[1, 1000]$.

3.1 EED graphs

In order to generate graphs that contain non trivial PEDSs, we rely on the alternative and frequently used description of an efficient EDS as a Dominating Induced Matching (DIM) (see [1], for instance).

An induced matching of a graph $G = (V, E)$ is a matching $D \subseteq E$ such that no two edges of D are joined by an edge of $E \setminus D$. Additionally, D is a DIM if every edge of $E \setminus D$ shares exactly one vertex with an edge of D. Accordingly, a DIM and an EEDS define a same graph theoretical structure and the problem of determining whether or not G contains an EEDS is frequently cast in the literature in DIM terms. As indicate in [1], G contains a DIM if there exists a partition of V into vertex subsets V_1 and V_2 such that V_1 is an independent set and V_2 induces a matching of G. Furthermore, provided such a partition exists, a DIM is defined by the matching induced by V_2.

In addition to the conditions above, we also impose, for simplicity, that $n_2 = |V_2|$ is even, so that the resulting DIM is a perfect matching for the vertices of V_2. An EED graph, $G = (V, E)$, is thus defined as follows: (a) vertex set V that partitions into two non empty subsets V_1 and V_2, (b) $n_2 = |V_2|$ is even, (c) edge set E partitions into two disjoint subsets E_1 and E_2, (d) E_1 edges, numbering $m_1 = |E_1|$, all have an end vertex in V_1 and the other in V_2, and (e) E_2 edges, numbering $m_2 = |E_2|$, all have both end vertices in V_2. Finally, we will only consider connected EED graphs since, otherwise, one could simply decompose them into connected components and solve a separate PEDP for every component.

3.1.1 A Generation Procedure for Connected EED Graphs

A description follows of the procedure we use to generate connected EED graphs. It takes $n = |V|$ as an input and then randomly selects positive values for n_1 and n_2, with n_2 even. In doing so the number of edges in E_2, i.e., $m_2 = \frac{|V|}{2}$, is automatically fixed and the edges of E_2 are generated by randomly selecting the pairs of vertices of V_2 to be matched (see Figures 1(a) and 1(b) for an example in which $n = 10$, $n_1 = 6$, $n_2 = 4$ and $m = 12$). Next, the number of edges of G, i.e., $m = |E|$ with $m = m_1 + m_2$, is fixed by randomly selecting a positive integer m_1 in the range $[2 \cdot \min\{n_1, n_2\} + |n_1 - n_2| - 1, n_1 \cdot n_2]$. At that stage, the generation of E_1 edges is initiated with an initial focus on enforcing connectivity for G. Accordingly, $2 \cdot \min\{n_1, n_2\}$ edges are generated in association with an (inclusion-wise) maximal simple alternating path, every edge of it with an end vertex in V_1 and the other in V_2. If $|n_1 - n_2| \geq 2$ holds, some *leftover vertices* in either V_1 or V_2, whatever applies, would not belong to the alternating path. In that case, additional E_1 edges are generated, one edge for every leftover vertex, all edges with a same end point in the opposite vertex set. See Figure 1(a) for the required 8 simple path edges for our example plus an additional edge, highlighted in dotted lines, associated with the single existing leftover vertex. Notice that the E_1 edges generated so far suffice to ensure that G is connected. Finally the procedure terminates by generating the remaining edges of E_1, so that $|E_1| = m_1$ is attained. See Figure 1(b) for the three additional E_1 edges required by the example, all of them highlighted in dotted lines.

A total of 363 EED graphs were generated, with $|E|$ ranging from 30 to 300 and densities ranging from either 50% to 10% or 50% to 1%, depending on the values of n_1, n_2 and m.

(a) V_2 matching + path + leftover edge.

(b) Remaining edges of E_1.

Figure 1: EED graph generation.

3.2 Bipartite graphs

The (connectred) bipartite graphs we consider conform with those found within the EED graphs described above. More specifically, they are defined by the E_1 edges found within an EED graph. Accordingly, we partially follow the EED graph generation scheme to obtain them. The only difference between the two is that no edges with both end vertices in V_2 are now generated.

A total of 363 bipartite graphs were generated, with $|E|$ ranging from 30 to 300 and densities ranging from either 50% to 10% or 50% to 1%, depending on the values of n_1, n_2 and m.

3.3 Connected K_3-free 3-regular Graphs

We denote *CR graphs* the connected K_3-free 3-regular graphs $G = (V, E)$ considered in this subsection. Likewise any 3-regular graph, a CR graph must contain an even number of vertices, $n = |V|$. However, contrary to general 3-regular graphs, they decompose into three particular components. Among these, the most important is a certain type of Binary Tree (BT), to be called an *OBT*. The other two are: (a) an additional edge incident to two edge degree 2 OBT vertices and (b) a simple cycle restricted to containing only all OBT leaves. Cycles conforming to (b) are required to satisfy an additional constraint, to be described later on, and will be called *leaf-restricted cycles*. A description of these three components follows and will serve as a description for a CR graph.

Any OBT must contain all vertices of G. Additionally, as it will be explained next, the OBT topology is tied to the topology of the minimal Complete BT (CBT) that contains it. A $p \geq 0$ levels CBT corresponding to a BT with 2^k vertices at every level $0 \leq k \leq p$. Accordingly, the standard graphical representation of a BT, in terms of father vertices and their left an right sons, thus applies to a CBT and will also be enforced here for OBTs.

For reasons that will become evident later on, OBTs are also required to have $p \geq 4$ levels, all of them complete up to level $p - 1$. That condition, in conjunction with n even, implies that OBT level p is incomplete and contains an odd number of vertices, t, with $1 \leq t \leq 2^p - 1$. Finally, these t vertices are required to correspond exactly to the t leftmost p-level vertices at a p levels CBT. Accordingly, all definitions put together, an OBT, as we shall see, is uniquely defined by its corresponding number of vertices, n. For simplicity, assume that the vertices of G are indexed so that: (a) vertices 1 and n are respectively the OBT root and the right most vertex found at OBT level p, (b) vertex indices increase with OBT level and within a level, from left to right. A 30 vertices OBT is depicted in Figure 2, together with and additional edge connecting its two edge degree 2 vertices and a leaf-restricted cycle, both components highlighted in dotted lines.

From the definitions above, the following remarks would apply to an OBT: (a) n is an OBT leaf and is the rightmost vertex found at level p, (b) n has no brother and is the left son of f, a vertex located at level $p - 1$, (c) f and the OBT *root vertex*, 1, are the only two edge degree 2 vertices found in the OBT, and (d) remaining OBT vertices either have edge degree 1, as it applies to OBT leaves, or else, edge degree 3.

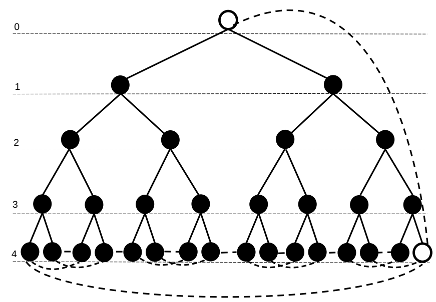


Figure 2: CR graph = OBT + single edge + leaf-restricted cycle.

Edge $\{1, f\}$ ensures that vertices 1 and f have edge degrees 3, in G, as required by a 3-regular graph such as G. It also explains why an OBT is required to have $p \geq 4$ levels. Notice that vertices 1 and f respectively belong to OBT levels 0 and $(p-1) \geq 3$. They are therefore more than 3 levels apart, what guarantees that no K_3 subgraph is thus induced by $\{1, f\}$.

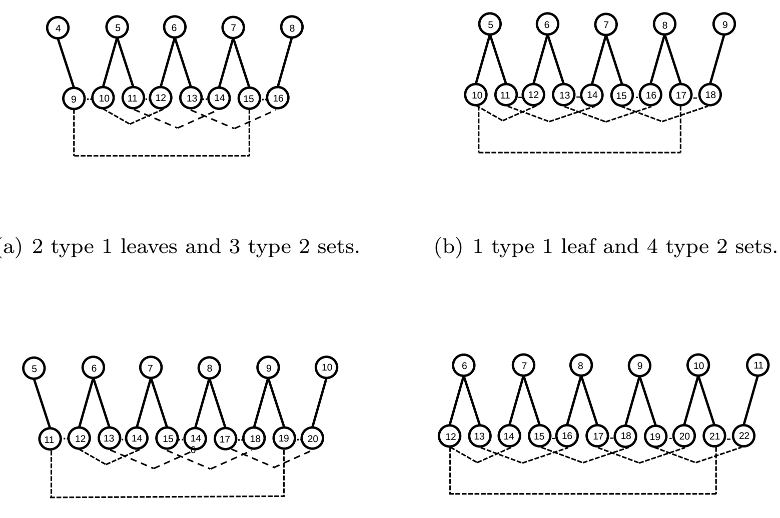
Let us now turn to leaf-constrained cycles. They ensure that OBT leaves have edge degrees 3, in G. Furthermore, they must not contain edges between leaves of a same father, what prevents K_3 subgraphs being formed. At this point, it only remains to be shown that a leaf-restricted cycle always exists for an OBT. We will show that through an example.

Under a conveniently chosen layout, Figure 3 displays the leaves for 16, 18, 20 and 22 vertices OBTs. In particular, 16 is the least possible number of vertices for an OBT. Examples of accompanying leaf-restricted cycles are also depicted in that figure, highlighted in dotted lines. Leaves are displayed side-by-side in increasing order of their indices, as if they all belonged to a same level. The 16 vertices OBT contains two *type 1* leaves and 3 sets of *type 2* leaves. A type 1 leaf defined as a son of f while a type 2 set is formed by exactly the two sons of a father other than f. Corresponding type 1 and type 2 figures for the 18 vertices OBT are respectively 2 and 4. On the other hand, the 20 vertices OBT, that conforms with the 16 vertices OBT, contains one type 1 leaf and four type 2 sets. Corresponding figures for the 22 vertices OBT, that conforms with the 18 vertices OBT, are respectively 1 and 5. As one may then infer from the cycle patterns we present, leaf-restricted cycles would always exist for OBTs with $n \geq 24$.

3.3.1 A CR Graph Generator

Our CR graph generator relies on leaf-restricted cycles similar to those displayed in Figure 3. It take n as an input, $n \geq 16$, and firstly generates an n vertices OBT. In doing so, edge $\{1, f\}$ is automatically defined. Additionally, depending on the number of type 1 leaves involved, one out of two leaf-restricted cycle building procedures is activated. If a single type 1 leaf exists, the cycle would involve the following edges: $\{\{f+2k, f+2k+1\} : k = 1, \dots l_2\}$, where l_2 is the number of type 2 brother pairs, $\{f+1, f+3\}$, $\{\{f+2k, f+2k+3\} : k = 1, \dots l_2 - 1\}$, and $\{f+1, f+2l_2\}$. Otherwise, if two type 1 leaves exist, one would then have: $\{\{f+2k-1, f+2k\} : k = 1, \dots, l_2+1\}$, $\{f+2, f+4\}$, $\{\{f+2k+1, f+2k+4\} : k = 1, \dots, l_2-1\}$, and $\{f+1, f+2l_2+1\}$. To introduce a degree of randomness into the procedure, a permutation of type 2 sets is randomly selected and the ordering it implies is then followed by the generation procedure. Namely, the positioning of type 1 leaves would be taken unaltered. However, type 2 sets would be taken in their permutation implied positions, and not in their true ones.

Ten CR graphs were generated, for $n \in \{20, 40, \dots, 200\}$ and $|E|$ ranging from 30 to 300. Instances are identified by their corresponding parameters $\{|V|, d, |E|\}$, where d stands for percentage graph density: $\{20, 15.79, 30\}$, $\{40, 7.69, 60\}$, $\{60, 5.08, 90\}$, $\{80, 3.8, 120\}$, $\{100, 3.03, 150\}$, $\{120, 2.52, 180\}$, $\{140, 2.16, 210\}$, $\{160, 1.89, 240\}$, $\{180, 1.68, 270\}$, and $\{200, 1.51, 300\}$.



(a) 2 type 1 leaves and 3 type 2 sets.

(b) 1 type 1 leaf and 4 type 2 sets.

(c) 2 type 1 leaves and 4 type 2 sets.

(d) 1 type 1 leaf and 5 type 2 sets.

Figure 3: Leaves and leaf-restricted cycles for $\{16, 18, 20, 22\}$-vertices OBTs.

4 Computational Experiments

Computational experiments are reported in this section for the BC algorithm described in Section 2, denoted BCA, and also for CPLEX IP solver, version 12.6 [15], operating as a stand alone algorithm and denoted CPLEX-BC, here. BCA was implemented in C, uses the LP module of CPLEX, and also relies on that solver for the management of the BC trees. However, it makes no use of the pre-processing, cutting plane, and heuristic modules CPLEX offers. BCA separates inequalities (8) at every node of the BC tree, under the procedure described in Subsection 2.0.1. An Intel Xeon X5675 based machine with 48 Gb of RAM memory and running at 3.07 GHz was used in the experiments. A CPU time limit of 7,200 seconds was imposed on every run and to simplify eventual future comparisons, the experiments were performed on a single CPU thread.

Results are presented by graph class and problem variant, i.e., PEDP or WPEDP. Furthermore, due to space limitation, we only show detailed results for some particular representative graphs for either variant. These are packed in two groups of five graphs, each. Each group highlighting one of the following BC performance parameters: (a) CPU time and (b) percentage *LP relaxation gap*, given by (optimal solution value - LP relaxation value)/(optimal solution value). In particular, for every different graph class, results are presented for the five PEDP and the five WPEDP graphs for which CPLEX-BC performed the best (resp. the worst). These results are accompanied by BCA results for the same set of instances. Additionally, some complementary information is also included. Namely, we replicate these experiments for the same graphs, but now tested for the alternative problem variant, PEDSP or WPEDP, whatever applies. In doing so one is able to compare, at least for extreme cases, the differences involved in solving the two problem variants over a same set of graphs.

For any of the tables that follow, a graph $G = (V, E)$ is identified by its corresponding $|E|$, density, d, and $|V|$. Algorithm statistics come next and indicate: optimal solution value, OPT; number of BC nodes, NN; CPU time in seconds, $t(s)$; LP relaxation value at the root node of the enumeration tree, LB_0; LP relaxation gap at the root node of the enumeration tree, GAP, and number of cuts separated by the BC algorithm, NUC. Following that, as complementary information, statistics are also presented for these same graphs, but solved for the alternative problem variant.

4.1 Results for EED graphs

Under the $7,200$ seconds CPU time limit imposed, both algorithms obtained optimal PEDP and WPEDP solutions for all 363 EED graphs. The average CPU time taken by CPLEX-BC to solve a PEDP instance was 0.52 seconds while its average LP relaxation gap was 0%. Corresponding figures for BCA are respectively 0.823 and 0%. For WPEDP instances, the corresponding CPLEX-BC figures are 0.91 and 15.8% while those for BCA are 5.01 and 37.6%. From the average results obtained, CPLEX-BC outperforms BCA for EED graphs.

Tables 1, 2 and 3 show extreme CPU time and LP relaxation gaps for EED graph instances. The first table applies to PEDP instances while the other two are for WPEDP.

Naturally integral LP relaxation solutions were obtained for all PEDP instances, either by CPLEX-BC or BCA. Accordingly, no table is used to display these results. Differently from that, fractional LP relaxation solutions were obtained by CPLEX-BC for all but 8 WPEDP instances. The corresponding figure for BCA is 9.

From the average results obtained and also from an analysis of the extreme cases shown in the tables, PEDP instances appear to be much easier to solve than their WPEDP counterparts, at least as far as EED graphs are concerned.

				PEDP						WPEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	290	33.68%	42	5	0	1.027	5	0	0%	3546	0	2.491	3544.63	0	0.04%
	220	39.22%	34	8	0	1.021	8	0	0%	4461	0	1.219	4415.57	0	1.02%
	240	34.14%	38	6	0	1.009	6	0	0%	3524	0	2.124	3470.42	0	1.52%
	70	40.94%	19	4	0	1.002	4	0	0%	1735	3	0.605	1610.91	0	7.15%
	260	19.61%	52	8	0	1.001	8	0	0%	4305	0	2.612	4261.14	0	1.02%
BCA results for the same instances above	290	33.68%	42	5	0	2.672	5	0	0%	3546	66	101.56	1411.13	1049	60.21%
	220	39.22%	34	8	0	1.167	8	0	0%	4461	72	14.678	1970.1	331	55.84%
	240	34.14%	38	6	0	2.267	6	0	0%	3524	3	6.664	1798.37	0	48.97%
	70	40.94%	19	4	0	0.369	4	0	0%	1735	6	0.467	1195.67	0	31.09%
	260	19.61%	52	8	0	0.977	8	0	0%	4305	49	16.255	2715.94	100	36.91%
Five lowest CPU times for CPLEX-BC	70	45.75%	18	3	0	0.019	3	0	0%	1338	0	0.638	1193.17	0	10.82%
	120	24.19%	32	4	0	0.021	4	0	0%	1709	0	0.743	1690.67	0	1.07%
	180	24.29%	39	8	0	0.027	8	0	0%	2341	0	0.324	2303.84	0	1.59%
	150	34.48%	30	6	0	0.027	6	0	0%	2692	0	0.957	2663.24	0	1.07%
	70	51.47%	17	4	0	0.031	4	0	0%	2177	5	0.786	1751.73	0	19.53%
BCA results for the same instances above	70	45.75%	18	3	0	0.403	3	0	0%	1338	5	0.643	937.261	48	29.95%
	120	24.19%	32	4	0	0.595	4	0	0%	1709	3	1.085	1293.56	43	24.31%
	180	24.29%	39	8	0	1.01	8	0	0%	2341	3	1.923	1915.72	0	18.17%
	150	34.48%	30	6	0	1.054	6	0	0%	2692	34	3.626	1866.73	90	30.66%
	70	51.47%	17	4	0	0.438	4	0	0%	2177	18	0.847	1213.77	0	44.25%

Table 1: Extreme PEDP cases and WPEDP complementary information.

				WPEDP						PEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	280	24.82%	48	4754	3	3.819	4453.95	0	6.31%	9	0	0.257	9.00	0	0%
	270	24.98%	47	4513	3	3.715	4235.26	0	6.15%	10	0	0.619	10.00	0	0%
	270	38.41%	38	4323	0	3.621	4258.22	0	1.5%	8	0	0.780	8.00	0	0%
	300	14.88%	64	5971	1	3.556	5604.03	0	6.15%	11	0	0.545	11.00	0	0%
	290	24.66%	49	4896	1	3.320	4663.22	0	4.75%	11	0	0.917	11.00	0	0%
BCA results for the same instances above	280	24.82%	48	4754	39	19.695	2449.23	71	48.48%	9	0	2.168	9.00	0	0%
	270	24.98%	47	4513	38	18.508	2268.39	83	49.74%	10	0	0.255	10.00	0	0%
	270	38.41%	38	4323	42	16.719	1691.68	28	60.87%	8	0	3.085	8.00	0	0%
	300	14.88%	64	5971	41	21.561	3464.94	272	41.97%	11	0	0.585	11.00	0	0%
	290	24.66%	49	4896	37	17.887	3274.22	50	33.12%	11	0	2.568	11.00	0	0%
Five lowest CPU times for CPLEX-BC	220	0.99%	211	4841	0	0.023	2207.91	0	54.39%	10	0	0.655	10.00	0	0%
	100	24.63%	29	842	0	0.033	799.35	0	5.07%	2	0	0.492	2.00	0	0%
	190	9.73%	63	4996	0	0.063	4614.37	0	7.64%	10	0	0.344	10.00	0	0%
	230	1%	215	6291	0	0.086	2498.20	0	60.29%	11	0	0.121	11.00	0	0%
	170	24.18%	38	1787	0	0.096	1725.17	0	3.46%	5	0	0.661	5.00	0	0%
BCA results for the same instances above	220	0.99%	211	4841	63	0.709	2521.84	21	47.91%	10	0	0.131	10.00	0	0%
	100	24.63%	29	842	3	0.543	317.57	0	62.28%	2	0	0.792	2.00	0	0%
	190	9.73%	63	4996	29	2.385	3393.05	33	32.08%	10	0	0.967	10.00	0	0%
	230	1%	215	6291	91	0.687	3126.17	28	50.31%	11	0	0.831	11.00	0	0%
	170	24.18%	38	1787	3	1.790	1464.75	0	18.03%	5	0	0.931	5.00	0	0%

Table 2: Extreme WPEDP cases and PEDP complementary information.

				WPEDP						PEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest gaps for CPLEX-BC	280	0.99%	238	2974	0	0.586	204.85	0	93.11%	4	0	0.937	4.00	0	0%
	140	2.95%	98	1182	0	0.482	157.71	0	86.66%	2	0	0.494	2.00	0	0%
	280	3.99%	119	2558	0	0.854	342.20	0	86.62%	4	0	0.045	4.00	0	0%
	260	1.99%	162	1759	0	0.593	254.47	0	85.53%	3	0	0.772	3.00	0	0%
	300	4%	123	3736	0	0.281	566.61	0	84.83%	5	0	0.346	5.00	0	0%
BCA results for the same instances above	280	0.99%	238	2974	131	1.795	382.24	55	87.15%	4	0	0.123	4.00	0	0%
	140	2.95%	98	1182	60	0.590	224.01	24	81.05%	2	0	0.238	2.00	0	0%
	280	3.99%	119	2558	61	4.995	411.88	43	83.9%	4	0	0.711	4.00	0	0%
	260	1.99%	162	1759	75	1.662	320.46	42	81.78%	3	0	0.923	3.00	0	0%
	300	4%	123	3736	87	6.182	710.71	57	80.98%	5	0	0.385	5.00	0	0%
Five lowest gaps for CPLEX-BC	30	19.61%	18	1480	0	0.133	1480.00	0	0%	4	0	0.443	4.00	0	0%
	40	23.39%	19	1475	0	0.233	1475.00	0	0%	4	0	0.519	4.00	0	0%
	40	9.85%	29	206	0	0.197	206.00	0	0%	1	0	0.493	1.00	0	0%
	50	29.24%	19	867	0	0.406	867.00	0	0%	4	0	0.651	4.00	0	0%
	70	9.96%	38	11	0	0.150	11.00	0	0%	1	0	0.900	1.00	0	0%
BCA results for the same instances above	30	19.61%	18	1480	0	0.109	1480.00	0	0%	4	0	0.332	4.00	0	0%
	40	23.39%	19	1475	0	0.314	1475.00	0	0%	4	0	0.463	4.00	0	0%
	40	9.85%	29	206	0	0.237	206.00	0	0%	1	0	0.409	1.00	0	0%
	50	29.24%	19	867	0	0.646	867.00	0	0%	4	0	0.648	4.00	0	0%
	70	9.96%	38	11	0	0.782	11.00	0	0%	1	0	0.108	1.00	0	0%

Table 3: Extreme WPEDP cases and PEDP complementary information.

4.2 Results for bipartite graphs

Under the 7,200 seconds CPU time limit imposed, optimal PEDP and WPEDP solutions were obtained for all 363 EED graphs by both algorithms. The average CPU time taken by CPLEX-BC to solve a PEDP instance was 1.06 seconds while the average LP relaxation gap it attained was 40.7%. Corresponding figures for BCA are respectively 56.3 and 43.3%. For WPEDP instances, the corresponding CPLEX-BC figures are 1.68 and 43.3%, while those for BCA are 25.3 and 64.3%. From average bipartite graph results, CPLEX-BC clearly outperforms BCA for these graphs.

Tables 4, 5, 6 and 7 show extreme CPU time and LP relaxation gaps for bipartite graph instances. From average bipartite graph results and also from an analysis of the extreme cases shown in the tables, WPEDP instances appear to be easier to solve than their PEDP counterparts. Accordingly, the picture that emerges here is the opposite of that we have for EED graphs.

				PEDP						WPEDP					
	$\|E\|$	*dens.*(%)	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	270	1%	233	82	1791	4.782	75.63	0	7.77%	36490	661	3.803	34228.50	0	6.2%
	260	1.99%	162	256	427	4.617	46.98	0	81.65%	126532	133	2.832	21867.50	0	82.72%
	290	2.98%	140	42	421	3.624	34.42	0	18.04%	20464	628	4.907	13380.10	0	34.62%
	290	24.66%	49	290	129	3.211	12.47	0	95.7%	153572	95	5.159	3909.79	0	97.45%
	300	4.91%	111	300	35	3.137	29.40	0	90.2%	152468	126	4.075	10545.50	0	93.08%
BCA results for the same instances above	270	1%	233	82	27370	45.390	71.07	231	13.33%	36490	20848	29.980	27352.10	204	25.04%
	260	1.99%	162	256	6512	51.136	46.03	641	82.02%	126532	11076	65.697	16482.40	591	86.97%
	290	2.98%	140	42	8711	209.645	32.62	829	22.33%	20464	14254	230.704	9792.03	744	52.15%
	290	24.66%	49	290	409	517.736	12.15	1922	95.81%	153572	253	192.540	3166.54	1875	97.94%
	300	4.91%	111	300	1222	113.732	29.15	750	90.28%	152468	863	65.100	9121.98	785	94.02%
Five lowest CPU times for CPLEX-BC	30	25%	16	7	21	0.037	4.43	0	36.66%	4313	17	0.613	2761.13	0	35.98%
	30	28.57%	15	3	0	0.047	2.85	0	4.91%	1173	0	0.623	1172.71	0	0.02%
	50	36.76%	17	4	0	0.057	3.40	0	15%	1220	0	0.120	1108.21	0	9.16%
	300	1.99%	174	15	0	0.058	15.00	0	0%	3597	0	0.370	3390.92	0	5.73%
	190	4.96%	88	11	0	0.089	11.00	0	0%	1796	0	0.109	1353.93	0	24.61%
BCA results for the same instances above	30	25%	16	7	19	0.354	4.02	16	42.52%	4313	17	0.603	1593.24	24	63.06%
	30	28.57%	15	3	5	0.370	2.67	3	11.11%	1173	17	0.636	799.02	32	31.88%
	50	36.76%	17	4	26	0.648	3.26	0	18.44%	1220	13	0.933	620.82	5	49.11%
	300	1.99%	174	15	0	0.302	15.00	1	0%	3597	4160	22.286	1856.22	64	48.4%
	190	4.96%	88	11	15	0.734	10.15	15	7.73%	1796	105	1.622	914.59	28	49.08%

Table 4: Extreme PEDP cases and WPEDP complementary information.

				PEDP						WPEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest gaps for CPLEX-BC	260	33.33%	40	260	80	2.141	8.91	0	96.57%	134029	69	2.916	2545.52	0	98.1%
	300	38.46%	40	300	15	2.065	10.61	0	96.46%	156804	75	3.039	2338.67	0	98.51%
	260	39.04%	37	260	23	1.050	9.26	0	96.44%	125767	121	3.456	2354.67	0	98.13%
	300	34.84%	42	300	15	2.077	10.78	0	96.41%	149764	15	4.890	2910.39	0	98.06%
	290	29.29%	45	290	12	1.483	10.68	0	96.32%	140961	261	6.251	2885.52	0	97.95%
0.00 0.00 BCA results for the same instances above	260	33.33%	40	260	219	509.720	8.69	953	96.66%	134029	125	103.271	1848.28	1944	98.62%
	300	38.46%	40	300	71	250.260	10.33	822	96.56%	156804	97	148.434	2183.34	1413	98.61%
	260	39.04%	37	260	137	362.016	9.01	1350	96.53%	125767	87	97.805	1699.47	1481	98.65%
	300	34.84%	42	300	259	745.003	10.50	2103	96.5%	149764	81	130.293	2206.02	1259	98.53%
	290	29.29%	45	290	133	534.155	10.39	2207	96.42%	140961	163	147.143	1869.58	1552	98.67%
Five lowest gaps for CPLEX-BC	30	10%	25	6	0	0.917	6.00	0	0%	1749	0	0.415	1719.79	0	1.67%
	40	51.28%	13	5	0	0.548	5.00	0	0%	1707	16	0.459	1306.72	0	23.45%
	40	14.49%	24	2	0	0.183	2.00	0	0%	316	0	0.854	150.48	0	52.38%
	50	14.25%	27	4	0	0.666	4.00	0	0%	1269	0	0.613	1143.42	0	9.9%
	70	3.95%	60	3	0	0.897	3.00	0	0%	240	0	0.140	240.00	0	0%
BCA results for the same instances above	30	10%	25	6	10	0.293	5.25	8	12.5%	1749	5	0.533	1509.79	0	13.68%
	40	51.28%	13	5	17	0.907	3.33	3	33.33%	1707	16	0.162	943.11	30	44.75%
	40	14.49%	24	2	0	0.521	2.00	0	0%	316	7	0.790	211.14	0	33.18%
	50	14.25%	27	4	5	0.042	3.73	1	6.82%	1269	34	0.310	646.31	2	49.07%
	70	3.95%	60	3	2	0.608	3.00	0	0%	240	0	0.960	240.00	0	0%

Table 5: Extreme PEDP cases and WPEDP complementary information.

				WPEDP						PEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	300	24.49%	50	150578	167	6.801	3535.04	0	97.65%	300	209	2.590	11.62	0	96.13%
	300	19.48%	56	148208	77	6.605	4367.37	0	97.05%	300	59	2.583	14.64	0	95.12%
	270	3.98%	117	11380	573	6.455	8032.53	0	29.42%	27	46	1.334	23.13	0	14.32%
	290	29.29%	45	140961	261	6.251	2885.52	0	97.95%	290	12	1.483	10.68	0	96.32%
	290	14.85%	63	147212	161	6.166	4593.26	0	96.88%	290	55	2.379	15.22	0	94.75%
BCA results for the same instances above	300	24.49%	50	150578	225	192.844	2426.91	1179	98.39%	300	263	625.378	11.33	1691	96.22%
	300	19.48%	56	148208	235	139.109	3436.92	1632	97.68%	300	325	412.606	14291.00	1976	95.24%
	270	3.98%	117	11380	2246	65.594	5344.95	567	53.03%	27	2266	64.464	22.45	470	16.87%
	290	29.29%	45	140961	163	147.143	1869.58	1552	98.67%	290	133	534.155	10.39	2207	96.42%
	290	14.85%	63	147212	261	118.512	3383.94	953	97.7%	290	593	456.276	14.85	1422	94.88%
Five lowest CPU times for CPLEX-BC	40	29.41%	17	575	0	0.036	451.97	0	21.4%	3	0	0.334	2.99	0	0.38%
	180	2.95%	111	2146	0	0.051	2081.77	0	2.99%	12	0	0.164	12.00	0	0%
	220	0.99%	211	3957	0	0.074	3876.50	0	2.03%	26	0	0.715	26.00	0	0%
	120	9.8%	50	434	0	0.084	429.76	0	0.98%	3	0	1.010	2.97	0	1.15%
	90	3%	78	8620	0	0.095	8500.88	0	1.38%	22	0	0.957	21.74	0	1.16%
BCA results for the same instances above	40	29.41%	17	575	4	0.918	505.87	1	12.02%	3	5	0.659	2.64	12	11.97%
	180	2.95%	111	2146	209	1.056	1524.03	11	28.98%	12	0	0.491	12.00	4	0%
	220	0.99%	211	3957	23	0.416	3744.02	0	5.38%	26	1	0.386	26.00	0	0%
	120	9.8%	50	434	20	0.863	110.40	1	74.56%	3	10	0.840	2.91	1	2.9%
	90	3%	78	8620	143	0.237	6559.05	19	23.91%	22	53	0.897	19.82	28	9.91%

Table 6: Extreme WPEDP cases and PEDP complementary information.

	$\lvert E\rvert$	$dens.(\%)$	$\lvert V\rvert$	WPEDP						PEDP					
				OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest gaps for CPLEX-BC	300	38.46%	40	156804	75	3.039	2338.67	0	98.51%	300	15	2.065	10.61	0	96.46%
	280	19.57%	54	139915	95	3.645	2490.39	0	98.22%	280	179	2.932	10.47	0	96.26%
	260	39.04%	37	125767	121	3.456	2354.67	0	98.13%	260	23	1.050	9.26	0	96.44%
	260	33.33%	40	134029	69	2.916	2545.52	0	98.1%	260	80	2.141	8.91	0	96.57%
	300	34.84%	42	149764	15	4.890	2910.39	0	98.06%	300	15	2.077	10.78	0	96.41%
BCA results for the same instances above	300	38.46%	40	156804	97	148.434	2183.34	1413	98.61%	300	71	250.260	10329.00	822	96.56%
	280	19.57%	54	139915	453	144.288	1717.67	695	98.77%	280	549	434.465	10.26	1087	96.34%
	260	39.04%	37	125767	87	97.805	1699.47	1481	98.65%	260	137	362.016	9.01	1350	96.53%
	260	33.33%	40	134029	125	103.271	1848.28	1944	98.62%	260	219	509.720	8.69	953	96.66%
	300	34.84%	42	149764	81	130.293	2206.02	1259	98.53%	300	259	745.003	10496.00	2103	96.5%
Five lowest gaps for CPLEX-BC	70	3.95%	60	240	0	0.140	240.00	0	0%	3	0	0.897	3.00	0	0%
	110	1.98%	106	131	0	0.221	131.00	0	0%	4	0	0.877	4.00	0	0%
	120	19.05%	36	701	0	0.889	701.00	0	0%	4	0	0.309	3.74	0	6.62%
	120	1.97%	111	291	0	0.481	291.00	0	0%	7	0	0.772	7.00	0	0%
	140	4.91%	76	978	0	1.036	978.00	0	0%	6	0	0.246	6.00	0	0%
BCA results for the same instances above	70	3.95%	60	240	0	0.960	240.00	0	0%	3	2	0.608	3.00	0	0%
	110	1.98%	106	131	0	0.135	131.00	0	0%	4	0	0.654	4.00	0	0%
	120	19.05%	36	701	25	0.927	302.47	4	56.85%	4	63	1.463	3.67	0	8.32%
	120	1.97%	111	291	0	0.457	291.00	0	0%	7	4	0.699	7.00	0	0%
	140	4.91%	76	978	58	0.728	369.66	6	62.2%	6	0	0.871	6.00	1	0%

Table 7: Extreme WPEDP cases and PEDP complementary information.

4.3 Results for 3-regular graphs

Under the 7,200 seconds CPU time limit imposed, optimal PEDP and WPEDP solutions were obtained for all 10 CR graphs by both algorithms. The average CPU time taken by CPLEX-BC to solve a PEDP instance was 11.85 seconds while the average LP relaxation gap it attained was 63%. Corresponding figures for BCA are respectively 18.95 and 64.85%. For WPEDP instances, the corresponding CPLEX-BC figures are 17.68 and 65.93%, while those for BCA are 38.80 and 72.80%. From average CR graph results, CPLEX-BC clearly outperforms BCA for these graphs. Furthermore, CR graph instances appear to be the hardest to solve in our test set.

Tables 8, 9, 10 and 11 show extreme CPU time and LP relaxation gaps for CR graph instances.

				PEDP						WPEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	270	1.68%	180	108	13729	81.469	54.94	0	49.13%	54409	9178	72.969	25195.00	0	53.69%
	240	1.89%	160	108	2096	12.329	50.90	0	52.87%	58105	3728	22.881	24481.40	0	57.87%
	300	1.51%	200	222	1418	10.968	65.78	0	70.37%	113030	4935	64.715	30576.60	0	72.95%
	210	2.16%	140	210	762	4.531	44.52	0	78.8%	110808	825	6.474	21488.10	0	80.61%
	150	3.03%	100	60	795	2.883	31.72	0	47.14%	28504	708	3.297	15289.30	0	46.36%
BCA results for the same instances above	270	1.68%	180	108	28307	80.490	54.00	487	50%	54409	32959	110.797	20764.00	506	61.84%
	240	1.89%	160	108	9615	27.853	48.00	398	55.56%	58105	59569	151.613	20090.60	452	65.42%
	300	1.51%	200	222	8591	35.889	60.00	521	72.97%	113030	20796	92.619	24740.30	560	78.11%
	210	2.16%	140	210	6071	20.050	42.00	374	80%	110808	5624	14.967	17072.60	371	84.59%
	150	3.03%	100	60	6523	12.291	30.00	262	50%	28504	2725	4.692	11912.20	253	58.21%
Five lowest CPU times for CPLEX-BC	90	5.08%	60	90	728	0.735	19.03	0	78.86%	45141	484	1.222	8389.91	0	81.41%
	30	15.79%	20	30	53	0.869	6.48	0	78.39%	15218	85	0.324	2873.61	0	81.12%
	60	7.69%	40	60	269	1.046	12.71	0	78.81%	31511	389	0.604	5930.93	0	81.18%
	120	3.8%	80	60	502	1.581	24.76	0	58.74%	29526	529	1.398	10872.90	0	63.18%
	180	2.52%	120	60	159	2.057	37.80	0	37%	28055	424	2.920	16560.00	0	40.97%
BCA results for the same instances above	90	5.08%	60	90	2427	2.707	18.00	172	80%	45141	1277	2.102	6964.47	159	84.57%
	30	15.79%	20	30	53	0.854	6.00	31	80%	15218	71	0.421	2310.42	36	84.82%
	60	7.69%	40	60	443	1.192	12.00	99	80%	31511	522	0.794	4550.62	105	85.56%
	120	3.8%	80	60	1532	3.164	24.00	205	60%	29526	2327	3.379	8397.46	217	71.56%
	180	2.52%	120	60	1907	4.987	36.00	267	40%	28055	2572	6.600	13085.50	289	53.36%

Table 8: Extreme PEDP cases and WPEDP complementary information.

				PEDP						WPEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest gaps for CPLEX-BC	90	5.08%	60	90	728	0.735	19.03	0	78.86%	45141	484	1.222	8389.91	0	81.41%
	60	7.69%	40	60	269	1.046	12.71	0	78.81%	31511	389	0.604	5930.93	0	81.18%
	210	2.16%	140	210	762	4.531	44.52	0	78.8%	110808	825	6.474	21488.10	0	80.61%
	30	15.79%	20	30	53	0.869	6.48	0	78.39%	15218	85	0.324	2873.61	0	81.12%
	300	1.51%	200	222	1418	10.968	65.78	0	70.37%	113030	4935	64.715	30576.60	0	72.95%
BCA results for the same instances above	90	5.08%	60	90	2427	2.707	18.00	172	80%	45141	1277	2.102	6964.47	159	84.57%
	60	7.69%	40	60	443	1.192	12.00	99	80%	31511	522	0.794	4550.62	105	85.56%
	210	2.16%	140	210	6071	20.05	42.00	374	80%	110808	5624	14.967	17072.60	371	84.59%
	30	15.79%	20	30	53	0.854	6.00	31	80%	15218	71	0.421	2310.42	36	84.82%
	300	1.51%	200	222	8591	35.889	60.00	521	72.97%	113030	20796	92.619	24740.30	560	78.11%
Five lowest gaps for CPLEX-BC	180	2.52%	120	60	159	2.057	37.80	0	37%	28055	424	2.92	16560.00	0	40.97%
	150	3.03%	100	60	795	2.883	31.72	0	47.14%	28504	708	3.297	15289.30	0	46.36%
	270	1.68%	180	108	13729	81469	54.94	0	49.13%	54409	9178	72.969	25195.00	0	53.69%
	240	1.89%	160	108	2096	12.329	50.90	0	52.87%	58105	3728	22.881	24481.40	0	57.87%
	120	3.8%	80	60	502	1.581	24.76	0	58.74%	29526	529	1.398	10872.90	0	63.18%
BCA results for the same instances above	180	2.52%	120	60	1907	4.987	36.00	267	40%	28055	2572	6.6	13085.50	289	53.36%
	150	3.03%	100	60	6523	12.291	30.00	262	50%	28504	2725	4.692	11912.20	253	58.21%
	270	1.68%	180	108	28307	80.49	54.00	487	50%	54409	32959	110.797	20764.00	506	61.84%
	240	1.89%	160	108	9615	27.853	48.00	398	55.56%	58105	59569	151.613	20090.60	452	65.42%
	120	3.8%	80	60	1532	3.164	24.00	205	60%	29526	2327	3.379	8397.46	217	71.56%

Table 9: Extreme PEDP cases and WPEDP complementary information.

				WPEDP						PEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest CPU times for CPLEX-BC	270	1.68%	180	54409	9178	72.969	25195.00	0	53.69%	108	13729	81.469	54.94	0	49.13%
	300	1.51%	200	113030	4935	64.715	30576.60	0	72.95%	222	1418	10.968	65.78	0	70.37%
	240	1.89%	160	58105	3728	22.881	24481.40	0	57.87%	108	2096	12.329	50.90	0	52.87%
	210	2.16%	140	110808	825	6.474	21488.10	0	80.61%	210	762	4.531	44.52	0	78.8%
	150	3.03%	100	28504	708	3.297	15289.30	0	46.36%	60	795	2.883	31.72	0	47.14%
BCA results for the same instances above	270	1.68%	180	54409	32959	110.797	20764.00	506	61.84%	108	28307	80.49	54.00	487	50%
	300	1.51%	200	113030	20796	92.619	24740.30	560	78.11%	222	8591	35.889	60.00	521	72.97%
	240	1.89%	160	58105	59569	151.613	20090.60	452	65.42%	108	9615	27.853	48.00	398	55.56%
	210	2.16%	140	110808	5624	14.967	17072.60	371	84.59%	210	6071	20.05	42.00	374	80%
	150	3.03%	100	28504	2725	4.692	11912.20	253	58.21%	60	6523	12.291	30.00	262	50%
Five lowest CPU times for CPLEX-BC	30	15.79%	20	15218	85	0.324	2873.61	0	81.12%	30	53	0.869	6.48	0	78.39%
	60	7.69%	40	31511	389	0.604	5930.93	0	81.18%	60	269	1.046	12.71	0	78.81%
	90	5.08%	60	45141	484	1.222	8389.91	0	81.41%	90	728	0.735	19.03	0	78.86%
	120	3.8%	80	29526	529	1.398	10872.90	0	63.18%	60	502	1.581	24.76	0	58.74%
	180	2.52%	120	28055	424	2.920	16560.00	0	40.97%	60	159	2.057	37.80	0	37%
BCA results for the same instances above	30	15.79%	20	15218	71	0.421	2310.42	36	84.82%	30	53	0.854	6.00	31	80%
	60	7.69%	40	31511	522	0.794	4550.62	105	85.56%	60	443	1.192	12.00	99	80%
	90	5.08%	60	45141	1277	2.102	6964.47	159	84.57%	90	2427	2.707	18.00	172	80%
	120	3.8%	80	29526	2327	3.379	8397.46	217	71.56%	60	1532	3.164	24.00	205	60%
	180	2.52%	120	28055	2572	6.6	13085.50	289	53.36%	60	1907	4.987	36.00	267	40%

Table 10: Extreme WPEDP cases and PEDP complementary information.

				WPEDP						PEDP					
	$\|E\|$	$dens.(\%)$	$\|V\|$	OPT	NN	$t(s)$	LB_0	NUC	GAP	OPT	NN	$t(s)$	LB_0	NUC	GAP
Five highest gaps for CPLEX-BC	90	5.08%	60	45141	484	1.220	8389.91	0	81.41%	90	728	0.730	19.03	0	78.86%
	60	7.69%	40	31511	389	0.600	5930.93	0	81.18%	60	269	1.050	12.71	0	78.81%
	30	15.79%	20	15218	85	0.320	2873.61	0	81.12%	30	53	0.870	6.48	0	78.39%
	210	2.16%	140	110808	825	6.470	21488.10	0	80.61%	210	762	4.530	44.52	0	78.8%
	300	1.51%	200	113030	4935	64.720	30576.60	0	72.95%	222	1418	10.970	65.78	0	70.37%
BCA results for the same instances above	90	5.08%	60	45141	1277	2.102	6964.47	159	84.57%	90	2427	2.707	18.00	172	80%
	60	7.69%	40	31511	522	0.794	4550.62	105	85.56%	60	443	1.192	12.00	99	80%
	30	15.79%	20	15218	71	0.421	2310.42	36	84.82%	30	53	0.854	6.00	31	80%
	210	2.16%	140	110808	5624	14.967	17072.60	371	84.59%	210	6071	20.05	42.00	374	80%
	300	1.51%	200	113030	20796	92.619	24740.30	560	78.11%	222	8591	35.889	60.00	521	72.97%
Five lowest gaps for CPLEX-BC	180	2.52%	120	28055	424	2.920	16560.00	0	40.97%	60	159	2.060	37.80	0	37%
	150	3.03%	100	28504	708	3.300	15289.30	0	46.36%	60	795	2.880	31.72	0	47.14%
	270	1.68%	180	54409	9178	72.970	25195.00	0	53.69%	108	13729	81.470	54.94	0	49.13%
	240	1.89%	160	58105	3728	22.880	24481.40	0	57.87%	108	2096	12.330	50.90	0	52.87%
	120	3.8%	80	29526	529	1.400	10872.90	0	63.18%	60	502	1.580	24.76	0	58.74%
BCA results for the same instances above	180	2.52%	120	28055	2572	6.6	13085.50	289	53.36%	60	1907	4.987	36.00	267	40%
	150	3.03%	100	28504	2725	4.692	11912.20	253	58.21%	60	6523	12.291	30.00	262	50%
	270	1.68%	180	54409	32959	110.797	20764.00	506	61.84%	108	28307	80.49	54.00	487	50%
	240	1.89%	160	58105	59569	151.613	20090.60	452	65.42%	108	9615	27.853	48.00	398	55.56%
	120	3.8%	80	29526	2327	3.379	8397.46	217	71.56%	60	1532	3.164	24.00	205	60%

Table 11: Extreme WPEDP cases and PEDP complementary information.

5 Conclusions

The paper suggests and investigates what appears to be the the very first formulation proposed for the Perfect Edge Domination Problem. So far, the problem has been studied mostly in computational complexity terms and no exact algorithm, Combinatorial or IP based, was previously suggested for it. Based on our formulation, two Branch-and-Cut algorithms were designed, implemented and tested for the problem. Extensive computational tests are reported for the best performing of the two, together with results obtained by IP solver CPLEX, used as a stand alone Branch-and-Cut algorithm. Test instances used in these experiments originate from quite elaborated procedures aimed at generating challenging instances for the problem.

From the computational results obtained, one conclusion that may be drawn is that there exists plenty of room for investigating valid inequalities to reinforce our PEDP formulation. With the single exception of non weighted EED graph test instances, LP relaxation gaps for the remaining instances are, on the average, quite high. Furthermore, the tailor made Branch-and-Cut algorithms we tested lagged well behind CPLEX in terms of CPU time and, to a lesser extent, LP relaxation gaps. The first part of the remark should probably be taken with caution since all CPLEX pre-processing, cutting plane, and heuristic modules were kept switched off for our algorithms. However, it is quite surprising that the tailor-made cutting planes we use are outperformed by the generic ones used by CPLEX. Accordingly, attempting to lift known inequalities for the Set Covering polytope [3] is an alternative one should probably investigate. However, these inequalities have been of limited use, so far, even for the Set Covering Problem itself. The investigation of PEDP heuristics is also another candidate research topic to pursue.

Finally, taking a broader view of the subject, we also plan to investigate formulations and exact algorithms for the Minimum Edge Dominating Set Problem and the Efficient Edge Domination Problem.

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