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Approximating weighted induced matchings

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ABSTRACT

An induced matching is a matching where no two edges are connected by a third edge. Finding a maximum induced matching on graphs with maximum degree Δ, for $\Delta \geq 3$, is known to be NP-complete. In this work we consider the weighted version of this problem, which has not been extensively studied in the literature. We devise an almost tight fractional local ratio algorithm with approximation ratio Δ, which proves to be effective also in practice. Furthermore, we show that a simple greedy algorithm applied to $K_{1,k}$-free graphs yields an approximation ratio $2k-3$. We explore the behavior of this algorithm on subclasses of chair-free graphs and we show that it yields an approximation ratio k when restricted to $(K_{1,k}$, chair)-free graphs.

1. Introduction

Let $G = (V, E)$ be a simple undirected graph. A subset $\mathcal{M} \subseteq E$ is an *induced matching* if $\mathcal{M}$ is a matching and no edge of E connects two edges of $\mathcal{M}$. The Maximum Induced Matching (MIM) problem is to find an induced matching of maximum cardinality. Note that the size of a MIM is the same as the size of a maximum independent set of $L(G)^2$, the square of the line graph of G. To our best knowledge, MIM was introduced in [30] among some generalizations of the classical maximum matching problem, although it was referred to as "risk-free" marriage problem. The authors proved it to be NP-complete when restricted to bipartite graphs of degree at most 4. Independently, this problem was introduced in [3], where a polynomial time algorithm for chordal graphs was given. Thereafter MIM was polynomially solved on trees [12], circular-arc graphs [14], trapezoid, k-interval dimension, cocomparability graphs [15], $(\mathrm{Star}_{1,2,3}, \mathrm{Sun}_4)$-free bipartite graphs [26], weakly chordal graphs [5] and on bounded treewidth graphs [27]. The relations between the families of G and $L(G)^2$ were exploited to conclude that MIM is polynomially solvable for polygon–circle, AT-free, and filament-interval graphs; where the latter contains cocomparability, circle, circular-arc, chordal and outerplanar graphs [4]. A polynomial time algorithm for hhd-free graphs and a linear time algorithm for a subclass of hhd-free graphs which is a more general class than chordal graphs were given in [24]. The problem was proven to be NP-complete on bipartite graphs of maximum degree 3, C_4-free bipartite graphs [26], d-regular graphs for $d \geq 5$, line graphs (which implies that MIM is NP-complete on claw-free graphs and chair-free graphs) [23] and on cubic planar graphs [11,22]. Note that since any claw-free graph is $K_{1,k}$-free for $k \geq 3$ and cannot contain a chair, it follows that MIM is NP-complete on $(K_{1,k}$, chair)-free graphs — a class that we address in this work. Several algorithms were given for classes related to AT-free graphs, in particular it was shown that the Maximum Weight Induced Matching (MWIM) problem is polynomially solvable on graphs with bounded asteroidal index [8]. It was observed that orthogonal ray graphs have bounded asteroidal index [31], which by [8] implies a polynomial time algorithm for MWIM for this class. MIM was solved in polynomial time for line graphs of Hamiltonian graphs, and it

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was shown that the problem remains NP-complete on Hamiltonian graphs [23]. In the same paper the authors gave some polynomial time algorithms for subclasses of P_5-free graphs and they noted that if G is P_5-free, then $L(G)^2$ is P_5-free. In those years was still an open question whether the maximum independent set on P_5-free graphs was polynomial time solvable, and therefore this observation did not directly yield a polynomial time algorithm for MIM on P_5-free graphs. Remarkably, this long standing question was affirmatively answered a decade later in [25]; which implies that MIM is polynomial on P_5-free graphs. In [23] a polynomial time algorithm was given for recognizing graphs where the size of a maximum induced matching is the same as the size of a maximum matching, and a polynomial time algorithm was given to find a MIM in such class. A simpler recognition algorithm for this class was given in [6]. The line of research of [6,23] was continued in [10], where the authors simplified further the proofs of [6] and gave a polynomial time algorithm for recognizing graphs where the maximum induced matching differs in at most k with the maximum matching. MWIM was shown to be polynomially solvable on chordal graphs, (claw, net)-free graphs and some other subclasses of claw-free graphs [2].

Regarding approximability, for graphs of degree at most Δ only a $2(\Delta-1)$-approximation is known [32]. However, several approximations were given for d-regular graphs with $d \geq 3$, which all used the same theoretical upper bound for proving their approximation ratios — namely, that any induced matching of a d-regular graph has at most $m/(2d-1)$ edges. This bound was first introduced in [32], where the author gave a simple greedy algorithm with performance ratio $d - \frac{1}{2} + \frac{1}{4d-2}$. This was improved in [11], where the authors gave an asymptotic approximation ratio $d-1$. In the same work they gave a PTAS for planar graphs of degree at most 3. In the following year, a $0.75d + 0.15$ approximation ratio for d-regular graphs was given [16]. This ratio was further improved to $0.708\overline{3}d + 0.425$ on (C_3, C_5)-free d-regular graphs [29]. An algorithm for cubic graphs with performance ratio 9/5 appears in [20].

Several results were given regarding lower bounds and algorithms attaining them. Any subcubic planar graph has an induced matching of size at least $m/9$ and it is possible to find such induced matching [21]. It is known that in subcubic graphs without short cycles there must be an induced matching of size at least $(n-1)/5$ [17]. For bounded degree graphs there is a polynomial time algorithm that computes an induced matching of size at least $\frac{n}{(\lceil \Delta/2 \rceil+1)(\lfloor \Delta/2 \rfloor+1)}$ for graphs with sufficiently large Δ and with no isolated vertices [18]. One can find an induced matching in polynomial time with at least $m/20$ edges for graphs with degree at most 4, and at least $m/18$ edges for a subclass of these graphs [19].

On the negative side, MIM cannot be approximated on general graphs with a constant performance ratio unless P = NP [32]. Furthermore, the problem cannot be approximated within a factor $n^{1/2-\epsilon}$ for some $\epsilon > 0$, unless P = NP [28]. Moreover, MIM is APX-complete for d-regular bipartite graphs for $d \geq 3$ [9].

Despite the vast amount of research done for induced matchings, not much has been achieved for the weighted version of this problem besides [2,8,31]. To our best knowledge, no approximation algorithm was given using a linear program as an upper bound. However, a generalization of MWIM with edge capacities was considered in [13], and for some particular cases (that excluded the classical induced matching) the authors gave some constant approximation ratios by relating a natural linear programming formulation with a capacitated b-matching polytope.

In this work we propose a fractional local ratio algorithm for MWIM with performance ratio Δ. For an overview on local ratio algorithms we suggest the survey [1]. Furthermore, we show that a simple greedy algorithm yields an approximation ratio $2k-3$ for $K_{1,k}$-free graphs and k for $(K_{1,k}$, chair)-free graphs.

2. A Δ-approximation algorithm

Let $G = (V, E)$ be an edge-weighted graph with weights $w_e \in \mathbb{Q}_{\geq 0}$. For an edge $uv = e \in E$ we define $N(e) = N(v) \cup N(u)$ and $\delta(e) = \delta(u) \cup \delta(v)$, where $N(v)$ is the open neighborhood of v and $\delta(v)$ is the set of edges incident to v. We denote $C(e) \subseteq E$ to be the set of edges which are *in conflict* with e; more formally, $C(e) = \bigcup_{w \in N(e)} \delta(w)$. Note that in our definition $e \in C(e)$.

We can model the MWIM problem with an integer linear program. We define the binary variables x_e for $e \in E$ such that e is included in the solution if and only if $x_e = 1$. Consider the following formulation, where $x(A)$ denotes $\sum_{a \in A} x_a$.

$$\begin{array}{lrcll} \max & \displaystyle\sum_{e \in E} w_e x_e & & & \\ \text{s.t.} & x(\delta(e)) & \leq & 1 & \forall e \in E, \\ & x_e & \in & \{0, 1\} & \forall e \in E. \end{array}$$

Our algorithm uses linear relaxations of the above program restricted to different sets of variables. Formally, for a subset $F \subseteq E$, we denote LP_F to be the following linear program restricted to the variables x_f for $f \in F$.

$$\begin{array}{lrcll} \max & \displaystyle\sum_{e \in E} w_e x_e & & & \\ \text{s.t.} & x(\delta(e)) & \leq & 1 & \forall e \in E, \\ & x_e & \geq & 0 & \forall e \in F. \end{array}$$

In what follows we show that Algorithm 1 has performance ratio Δ provided that in each recursive call one can find an edge $e \in F$ such that $x(C(e)) \leq \Delta$. Note that there is at most one recursive call in each call, and in each step $|F|$ decreases by at least one. Therefore, the algorithm ends after at most $|E|$ recursive calls. For the sake of completeness, we include the proof of the following theorem.

Algorithm 1 IM(F, w)

1: **if** $F = \emptyset$ **then**
2: **return** $\emptyset$
3: Compute an optimal solution x to LP_F
4: Let $F_0 = \{e : x_e = 0\}$
5: **if** $F_0 \neq \emptyset$ **then**
6: **return** IM($F \setminus F_0$, w)
7: Let $e \in F$ such that $x(C(e)) \leq \Delta$
8: For each $f \in F$, let $\widehat{w}_f = \begin{cases} w_e & \text{if } f \in C(e), \\ 0 & \text{otherwise.} \end{cases}$
9: $\mathcal{M} \leftarrow$ IM($F \setminus \{e\}$, $w - \widehat{w}$)
10: **if** $\mathcal{M} \cup \{e\}$ is an induced matching (i.e. if $C(e) \cap \mathcal{M} = \emptyset$) **then**
11: $\mathcal{M} \leftarrow \mathcal{M} \cup \{e\}$
12: **return** $\mathcal{M}$

Theorem 1 ([1,7]). *If for any non empty subset $F' \subseteq F$ and any feasible solution y to $LP_{F'}$ there is some $g \in F'$ such that $y(C(g)) \leq \Delta$, then Algorithm 1 computes an induced matching $\mathcal{M}$ such that $w(\mathcal{M}) \geq \frac{1}{\Delta}\sum_{f\in F} w_f x_f$, where x is the solution to LP_F computed in Line 3.*

Proof. We prove this by induction in the number of iterations. The base case is handled in Line 2, which trivially holds.

Suppose the algorithm returns on Line 6. Let x' be the solution to $LP_{F\setminus F_0}$ computed in the recursive call. Then

$$w(\mathcal{M}) \geq \frac{1}{\Delta}\sum_{f\in F\setminus F_0} w_f x'_f \geq \frac{1}{\Delta}\sum_{f\in F\setminus F_0} w_f x_f = \frac{1}{\Delta}\sum_{f\in F} w_f x_f,$$

where the first inequality holds by inductive hypothesis; the second holds because x' is an optimal solution for $LP_{F\setminus F_0}$ and x restricted to $F \setminus F_0$ is feasible for $LP_{F\setminus F_0}$; and the last equality holds because $x_f = 0$ for each $f \in F_0$.

We now consider the case when the algorithm returns on Line 12. Let $\widetilde{w} = w - \widehat{w}$. Denote x' to be the computed optimal solution to $LP_{F\setminus\{e\}}$ with weights $\widetilde{w}$. On the one hand, we have

$$\widetilde{w}(\mathcal{M}) \geq \frac{1}{\Delta}\sum_{f\in F\setminus\{e\}} \widetilde{w}_f x'_f \geq \frac{1}{\Delta}\sum_{f\in F\setminus\{e\}} \widetilde{w}_f x_f = \frac{1}{\Delta}\sum_{f\in F} \widetilde{w}_f x_f,$$

where the first inequality follows from the inductive hypothesis; the second from the fact that x' is an optimal solution for $LP_{F\setminus\{e\}}$ (with weights $\widetilde{w}$) and x restricted to $F \setminus \{e\}$ is feasible for $LP_{F\setminus\{e\}}$; and the last equality holds because $\widetilde{w}_e = 0$, regardless of whether $e \in \mathcal{M}$ or not. On the other hand, we have

$$\widehat{w}(\mathcal{M}) = \widehat{w}_e|\mathcal{M} \cap C(e)| \geq \widehat{w}_e \geq \widehat{w}_e\frac{x(C(e))}{\Delta} = \frac{1}{\Delta}\sum_{f\in C(e)} \widehat{w}_e x_f = \frac{1}{\Delta}\sum_{f\in F} \widehat{w}_f x_f,$$

where the first equality follows from the definition of $\widehat{w}$; the first inequality follows since $\mathcal{M} \cap C(e)$ is always non-empty; the second inequality because $x(C(e)) \leq \Delta$; and the last equality follows because $\widehat{w}_f = 0$ for any $f \in F \setminus C(e)$ and $\widehat{w}_f = \widehat{w}_e$ for each $f \in C(e)$.

Therefore, we have

$$w(\mathcal{M}) = \widetilde{w}(\mathcal{M}) + \widehat{w}(\mathcal{M}) \geq \frac{1}{\Delta}\sum_{f\in F} \widehat{w}_f x_f + \frac{1}{\Delta}\sum_{f\in F} \widetilde{w}_f x_f = \frac{1}{\Delta}\sum_{f\in F} w_f x_f. \quad \square$$

Lemma 2. *Let x be a solution to LP_F, then there is some edge $e \in F$ such that $x(C(e)) \leq \Delta$.*

Proof. Let $v \in V$ be a vertex that maximizes $\beta = x(\delta(v))$. Take any edge $uv = e \in \delta(v) \cap F$. For each $w \in N(v) \setminus \{u\}$, we have that $x(\delta(vw)) = x(\delta(v) \cup \delta(w)) \leq 1$, and therefore $x(\delta(w) \setminus \delta(v)) \leq 1 - x(\delta(v)) = 1 - \beta$. Hence,

$$\begin{aligned} x(C(e)) &\leq x(\delta(e)) + \sum_{w\in N(v)\setminus\{u\}} x(\delta(w) \setminus \delta(v)) + \sum_{w\in N(u)\setminus\{v\}} x(\delta(w)) \\ &\leq 1 + (\Delta - 1)(1 - \beta) + (\Delta - 1)\beta \\ &= \Delta, \end{aligned}$$

where the first inequality holds because each variable of $x(C(e))$ appears at least once in the sum; and the second inequality holds since $x(\delta(w)) \leq \beta$ for each $w \in N(u) \setminus \{v\}$ by definition of β. $\square$

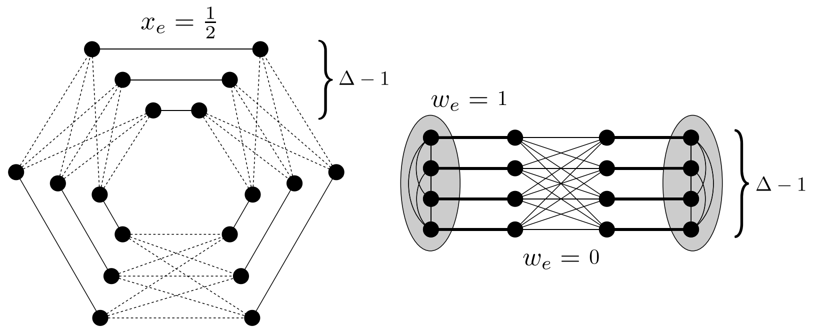

Fig. 1. Tightness of the Algorithm 1 for graphs of degree at most Δ.

Corollary 1. *Algorithm 1 has performance ratio Δ.*

Remark 1. For any Δ there is a Δ-regular graph G and a subset of edges F such that the optimal solution x to LP_F is such that $x(C(e)) = \Delta - 1/2$ for each edge $e \in F$. In other words, Lemma 2 is almost tight.

Proof. Connect three matchings of $\Delta - 1$ edges by three complete bipartite subgraphs. Define the set F as the matchings. An example for $\Delta = 4$ is depicted in the first graph of Fig. 1, where the solid edges are those in F. An optimal solution to LP_F is given by assigning $1/2$ to each edge in F, which implies $x(C(e)) = \Delta - 1/2$ for each $e \in F$. □

Remark 2. The integrality gap is at least $\Delta - 1$. This means that unless we use a stronger linear formulation, we cannot hope to improve much the approximation ratio using the linear program as upper bound.

Proof. Consider two matchings of $\Delta - 1$ edges connected by a complete bipartite graph. With the remaining vertices of each matching form a clique. Assign a zero weight to the edges in the complete bipartite subgraph and the cliques, and a unitary weight to the edges of the matchings. An example for $\Delta = 5$ is depicted in the second graph of Fig. 1, where the bold edges have weight 1. Clearly, an optimal induced matching has weight 1, while assigning $1/2$ to each edge of the matchings yields a fractional solution of weight $\Delta - 1$. □

3. A greedy algorithm

Consider the natural greedy approach: sort the edges by their weights in a non-increasing order and iteratively keep adding the heaviest possible edge to the solution, making sure that at every moment the solution is an induced matching. When no more edges can be added, return the constructed solution.

Observe that this algorithm can be implemented in $O(m \log n)$ time: sorting is $O(m \log n)$; then we iterate through the edge set and we take the first edge uv whose endpoints are non-marked; we add this edge and we mark $N(u) \cup N(v)$ — this second phase takes $O(m)$ time.

Note that if $\mathcal{M}$ is a maximum weight induced matching and $|\mathcal{M} \cap C(e)| \leq \alpha$ for each $e \in E$, then in each iteration of the greedy algorithm we pay at most α times what we receive for the edge we pick, and thus the approximation ratio is α. Formally, if $S = \{e_1, \ldots, e_t\}$ is the induced matching constructed by the algorithm, where e_i was added before e_{i+1}, then

$$\begin{aligned} w(\mathcal{M}) &= w\left(\bigcup_{i=1}^{t} C(e_i) \cap \mathcal{M}\right) \\ &= w\left(\bigcup_{i=1}^{t} C(e_i) \cap \mathcal{M} \setminus \bigcup_{j=1}^{i-1} C(e_j)\right) \\ &= \sum_{i=1}^{t} w\left(C(e_i) \cap \mathcal{M} \setminus \bigcup_{j=1}^{i-1} C(e_j)\right) \\ &\leq \sum_{i=1}^{t} \alpha w_{e_i} \\ &= \alpha w(S), \end{aligned}$$

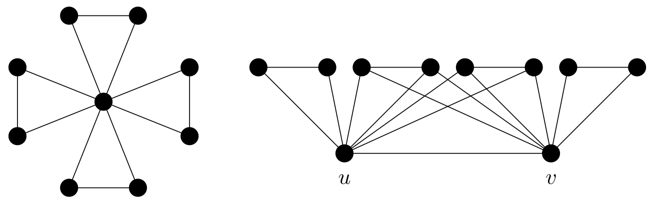

Fig. 2. Graphs G_4 and H_4.

where the first equality holds because $\bigcup_{e\in S} C(e) = E$; the second is a rewriting of the edges of $\mathcal{M}$ in the order of their appearances in $C(e_1), \ldots, C(e_t)$; the third equality holds because the above sets are disjoint; the inequality holds since $w_{e_i} \geq w_f$ for each $f \in \mathcal{M} \setminus \bigcup_{j=1}^{i-1} C(e_j) \subseteq E \setminus \bigcup_{j=1}^{i-1} C(e_j)$, and since $|C(e_i) \cap \mathcal{M} \setminus \bigcup_{j=1}^{i-1} C(e_j)| \leq |C(e_i) \cap \mathcal{M}| \leq \alpha$; and the final equality is simply the definition of $w(S)$. We thus proved the following.

Theorem 3. *If for any induced matching $\mathcal{M}$ we have $|\mathcal{M} \cap C(e)| \leq \alpha$ for each $e \in E$, then the greedy algorithm has performance ratio α.*

3.1. $K_{1,k}$-free graphs

Lemma 4. *If G is $K_{1,k}$-free and $\mathcal{M}$ is an induced matching of G, then $|\mathcal{M} \cap C(e)| \leq 2k - 3$ for any edge $uv = e \in E$.*

Proof. If $e \in \mathcal{M}$, then $|C(e) \cap \mathcal{M}| = 1 \leq 2k - 3$. Suppose that $e \notin \mathcal{M}$. If $\delta(u) \cap \mathcal{M} \neq \emptyset$, then since $\mathcal{M}$ is an induced matching, it follows that $\delta(v) \cap \mathcal{M} = \emptyset$. Since G is $K_{1,k}$-free, there are at most $k - 1$ pairwise non-adjacent neighbors of v, meaning that $|\delta(N(v)) \cap \mathcal{M}| \leq k - 1$, where $\delta(N(v)) = \bigcup_{w\in N(v)} \delta(w)$. Note that $\delta(N(u)) \cap \mathcal{M} \subseteq \delta(N(v))$, and therefore $|C(e) \cap \mathcal{M}| \leq k - 1 < 2k - 3$. Suppose now $\delta(e) \cap \mathcal{M} = \emptyset$. If $|C(e) \cap \mathcal{M}| \geq 2k - 2$, then it must be the case that $|\delta(N(u)) \cap \mathcal{M}| = |\delta(N(v)) \cap \mathcal{M}| = k - 1$ and these sets are disjoint, but this is not possible because in $N[u]$ there must be an induced $K_{1,k}$ centered on u with k pairwise disjoint vertices given by $k - 1$ endpoints of $\mathcal{M} \cap \delta(N(u))$ and v. $\square$

Corollary 2. *If G is $K_{1,k}$-free ($k \geq 3$), then the greedy algorithm has performance ratio $2k - 3$.*

3.2. Subclasses of chair-free graphs

Suppose now that G is chair-free and $\mathcal{M}$ is an induced matching of G. In what follows we show that if there is some edge $e \in E$ such that $|\mathcal{M} \cap C(e)| = k \geq 4$, then G must have at least one of two possible graphs as induced subgraph.

Define G_k ($k \geq 3$) to be the graph given by an induced matching of k edges with a universal vertex. Define H_k ($k \geq 4$) to be the graph given by an induced matching of size k and two additional adjacent vertices, u and v, such that u is connected to all the vertices of the first $k-1$ edges in the matching, and v is connected to all the vertices of the last $k-1$ edges in the matching (see Fig. 2 for an example). In what follows we say that an edge uv neighbors with a vertex w if $w \in N(v) \cup N(u) \setminus \{u, v\}$.

Lemma 5. *Let G be a chair-free graph and $\mathcal{M}$ be an induced matching of G. If $v \in V$ is not an endpoint of some edge in $\mathcal{M}$ and v neighbors with $k \geq 3$ edges from $\mathcal{M}$, then v and its neighboring edges in $\mathcal{M}$ induce a G_k.*

Proof. Suppose for contradiction that there is some edge $uw \in \mathcal{M}$ such that $uv \in E$ but v and w are not adjacent. Then, since $k \geq 3$, there must be at least two non-adjacent vertices, x and y, neighboring v that are endpoints of edges in $\mathcal{M}$. But this is not possible, because $\{x, y, v, u, w\}$ induces a chair. $\square$

Theorem 6. *Let G be a chair-free graph and $\mathcal{M}$ be an induced matching of G. If there is some edge $uv = e \in E$ such that $|\mathcal{M} \cap C(e)| = k \geq 4$, then there is an induced G_k or an induced H_k in G.*

Proof. Clearly, $e \notin \mathcal{M}$ because $k \geq 4$. If u is an endpoint of some edge in $\mathcal{M}$, then v must be neighboring k edges of $\mathcal{M}$, and therefore by Lemma 5 there must be an induced G_k centered on v. Suppose now that e does not share any endpoint with edges from $\mathcal{M}$. Clearly, if u or v neighbors k edges from $\mathcal{M}$, then by Lemma 5 there is an induced G_k. Suppose that there is an edge $xy \in \mathcal{M}$ neighboring u but not v, and an edge $pq \in \mathcal{M}$ neighboring v but not u. Suppose w.l.o.g. that $xu \in E$ and $vq \in E$. It thus follows that $\{x, u, v, q\}$ induces a P_4, which means that any endpoint of an edge in $\mathcal{M}$ (other than xy and pq) adjacent to u must be adjacent to v and vice versa (because G is chair-free). Therefore, there are $k - 2$ edges in $\mathcal{M}$ neighboring both, u and v. It follows that there are exactly $k - 1$ edges in $\mathcal{M}$ neighboring u and $k - 1$ edges in $\mathcal{M}$ neighboring v. Since $k - 1 \geq 3$, by Lemma 5 there is a G_{k-1} centered on u and a G_{k-1} centered on v. It follows that the vertex set of $\mathcal{M} \cap C(e)$ induces H_k. $\square$

Table 1
Random graphs with $n = 100$ and $100 \leq m \leq 1000$.

Apx	OPT	OPT/Apx	LP/Apx	Δ
994.235	1313.783	1.321	1.981	16
586.682	1046.750	1.784	3.730	20
569.141	987.044	1.734	3.900	23
779.096	1071.847	1.376	2.735	20
531.683	979.944	1.843	4.280	29

Table 2
20-regular graphs with $n = 40$.

Apx	OPT	OPT/Apx	LP/Apx	Δ
166.859	291.569	1.747	5.669	20
180.933	268.522	1.484	5.108	20
187.330	288.790	1.542	5.012	20
99.695	286.538	2.874	9.232	20
187.891	279.431	1.487	4.971	20

Table 3
5-regular graphs with $n = 40$.

Apx	OPT	OPT/Apx	LP/Apx	Δ
490.066	625.157	1.276	1.548	5
418.108	628.044	1.502	1.905	5
436.627	568.391	1.302	1.724	5
459.120	654.537	1.426	1.717	5
336.008	587.630	1.749	2.294	5

Table 4
Bipartite graphs with $n_1 = 20$, $n_2 = 30$ and $Prob[uv \in E] = 0.2$.

Apx	OPT	OPT/Apx	LP/Apx	Δ
470.472	745.622	1.585	1.735	9
540.191	701.684	1.299	1.501	10
473.746	719.346	1.518	1.839	9
647.261	647.896	1.001	1.224	9
430.503	667.672	1.551	1.988	13

Table 5
Bipartite graphs with $n_1 = 20$, $n_2 = 30$ and $Prob[uv \in E] = 0.8$.

Apx	OPT	OPT/Apx	LP/Apx	Δ
175.597	287.360	1.636	6.260	29
196.746	287.731	1.462	5.588	27
184.185	268.497	1.458	5.802	27
185.272	352.590	1.903	5.929	28
198.288	266.653	1.345	5.671	29

Corollary 3. *If G is $(G_k, H_k, chair)$-free $(k \geq 4)$, then the greedy algorithm has performance ratio $k - 1$.*

Corollary 4. *If G is $(G_k, chair)$-free $(k \geq 3)$, then the greedy algorithm has performance ratio k.*

Corollary 5. *If G is $(K_{1,k}, chair)$-free $(k \geq 3)$, then the greedy algorithm has performance ratio k.*

4. Experimental results

Because of its simple structure, it is of interest to analyze the practical behavior of Algorithm 1. The first thing to observe is that the linear program solved at the first iteration is of size $|E| \times |E|$, which is the most expensive step in the computation. This imposes a practical limit on the size of the problem to be solved.

We explored the approximation behavior for randomly generated graphs, bipartite graphs and regular graphs. The weights were picked randomly with a uniform distribution between 1 and 100. The results are shown in Tables 1–5.

The gap between our solution values and the optimal ones is much smaller than the approximation factor. This suggests that the algorithm performs very well in practice, disregarding the gap with respect to the linear relaxation. It is worth noting that we computed the optimal values only for small instances, which means we cannot claim good behavior with respect to the optimal value for large instances. Furthermore, we observe that the denser the graph, the bigger is the integrality gap. This is expected to be so due to the local nature of the constraints in the linear program. Finally, we note that the gap with respect to the linear relaxation for random graphs is usually much smaller than Δ. However, as we have seen in Remark 2, there are instances where this gap is $\Delta - 1$.

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