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Approximating weighted neighborhood independent sets

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ABSTRACT

A neighborhood independent set (NI-set) is a subset of edges in a graph such that the closed neighborhood of any vertex contains at most one edge of the subset. Finding a maximum cardinality NI-set is an NP-complete problem. We consider the weighted version of this problem. For general graphs we give an algorithm with approximation ratio Δ, and for diamond-free graphs we give a ratio $\Delta/2+1$, where Δ is the maximum degree of the input graph. Furthermore, we show that the problem is polynomially solvable on cographs. Finally, we give a tight upper bound on the cardinality of a NI-set on regular graphs.

1. Introduction

Let $G=(V,E)$ be a simple undirected graph. For $v \in V$, let $N(v)=\{u : uv \in E\}$ be the open neighborhood of v; and let $N[v]=N(v)\cup\{v\}$ be the closed neighborhood of v. Let $\Delta(G)=\max_{v\in V}|N(v)|$ be the maximum degree of a vertex in G, if the context is clear we write $\Delta=\Delta(G)$. A subset $S\subseteq E$ is *neighborhood independent* if $|E[v]\cap S|\leq 1$ for any vertex $v\in V$, where $E[v]$ denotes the set of edges in the subgraph induced by $N[v]$. The goal of the maximum NI-set problem is to find a NI-set S of maximum cardinality. The decision version of the problem is formulated as follows: given an integer k and a graph G, decide whether G contains a NI-set of size at least k.

In 1986, Lehel and Tuza [1] gave a linear time algorithm for interval graphs. Wu [2] gave a $O(n^3)$ algorithm for strongly chordal graphs. Tuza et al. [3] proved the problem to be NP-complete on split graphs whose vertices of the independent set have degree 3; and gave a linear time algorithm for strongly chordal graphs if a strong elimination order is given as input. Guruswami and Rangan [4] proved the problem to be NP-complete for diamond-free planar graphs with $\Delta=3$. In the same work it is shown that the problem is NP-complete on line graphs with $\Delta=3$. Warnes [5] gave a linear time algorithm for tree-cographs and P_4-tidy graphs, and proved the problem to be NP-complete on co-bipartite graphs. Other non-algorithmic results related to this problem can be found in [6,7]. A natural generalization of the NI-set problem was considered in [8]. We remark that the above mentioned results are for the unweighted version of the problem. To our best knowledge, no approximation algorithms for this problem were explored before.

In this work we consider the weighted version of this problem, which we call *Maximum Weighted NI-set (MWNI)*. Formally, given an edge-weighted graph we are to find a NI-set that maximizes its total weight. To our best knowledge, MWNI was not studied before. First we argue that this problem is hard to approximate. Then we show that a simple greedy algorithm yields an approximation ratio Δ for general graphs. Furthermore, we propose a fractional local ratio algorithm for diamond-free graphs with approximation ratio $\Delta/2+1$. We give a polynomial time algorithm for cographs. Finally, a tight bound on the cardinality of a NI-set on d-regular graphs is given.

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We close the introduction with some definitions and notation used in this paper. Let $dist_G(u, v)$ be the distance between the vertices u and v in G. A maximal complete subgraph $K = (V(K), E(K))$ of G with at least two vertices is a *clique* of G. Let $K_{i,j}$ be the complete bipartite graph of i vertices in one partition and j in the other. Given a vector x over E and a subset $F \subseteq E$, we define $x(F) = \sum_{f \in F} x_f$. For an edge $uv = e \in E$, let $C(e) \subseteq E$ be the set of edges which are *in conflict* with e; more formally, $C(e) = \bigcup_{w \in N[u] \cap N[v]} E[w]$. Note that $e \in C(e)$.

2. Inapproximability

Let us first briefly observe that the NI-set problem is hard to approximate within a ratio $O(\Delta^{1-\epsilon})$ for any $\epsilon > 0$. The implications of this are twofold: on the one hand, it rules out any constant approximation ratio; and on the other hand, it proves that our algorithms are in a sense tight.

Theorem 1. *For $\Delta \geq 4$, MWNI is NP-hard to approximate within a ratio*

$$\frac{\Delta - 1}{2^{O\left(\sqrt{\log(\Delta-1)}\right)}}.$$

Proof. To show this we give a reduction from independent set that preserves approximability and use a hardness result given by Trevisan in [9].

Let G be a graph with $\Delta(G) \geq 3$ and suppose we are to compute an independent set of G. Construct the graph H as follows. For each vertex $u \in V(G)$ add two adjacent vertices u and u' to H. For each edge $uv \in E(G)$ add a new vertex c_{uv} in H adjacent to u, u', v and v'. For an edge of type uu' we set $w_{uu'} = 1$; and $w_e = 0$ for any edge e incident to some c_{uv}. There is a direct correspondence between NI-sets of H with no zero-weight edges and independent sets in G preserving their sizes: for an independent set I of G we define the NI-set $\{uu' : u \in I\}$ in H; and for an NI-set S of H we define the independent set $\{u : uu' \in S\}$ in G. It is clear that a β-approximate NI-set of H amounts to a β-approximate independent set of G.

A key observation is that $\Delta(H) = \max\{\Delta(G) + 1, 4\} = \Delta(G) + 1$. Trevisan [9] observed that it is NP-hard to approximate the maximum independent set problem within a ratio $\Delta(G)/2^{O\left(\sqrt{\log \Delta(G)}\right)}$. Therefore, it is NP-hard to approximate MWNI within a ratio

$$\frac{\Delta(H) - 1}{2^{O\left(\sqrt{\log(\Delta(H)-1)}\right)}}. \quad \square$$

Remark 1. For any $\epsilon > 0$ and $\Delta \geq 4$, MWNI cannot be approximated within a ratio $O(\Delta^{1-\epsilon})$, unless P = NP.

Proof. This follows because for any constant $c > 0$ we have

$$\Delta^{1-\epsilon} = \frac{\Delta}{2^{\epsilon \log \Delta}} = o\left(\frac{\Delta - 1}{2^{c\sqrt{\log(\Delta-1)}}}\right). \quad \square$$

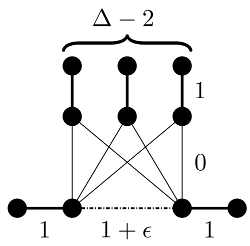

Fig. 1. Tightness of the greedy algorithm.

3. A Δ-approximation algorithm

If $\Delta \leq 2$, then computing a maximum weighted NI-set amounts to finding a maximum weighted matching; the set S is the maximum weighted matching. We thus consider graphs with $\Delta \geq 3$. We first introduce a technical result.

Lemma 2. *If S is a NI-set of G and $\Delta \geq 3$, then $|S \cap C(e)| \leq \Delta$ for each $uv = e \in E$.*

Proof. Let $W = N[u] \cap N[v] \setminus \{u, v\}$. Observe that $|W| \leq \Delta - 1$. If $|W| = \Delta - 1$, then $S \cap C(e) = S \cap \bigcup_{w \in W} E[w]$, and therefore $|S \cap C(e)| \leq \sum_{w \in W} |E[w] \cap S| \leq |W| = \Delta - 1$. If $|W| \leq \Delta - 2$, then $S \cap C(e) = S \cap (E[u] \cup E[v] \cup \bigcup_{w \in W} E[w])$, and therefore $|S \cap C(e)| \leq 2 + |W| \leq \Delta$. $\square$

Consider the natural greedy approach: begin with an empty solution S, and in each iteration add to S the edge $e \in E \setminus S$ of maximum weight such that $|(S \cup \{e\}) \cap E[v]| \leq 1$ for each $v \in V$. Return the constructed set once no more edges can be added.

Let S^* be an optimal NI-set of G. Each time we add an edge e to S, the edges in $C(e)$ cease to be candidates for future iterations—they become blocked. Among the edges in $C(e)$ some may have been blocked in a previous iteration while some in the current. Those in $S^* \cap C(e)$ blocked in a previous iteration are already accounted for by some $e' \in S$. And those in $S^* \cap C(e)$ blocked in the current iteration are at most Δ by the above lemma, and each of them has weight at most w_e by the choice of e. When the algorithm ends, all edges in S^* are blocked by S, and it follows that $w(S^*) \leq \Delta w(S)$. This implies an approximation ratio Δ.

Remark 2. The analysis of the algorithm is tight.

Proof. Consider the graph of Fig. 1. The greedy algorithm outputs the sole edge of weight $1 + \epsilon$, while the optimum corresponds to the bold edges of total weight Δ. Therefore, the ratio between the optimum and the computed solution is $\Delta/(1 + \epsilon)$. $\square$

4. A (Δ/2 + 1)-approximation algorithm for diamond-free graphs

In this section we give a fractional local ratio approximation algorithm for the MWNI problem on diamond-free graphs. To the unfamiliarized reader, the intuition behind the local ratio approach is as follows: given some feasible solution, one can decompose the weight vector into a sum of components and analyze the approximation ratio with respect to (w.r.t.) each of them. Under very general conditions, if the solution has an approximation ratio α w.r.t. each of these components, then it is an α-approximation w.r.t. the original weight vector. In practice, the feasible solution is constructed recursively decomposing the weight vector into two vectors: the current (or local) and the recursive. By proving that the built solution is α-approximate w.r.t. the local vector and using an inductive reasoning for the recursive vector, one obtains an α-approximate solution w.r.t. the original vector. Sometimes it is helpful to use fractional solutions to linear programs to make the recursive decompositions: on the one hand, the approximation ratio may be improved; and on the other hand, one has a natural bound for proving a ratio. This last approach is usually called *fractional* local ratio. For an in depth overview on local ratio algorithms we suggest the survey by Bar-Yehuda et al. [10].

Let $G = (V, E)$ be an edge-weighted diamond-free graph with weights $w_e \in \mathbb{Q}_{\geq 0}$. Consider the following linear relaxation of the MWNI problem.

$$\begin{array}{llcll} \max & \sum_{e \in E} w_e x_e & & & \\ \text{s.t.} & x(E[v]) & \leq & 1 & \forall v \in V, \\ & x_e & \geq & 0 & \forall e \in E. \end{array}$$

For a subset $F \subseteq E$, we denote LP_F to be the above linear program restricted to the variables x_f for $f \in F$.

Algorithm 1 NI(F, w).

1: **if** $F = \emptyset$ **then**
2:     **return** $\emptyset$
3: Compute an optimal solution x to LP_F
4: Let $F_0 = \{e \in F : x_e = 0\}$
5: **if** $F_0 \neq \emptyset$ **then**
6:     **return** NI$(F \setminus F_0, w)$
7: Let $e \in F$ such that $x(C(e)) \leq \Delta/2 + 1$
8: For each $f \in F$, let $\widehat{w}_f = \begin{cases} w_e & \text{if } f \in C(e), \\ 0 & \text{otherwise.} \end{cases}$
9: $S \leftarrow$ NI$(F \setminus \{e\}, w - \widehat{w})$
10: **if** $S \cup \{e\}$ is an NI-set (i.e. if $C(e) \cap S = \emptyset$) **then**
11:     $S \leftarrow S \cup \{e\}$
12: **return** S

In what follows we show that Algorithm 1 has approximation ratio $\Delta/2 + 1$ provided that in each recursive call one can find an edge $e \in F$ such that $x(C(e)) \leq \Delta/2 + 1$. Note that there is at most one recursive call in each step, and in each iteration $|F|$ decreases by at least one. Therefore, the algorithm ends after at most $|E|$ recursive calls. For the sake of completeness, we include the proof of the following theorem, which is essentially the same proved in [10,11]; although for a different problem, ratio α and method of finding the decomposition of the weight vector.

Theorem 3. *([10,11]). Let $\alpha = \Delta/2 - 1$ and $F \subseteq E$. If for any non empty subset $F' \subseteq F$ and any feasible solution y to $LP_{F'}$ there is some $g \in F'$ such that $y(C(g)) \leq \alpha$, then Algorithm 1 computes a NI-set S such that $w(S) \geq \frac{1}{\alpha}\sum_{f \in F} w_f x_f$, where x is the solution to LP_F computed in Line 3.*

Proof. We prove this by induction in the number of iterations. The base case is handled in Line 2, which trivially holds.

Suppose the algorithm returns on Line 6. Let x' be the solution to $LP_{F \setminus F_0}$ computed in the recursive call. Then

$$w(S) \geq \frac{1}{\alpha}\sum_{f \in F \setminus F_0} w_f x'_f \geq \frac{1}{\alpha}\sum_{f \in F \setminus F_0} w_f x_f = \frac{1}{\alpha}\sum_{f \in F} w_f x_f,$$

where the first inequality holds by inductive hypothesis; the second holds because x' is an optimal solution for $LP_{F \setminus F_0}$ and x restricted to $F \setminus F_0$ is feasible for $LP_{F \setminus F_0}$; and the last equality holds because $x_f = 0$ for each $f \in F_0$.

We now consider the case when the algorithm returns on Line 12. Let $\widetilde{w} = w - \widehat{w}$. Denote x' to the computed optimal solution to $LP_{F \setminus \{e\}}$ with weights $\widetilde{w}$. On the one hand, we have

$$\widetilde{w}(S) \geq \frac{1}{\alpha}\sum_{f \in F \setminus \{e\}} \widetilde{w}_f x'_f \geq \frac{1}{\alpha}\sum_{f \in F \setminus \{e\}} \widetilde{w}_f x_f = \frac{1}{\alpha}\sum_{f \in F} \widetilde{w}_f x_f,$$

where the first inequality follows from the inductive hypothesis; the second from the fact that x' is an optimal solution for $LP_{F \setminus \{e\}}$ (with weights $\widetilde{w}$) and x restricted to $F \setminus \{e\}$ is feasible for $LP_{F \setminus \{e\}}$; and the last equality holds because $\widetilde{w}_e = 0$, regardless of whether $e \in S$ or not. On the other hand, we have

$$\widehat{w}(S) = \widehat{w}_e |S \cap C(e)| \geq \widehat{w}_e \geq \widehat{w}_e \frac{x(C(e))}{\alpha}$$
$$= \frac{1}{\alpha}\sum_{f \in C(e)} \widehat{w}_e x_f = \frac{1}{\alpha}\sum_{f \in F} \widehat{w}_f x_f,$$

where the first equality follows from the definition of $\widehat{w}$; the first inequality follows since $S \cap C(e)$ is always non-empty; the second inequality because $x(C(e)) \leq \alpha$; and the last equality follows because $\widehat{w}_f = 0$ for any $f \in F \setminus C(e)$ and $\widehat{w}_f = \widehat{w}_e$ for each $f \in C(e)$.

Therefore, we have

$$w(S) = \widetilde{w}(S) + \widehat{w}(S) \geq \frac{1}{\alpha}\sum_{f \in F} \widehat{w}_f x_f + \frac{1}{\alpha}\sum_{f \in F} \widetilde{w}_f x_f$$
$$= \frac{1}{\alpha}\sum_{f \in F} w_f x_f. \quad \square$$

In what follows we show that there is some edge $e \in F$ such that $x(C(e)) \leq \Delta/2 + 1$. Note that since G is diamond-free, each edge is contained in exactly one clique.

Lemma 4. *Let x be an optimal solution to LP_F. If each clique K such that $x(E(K)) > 0$ has at least $\Delta/2 + 1$ vertices, then there is some clique K' such that $x(E(K')) \geq 1/2$.*

Proof. Let v be a vertex associated to a tight constraint of LP_F, that is, such that $x(E[v]) = 1$. Let K be a clique containing v. If $x(E(K)) \geq 1/2$, then there is nothing to prove. Suppose $x(E(K)) < 1/2$. Then there must be some other clique K' such that $x(E(K')) > 0$ containing v because of the tightness of v. Observe that $K \cap K' = \{v\}$ because K and K' cannot share another vertex since G

is diamond-free. Since $\Delta \geq \deg(v) \geq |V(K)| + |V(K')| - 2 \geq \Delta$, it follows that we have equality throughout and the only cliques containing v are K and K', and therefore $x(E(K')) > 1/2$. □

Lemma 5. *Let x be an optimal solution to LP_F, then there is some edge $e \in F$ such that $x(C(e)) \leq \Delta/2 + 1$.*

Proof. Suppose that there is some clique K such that $x(E(K)) > 0$ and $|V(K)| < \Delta/2 + 1$. Take an edge $e \in F \cap E(K)$. Clearly, $x(C(e)) \leq \sum_{w \in V(K)} x(E[w]) \leq |V(K)| < \Delta/2 + 1$.

Suppose now that for any clique K such that $x(E(K)) > 0$ we have $|V(K)| \geq \Delta/2 + 1$. By Lemma 4 there is some clique K such that $x(E(K)) \geq 1/2$. Take an edge $uv = e \in F \cap E(K)$. We have that

$$\begin{aligned} x(C(e)) &\leq x(E[u]) + x(E[v]) + \sum_{w \in V(K)\setminus\{u,v\}} x(E[w] \setminus E(K)) \\ &\leq 2 + \frac{|V(K)| - 2}{2} \\ &\leq 2 + \frac{\Delta - 2}{2} \\ &= \Delta/2 + 1, \end{aligned}$$

where the first inequality holds because each variable x_f with $f \in C(e)$ appears in the sum; the second because $x(E[w] \setminus E(K)) \leq 1 - x(E(K)) \leq 1/2$; and the third because if $|V(K)| = \Delta + 1$, then K must be an isolated clique, and therefore $x(C(e))$ would trivially be at most 1, which means that we can assume $|V(K)| \leq \Delta$. □

Corollary 1. *Algorithm 1 has approximation ratio $\Delta/2 + 1$.*

5. A polynomial time algorithm on cographs

Given two graphs G and H, the *join* graph $G \oplus H$ is the graph obtained after connecting all the vertices of G with all the vertices of H. A *cograph* is a graph that can be constructed using the following rules.

- $(\{v\}, \emptyset)$ is a cograph.
- If G and H are cographs, then $G \oplus H$ is a cograph.
- If G and H are cographs, then $G \cup H$ is a cograph.

A tree representing the above decomposition is called *cotree*, and it can be computed in linear time due to an algorithm by Corneil et al. [12].

In what follows we give the recursive rules for building a polynomial time algorithm using the cotree of G for computing a NI-set of maximum weight. Note that the unweighted case is linear for P_4-tidy graphs [5], which is a superclass of cographs. However, it is not clear how to generalize that algorithm for the weighted case.

The base and union cases are trivial. We are to analyze the $G \oplus H$ case. Let F denote the set of edges introduced when joining G and H.

Lemma 6. *If S is a NI-set of $G \oplus H$ and it contains an edge $e \in E(G) \cup E(H)$, then it is the only edge in S.*

Proof. Suppose $e \in E(G) \cap S$ and that there is another edge $f \in S$. If $f \in E(G)$, then for any vertex $v \in V(H)$ we have $|E[v] \cap S| \geq 2$. If $f \in E(H)$, then $|E[v] \cap S| \geq 2$ for each endpoint v of e and f. Finally, if $uv = f \in F$ with $v \in V(H)$, it follows that $|E[v] \cap S| \geq 2$ because v is adjacent to both endpoints of e. □

The optimum of $G \oplus H$ will thus be either a sole edge from $E(G) \cup E(H)$ or some subset of F. In what follows we show how to find the best possible NI-set contained in F.

Let G' be the complete bipartite graph given by vertex sets $\{c_G(v) : v \in V(G)\}$ and $\{c_H(u) : u \in V(H)\}$, where $c_G(v)$ is the connected component of G containing v, and $c_H(u)$ the connected component of H containing u. Define the weight of an edge $c_G(v)c_H(u)$ as $\max\{w_{xy} : x \in c_G(v) \text{ and } y \in c_H(u)\}$.

Lemma 7. *([13]) Each connected component of a cograph has diameter at most 2.*

Lemma 8. *If M is a maximum matching of G' and N a maximum weighted NI-set contained in F, then $w(M) = w(N)$.*

Proof. We first prove $w(M) \leq w(N)$. To see this it is enough to prove that M is a NI-set contained in F, where M in the edge set of $G \oplus H$ is given by the weight definition of G'. Under this interpretation it is clear that $M \subseteq F$. Suppose M is not a NI-set. Then, without losing overall generality, there must be some vertex $v \in V(G)$ such that $|E[v] \cap M| \geq 2$. Suppose $\{e, f\} \subseteq E[v] \cap M$. Note that if e has no endpoint in $c_G(v)$, then $e \notin E[v]$. In the same way, if f has no endpoint in $c_G(v)$, then $f \notin E[v]$. Therefore, e and f have both at least one endpoint in $c_G(v)$—a contradiction because M is a matching.

We now prove $w(M) \geq w(N)$. First suppose that there are two edges $e, f \in N$ with endpoints in one connected component of G. Suppose these endpoints are u and v, respectively. Clearly, $u \neq v$. If $dist_G(u, v) = 1$, then $|E[v] \cap \{e, f\}| = |E[u] \cap \{e, f\}| = 2$. Finally, if $dist_G(u, v) = 2$, then for any vertex $w \in V(G)$ adjacent to u and v we have $|E[w] \cap \{e, f\}| = 2$. Note that by Lemma 7 it cannot be the case that $dist_G(u, v) > 2$. Therefore, each connected component of G and H contains at most one endpoint of N. In other words, N is a matching in G'. By the weight definition of G', it follows that $w(M) \geq w(N)$. □

Theorem 9. *Weighted NI-set on cographs is polynomially solvable.*

6. Unweighted NI-set

In this section we present an asymptotically tight upper bound on the cardinality of a NI-set for d-regular graphs.

Theorem 10. *Let G be a d-regular graph and S a NI-set. Then $|S| < \frac{m}{d-1}$.*

Proof. First note that in a d-regular graph $2m = nd$. Suppose that $|S| \geq \frac{m}{d-1}$. Then, multiplying by 2 on each side

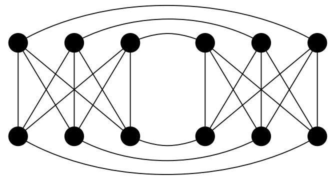

Fig. 2. Tightness of Theorem 10 for $d = 4$.

we get $\sum_{v \in V} d_S(v) = 2|S| \geq \frac{2m}{d-1}$, where $d_S(v)$ is the number of edges in S incident to v. Therefore, by the pigeonhole principle, there must be some vertex $v \in V$ such that $d_S(v) \geq \frac{2m}{n(d-1)} = \frac{d}{d-1} > 1$, which is a contradiction since $|E[v] \cap S| \leq 1$. □

Remark 3. The bound is asymptotically tight. Indeed, for any $d \geq 2$ there is a d-regular graph G such that for a maximum NI-set S,

$$\frac{\frac{m}{d-1}}{|S|} = \frac{d}{d-1}.$$

Proof. Consider the graph given by two copies of $K_{(d-1),(d-1)}$ where each vertex is adjacent to its copy as in Fig. 2. Clearly, $m = 2d(d-1)$ and $|S| = 2(d-1)$. □

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