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Generic profit singularities of one-parameter cyclic processes with discount.
 (English summary)

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This article studies the generic behavior of a one-parameter family of pairs of functions (v, f) , where v is the velocity of a periodic motion of some state $x \in [0, 2\pi]$, i.e.,

$$\dot{x} = v(x, p),$$

p is a parameter, f is interpreted as a profit function (the Lagrangian), whose discounted time average

$$J = \frac{1}{T} \int_0^T e^{-\sigma t} f(x(t), p(t)) dt$$

is to be optimized over all admissible (control) trajectories $p(\cdot)$, T is the duration of the period, and σ is the discount rate.

The subject matter in the article has been motivated by [V. I. Arnol'd, in *Nonlinear problems in mathematical physics and related topics, I*, 23–34, Int. Math. Ser. (N. Y.), 1, Kluwer/Plenum, New York, 2002; [MR1970602 \(2004b:49027\)](#)], and some related previous results can be tracked in the references. Physics oriented mathematicians sometimes assume that the system (process) will always work under optimal conditions. Perhaps because of this, the authors seem to be not very concerned about how the parameter exactly changes over time in each particular case, nor in the material possibilities of their implementation, but rather they elucidate and classify the generic properties of the optimal solutions.

The subjacent optimal control problem is, however, nontrivial, since the admissible control values are restricted to range over a closed manifold, and so its Hamiltonian may potentially lack regularity. Then, from the mathematical control theory and applied mathematics standpoints it would be worthy to explore the time behavior of the optimal control strategy in individual different situations (economics crisis, cardiac diseases). In light of the Pontryagin Maximum Principle, the evolution of the optimal costate may also help to clarify further geometric aspects of nonautonomous Hamiltonian dynamical/control systems.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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