A Fuzzy Programming Approach for the Multi-objective Patient Appointment Scheduling Problem under Uncertainty in a Large Hospital

Abstract

Inspired by a real case of a large regional hospital in Argentina, this research presents an integer linear programming (ILP) model for the patient appointment scheduling problem. The mathematical formulation allows to plan the admission of patients belonging to a waiting list according to a week-hospital modality where all the prescribed clinical procedures to a patient have to be performed during a short hospitalization time no longer than one week. For modeling purposes, each day of the planning horizon is divided in two blocks which, in turn, are discretized into a number of time slots where the different clinical services are carried out. Key features of the model include limited clinical services availability in each time slot and a reduced number of beds and armchairs for hospitalization of patients. The two main goals pursued in the scheduling problem are to provide early patients' admissions, especially for those with high clinical priority (i.e., minimize the admission dates), and reducing the hospital stay per patient (i.e., minimize the length of stay). Since there exists a significant uncertainty in the available amount of clinical services in each time slot, a fuzzy programming approach was adopted to solve the resulting multi-objective problem. According to the obtained results, such an approach

conciliates in a natural way decision maker expectations and constraint fuzziness conducting to balanced schedules.

Keywords: Appointment Scheduling; Week Hospital; ILP Model; Fuzzy Programming

1. Introduction

Current health care paradigm is increasingly focused on patients and aimed at reducing their waiting times as much as possible. Therefore, hospitals and clinics have evolved from an attention scheme based on the order of arrival to a more planned approach based on appointment scheduling. By using appointment scheduling, a convenient balance between demand and capacity can be achieved, as well as an improved patient satisfaction and an efficient use of the available resources.

In the last decades, the appointment scheduling problem has been intensively addressed by the industrial engineering research community using different approaches, such as design of scheduling rules, use of discrete event simulation, application of queuing theory, and development of mathematical programs The interested reader is invited to consult the excellent reviews of (Cayirli & Veral, 2003) and (Gupta & Denton, 2008) where several methodologies and practical issues associated with appointment scheduling systems are given.

A challenging feature related to the patient appointment scheduling problem is that each health care institution around the world (hospitals, health centers, specialized clinics, laboratories, etc.) has unique characteristics regarding aims, management modality (private/ public/ mixed), people idiosyncrasies (patients and medical staff), performance targets, etc., making practically impossible to design a unique approach of universal validity. For this reason, most studies are based on specific systems. For instance, the chemotherapy scheduling problem in outpatient clinics was addressed by (Condotta & Shakhlevich, 2014) via multilevel template, while (Anderson, Zheng, Yoon, & Khasawneh, 2015) have been focused on the analysis of the overlapping appointments, aimed at minimizing patient waiting time and doctor idle time in an outpatient clinic. (Bhattacharjee & Ray, 2016) modelled the patient flows using discrete-event simulation under several sequencing and appointment rules at the Magnetic Resonance Imaging (MRI) section of the Radiology department of a hospital in India. (Baril et al., 2014) also employed appointment scheduling rules for improving the outpatient orthopaedic clinic performance. (Peng, Qu, & Shi, 2014) developed a mathematical programing model and a solution method based on both discrete-event simulation and genetic algorithm in order to find the heuristic scheduling template for open access clinic that admits walk-in patients. (Conforti, Guerriero, Guido, Cerinic, & Conforti, 2011) proposed an integer linear programming formulation for the optimal management of week hospital patients. The posed decision-making model was evaluated using real data from the Rheumatology division of a University Hospital. (Oddoye, Jones, Tamiz, & Schmidt, 2009) studied the performance of medical assessment units of a general hospital in the UK via simulation and goal programming. They have been focused on identifying the impact of reducing the number of available doctors, nurses, and beds on the length of stay of each patient, queue lengths, and waiting times. In (Zhou, Li, Guo, & Lin, 2017) the booking problem for computed tomography scans in a large hospital in China is modeled as a finite horizon Markov decision process. Finally, there is also a huge body of literature devoted to the operating room planning and scheduling problem ((Cardoen, Demeulemeester, & Beliën, 2010), (Wang, Guo, Bakker, & Tsui, 2018)), which might be considered as a particular case of the patient appointment problem. It should be emphasized that the previous review is by no means exhaustive, but simply an overview of approaches and applications on the appointment scheduling problem.

From an operating point of view, a complicating issue of the appointment scheduling problem is the existence of a significant level of uncertainty in the various elements of the process (appointment cancelations, patient arrival times, patient no-shows, etc.). Although uncertainty is recognized as a major issue, it is very difficult to deal with. The discrete simulation approach (Bhattacharjee & Ray, 2016) is possibly the best way to expose its influence on the patient flow while the rolling horizon framework is the most practical approach to implement the schedules in practice (Addis, Carello, Grosso, & Tànfani, 2016).

However, according to our knowledge, the medical appointment scheduling problem has not been simultaneously addressed from a multi-objective perspective under parametric uncertainty in the open literature. This work investigates such an approach considering two typical patient-focused objective functions and the inherent fuzziness in certain parameters. Additionally, several features of a realistic week-hospital department are also considered.

Therefore, the aim of this work is to develop and evaluate a scheduling approach to manage appointments in a week-hospital division in a fuzzy environment. The problem is motivated by a consulting work for an internal division of typical large regional hospital in Argentina. This division deals with the appointment scheduling problem where all the clinical tests prescribed to an admitted patient have to be performed in no more than a week. Moreover, limited clinical services availability and a reduced number of beds and armchairs for hospitalization of patients are accounted for. The proposed approach is based on a mathematical programming model to support "week hospital" type decision making. The analysis is focused on the tradeoffs between two relevant objectives regarding patient satisfaction (i.e., early admission vs. minimum length of stay) under uncertainty in the clinical service availability.

The rest of the article is structured as follows. In section 2, the relevant features of the system under study are described. The problem statement and the corresponding mathematical model are presented in Section 3. A patient waiting list scenario is analyzed in Section 4. In section 5, the adopted fuzzy programming approach to deal with the uncertain sources is described and illustrated with results related to the previous case study. Finally, conclusions and future work directions are drawn in Section 6.

2. Problem background

In the last years, hospital managers have been making efforts to reduce the inpatient length of stay as a way of improving patient satisfaction as well as minimizing patient exposure to nosocomial infections, and increasing the efficiency of the available resources for health care delivery. This managerial improvement can be achieved because there exists not only the technological possibility of completing complex medical procedures in short times (in less than one day), but also decision support systems that facilitate the planning and monitoring of large patient flows within the institutions.

The concepts of day hospital and week hospital refer to organizational models for the patient clinical management where all the prescribed diagnostic and/or therapeutic procedures are scheduled within a hospitalization time no longer than a determined period (one day and one week, respectively).

In Argentina, most medical institutions are adopting some variation of day/week-hospital organization. In particular, this type of management modality is considered to be especially beneficial for the many large regional hospitals existing in the country. These are public hospitals funded by the state that provide health care to a large proportion of the population with no medical insurance in every clinical specialty, including even organ transplant surgery. These institutions are located in medium/large cities and also receive patients from other small cities and towns. Since several patients might spend more than one day in the institution the management modality is in fact closer to the week-hospital organizational model but it still has day-hospital features.

Thus, the week hospital is therefore a management modality within a hospital aimed at organizing admissions of patients with a prescribed number of clinical services (such as MRI scan, endoscopy, mammography, X-ray, etc.) to be carried out for diagnostic purposes. The goal of this organization is to ensure an as short as possible permanence of the patient in the hospital, with a maximum stay of one week. It works as a department/division within the institution that operates with an amount of assigned resources (e.g., beds, armchairs, nursing and medical staff) and shares others with other divisions of the hospital (e.g., X-ray service). The accommodation of the patients depends on the number of available beds and chairs: beds allow overnight stays while armchairs are used for those patients that only remain few hours into this hospital division.

It should be mentioned that while the week-hospital division constitutes a large part of the hospital itself, it has a limited amount of assigned resources, since there also exist other departments/divisions/wards within the institution. For example, on a daily basis the hospital receives a significant amount of external consultations which also require clinical services.

Moreover, there is a large number of inpatients in the hospital under treatment or in recovery. Finally, the institution has also to deal with emergencies.

At these hospitals, the typical phases in order to admit a patient to the week-hospital division are the following:

i. Patient assessment: A physician performs the clinical assessment of the patient and prescribes a list of diagnostic tests and other procedures to be performed. He also assigns a clinical priority to the patient based on his/her general condition and, if appropriate, he could indicate a minimum length of stay. The patient can be assessed by a doctor of the regional hospital if he/she resides in the city where the hospital is located. If the patient lives in other city or town, he/she is checked-up by a local clinician and referred to the regional hospital. In this case, the medical professional telephonically contacts the week-hospital manager to notify the appointment requirement. The patient is then inserted into a waiting list.

ii. Appointment assignment: Based on the availability of armchairs and beds as well as clinical services assigned to the week-hospital department in the near future, the manager assigns, on a daily basis, an appointment for each patient of the waiting list. Such appointment usually lays within the next 30 days. The proximity of the appointment is based on the patient’s clinical priority category, but also considering that it is desirable that each patient spends the least possible time in the hospital.

Currently, the appointment assignment phase is performed manually by an experienced professional (doctor or nurse) of the week-hospital staff. This manager receives on a weekly basis the capacity, in each time slot, of the different clinical services (i.e., the maximum number of patients that can carry out each clinical service per time slot) for the following month.

Moreover, each day, waiting lists of about 10 to 20 patients are generated. Thus, producing sharp schedules is indeed a challenging task due to, at least, the following reasons:

- There is a large number of patients, each one with different prescribed clinical tests to be scheduled within the time horizon.
- Two different objective functions have to be balanced: (i) waiting time for patient admission (especially for those patients with a high clinical priority) and (ii) length of stay per patient, both to be minimized.
- Several sources of uncertainty can produce disruptive events, making rescheduling necessary or convenient (changes in the availability of clinical services due to equipment out of service, shortages in supplies and in health personnel, cancellations, patient no-shows, etc.).

Due to the above reasons, the produced schedules tend to be conservative (later admissions in the scheduling horizon) to better deal with the uncertainty and ensure short stays. Thus, the development of a decision support system aimed at automating the generation of appointment schedules is identified as a desirable tool for the week-hospital management. On one hand, even small improvements in the use of available resources should increase the number of patients treated over a certain time period. On the other hand, the possibility of quickly generating schedules and re-schedules should alleviate a time-consuming activity of the division staff. Furthermore, an objective assessment of the overall performance of the week hospital division could be conducted.

As previously mentioned, the planned appointment modality in large regional hospitals in Argentina resembles that of a week-hospital organizational model. To the best of our knowledge, only (Conforti et al., 2011) have specifically addressed this problem from a mathematical programming approach, although, of course, it shares many common features with other variations of the outpatient appointment scheduling problem.

In the next section, the proposed model for the appointment scheduling problem in a week-hospital division is presented. This model is inspired from the one earlier introduced by (Conforti et al., 2011). It is an extended and adapted version accounting for specific features such as simultaneous day and week-hospital management modalities, different available resources, a longer planning horizon, and problem multi-criteria, to better represent the system under study. For instance, regarding the planning horizon in (Conforti et al., 2011) one week is considered and therefore, the waiting lists are weekly updated. At the beginning of the week, the system decides which patients of the waiting list are to be hospitalized that week in order to maximize the number of admitted patients, each one weighted by a score based on clinical priority and elapsed waiting time. No admitted patients are kept in the waiting list for future assignment in subsequent weeks. In the regional hospitals under analysis, on the other hand, an appointment has to be (almost) immediately assigned. Hence, in order to be able to accommodate the patient somewhere in the near future, a larger planning horizon is considered. In particular, a 4-week planning horizon was adopted in this work with a daily update of waiting lists and scheduling frequency.

3. Problem description and model formulation

The problem addressed in this article deals with the patient admission and scheduling into a week-hospital division belonging to a large regional hospital. The problem is formulated as an integer linear programming (ILP) model and the basic decisions are related to when and for how long every waiting patient is hospitalized.

Each weekday of the 4-week planning horizon is split into two blocks: morning and afternoon. Also, each block is divided into nine half hour slots during which the different clinical services can be delivered. In this way, the planning horizon is organized in a flexible manner closely resembling current practice. It should be mentioned that the week-hospital division works from Monday morning to Saturday midday which makes an 11-block week.

The basic data for this problem are a waiting list of patients to be scheduled, the number of available beds and armchairs at the week-hospital division, and a prediction of the capacities of each clinical service (expressed as number of patients) in each time slot of every block of the planning horizon. In addition, for each patient on the waiting list, the set of prescribed clinical services, the assigned clinical priority, and the minimum length of stay (if required by the physician) are known. It is worth observing that, in this work, the waiting list is assumed as a dynamic one, meaning that it is daily updated.

Each patient on the waiting list is admitted only in one day of the planning horizon and, if this happens, all the prescribed clinical tests are carried out during the week of the admission date. Since the services end on Saturday midday, the patient has to be discharged before this time. As mentioned earlier, armchairs can only be assigned to a patient if all required procedures are performed in one day. Beds are assigned if overnight stays are required and also for a one day stay. It should be noted that the list of patients is updated in a daily basis, thus the schedule is continuously revised.

For the case under study, since all patients on the waiting list have to be scheduled in the near future, two conflicting objectives arise: (i) provide an as soon as possible admission date for each patient, and (ii) minimize the length of stay of each patient.

3.1 Model formulation

3.1.1 Constraints

Since the patients can be admitted to the week-hospital division at any block, a binary variable adm_{pb} is used. Each patient p can only be admitted in one block b of the planning horizon:

$$\sum_{b \in B} adm_{pb} = 1 \qquad \forall p \in P \tag{1}$$

A patient can occupy either a bed or an armchair in the week-hospital division.

$$y_{pb} + z_{pb} \leq 1 \qquad \forall p \in P, b \in B \tag{2}$$

Constraints (3) and (4) represent the upper bound for beds and armchairs, respectively. In other words, the number of patients that can be simultaneously hospitalized in each block b is upper bounded by the number of beds (AB) and armchairs available (AC) in the week-hospital division.

$$\sum_{p \in P} y_{pb} \leq AB \qquad \forall b \in B \tag{3}$$

$$\sum_{p \in P} z_{pb} \leq AC \qquad \forall b \in B \tag{4}$$

Furthermore, a bed or an armchair can be occupied only after the patient's admission block:

$$\sum_{\substack{j\in B\\ j\le b}} y_{pj} \le b \sum_{\substack{j\in B\\ j\le b}} adm_{pj} \qquad \forall p \in P, b \in B \tag{5}$$

$$\sum_{\substack{j\in B\\ j\le b}} z_{pj} \le b \sum_{\substack{j\in B\\ j\le b}} adm_{pj} \qquad \forall p \in P, b \in B \tag{6}$$

Each patient p should undergo every prescribed clinical service i during some time slot k of his/her hospitalization:

$$\sum_{b\in B} \sum_{k\in K} x_{pikb} = s_{ip} \qquad \forall i \in I, p \in P \tag{7}$$

Furthermore, each patient p can undergo at most only one clinical service i during time slot k of block b if either a bed or an armchair is assigned.

$$\sum_{i\in I} x_{pikb} \le y_{pb} + z_{pb} \qquad \forall p \in P, k \in K, b \in B \tag{8}$$

The number of patients undergoing a specific procedure i during slot k of block b is upper bounded by the clinical service capacity in that time slot, η_{ikb}, namely the number of studies (patients) that can be performed by the specific service during that time slot.

$$\sum_{p\in P} x_{pikb} \le \eta_{ikb} \qquad \forall i \in I, k \in K, b \in B \tag{9}$$

Moreover, if the patient p was not admitted yet, he/she cannot undergo any clinical service:

$$\sum_{\substack{j\in B\\ j\le b}} adm_{pj} \le \sum_{i\in I} \sum_{k\in K} \sum_{\substack{j\in B\\ j\le b}} x_{pikj} \qquad \forall p \in P, b \in B \tag{10}$$

A patient p is admitted only if it is possible to deliver all the procedures prescribed by the physician, ns_p, in less than one week of the planning horizon.

$$\sum_{i \in I} \sum_{k \in K} \sum_{b \in B} x_{pikb} = ns_p \qquad \forall p \in P \tag{11}$$

In order to ensure that each scheduled patient p occupies the assigned bed or armchair every block he/she is hospitalized, constraints (12) to (16) are included. Note that subscript f and j are also used to represent elements in the set of blocks.

$$o_{pb} \geq \sum_{i \in I} x_{pikb} \qquad \forall p \in P, k \in K, b \in B \tag{12}$$

$$o_{pb} \leq k \sum_{k \in K} \sum_{i \in I} x_{pikb} \qquad \forall p \in P, b \in B \mid nb_p = 0 \tag{13}$$

$$y_{pb} + z_{pb} \geq o_{pj} - \left(1 - \sum_{\substack{f \in B \\ f \leq b}} adm_{pf} \right) \qquad \forall p \in P, b \in B, b \leq j \leq NB \tag{14}$$

$$y_{pb} \leq \sum_{\substack{j \in B \\ j \geq b}} o_{pj} \qquad \forall p \in P, b \in B \tag{15}$$

$$z_{pb} \leq \sum_{\substack{j \in B \\ j \geq b}} o_{pj} \qquad \forall p \in P, b \in B \tag{16}$$

Constraints (17) and (18) guarantee that if a bed is assigned to patient p, he/she remains in bed during all the length of his/her stay. In other words, when a bed is assigned to a patient, he/she cannot change to an armchair during his/her stay.

$$\sum_{\substack{j \in B \\ j \geq b}} y_{pj} \leq NB(2 - z_{pb} - adm_{pb}) \qquad \forall p \in P, b \in B \tag{17}$$

$$\sum_{\substack{j\in B\\ j\geq b}} z_{pj} \leq NB(2-y_{pb}-adm_{pb}) \qquad \forall p\in P, b\in B \tag{18}$$

As already mentioned, an armchair is assigned to a patient if he/she undergoes all the clinical services in less than one day. If an armchair is assigned, Eq. (19) allows a length of stay of one day, i.e., the patient can stay during the morning and the afternoon of a day if he/she was admitted to the division in the morning. On the other hand, if a patient is admitted in the afternoon, Eq. (20) avoids he/she stays more than one block.

$$z_{pb} + z_{pb+1} \leq 1 + adm_{pb} \qquad \forall p\in P, b=1, b=2b+1 \tag{19}$$

$$z_{pb} + z_{pb+1} \leq 1 \qquad \forall p\in P, b=2b \tag{20}$$

Since a minimum length of stay (i.e., nb_p>0) could be required by the physician for some patients, constraints (21) to (23) are included in the model to force that those patients remain hospitalized during at least nb_p blocks.

$$rb_{p,b1} = nb_p\, adm_{p,b1} \qquad \forall p\in P \mid nb_p > 0 \tag{21}$$

$$rb_{pb} \leq nb_p \sum_{\substack{j\in B\\ j\leq b}} adm_{pj} - \sum_{\substack{j\in B\\ j\leq b-1}} y_{pj} \qquad \forall p\in P, 2\leq b\leq NB \mid nb_p > 0 \tag{22}$$

$$\sum_{\substack{j\in B\\ j\geq b}}^{b+(nb-1)} y_{pj} \leq nb_p\, adm_{pb} \qquad \forall p\in P, 1\leq b\leq NB-(nb+1) \mid nb_p > 0 \tag{23}$$

No patient shall be admitted on Saturday afternoons and Sundays. This is posed in Eqs. (24) and (25). The parameter $SA \in B$ represents those blocks corresponding to Saturday afternoon in the adopted time horizon.

$$adm_{pb} = 0 \qquad \forall p \in P, SA - (nb_p + 1) \le b \le SA + 2 \tag{24}$$

$$y_{pb} + z_{pb} = 0 \qquad \forall p \in P, SA \le b \le SA + 2 \tag{25}$$

It is worth observing that some patient may have some preferences or constraints about the time he/she could start the hospitalization to carry out the prescribed procedures and tests. The parameter pb_p refers to the date (block) from which the patient p can be scheduled into the week division. Eq. (26) imposes the no admission of patient p for the previous blocks to pb_p:

$$adm_{pb} = 0 \qquad \forall p \in P, 1 \le b < pb_p \tag{26}$$

3.1.2 Objective Functions

As mentioned before, there are two relevant and conflicting objective functions in the week-hospital appointment scheduling problem under study:

(i) provide an as early as possible admission date for each patient (*EA*),

(ii) minimize the length of stay of each patient (*LS*).

The first objective function aims to maximize the admission of patients by considering the priority levels and the time of admission. It is formulated mathematically as follows:

$$EA = \sum_{p \in P} \sum_{b \in B} \left(\frac{pr_p \, adm_{pb}}{b} \right) \tag{27}$$

where pr_p is an integer value related to the patient's clinical priority (high, medium, low) assigned by the physician based on the assessment of the person in the first consultation, and b is the number of the admission block. Thus, the patients with the highest priority values are prone to be admitted first. The factor (1/*b*) has been introduced in Eq. (27) in order to favor early admission. In this way, the waiting time for patient admission is minimized. This performance index explicitly accounts for hospitalization date.

The second objective function consists in minimizing the stay into the week-hospital division of the patients belonging the waiting list.

$$LS = \sum_{p \in P} \sum_{b \in B} \left(y_{pb} + z_{pb} \right) \tag{28}$$

The model was implemented in the GAMS modeling platform (Brooke, Kendrick, Meeraus, & Raman, 2015) and all instances were solved with the CPLEX 12.5.1 solver (CPLEX 12, The solver manuals) using an optimality gap of 0% on an AMD A6-3620 APU CPU with 2.20 GHz and 8 GB of RAM. The GAMS program was interfaced with Microsoft Excel to facilitate data input and visualization of results. In the next section, a case study is developed in order to illustrate the use of the proposed model and evaluate the tradeoff between both conflicting objectives.

4. Model results

A case study inspired by a large regional hospital in Argentina is presented to assess the features of the posed week-hospital model. The week-division can perform a total of twenty-four diagnostic tests. A typical list of these clinical services is reported in Table 1.

As mentioned before, a planning horizon of four weeks (i.e., 28 days) is considered. It is assumed that each day is divided into two blocks (i.e., morning and afternoon). In this case, the morning block starts at 8:00 a.m. and ends at 12:30 p.m. and the afternoon block starts at 1:30 p.m. and ends at 18:00 p.m. In addition, every block is partitioned into nine 30 min slots (NK = 9) that correspond to time intervals where a given number of diagnostic tests can be carried out. It is also assumed that, for the sake of simplicity, the duration of each slot is the same in all the blocks.

Five armchairs (AC = 5) and four beds (AB = 4) are available in the hospital division.

Table 1. Clinical services of the week-hospital division (NI = 24)

1. Laboratory tests	13. Otorhinolaryngology
2. X-rays	14. Endoscopy
3. Magnetic resonance imaging	15. Ophthalmology
4. Biopsy	16. Mammography
5. Ultrasound	17. Angiography
6. Echo doppler	18. Colonoscopy
7. Computed tomography scan	19. Electroencephalogram
8. Pap test	20. Cystoscopy
9. Pulmonary function testing	21. Hysterosalpingogram
10. Scintigraphy	22. Dentistry
11. Echocardiogram	23. Lumbar Puncture
12. Holter monitor	24. Thoracentesis

The clinical priority assigned to each waiting patient is reflected in Eq. (27) by considering the following three priority levels: High priority pr_p = 100, Medium priority pr_p = 10, and Low priority pr_p = 1.

Table 2 shows a possible waiting list of unscheduled patients, reporting the corresponding prescribed clinical tests, their priority values, the minimum length of stay, and preference admission dates.

Table 2. Waiting list (NP = 22)

Patient, p	Priority, pr_p	Minimum stay, nb_p	Patient's preferences, pb_p	Prescribed services, s_{ip}
1	100	2	0	1, 2, 9, 11
2	10	0	4	1, 5, 11, 14
3	10	2	2	1, 5, 8, 11, 16, 21
4	1	0	16	1, 13, 15, 22
5	100	0	0	1, 3, 10, 18
6	10	0	0	1, 2, 3, 5, 6, 7, 10
7	100	2	0	1, 2, 3, 7, 10
8	1	0	14	1, 5, 6, 8, 16, 20
9	10	3	0	11, 12
10	10	0	6	1, 19
11	100	0	0	1, 6, 11
12	10	0	20	5, 24
13	10	0	0	1, 19
14	1	0	30	2, 11
15	10	0	28	1, 23
16	1	2	18	1, 5, 8, 16, 21
17	1	0	44	22
18	100	2	0	1, 4, 5
19	10	0	0	1, 3, 6, 9, 17
20	100	3	0	1, 5, 11, 12
21	100	0	0	1, 2, 3, 7
22	1	0	32	1, 2, 7, 10

Table 3 presents the number of clinical services available per time slot in every week of the scheduling horizon. It is important to point out that Table 3 considers a 14-block week (i.e., seven days) but only in the first 11 blocks there are clinical services available, since the week-hospital division works from Monday morning to Saturday midday. Although the week has only 11 active blocks, the 14 blocks were maintained in the formulation because, though unlikely, additional time slots might theoretically be enabled on Saturday afternoons and Sundays. From a modelling point of view, the resulting variables and equations of these three inactive additional blocks can be straightforwardly programmed and solved within the adopted modelling platform with almost no computational expense.

Furthermore, the maximum capacity η_{ikb} is arbitrarily set at 2 in those time slots where the clinical service is available. This might be considered a reasonable value for tests/procedures with current clinical technology. Of course, this number depends on staff and equipment available to perform the procedure in each case, making this approach customizable.

Table 3. Capacity of the clinical service in each time slot (Parameter η_{ibk})

η_{ikb}	b_1								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1	2	2	2						
i_2			2						
i_3		2				2			
i_4				2					
i_5				2	2				
i_6		2	2						
i_7							2		
i_8				2	2				
i_9									
i_{10}				2					
i_{11}							2	2	
i_{12}									
i_{13}					2				
i_{14}	2	2							
i_{15}									
i_{16}							2		
i_{17}									2
i_{18}					2	2			
i_{19}									
i_{20}									
i_{21}								2	
i_{22}									
i_{23}									
i_{24}									

η_{ikb}	b_2								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1									
i_2									
i_3									
i_4									
i_5									
i_6						2			
i_7									
i_8		2							
i_9									
i_{10}									
i_{11}									
i_{12}									
i_{13}									
i_{14}									
i_{15}									
i_{16}			2						
i_{17}									
i_{18}									
i_{19}									
i_{20}									
i_{21}									
i_{22}									
i_{23}		2							
i_{24}									2

η_{ikb}	b_3								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1					2	2			
i_2									
i_3									
i_4				2	2				
i_5									
i_6	2				2				
i_7									
i_8									
i_9									
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i_{11}					2	2			
i_{12}									
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i_{19}	2								
i_{20}									
i_{21}									
i_{22}						2	2		
i_{23}									
i_{24}									

η_{ikb}	b_4								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1									
i_2									
i_3									
i_4									
i_5	2	2							
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i_{21}					2				
i_{22}									
i_{23}									
i_{24}									

η_{ikb}	b_5								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1	2	2	2						
i_2			2						
i_3									
i_4									
i_5									
i_6									
i_7							2		
i_8								2	
i_9									
i_{10}									
i_{11}									
i_{12}									
i_{13}									
i_{14}					2				
i_{15}									
i_{16}									
i_{17}									2
i_{18}									
i_{19}									
i_{20}									
i_{21}									
i_{22}									
i_{23}									
i_{24}									

η_{ikb}	b_6								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1									
i_2									
i_3									
i_4									
i_5									
i_6									
i_7			2						
i_8									
i_9									
i_{10}									
i_{11}									
i_{12}			2	2					
i_{13}									
i_{14}									
i_{15}									
i_{16}									
i_{17}									
i_{18}									
i_{19}									
i_{20}					2	2			
i_{21}									
i_{22}									
i_{23}									
i_{24}									

η_{ikb}	b_7								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1	2	2	2	2					
i_2									
i_3									
i_4				2					
i_5									
i_6									
i_7									
i_8									
i_9					2				
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i_{11}					2	2			
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i_{18}									
i_{19}						2			
i_{20}									
i_{21}									
i_{22}									
i_{23}									
i_{24}			2						

η_{ikb}	b_8									b_9								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1													2	2				
i_2				2														
i_3																2		
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i_{18}												2	2					
i_{19}																		
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i_{21}															2			
i_{22}			2	2														
i_{23}																		2
i_{24}																		

η_{ikb}	b_{10}									b_{11}								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1												2	2					
i_2	2												2					
i_3			2							2								
i_4	2												2					
i_5			2											2				
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i_{20}								2	2									
i_{21}																		
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i_{23}																		
i_{24}												2						

η_{ikb}	b_{12}									b_{13}									b_{14}								
	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9
i_1																											
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This 7-day (14 blocks) timetable is repeated for each of the 4 weeks. Blocks b_{12}, b_{13} and b_{14} stand for Saturday afternoon, Sunday morning, and Sunday afternoon, respectively. Each number indicates the maximum number of patients that can undergo service i, in block b, in slot k.

In order to evaluate the behavior of the proposed model, two computational experiments have been carried out by considering, independently, each one of the above-mentioned objective functions. These models involve 83,882 constraints and 271,433 variables, 269,808 of which are binary variables.

In the first instance, the admission of patients, *EA*, is maximized in Eq. (27) with the aim of hospitalizing the patients as soon as possible prioritizing their clinical state. In the optimal solution, variable *EA* takes value 724.27 in a CPU time of 43.18 s. The corresponding appointment schedule is shown in Fig. 1. In this figure, the use of a bed is highlighted with gray color while the use of an armchair is indicated with borders.

As it can be observed from Fig. 1, the whole group of patients with high priority (pr_p = 100) is admitted at the beginning of the planning horizon, that is, in the first block. A bed is assigned to patients p_1, p_7, p_{18} and p_{20} whereas an armchair is allocated to patients p_5, p_{11} and p_{21}. Also, note in Table 2 that none of these patients have an admission preference (pb_p=0) so, they can be admitted with no constraints. In addition, patients with lower priority values are admitted later in the scheduling horizon with the exception of patient 6 (pr_p = 10) that was also admitted in the first block. It should be pointed out that many patients have appointment-booking preferences ($pb_p \neq 0$), therefore they are not admitted before that dates.

It is worth observing that patient 6, who has 7 prescriptions, is hospitalized for only one block (b_1), since all the required clinical tests were available during that period. This situation is highly desirable. On the other hand, patient 7 has to stay 11 blocks (6 days!) to complete his/her 5 prescriptions, which is clearly unsatisfactory. This situation reflects the conflicting nature between early hospitalizations and short length stays. For this solution, the minimum stay *LS* (Eq. 28) has a value of 86 blocks.

Then, the model was solved aiming at minimizing the length of stay, *LS*, in Eq. (28). The optimal schedule, shown in Fig. 2, involves an objective value of 32 blocks, which corresponds to an improvement of 63% respect to the previous one (i.e., 86 blocks). This result is achieved after a CPU time of 19.98 s. This improvement arises at the expense of a significant deterioration in the value of *EA* of about 69% (224.87 compared with 724.27). This situation is mostly explained by late admissions of patients with high and medium level priorities, in particular patients 1, 11, 20, 21, 6, 13, and 15. It can also be observed that the schedule of Fig. 2 expands over the whole available horizon, from block 1 to block 53, while in the previous case all patients were hospitalized within the first 46 blocks.

Considering the results presented above, there is a clear need to deal with the multi-objective nature of the problem in order to achieve a balance between early admissions and short hospitalizations.

		Block																																									
Patient	Priority	b_{1}	b_{2}	b_{3}	b_{4}	b_{5}	b_{6}	b_{7}	b_{8}	b_{9}	b_{10}	b_{11}	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}	b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{30}	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}	b_{36}	b_{37}	b_{38}	b_{39}	b_{43}	b_{44}	b_{45}	b_{46}	b_{47}	b_{48}	b_{49}	b_{50}	b_{51}	b_{52}	b_{53}
p_{1}	**100**	1	1	1	1	1																																							
p_{2}	10					1	1	1	1	1	1	1																																	
p_{3}	10			1	1	1																																							
p_{4}	1														1	1	1	1	1	1																									
p_{5}	**100**	1																																											
p_{6}	10	1																																											
p_{7}	**100**	1	1	1	1	1	1	1	1	1	1	1																																	
p_{8}	1												1	1	1	1	1	1	1	1	1	1																							
p_{9}	10						1	1	1	1	1	1																																	
p_{10}	10							1																																					
p_{11}	**100**	1	1																																										
p_{12}	10																		1	1																									
p_{13}	10			1																																									
p_{14}	1																									1	1	1																	
p_{15}	10																							1	1																				
p_{16}	1																1	1	1	1	1	1																							
p_{17}	1																																				1	1							
p_{18}	**100**	1	1																																										
p_{19}	10							1	1	1																																			
p_{20}	**100**	1	1	1	1																																								
p_{21}	**100**	1																																											
p_{22}	1																											1	1	1	1	1	1	1											

Fig. 1. Optimal schedule of patients by maximizing *EA*

		Block																																									
Patient	Priority	b_{1}	b_{2}	b_{3}	b_{4}	b_{5}	b_{6}	b_{7}	b_{8}	b_{9}	b_{10}	b_{11}	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}	b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{30}	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}	b_{36}	b_{37}	b_{38}	b_{39}	b_{43}	b_{44}	b_{45}	b_{46}	b_{47}	b_{48}	b_{49}	b_{50}	b_{51}	b_{52}	b_{53}
p_{1}	**100**																																											1	1
p_{2}	10												1																																
p_{3}	10												1	1																															
p_{4}	1																																												1
p_{5}	**100**												1																																
p_{6}	10																																		1										
p_{7}	**100**	1	1																																										
p_{8}	1																																											1	1
p_{9}	10			1	1	1																																							
p_{10}	10							1																																					
p_{11}	**100**																							1																					
p_{12}	10																																	1											
p_{13}	10																																								1				
p_{14}	1																																												1
p_{15}	10																																										1		
p_{16}	1																																					1	1						
p_{17}	1																																												1
p_{18}	**100**	1	1																																										
p_{19}	10																																										1		
p_{20}	**100**																																							1	1	1			
p_{21}	**100**																							1																					
p_{22}	1																																		1										

Fig. 2. Optimal schedule of patients by minimizing *LS*

5. Fuzzy programming approach for multi-objective optimization under uncertainty

As was shown in the previous section, the patient appointment scheduling problem addressed in this article is a bi-objective problem from a patient-focused perspective. Uncertainty is also present in several elements of the problem at hand. In particular, there exists a considerable uncertainty in the availability of clinical services in every slot. Although an estimation of these parameters is provided by the hospital manager for the near future, some of them can be cancelled because of equipment out of service, supply and/or staff shortages, among other reasons. In this work, this forecast parameter is assumed to be imprecise (i.e., fuzzy).

In practice, when an appointment cancellation occurs, a rescheduling has to be done to reassign appointments with the minimum possible disturbance of the original schedule. Moreover, the decision maker in the week-hospital division wants to improve the situation presented in previous section by reaching his aspiration levels (preferences) for each goal.

Multi-criterion optimization under uncertainty has received considerable attention during the last decades since practically every decision-making problem in the real word is multi-objective and uncertain in nature. Among the different approaches dealing with uncertainty, the fuzzy mathematical programming provides a practical methodology to address this type of problems. One of the main advantages of the fuzzy approach over stochastic programming is that the uncertain parameters do not have to follow any statistical distribution and that the size of the deterministic formulation equivalent to the uncertain model does not blow up with the number of parameters.

In 1970, (Bellman & Zadeh, 1970) distinguished between random and fuzzy uncertainty. They defined *randomness* as a type of uncertainty concerning membership or non-membership of an object in a set and *fuzziness* as a type of imprecision associated with fuzzy sets, i.e., it refers to classes of objects that admit grades of membership intermediate between full-membership and non-membership. Furthermore, a decision in a fuzzy environment is described by these authors as the confluence of goals and constraints, which, applied to linear programming, can be viewed as the intersection of the fuzzy sets describing the constraints and the objective functions.

Later on, using the above concept, (Zimmermann, 1978) demonstrated that linear multi-objective problems with fuzzy goals and/or fuzzy constraints can be transformed into a crisp linear programming formulation called mix-max approach. Since then, several formulations have used this approach in different decision-making applications. For example, (Kumar, Vrat, & Shankar, 2004) and (Kumar, Vrat, & Shnakar, 2006) have applied this method to the capacitated vendor selection problem, a relevant application in industrial engineering and management science. Also, it has been applied to process planning and supply chain planning in (Liu & Sahinidis, 1997) and (Mitra, Gudi, Patwardhan, & Sardar, 2009), respectively. The reader is referred to (Zimmermann, 2010) for a comprehensive review about fuzzy set theory and its mathematical framework.

In what follows, a brief overview of the fuzzy programming method is given in a separate subsection. After that, the basic multi-objective patient appointment scheduling problem is extended using the fuzzy set theory.

5.1. Fuzzy linear programming

The conventional multi-objective linear program (LP) can be expressed as:

$$
\begin{array}{lll}
\min(or\max) & Z = \left[\mathbf{c}_1^T\mathbf{x}, \mathbf{c}_2^T\mathbf{x}, \ldots, \mathbf{c}_J^T\mathbf{x}\right] & \\
s.t. & A\mathbf{x} \le \mathbf{b} & \quad (29) \\
 & \mathbf{x} \ge \mathbf{0} &
\end{array}
$$

In formulation (29), $\mathbf{x}$ is a n-dimensional vector, $\mathbf{c}^T{}_j\mathbf{x}$ represents each objective function j, A is the $m\times n$ matrix of coefficients of inequality constraints, and $\mathbf{b}$ the right-hand side vector of constants. The corresponding fuzzified form of the above model is:

$$
\begin{array}{lll}
\mathit{find} & \mathbf{x} & \\
st. & \mathbf{c}_j^T\mathbf{x} \,\tilde{\le}\, Z_j^0 \qquad \forall j \in J & \\
 & A\mathbf{x} \,\tilde{\le}\, \mathbf{b} & \quad (30) \\
 & \mathbf{x} \ge \mathbf{0} &
\end{array}
$$

where symbol $\tilde{\le}$ indicates the fuzzified form of $\le$ and represents the linguistic term "essentially smaller than or equal to". The parameter Z_j^0 represents the aspiration level for the value of each objective j that the decision maker wants to achieve.

In this study, linear membership functions are assumed for all fuzzy sets involved in the problem. They are defined for the fuzzy goals, μ_Z, and the fuzzy constraints, μ_G, as follows:

$$
\mu_{Zj}(\mathbf{x}) = \begin{cases} 1 & \text{if } Z_j(\mathbf{x}) < Z_j^{\min} \\ [Z_j^{\max} - Z_j(\mathbf{x})]/[Z_j^{\max} - Z_j^{\min}] & \text{if } Z_j^{\min} \le Z_j(\mathbf{x}) \le Z_j^{\max} \qquad \forall j \in J \\ 0 & \text{if } Z_j(\mathbf{x}) > Z_j^{\max} \end{cases} \qquad (31)
$$

$$
\mu_{Gq}(\mathbf{x}) = \begin{cases} 1 & \text{if } g_q(\mathbf{x}) < b_q \\ 1 - [g_q(\mathbf{x}) - b_q]/d_q & \text{if } b_g \le g_q(\mathbf{x}) \le b_q + d_q \qquad \forall q \in Q \\ 0 & \text{if } g_q(\mathbf{x}) > b_q + d_q \end{cases} \qquad (32)
$$

In (31) Z_j^{max} and Z_j^{min} are the maximum and minimum values of each individual objective function, respectively. They are evaluated by solving independently the problem in (33) for each goal j.

$$
\begin{aligned}
\min/\max \quad & Z_j = \mathbf{c}_j^T \mathbf{x} \\
s.t. \quad & g_q(\mathbf{x}) \leq b_q + d_q \qquad \forall q \in Q \\
& D\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned} \tag{33}
$$

In (32) and (33), $g_q(\mathbf{x})$ represents the fuzzy constraint and d_q are the admissible violations for the fuzzy constraints. In (33), D is the matrix of coefficients of the deterministic constraints.

The fuzzy set "decision" (solution) of model (30), represented by a membership function of the solution set, is given by the intersection of all the fuzzy sets describing the goals and constraints, $\mu_S(\mathbf{x}) = \min\{\mu_z(\mathbf{x}), \mu_G(\mathbf{x})\}$(Bellman & Zadeh, 1970). The optimal solution is the one that has the maximal grade of membership to the fuzzy decision set, in other words, it is defined as a maximizing decision. Thus, by introducing an auxiliary variable λ, the equivalent crisp formulation (Kumar et al., 2006) to the fuzzy optimization problem (30) is:

$$
\begin{aligned}
\max \quad & \lambda \\
s.t. \quad & \lambda\left(Z_j^{\max} - Z_j^{\min}\right) + Z_j(\mathbf{x}) \leq Z_j^{\max} \qquad \forall j \in J \\
& \lambda\, d_q + g_q(\mathbf{x}) \leq b_q + d_q \qquad \forall q \in Q \\
& D\mathbf{x} \leq \mathbf{b} \\
& \mathbf{x} \geq \mathbf{0} \\
& 0 \leq \lambda \leq 1
\end{aligned} \tag{34}
$$

where λ represents the degree of membership to be maximized.

5.2. Application of the fuzzy approach to the patient appointment scheduling problem

As mentioned above, the source of fuzziness of the appointment scheduling model is present in Eq. (9), which establishes the upper bound for the available capacity of clinical services in each time slot (i.e., η_{ikb}). From Table 3, it can be seen that the nominal value was arbitrarily set at 2 in those cases where the service is available. Due to previously mentioned reasons, some of these capacities might be reduced. In order to illustrate the approach, it was considered that the available capacity could be reduced by one (d_q = -1) in all cases. Therefore, the corresponding constraint takes the following form in formulation (34):

$$\lambda(-1)+\sum_{p\in P} x_{pikb} \leq \eta_{ikb} - 1 \qquad \forall i \in I, k \in K, b \in B \middle| \eta_{ikb} > 0 \tag{35}$$

In order to complete the data, the bounds on each objective function were calculated by solving problem (33) for each individual objective. The corresponding results are presented in Table 4.

Table 4. Bounds for objective functions

Objective function	$Z_j^{\min}$	$Z_j^{\max}$
Early admission (*EA*)	15.98	646.74
Length of stay (*LS*)	32	183

Then, the formulation (34) is solved in order to obtain the solution with the highest degree of membership λ. This model comprises 83,883 equations and 271,433 variables, and the solution was found in a CPU time of 1937.16 s. The optimal schedule, which corresponds to λ = 0.96, is shown in Fig. 3. The first goal has a value of EA = 621.74, while the minimum length of stay (the minimum number of blocks to accommodate all patients), corresponds to LS = 38 blocks. In Table 5 the results of the three experiments are summarized for comparison purposes.

Table 5. Results summary

Objective function	Problem	Early admission	Length of stay	λ	CPU time (s)
Early admission (EA)	(1)-(26), (27)	724.27	86	-	43.18
Length of stay (LS)	(1)-(26), (28)	224.87	32	-	19.98
λ	(34)	621.74	38	0.96	1937.16

As it can be observed, the solution of the crisp formulation in Eq. (34) represents indeed a tradeoff between the two conflicting objective functions analyzed in Section 4. This result is evident from the values of the objective functions. For instance, the minimum number of blocks required to accommodate the whole waiting list shows a higher value than the one corresponding to the balanced schedule (38 vs. 32).

From Fig. 3 it can be inferred that the achieved balance is mostly explained by the fact that most patients with the highest priority value (pr_p =100) are admitted in the first block and the lengths of their stay in the hospital division are very short. Note that the longest hospitalization time corresponds to 5 blocks (i.e., two and a half days) for patients p_1 and p_{20}. Patients with value of priority 2 are scheduled next. Finally, most patients with low priority value are scheduled later in the planning horizon but with relative short hospitalization periods. Recall that patients' preferences related to admission dates have also been taken into account in the problem.

		Block																																											
Patient	Priority	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}	b_{20}	b_{21}	b_{22}	b_{23}	b_{24}	b_{25}	b_{26}	b_{30}	b_{31}	b_{32}	b_{33}	b_{34}	b_{35}	b_{36}	b_{37}	b_{38}	b_{39}	b_{43}	b_{44}	b_{45}	b_{46}	b_{47}	b_{48}	b_{49}	b_{50}	b_{51}	b_{52}	b_{53}
p_1	**100**	1	1	1	1	1																																							
p_2	10												1																																
p_3	10												1	1																															
p_4	1																						1																						
p_5	**100**	1																																											
p_6	10																							1																					
p_7	**100**										1	1																																	
p_8	1																				1	1																							
p_9	10			1	1	1																																							
p_{10}	10							1																																					
p_{11}	**100**	1																																											
p_{12}	10																						1																						
p_{13}	10			1																																									
p_{14}	1																																	1											
p_{15}	10																															1													
p_{16}	1																							1	1																				
p_{17}	1																																												1
p_{18}	**100**	1	1	1																																									
p_{19}	10									1																																			
p_{20}	**100**	1	1	1	1	1																																							
p_{21}	**100**	1																																											
p_{22}	1																																		1										

Fig. 3. Optimal schedule of patients by maximizing the degree of membership, λ.

6. Conclusions and future work

In this work, the patient appointment scheduling problem in a week-hospital division belonging to a large regional hospital is addressed by formulating an integer linear programming model. This hospital department combines both the day- and week-hospital organizational models for the clinical management of patients. It operates with a limited capacity of clinical services as well as a reduced number of beds and armchairs. A relevant operational feature of this division is the requirement to assign an appointment (almost) as soon as requested, with the aim of balancing two conflicting objectives, namely early admissions and short hospitalizations.

Since the process is subject to significant uncertainty in the available capacity of clinical services in each time slot, a multi-objective fuzzy programming approach was applied, whose solution provides a convenient trade-off between the relevant performance criteria.

The major contribution of this study is the simultaneous consideration of multiple objective functions, parameter uncertainty, and distinctive features of the division operations in the hospital such as bed and armchairs availability and mandatory admission of every patient in the waiting list. According to our knowledge, no studies addressing these issues have been presented so far in the open literature.

The numerical results suggest that the proposed model is a promising approach to help in the organization of such departments within the hospitals.

The managers of these services face a challenging decision-making process aimed at providing good quality schedules while minimizing cancellations. It is considered that these models, complemented with appropriate user interfaces, are a valuable tool to replace current time-consuming hand-made scheduling activity which typically produces suboptimal appointment plans.

Although the proposed models are complex in terms of number of variables and constraints since they represent the system operations in a realistic manner, their solutions are obtained in acceptable CPU times compatible with those required by a daily based decision-making process.

Future research will focus on the rescheduling approach once disruptive episodes occur. The basic objective is to repair the existing appointment schedule with the minimum possible disturbance. This activity is facilitated by the fact that the system naturally operates in a rolling horizon fashion (daily scheduling of waiting lists) making the introduction of changes operationally simple. However, since there exist different types of disruptions, there are also many repairing options. For example, if the disruption has to do with the cancellation of a specific clinical service in a single slot, probably only one patient had to be reassigned. On the other hand, if a clinical service is cancelled along several slots due to, for example, equipment out of service during several days, probably the new assignment of all the patients (i.e., current waiting list plus patients already scheduled) might be beneficial in order to obtain a more efficient use of the available resources. Additionally, appointment cancellations or patient no-shows might also occur. In these cases, the simplest solution is to reset the parameter η_{ikb} to make the time slots available for the accommodation of patients.

NOTATION

Sets

B	Blocks:	$B = \{ b, j, f = 1, 2, \ldots, NB \}$
I	Clinical services:	$I = \{ i = 1, 2, \ldots, NI \}$
J	Objective functions:	$J = \{ i = 1, 2, \ldots, NJ \}$
K	Time slots:	$K = \{ k = 1, 2, \ldots, NK \}$
P	Patients:	$P = \{ p = 1, 2, \ldots, NP \}$
Q	Constraints	$Q = \{ q = 1, 2, \ldots, NQ \}$

Parameters

A	$m \times n$ matrix of coefficients of inequality constraints
AB	number of available beds at the hospital division
AC	number of available armchairs at the hospital division
$\mathbf{b}$	right-hand side vector of inequality constraints
$\mathbf{c}$	vector of coefficients
D	matrix of coefficients of the deterministic constraints
NB	maximum number of blocks
NI	number of clinical services
NJ	number of problem objective functions
NK	number of slots in each block
NP	number of patients on the waiting list
NQ	number of problem constraints
nb_p	minimum length of stay expressed in number of blocks

s_{ip} 1 if the clinical service i is prescribed to patient p, 0 otherwise

ns_p number of services prescribed to patient p $\left(ns_p = \sum_i s_{ip} \right)$

pr_p priority value for patient p assigned by the physician according to his/her health condition

η_{ikb} capacity of the clinical service i in time slot k of the block b

d_q admissible violations for the fuzzy constraints q

$\mathbf{x}$ vector of n decision variables

$Z_j^{\min}$ minimum value of objective function j

$Z_j^{\max}$ maximum value of objective function j

Z_j^0 decision maker's aspiration level for objective function j

$\mu_S(\mathbf{x})$ membership function of the solution set

$\mu_z(\mathbf{x})$ membership function of the goals

$\mu_G(\mathbf{x})$ membership function of the constraints

Decision variables

adm_{pb} 1 if patient p is admitted during block b, 0 otherwise

o_{pb} 1 if patient p undergoes some clinical service in block b, 0 otherwise

x_{pikb} 1 if patient p undergoes clinical service i during slot k of block b, 0 otherwise

y_{pb} 1 if patient p occupies a bed during block b, 0 otherwise

z_{pb} 1 if patient p occupies an armchair during block b, 0 otherwise

rb_{pb} number of remaining blocks with respect to the minimum length of stay

EA early admission

LS length of stay

λ degree of membership

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