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# Disjunctive Optimization Model for the Production Planning and Blending of Crude Oil in a Conventional Oil Field

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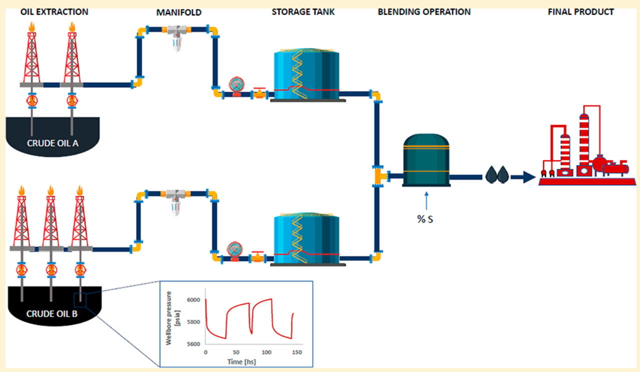
*Supporting Information*

**ABSTRACT:** This work presents a multiperiod nonlinear formulation based on generalized disjunctive programming (GDP) that integrates the production planning and pooling problems in a conventional onshore oil reservoir with fixed topology and a surface interconnection scheme. The model optimizes the well production and blending decisions, while complying with product specifications, such as the sulfur content required by refineries downstream of the oil field. It considers wells from different reservoirs belonging to a given oil field, which are interconnected through manifolds at a network of pipes. The nonlinear behavior of well pressure as a function of time and bilinearities in mass balances are also taken into account. The objective function maximizes the accumulated production of the end products. Numerical results show that obtaining end products as result of blending operations is affected by production planning in wells, the capacity of intermediate tanks, and sulfur specification.




## 1. INTRODUCTION

Despite the growing importance of renewable energy sources for example wind, solar, and biofuels,[1−4] fossil fuels remain the main source of energy for modern society. Major economic activities such as industry and transportation, still have a strong dependence on gas and oil, and particularly the transportation sector relies in a large percentage on nonrenewable fossil fuels.[5,6]

Although hydrocarbons production from unconventional sources (mainly shale gas) have been under strong development in recent years, and in spite of the continuous diminishing of conventional reserves,[7] the latter are still of great importance for the oil industry at a global level.[8]

This situation, coupled with lower oil prices in recent years,[9,10] forces the companies producing from conventional sources to face the need of introducing continuous operational improvements to maintain production levels in line with the demands and their own need to ensure its economic sustainability.

The oil industry has a very complex value chain, for which geographic scales can reach global level. On one end are the hydrocarbon reservoirs (upstream), and on the other the refineries (downstream) that obtain a wide variety of products of economic interest. In particular, the conventional oil industry is currently experiencing a situation where the ever-increasing demand for oil, the decline of reserves, and the fall in the price of crude oil are combined. In this context, it is crucial for the companies to improve their operational decisions in all possible areas, to be sustainable in a market of such complexity. To achieve global efficiency, it is important to take a comprehensive look at all decisions from reservoirs to refineries.

At the downstream level, efficiency improvements depend strongly on the quality and composition stability of the raw materials that are processed. This can be achieved by the appropriate blending of crude oils of different qualities, carried out in storage and dispatch centers, which can be located in different places along the supply chain, from the reservoirs to the refineries themselves. However, the most usual situation is to make the blending at the refineries. This helps to continue working as usual, in the sense that mixing decisions are taken separately from well planning decisions.

To increase global efficiency of the oil business, it is also important to include the problems of pressure drops in wells, manifolds, tanks, etc., because the operative limitations that arise could alter the optimum planning and blending decisions obtained without taking into account these working considerations. Consequently, it is important to integrate planning, mechanical, and mixing decisions into a single problem to take advantage of any room for optimization. However, the entire problem could be quite complex, because it would be also be

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necessary to introduce the dynamic behavior of pipelines. Therefore, to narrow the problem to one that is feasible despite its complexity, this work addresses production, pressure drops, and pooling problems, without consideration of pipelines existing between upstream and downstream facilities.

The operational complexity resulting from combining all above-mentioned decisions makes it useful to develop and apply decision support tools based on mathematical modeling and optimization. An integrated view of the problem involves simultaneously considering planning, blending, and operational decisions, which obviously results in more complicated mathematical models. This is one of the reasons why, according to our knowledge, there are practically no works in the literature that address this integrated problem, but rather, individual problems are considered separately. This latter approach does not allow the evaluation of the trade-offs between both upstream and downstream decisions, which could lead to very different results and potentially achieve better company performance.

One complicating feature incorporated into the posed problem is the diminishing of wellbore pressure. This causes the need to plan cycles of opening/shutting-in of wells to increase the production obtained in a given time horizon. This behavior introduces discontinuities in the model because of the variation of wellbore pressure over time is different, depending on whether the well is open or not. In the present work, Generalized Disjunctive Programming[11] is used to address this type of decisions, with Boolean variables characterizing each phase of the well operation.

Pooling problems have been extensively addressed in the literature because of their practical importance in many industrial applications, particularly in the refining industry. The pooling problem was first presented by Haverly;[12] it was called the p-formulation and was based on recursive Linear Programing. In the following decades, numerous models and solution strategies have been reported to deal with the inherent nonlinearity and then local solutions of this type of problems. Ben-tal et al.[13] introduced the q-formulation where multiple liquid streams with several qualities are considered. Foulds et al.[14] developed a method based on convex approximation of the bilinear terms, and Adhya et al.[15] introduced a new lagrangian relaxation approach using tighter lower bounds than those produced by the standard linear programming relaxations. Audet et al.[16] developed a different approach based on a branch-and-cut quadratic programming algorithm.

Later on, Meyer and Floudas[17] proposed a generalization of the pooling problem considering both discrete and continuous variables for properly capturing the existence of pools and the interconnections in the network. Another generalized multiperiod scheduling version of the pooling problem considering demand and supply flows as a function of time was presented in Kolodziej et al.[18] More recently, Lotero et al.[19] introduced a bilevel decomposition algorithm that can be solved faster than alternative models.

Scheduling and planning problems for the oilfield upstream operation have been mainly considered for planning of offshore facilities. So, a series of works based on Mixed Integer Linear Programming (MILP) models have been presented to address the design and planning of drilling and operation of wells in offshore fields.[20,21] Iyer and Grossmann,[22] proposed an MILP formulation (solved by a bilevel decomposition technique) for the planning and scheduling of well and facility operations in offshore oil fields. Van den Heever et al.[23] introduced an MINLP model for offshore oilfield infrastructure planning, including discrete decision such as wells to be drilled/installed and the selection of the platforms. The same authors,[24] using a disjunctive formulation, integrated complex business rules, for example calculations of tariffs, taxes, and royalties in the design and planning of off-shore infrastructures.

However, as far as we know, only a few studies appearing in the literature deal with the planning problem in onshore oil fields. Usually, such works assume cycles of well operation with equal time period lengths. Ortiz-Gómez et al.[25,26] reported three different models of varying complexity for the planning of oilfield production with cyclic operation. Among them, the most important results are obtained with an MINLP with fixed topology, periods of uniform length, and a given demand.

Regarding the infrastructure in oil fields, from the reservoirs to the surface, several works can be found in the literature. Kosmidis et al.[27] presented a mixed-integer optimization formulation with a solution strategy for daily well operation and gas-lift allocation of integrated oil and gas production in offshore fields, considering the multiphase flow in pipelines. The same authors[28] also developed an MINLP model for the daily well scheduling in oil fields, where the optimal connectivity of wells to manifolds and separators is addressed simultaneously with the optimal well operation and gas-lift allocation. Barragán-Hernández et al.[29] considered process units such as wells, pipes, and manifolds in the gas and oil production system through a one-day time period planning. A model to calculate the amount of gas and oil produced by a network with a given interconnectivity scheme was presented in Flores-Salazar et al.[30] More recently, Lang and Zhao[31] formulated an improved particle swarm optimization (PSO) algorithm to solve the problem of interconnected oil wells production scheduling.

Based on the above considerations, the purpose of this paper is to introduce a new Generalized Disjunctive Programming (GDP) formulation that solves, simultaneously, onshore oilfield production planning and crude oil blending operations, considering a fixed scheme of interconnectivity. Nonlinear behavior on the wellbore pressure, besides the bilinear terms present in quality balances, are considered. The main constraints include not only the reservoir area and the geological aspects but also the interconnectivity between wells and storage tanks and the blending specifications. The production planning problem involves the operation of wells in the onshore oil field, i.e., the number of openings and closings in each well, the pressure at the beginning/end of each period, the lengths of these periods and the production volume of crude oil. The model also determines the number of intermediate tanks to be used in blending operations, along with the inflows required of each type of crude oil and the pressure drops involved in the interconnectivity scheme. The objective function maximizes the accumulated production of the end products within an acceptable range of sulfur content specified by the refineries. Big-M relaxation technique is applied to reformulate the disjunctive problem as an MINLP whose solution is obtained using standard solvers.

This paper is organized as follows. In the next section, a background description of the problem is posed. Section 3 presents a complete definition of the problem. The proposed multiperiod GDP model is described in detail in section 4. Results on case study and several examples are discussed in section 5, followed by conclusions presented in section 6.


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## 2. PROBLEM BACKGROUND

A typical onshore oil production system consists of an oil field where one or more reservoirs are located. A petroleum reservoir is a unit of subsurface formed by a porous permeable rock where oil, gas, and salt water accumulation occur. The oil field is the area where the wells are drilled to extract these fluids from the reservoir. The number of wells may vary from a few up to hundreds and they can be distributed over large areas, in many cases several square kilometers. Surface facilities are also included in the scheme of oil production system, interconnecting wells with manifolds, from where oil is transported to storage tanks.

During the stage of oil production, multiple wells open/shut-in in several times as a result of pressure variation on the wellbore, i.e., bottom hole pressure. According to Horne,[32] well flowing pressure presents a time dependent nonlinear behavior. At the beginning, when the well is open to flow and oil is extracted, the wellbore pressure decreases as the operation time increases, as shown by the following expression:

$$\Delta P_i^{\mathrm{ri}} = P_i^{\mathrm{in}} - P_i^{\mathrm{f}} = \frac{141.2 q_i^{\mathrm{up}} B_i \mu_i}{k_i h_i}\left(\frac{1}{2}\left[\ln \frac{0.000264 k_i t_i^{\mathrm{o}}}{\Theta_i \mu_i c_i^{\mathrm{t}} r_i^2} + 0.80907\right]\right) \quad \forall\, i \in I \tag{1}$$

where $B_i$, $\mu_i$, $k_i$, $h_i$, $\Theta_i$, $c_i^t$, and $r_i$ are geological properties characterizing the well surroundings such as porosity, permeability, viscosity, compressibility, formation volume factor, and thickness, which can vary significantly because of the reservoir formation or depth and are determined experimentally. $P_i^{\mathrm{in}}$ is the pressure of the wellbore $i$ at the initial time. The well produces oil with a constant volumetric flow rate $q_i^{\mathrm{up}}$ along the entire operation time $t_i^{\mathrm{o}}$. According to Flores-Salazar et al.[30] the production in each time period must be at constant rate to avoid the "heading" in wells. If the well has been closed long enough, it is assumed that $P_i^{\mathrm{in}}$ is equal to the reservoir pressure. Moreover, $P_i^{\mathrm{f}}$ is the pressure of the wellbore at the end of the period.

If the values of geological properties are known, eq 1 can be reformulated as follows:

$$P_i^{\mathrm{f}} = P_i^{\mathrm{in}} - c1_i q_i^{\mathrm{up}}[\ln(t_i^{\mathrm{o}}) + c2_i] \qquad \forall\, i \in I \tag{2}$$

where $c1_i$ and $c2_i$ are the result of combining the parameters from eq 1.

If the well is not producing, i.e., if it is shut-in, there will be a recovery in the wellbore pressure. As a result, the pressure is reestablished in the time $t_i^{\mathrm{c}}$. This process has been expressed in Ortiz-Gómez et al.[25] through eq 3:

$$P_i^{\mathrm{f}} = P_i^{\mathrm{in}} + c1_i^{\mathrm{rec}}[\ln(t_i^{\mathrm{c}}) + c2_i^{\mathrm{rec}}] \qquad \forall\, i \in I \tag{3}$$

Once the well is not producing, there are two different behaviors in the wellbore pressure. First, this pressure increases with time logarithmically as shown in eq 3. When the wellbore pressure reaches the reservoir pressure $P_i^{\mathrm{up}}$, it remains constant at that value regardless of how long it stays closed. In other words, when the well is shut-in there exist two behaviors: a logarithmic one and a constant one. Consequently, the nonsmooth intersection point between these functions causes some difficulties when derivative-based optimization methods are used for solving problems involving this discontinuity.

To cope with the above situation, previous works[25,30] have resorted to the use of binary variables associated with the different functions, but at the expenses of increasing the problem size and complexity and, therefore, the computational burden. In this work, instead of using binary variables, a smooth approximation is proposed to link the two aforementioned behaviors into a single continuous and derivable function. This expression,[33] presented in eq 4, enables a correct representation of the wellbore pressure behavior when the well is closed.

$$P_i^{\mathrm{f}} = \left\{P_i^{\mathrm{in}} + c1_i^{\mathrm{rec}}(\ln t_i^{\mathrm{c}} + c2_i^{\mathrm{rec}}) + P_i^{\mathrm{up}} - \sqrt{(P_i^{\mathrm{in}} + c1_i^{\mathrm{rec}}(\ln t_i^{\mathrm{c}} + c2_i^{\mathrm{rec}}) - P_i^{\mathrm{up}})^2 + \delta^2}\right\} \div 2 \quad \forall\, i \in I \tag{4}$$

The parameter $\delta$ is a small scalar, which can be used to control the accuracy of the approximation. In most cases, a value between $1 \times 10^{-3}$ and $1 \times 10^{-4}$ is appropriate. In this work, a value of $5.5 \times 10^{-4}$ is used.

A typical wellbore pressure profile can be observed in Figure 1. At the beginning, the well $i$ is closed and its pressure is the same as the one of the reservoir $P_i^{\mathrm{up}}$, assuming the well has been closed for some considerable length of time. When the well is opened, the wellbore pressure decreases with time because of the oil flow from the reservoir into the wellbore (i.e., there is a flow up to the surface). The Figure 1 shows a marked decline of the pressure, especially at the beginning of the well operation until it reaches the minimum allowed pressure $P_i^{\mathrm{low}}$. This value is determined by operational or economic reasons, when it is no longer convenient to continue with the oil extraction. The wellbore pressure starts to recover when the well shuts-in. First, the recovery curve presents a steep increment, becoming more gradual thereafter until it attains the reservoir pressure. As can be observed in Figure 1, to achieve this value can take a considerable time.

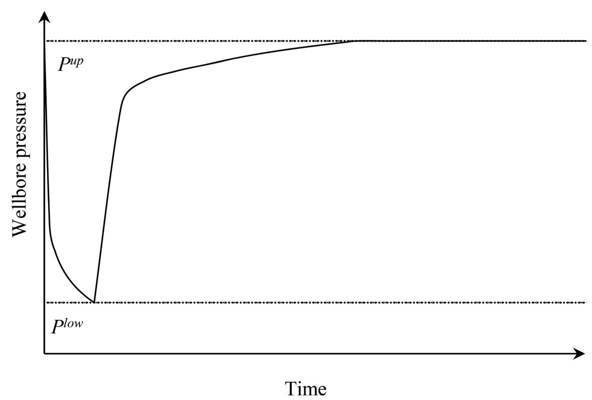


**Figure 1.** Typical behavior of the wellbore pressure.

As it was mentioned previously, the crude oil production system also contains the pipeline network that connects the wells with the surface facilities. However, in the past, this interconnectivity between oil wells has usually not been considered in the planning stage. Wells belonging to a reservoir are not isolated units but are interconnected to the manifolds by pipelines and valves, as shown Figure 2. This research assumes each manifold collects oil from wells with equal sulfur concentration belonging to the same reservoir. Because crude oil from different manifolds presents distinct properties, every


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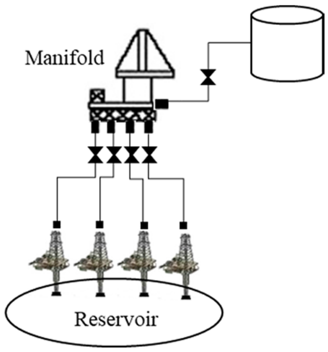


**Figure 2.** Oil production system.

petroleum type is sent to an assigned storage tank. Pressure constraints in surface facilities are included in the proposed model because well production is strongly affected by the pressure along the interconnection network.

To simplify the description the above-mentioned oil production network, the basic conceptual process units proposed by Barragán-Hernández et al.[29] have been used. As indicated by these authors, the conceptual well is considered as a single pipe where oil flows in from the reservoir and involves pressure drop between the reservoir and the wellbore. Considering that every well has a valve at the top of the pipe to allow individual flow rate control, the pressure balance is given by[30]

$$\Delta P_i = P_i^{\mathrm{up}} - P_i^{\mathrm{t}} = \Delta P_i^{\mathrm{r}} + \Delta P_i^{\mathrm{p}} + \Delta P_i^{\mathrm{v}} \qquad \forall\ i \in I \tag{5}$$

where $\Delta P_i$ is the total pressure drop in the well $i$, between the reservoir pressure $P_i^{\mathrm{up}}$ and the pressure at the top of the pipe $P_i^{\mathrm{t}}$. $\Delta P_i^{\mathrm{r}}$ is the pressure drop from the reservoir to the wellbore calculated with eq 2, and $\Delta P_i^{\mathrm{v}}$ is the pressure drop in the valve at the top of the pipe.

According to Barragán-Hernández et al.,[29] the models for pipes are represented by a set of differential-algebraic equations. Afterward, by considering some properties to be constant they come up with algebraic expressions. A homogeneous mass flow rate $\dot{m}_i$ flowing inside a pipe with diameter $D_i$ inclined a $\theta_i$ angle with total length $L_i$ is assumed, resulting in the next equation:

$$\Delta P_i^{\mathrm{p}} = \frac{\rho_i g L_i \sin \theta_i}{g_{\mathrm{c}}} + \frac{L_i f_i \dot{m}_i^{\,2}}{2 A_i^{\,2} D_i \rho_i g_{\mathrm{c}}} \qquad \forall\ i \in I \tag{6}$$

where $\rho_i$ is the crude oil density, $g$ is the acceleration due to gravity, $f_i$ is the friction factor, $A_i$ is the cross-sectional area of the pipe, and $g_{\mathrm{c}}$ is a conversion constant. In this work, friction factors are calculated with eq 7,[34]

$$f_i = \frac{0.079}{\sqrt[4]{\mathrm{Re}_i}} \qquad \forall\ i \in I \tag{7}$$

where $\mathrm{Re}_i$ is the Reynolds number.

Valves perform an important role in the oil production systems, allowing the opening and shutting-in of the wells in addition to regulating the crude oil flows at the desired levels. In this work, the conceptual valve is modeled using a simplified linear expression provided by Smith and Corripio,[35]

$$\Delta P_i^{\mathrm{v}} = \frac{C_i^{\mathrm{v}} \dot{m}_i}{\mathrm{AP}_i} \qquad \forall\ i \in I \tag{8}$$

where $C_i^{\mathrm{v}}$ is usually a constant parameter and $\mathrm{AP}_i$ is the opening of the valve, $\mathrm{AP}_i \in [0, 1]$. If the valve is closed, the flow is null even though there may be a pressure difference between the well bottom and head that must be taken into account.

To simplify the analysis, the energy balances are not included and isothermal profiles are supposed. Thus, by substituting eqs 2 and 6−8 into eq 5 and considering a multiperiod framework, the following single constraint is obtained:

$$P_{ij}^{\mathrm{t}} = P_{ij}^{\mathrm{f}} - P_{ij}^{\mathrm{in}} + P_i^{\mathrm{up}} - \frac{\rho_i g L_i \sin \theta_i}{g_{\mathrm{c}}} - \frac{L_i f_i {q_i^{\mathrm{up}}}^2}{2 A_i^{\,2} D_i \rho_i g_{\mathrm{c}}} - C_i^{\mathrm{v}} q_i^{\mathrm{up}} \qquad \forall\ i \in I, j \in J \tag{9}$$

where $q_i^{\mathrm{up}}$ is the volumetric flow rate in barrels per day.

A manifold is an arrangement of pipes and valves designed to control, distribute and monitor fluid flows. Manifolds are often configured for specific functions, such as well control operations or directing fluids. The conceptual manifold can be simply modeled as a single pipe plus a valve. Therefore, the following expression describes the pressure drop in the manifold:

$$\Delta P_m = \frac{\rho_m g L_m \sin \theta_m}{g_{\mathrm{c}}} + \frac{L_m f_m \dot{m}_m^{\,2}}{2 A_m^{\,2} D_m \rho_m g_{\mathrm{c}}} + \frac{C_m^{\mathrm{v}} \dot{m}_m}{AP_m} \qquad \forall\ m \in M \tag{10}$$

where $\Delta P_m$ is the pressure drop in the manifold $m$, and the rest of the variables are the same as described for wells but referred to manifolds. The pressure $P_m^{\mathrm{d}}$ is fixed within an acceptable range to guarantee the discharge of crude oils into the tanks.

If the multiperiod planning is considered as well as the volumetric flow rate at each period, eq 10 is reformulated in eq 11:

$$\Delta P_m = P_{mj}^{\mathrm{in}} - P_{mj}^{\mathrm{d}} = \frac{\rho_m g L_m \sin \theta_m}{g_{\mathrm{c}}} + \frac{L_m f_m {q_m^{\mathrm{up}}}^2}{2 A_m^{\,2} D_m \rho_m g_{\mathrm{c}}} + \frac{C_m^{\mathrm{v}} q_m^{\mathrm{up}}}{AP_{mj}} \qquad \forall\ m \in M, j \in J \tag{11}$$

where $P_{mj}^{\mathrm{in}}$ is the pressure at the inlet point of the manifold $m$ in period $j$. Constants of the valves are modeled following the classic definitions of valves given by Smith and Corripio,[35] as shown eq 12:

$$C_m^{\mathrm{v}} = \frac{s}{{CV_m^{\max}}^2 g_{\mathrm{c}}} q_m^{up} \qquad \forall\ m \in M \tag{12}$$

where $s$ is the specific gravity and $\mathrm{CV}_m^{\max}$ is a parameter provided by the valve supplier.

It is known that oil flowing from the reservoir is usually a complex mixture of hydrocarbons, which also contains sulfur, hydrogen, nitrogen, oxygen, and metallic compounds at low concentrations. Detecting these small component quantities is relevant to the petroleum industry because it provides information regarding the crude oil origin, and it can influence on its commercial value. Furthermore, different types of oil can coexist in a same sedimentary basin depending on their origin (lacustrine or marine).[36]

Because oil fields may contain different types of crude oil, it is difficult to characterize them accurately, so different methods have been developed. Vieira et al.[36] have used atomic absorption spectrometry to measure the concentration of nickel and vanadium in the crude oil samples. Instead, Ji et al.[37]


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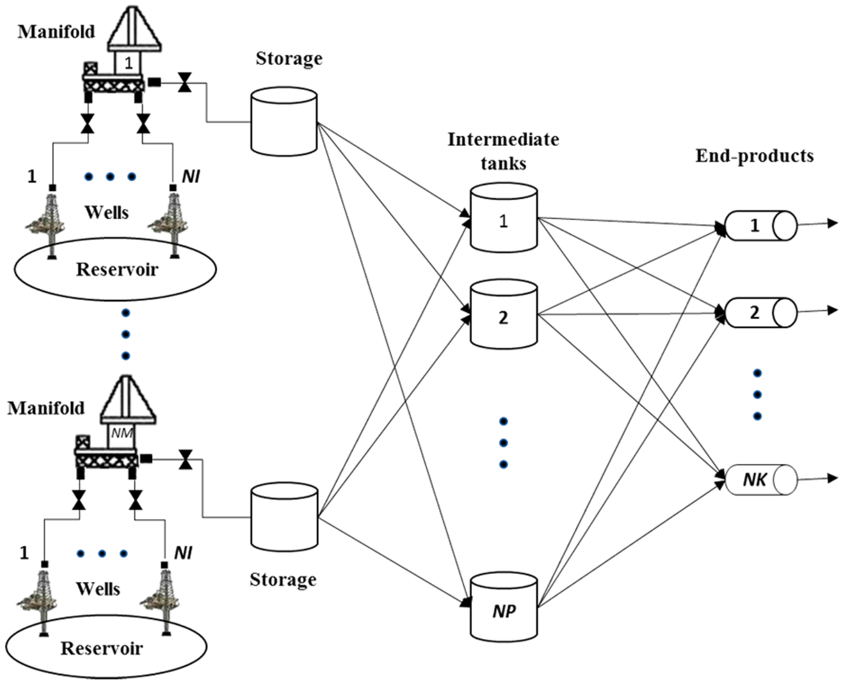


**Figure 3.** Oil production system representation considered in the proposed model.

used biomarker distributions of isoprenoid alkanes, steranes, and terpanes to identify two types of crude oils derived from different environments (estuarine and marine). However, the most commonly used method in the literature to characterize the crude oil is measuring the sulfur content because it is critical for the refining process. Therefore, the restrictions on sulfur content of the end products are dictated by the consumers (mainly refineries), which usually require strict quality specifications.[15]

Consequently, once the different types of crude oil are stored in tanks, an important remaining problem in the petroleum industry is the planning of crude oil blending operations to satisfy product specifications required by the consumers. The pooling problem determines the optimal scheme of blending of crude oils where several inflows with different attributes of quality are combined into intermediate tanks. Then, the outflows can be mixed again at the final node to obtain the end products within the specified range of sulfur concentration.

## 3. PROBLEM STATEMENT

The approach proposed in this work poses a multiperiod model for integrating the production planning and crude oil blending problems in an onshore oil field with known topology.

As shown in Figure 3, the complete oil production system studied in this paper consist of an oil field, reservoirs, wells, manifolds, storage and intermediate tanks, and transportation pipelines. The oil field consists of a number of reservoirs with a set $I$ ($i$ = 1, ..., NI) of drilled wells that are ready to produce over a short-term planning horizon $H$, divided into a number NJ ($j$ = 1, ..., NJ) of variable length time periods.

Because of production requirements, the wellbore pressure varies between a specified minimum operative pressure $P_i^{\text{low}}$ and the reservoir pressure $P_i^{\text{up}}$. Thus, procedures for opening or shutting-in the well are performed to maintain the wellbore pressure within this range. As mentioned earlier, the wellbore pressure follows a logarithmic behavior during the production process (eq 2). When the well $i$ is producing in period $j$ under a constant flow rate $q_i^{\text{up}}$, a total volume $Q_{ij}$ is obtained during an operation time $t_{ij}^{\text{o}}$. Each well $i$ has different geological properties $c1_i$, $c2_i$ that are known and are considered constant during the time horizon. These constants have been arbitrarily selected to represent different well behaviors.

The crude oil is gathered from wells with the same sulfur content and flows to manifold $m$ ($m$ = 1, ..., NM), which collects a volume of crude oil $C_{mj}$ with a sulfur concentration $\text{SQ}_m$. It is assumed that wells belonging to the same reservoir have equal sulfur concentration. The aperture and the CV associated with each manifold $m$, $\text{AP}_{mj}$ and $C_m^{\text{v}}$, are considered model variables to adjust valve operation according to the flow through the manifold $m$ and to accomplish the specified range of discharge pressure in the storage tank for period $j$. In each time period $j$ every type of crude oil is transported from manifolds to an assigned storage tank where a range of discharge pressure $[P_{mj}^{\text{d,low}} - P_{mj}^{\text{d,up}}]$ is fixed.

Once different types of crude oil are produced and stored, the pooling is performed in the first place in a set of $P$ ($p$ = 1, ..., NP) intermediate tanks, with an $S_p$ sulfur content each, and in the second place directly into the dispatch pipeline. Given a set $K$ ($k$ = 1, ..., NK) of end products demanded by the refineries and their quality specifications in an allowable range $[Z_k^{\text{low}} - Z_k^{\text{up}}]$, the problem determines the total volume of each end product $E_k$ and its sulfur concentration $Z_k$.

Then, the model consists of determining the following:

(a) Planning decisions: (i) the existence of periods; (ii) the operation of wells, i.e., if the well is producing or shut-in; (iii) the number of time periods; (iv) the pressure at the beginning/end of each time period; (v) the length of the periods; (vi) the produced volume of crude oil.

(b) Blending decisions: (i) the selection of the intermediate tank $p$; (ii) existence of an inflow from manifold $m$ to the intermediate tank $p$; (iii) the discharge pressure in storage tanks; (iv) the degree of opening in the valve of manifolds; (v) the volume of the inflow from manifolds to intermediate tanks; (vi) the volume of the outflows


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toward the end products; (vii) the pressure drops in pipes, valves, and manifolds; (viii) the sulfur content of oils in intermediate tanks; (ix) the volume of the end products; (x) the sulfur concentration in the end products.

The objective function OP maximizes the produced volume of the $k$ end products within the required range of sulfur, because there is no constraint in the demand and it is assumed that all production can be sold.

Finally, some assumptions have been made to model the problem:

(i) the wells in the reservoir are separated enough from each other; thus they produce independently
(ii) geological and physical properties in wells are known
(iii) the reservoir pressure remains constant throughout the planning horizon
(iv) nonlinear behavior exists on the wellbore pressure
(v) if the well is open, it operates at a constant flow rate $q_i^{\mathrm{up}}$
(vi) because of geological and physical reasons each well has a different pressure profile
(vii) wells only produce crude oil
(viii) only sulfur concentration is considered as quality parameter
(ix) each manifold collects crude oil from wells with equal sulfur concentration
(x) the process is isothermal
(xi) mixing times effects in the storage tanks are neglected

## 4. MULTIPERIOD GENERALIZED DISJUNCTIVE PROGRAMMING MODEL

To obtain a mathematical formulation of the aforementioned crude oil onshore optimization problem, the Generalized Disjunctive Programming (GDP) representation[38] is used in this work. In what follows, the objective function and constraints corresponding to the planning part of the well operation are explained first.

$$\mathrm{OP} = \sum_k E_k \tag{13}$$

$$\begin{bmatrix} W_{ij} \\ \mathrm{tr}_{ij} = \mathrm{tp}_{ij} \\ \begin{bmatrix} Y_{ij} \\ \mathrm{tp}_{ij} = t_{ij}^{\mathrm{o}} \\ Q_{ij} = q_i^{\mathrm{up}}\mathrm{tp}_{ij} \\ P_{ij}^{\mathrm{f}} = P_{ij}^{\mathrm{in}} - c1_i q_i^{\mathrm{up}}(\ln t_{ij}^{\mathrm{o}} + c2_i) \\ P_{ij}^{\mathrm{f}} \le P_{ij}^{\mathrm{in}} \\ P_{ij}^{\mathrm{t}} = P_{ij}^{\mathrm{f}} - P_{ij}^{\mathrm{in}} + P_i^{\mathrm{up}} - \dfrac{\rho_i g L_i}{g_{\mathrm{c}}} - \dfrac{L_i f_i (q_i^{\mathrm{up}})^2}{2A_i^2 D_i \rho_i} - C_i^{\mathrm{v}} q_i^{\mathrm{up}} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{ij} \\ \mathrm{tp}_{ij} = t_{ij}^{\mathrm{c}} \\ Q_{ij} = 0 \\ P_{ij}^{\mathrm{f}} = P_{ij}^{\mathrm{f}}(P_{ij}^{\mathrm{in}}, q_i^{\mathrm{up}}, t_{ij}^{\mathrm{c}}) \\ P_{ij}^{\mathrm{t}} = P_{ij}^{\mathrm{f}} - P_{ij}^{\mathrm{in}} + P_i^{\mathrm{up}} - \dfrac{\rho_i g L_i}{g_{\mathrm{c}}} \end{bmatrix} \end{bmatrix} \vee \begin{bmatrix} \neg W_{ij} \\ \mathrm{tr}_{ij} = 0 \end{bmatrix}$$
$$\forall\, i \in I, j \in J \tag{14}$$

$$W_{ij+1} \Rightarrow W_{ij} \qquad \forall\, i \in I, j \in J, j < \mathrm{NJ} \tag{15}$$

$$W_{ij} \wedge W_{ij+1} \Rightarrow (Y_{ij} \wedge \neg\, Y_{ij+1}) \vee (\neg\, Y_{ij} \wedge Y_{ij+1}) \quad \forall\, i \in I, j \in J \tag{16}$$

$$n_i = \sum_j w_{ij}\ \forall\, i \in I \tag{17}$$

$$\mathrm{QT}_i = \sum_j Q_{ij} \qquad \forall\, i \in I \tag{18}$$

$$H = \sum_j \mathrm{tr}_{ij} \qquad \forall\, i \in I \tag{19}$$

The objective function (13) consists of maximizing the total production of end products $E_k$. Although alternative performance criteria can be applied, e.g., minimizing production cost or maximizing the profit, the production goal selected here is more meaningful for the upstream production area of the oil industry.

Disjunctions (14) represent the behavior of wells $i$ in each time period $j$. In the first decision level, the existence or absence of time period $j$ for well $i$ is taken into account by Boolean variable $W_{ij}$. If the time period $j$ exists ($W_{ij}$ = True), there are two possible scenarios in the second decision level represented by the embedded disjunctions:

(i) If the Boolean variable $Y_{ij}$ is true, the well $i$ is open to flow at full capacity $q_i^{\mathrm{up}}$ through the period $j$. The processing time $\mathrm{tp}_{ij}$ will be the opening time $t_{ij}^{\mathrm{o}}$ and the pressure behavior is described by eq 2. The pressure drop through the pipe is calculated with eq 9.
(ii) However, if this variable is false, the well $i$ is not producing in the period $j$; thus the volume of crude oil $Q_{ij}$ is null and the recover pressure is calculated by eq 4. The processing time $\mathrm{tp}_{ij}$ will be the closing time $t_{ij}^{\mathrm{c}}$, and the pressure drop through the pipe is obtained only by considering the hydrostatic pressure (section 2).

On the contrary, if $W_{ij}$ is false, the period $j$ for well $i$ does not exist.

Logic proposition (15) ensures that if the time period $j$ + 1 exists, then the previous period $j$ also exists. Proposition (16)


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introduces the requirement of a cyclic operation; i.e., if a well operates in a number of periods, e.g., $n = 4$, the sequence will be open/shut-in/open/shut-in. In other words, if two consecutive periods $j$ and $j + 1$ exist, the well $i$ will be open in the period $j$ and will be closed in the period $j + 1$, or conversely. As already mentioned, the periods in the model are defined by the change of operation in the well.

The number of existing periods for each well, $n_i$, is a variable of the model and it is calculated in eq 17 as the number of periods in which well $i$ is open or shut-in. Equation 18 provides the total volume of crude oil produced by the well $i$ in period $j$, $QT_{ij}$, whereas the total sum of the real times in each period $j$ for the well $i$ must be equal to the planning horizon $H$, as stated by eq 19.

To ensure the continuity of wellbore pressures, eq 20 forces the final pressure of period $j$ to be equal to the initial pressure of period $j + 1$. Furthermore, it is assumed that at the beginning of the planning horizon (i.e., initial time $j = 1$) all wells are closed, with a bottom-hole pressure equal to the reservoir pressure (eq 21). However, this restriction could be easily changed if required.

$$P^{in}_{i,j+1} = P^{f}_{j} \qquad \forall\ i \in I,\ j \in J,\ j < NJ \tag{20}$$

$$P^{in}_{i,1} = P^{up}_{i} \qquad \forall\ i \in I \tag{21}$$

Equations corresponding to pressure continuity in the interconnection network and manifolds operation are following explained. Because several wells can feed a single manifold, a mass balance must be included at its inlet point. So, eq 22 represents for each period of time $j$ the mass balance in the manifold $m$, where $O_m$ is the set of wells $i$ that are connected, and $C_{mj}$ is the total volume of crude oil.

$$C_{mj} = \sum_{i \in O_m} Q_{ij} \qquad \forall\ m \in M,\ j \in J \tag{22}$$

Equation 23 calculates the total volume of crude oil collected in the manifold $m$ at the end of the planning horizon.

$$CT_m = \sum_{j} C_{mj} \qquad \forall\ m \in M \tag{23}$$

When several wells are connected to the same manifold, they may have different wellbore pressures. The resulting pressure, $P^{in}_{mj}$, must be equal to the minimum between the involved wells and is modeled by eq 24:

$$P^{in}_{mj} = \min_{i}\{P^{t}_{ij}\} \qquad \forall\ m \in M,\ j \in J \tag{24}$$

Equation 25 calculates the total production volume of the manifold $m$, as the summation of the volumes dispatched to the intermediate tanks. This expression represents the link between the production planning and pooling problems.

$$CT_m = \sum_{p} b_{mp} \qquad \forall\ m \in M \tag{25}$$

To complete the manifolds and tanks model, eqs 11 and 12 calculate the discharge pressure of the storage tanks.

Finally, the pooling itself is modeled through a new set of constraints presented in eqs 26–32, which describe blending operations in the intermediate tanks.

$$\begin{bmatrix} U_{mp} \\ b_{mp} \geq b^{min}_{mp} \end{bmatrix} \vee \begin{bmatrix} \neg U_{mp} \\ b_{mp} = 0 \\ \sum_{k} x_{pk} = 0 \end{bmatrix} \qquad \forall\ m \in M,\ p \in P \tag{26}$$

$$V_p \Rightarrow \bigvee_{m} U_{mp} \qquad \forall\ p \in P \tag{27}$$

$$\neg V_p \Rightarrow \neg\ U_{mp} \qquad \forall\ m \in M,\ p \in P \tag{28}$$

$$U_{mp} \Rightarrow \bigvee_{r \neq m} U_{r,p} \qquad \forall\ m \in M,\ p \in P \tag{29}$$

$$\sum_{m} b_{mp} - \sum_{k} x_{pk} = 0 \qquad \forall\ p \in P \tag{30}$$

$$\sum_{m} SQ_{m} b_{mp} - S_p \sum_{k} x_{pk} = 0 \qquad \forall\ p \in P \tag{31}$$

$$\sum_{m} b_{mp} \leq CP_p \qquad \forall\ p \in P \tag{32}$$

First, the decision process involves the selection of the available tanks that will be used in the blending operation, which is described by Boolean variable $V_p$. Then, disjunction in eq 26 determines if a stream from manifold $m$ is assigned to intermediate tank $p$. If through Boolean variable $U_{mp}$ is true, a minimum volume $b^{min}_{mp}$ (in barrels) must be sent to each blending tank $p$. In the negative case ($U_{mp}$ = False), the input flow from manifold $m$ to the intermediate tank $p$ is null, as well as the output flow toward end product $k$.

Equation 27 ensures that if $V_p$ is true, the intermediate tank has at least one incoming flow, whereas if $V_p$ is false, eq 28 establishes that no stream is assigned to that tank. Furthermore, as proposed in eq 29, when an incoming flow from manifold $m$ is assigned to the intermediate tank $p$ ($U_{mp}$ = True), there exists at least one stream from another manifold $r$, $b_{rp}$, assigned to it. The mass balance, the quality balance, and the capacity limit ($CP_p$) of each tank are given by eqs 30–32.

The total volume of end product $k$ obtained at the end of the planning horizon is calculated in eq 33 as the sum of the outflows of the intermediate tanks. Finally, eq 34 enforces the sulfur quality requirements for each end products in the final node.

$$E_k = \sum_{p} x_{pk} \qquad \forall\ k \in K \tag{33}$$

$$\sum_{p} S_p x_{pk} = Z_k \sum_{p} x_{pk} \qquad \forall\ k \in K \tag{34}$$

$$\begin{gathered} t^{o}_{ij},\ t^{c}_{ij},\ tp_{ij},\ tr_{ij},\ n_i,\ P^{in}_{ij},\ P^{f}_{ij},\ Q_{ij},\ QT_{ij},\ C_{mj},\ CT_m,\ P^{t}_{ij}, \\ P^{in\,m}_{ij},\ P^{d}_{mj},\ AP_{mj},\ b_{mp},\ x_{pk},\ S_p,\ Z_k,\ E_k \geq 0 \\ \forall\ i \in I,\ j \in J,\ m \in M,\ k \in K \end{gathered} \tag{35}$$

$$W_{ij},\ Y_{ij},\ V_p,\ U_{mp} \in \{\text{True, False}\} \tag{36}$$

In summary, the nonlinear multiperiod GDP model for integrating planning and pooling problems consists in maximizing total production represented in eq 13 subject to constraints (11), (12), and (14)–(36). There are two strategies widely used to solve this GDP model: big-M and convex


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hull.[11,38−40] Both strategies reformulate the problem as a mixed-integer nonlinear program (MINLP). In this paper, the big-M representation has been used to solve the proposed model. See the Supporting Information for details of the reformulation.

## 5. CASE STUDY RESULTS AND DISCUSSION

A case study is presented to demonstrate the scope and versatility of the proposed formulation. Also, a sensitivity analysis has been performed by varying the lower bound of sulfur quality specification to highlight its influence on the production of wells. Next, several problem instances are presented to demonstrate how the model performance and the results vary according to the values of different parameters. Finally, a comparison between the results of these examples is addressed. All examples have been implemented and solved through GAMS[33] version 24.1.3 on a PC Intel Core i7 with 3.4 GHz and 8 GB of RAM. The code DICOPT was employed for solving the reformulated MINLP model.

**5.1. Case Study Description.** The onshore production system consists of a single oil field where oil is produced from two reservoirs over a planning horizon of 6 days that can be divided into a maximum of 6 time periods (NJ = 6). The reservoir A contains two wells $i_1$ and $i_2$, drilled at 2500 m of depth, where the extracted oil presents a sulfur content of 3%. The oil is collected through the manifold $m_1$ and stored in an assigned tank. In the reservoir B, there are four wells $i_3$, $i_4$, $i_5$, and $i_6$ drilled at 2000 m of depth. The oil from these wells, with a sulfur content of 1%, is collected through manifold $m_2$ and stored in a different tank. Note that depending on the sulfur concentration the oil produced is stored in different tanks. A given desirable range of discharge pressure is assumed within the range 22−73 psia.

For both reservoirs, the pressure is considered uniform through the planning horizon at 6009 psia, whereas the minimum operative pressure allowed is 5650 psia. It is assumed that at the initial time, all wells are closed and their pressure is the same as the related reservoir pressure. Because of geological considerations indicating nonuniformity in the reservoir, wells present different wellbore pressure behavior. Table 1 shows the

**Table 1. Constants for Each Well for the Case Study**

| wells | $q_i^{up}$ [bbls[a]/day] | $c1_i$ | $c2_i$ | $c1_i^{rec}$ | $c2_i^{rec}$ |
|---|---|---|---|---|---|
| $i_1$ | 1050.00 | $4.39 \times 10^{-2}$ | 4.61 | 38.00 | 4.61 |
| $i_2$ | 900.00 | $4.39 \times 10^{-2}$ | 5.60 | 34.80 | 5.60 |
| $i_3$ | 900.00 | $4.38 \times 10^{-2}$ | 5.60 | 34.80 | 5.60 |
| $i_4$ | 600.00 | $6.10 \times 10^{-2}$ | 5.94 | 34.30 | 5.94 |
| $i_5$ | 900.00 | $4.38 \times 10^{-2}$ | 5.60 | 34.80 | 5.60 |
| $i_6$ | 600.00 | $6.10 \times 10^{-2}$ | 5.94 | 34.30 | 5.94 |

[a]bbls: blue barrels.

characteristic parameters of each well, which were selected to have different pressure profiles. The table also shows the maximum flow rate for each well, which is assumed constant over the time horizon.

For simplicity, the same tube diameter ($7.62 \times 10^{-2}$ m) and similar fluid properties are considered for all wells; therefore, the Reynolds number can be expressed as the product between a single constant and the corresponding well flow rate $i$, $q_i^{up}$. Manifolds are represented by a pipe of 100 m length.

Once crude oil is stored, the pooling is carried out in the intermediate tanks to achieve the sulfur quality specifications requested by the refineries. Two end products are required with a range of sulfur concentration between 2.4 and 2.8% for $k_1$ and 1.4−1.8% for $k_2$. Three intermediate tanks with a capacity of 5000 barrels each are available for blending operations.

**5.2. Case Study Results.** The entire model comprises 1056 equations, 453 continuous variables, and 81 binary variables, and it was solved in a CPU time of 5.9 s.

The optimum solution gives an objective function value of 13 608.4 barrels, with an oil volume of 4226.6 barrels ($CT_1$) collected through manifold $m_1$ and 9381.8 barrels ($CT_2$) in manifold $m_2$. This production is achieved with an optimal planning where wells $i_1$, $i_2$, $i_3$, $i_5$, and $i_6$ operate in five periods each, and well $i_4$ operates in four periods, as can be observed in Figure 4. Also, this figure shows the bottom-hole pressure

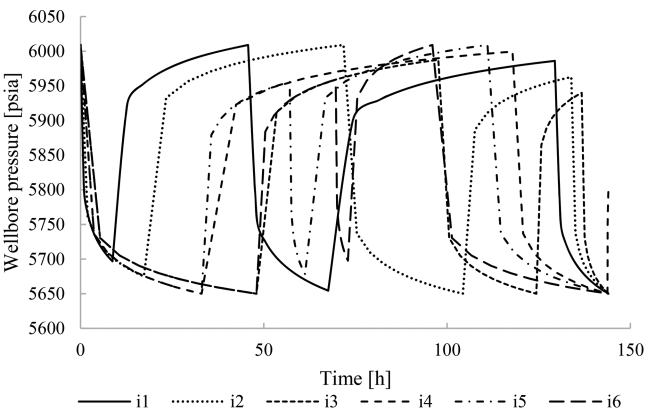


**Figure 4.** Wellbore pressure profile for the case study (NJ = 6).

profile per well in each time period. For example, well $i_2$ opens in the first period during a time $t_{21}^o$ of 17.3 h and its pressure decreases until 5675 psia, whereas in the second period it shuts-in over a time period $t_{22}^c$ of 54.5 h achieving the reservoir pressure $P^{up}$. Then, the well opens during $t_{23}^o$ = 32.5 h in the third period until the pressure reaches the value of $P^{low}$ where it shuts-in for second time ($t_{24}^c$ = 29.6 h) to recover a pressure of 5962.8 psia. Finally, it opens once again in the fifth and last period during $t_{25}^o$ = 10.1 h achieving the minimum allowed pressure at the end of the planning horizon.

Table 2 presents the number and length of the time periods as well as the total production time per well. As can be observed, the lengths of the open and shut-in periods of each well, $t_{ij}^o$ and $t_{ij}^c$, are not necessarily the same.

The optimal flowchart for this base case is shown in Figure 5. The blending of crude oils takes place through the three available intermediate tanks $p_1$, $p_2$, and $p_3$ to satisfy the quality

**Table 2. Length and Number of Time Periods for the Case Study**

| | length of time periods [h] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| well | $t_{i1}^o$ | $t_{i2}^c$ | $t_{i3}^o$ | $t_{i4}^c$ | $t_{i5}^o$ | $t_{i6}^c$ | no. of periods, $n_i$ | production time ($\sum_j t_{ij}^o$) [h] |
| $i_1$ | 8.7 | 37.0 | 21.9 | 61.8 | 14.6 | | 5 | 45.2 |
| $i_2$ | 17.3 | 54.5 | 32.5 | 29.6 | 10.1 | | 5 | 59.9 |
| $i_3$ | 48.0 | 49.6 | 26.7 | 12.4 | 7.3 | | 5 | 82.0 |
| $i_4$ | 33.0 | 84.9 | 25.9 | 0.3 | | | 4 | 58.9 |
| $i_5$ | 33.0 | 24.1 | 4.2 | 49.7 | 33.0 | | 5 | 70.2 |
| $i_6$ | 48.0 | 21.6 | 3.3 | 23.0 | 48.0 | | 5 | 99.3 |


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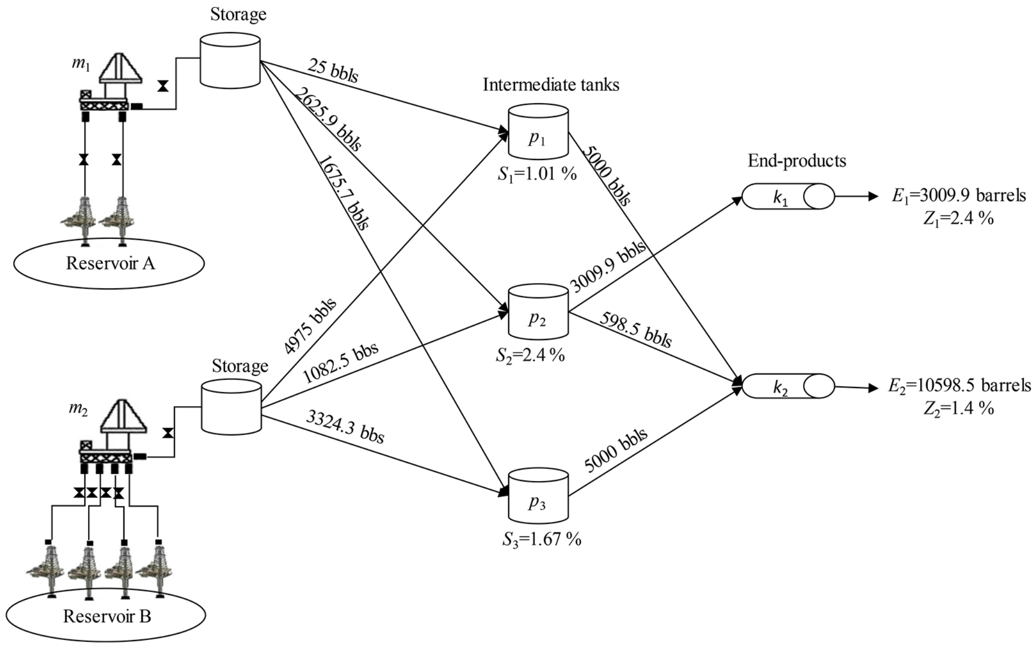


**Figure 5.** Optimal flowchart for the case study.

specification (sulfur content) of the end products requested by refineries. Note that the intermediate tanks $p_1$ and $p_3$ are at full capacity while $p_2$ still has storage capacity. At the solution, the content of sulfur of the oil in these tanks are 1.01%, 2.4%, and 1.67% for $p_1$, $p_2$, and $p_3$, respectively.

To obtain the 3009.9 barrels of end product $k_1$ with a sulfur concentration of 2.4%, 3009.9 barrels of crude from intermediate tank $p_2$ are sent directly to the final node. Also, the 10 598.5 barrels of $k_2$ with a sulfur concentration of 1.4% were obtained by blending 5000 barrels of oil containing 1.01% of sulfur from tank $p_1$, with 598.5 barrels with a sulfur content of 2.4% from $p_2$ and 5000 barrels with 1.67% sulfur content from $p_3$.

It is worth noting that although the objective function is the maximization of the volume of final products, one of the three intermediate tanks and the wells have idle capacity. This means that there must be active constraints limiting the increase in production. From Figure 5 can be observed that these constraints are the sulfur content of both end products, which are at the lower bound of their specified ranges. It can be inferred that wells are not producing more crude oil because the resulting blending would be out of specification. To evaluate the impact of this bottleneck, a sensitivity analysis was conducted on the lower bounds of variables $Z_1$ and $Z_2$, and the results are presented in Figures 6 and 7.

The study demonstrates that the objective function OP increases as the lower bound on sulfur concentration of both end products is relaxed. This allows that manifold $m_2$, which is the manifold with a larger number of wells, to produce more crude oil with 1% of sulfur content and, therefore, increases the accumulated production of the end products. This increment is related with the production time. In the base case, the wells connected to manifold $m_2$ present a lower production time than the corresponding values when the lower bound on sulfur concentration decreases. For example, in the case study the well $i_3$ has a production time of 82 h (Table 2), whereas when the lower bound of $Z_1$ is 2.2% the production time increases to 99.3 h. As a consequence of a higher volume of crude oil collected in

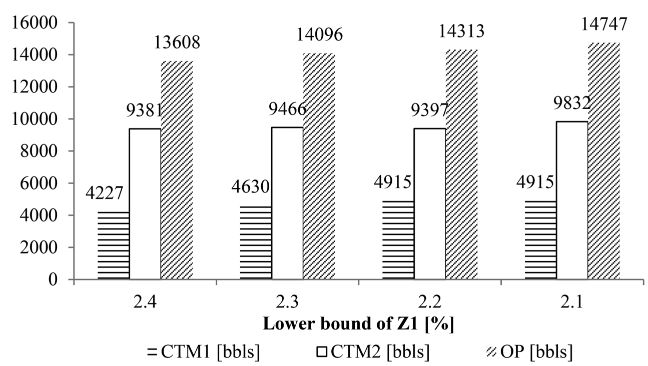


**Figure 6.** Sensitivity analysis of variable $Z_1$.

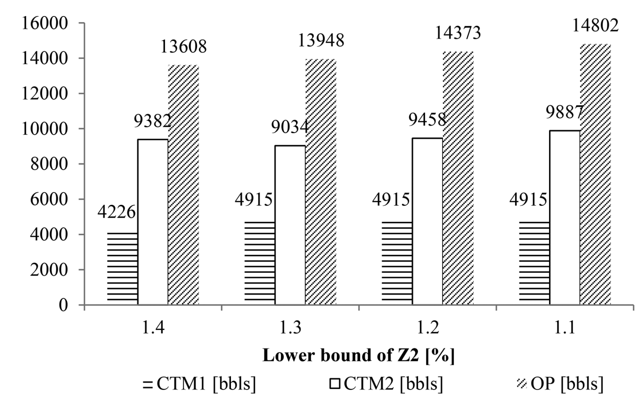


**Figure 7.** Sensitivity analysis of variable $Z_2$.

manifold $m_2$ a variation, generally an increment, on the volume collected in manifold $m_1$ is necessary to attain the quality required in the final blending. Note that, in fact, when the lower bound of end product $Z_1$ is 2.2%, the production of manifold $m_2$ decreases, but production of manifold $m_1$ increases in such proportion that the objective function is even higher than the previous case (14 313 vs 14 096 barrels). These results suggest that the sulfur specification for final products required by


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refineries significantly affects the objective function and the wells operation scheme.

**5.3. Other Results of the Case Study.** Next, three examples are presented to show how the changes in some model parameters can lead to significant variations in the results of the crude oil production system in the case study.

*5.3.1. Example 1: Variation in the Maximum Number of Periods.* To assess the effect of discretizing the planning horizon in different number of periods, two additional instances have been solved. In the first one, the planning horizon can be discretized in up to three time periods (NJ = 3), whereas in the second one, this number is increased up to nine periods (NJ = 9). Note that only the number of periods is modified; the rest of the constants and parameters remain the same.

The accumulated production of the end products, OP, increases significantly with the maximum number of periods. These increments can be explained by analyzing the wellbore pressure profile for each case, illustrated in the Figures 4, 8, and 9. Figure 8 illustrates pressure profiles when the wells can only operate/shut-in up to three time periods. Note that, for all wells there exist three periods, i.e., $n_i = 3\ \forall\ i$. For example, the well $i_2$ is open in the first period during a time $t^{o}_{21}$ of 32.5 h until the minimum allowed pressure $P^{low}$ = 5650 psia is reached. In the second period, the well shuts-in during a time $t^{c}_{22}$ of 85.7 h achieving the reservoir pressure $P^{up}$. In the third and last period, the well opens once again during 25.8 h until reaching the value of $P^{low}$ at the end of the time horizon. The same behavior is observed in the other wells, where only the length of their periods is different. It can be noted that in all the profiles that the second period ($j$ = 2), which is necessary to recover the pressure, requires a considerable time to reach the reservoir pressure. Therefore, the production time is lower and consequently the objective function diminishes. For instance, when $NJ$ = 3 the production time of the well $i_3$ is 66.6 h, whereas in the base case is 82 h (Table 2).

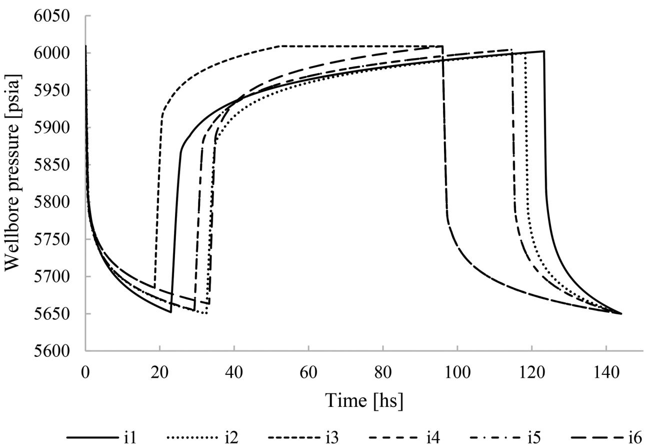


**Figure 8.** Wellbore pressure profile for example 1 (NJ = 3).

Figure 9 shows the bottom-hole pressure profiles when the planning horizon can be discretized in up to 9 periods (NJ = 9). As can be seen, some wells operate in more cycles (open/shut-in) than in the case study. Table 3 summarizes the number of periods in which each well operates and the different lengths of these time periods. For example, if the behavior of the well $i_2$ is observed, it is open during five periods and is closed in four periods. Also, note the fast switch of pressures in the fourth and sixth periods. This staggered behavior allows reaching the reservoir pressure in the shortest possible time.

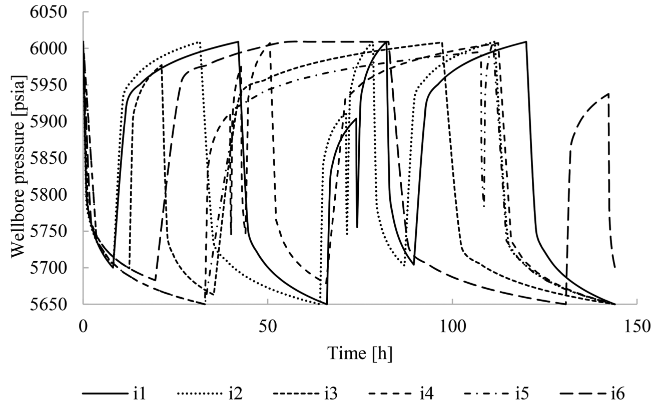


**Figure 9.** Wellbore pressure profile for example 1 (NJ = 9).

If a well operates in more cycles through the same planning horizon, its production will be higher. As stated previously, this is related to an increment in the production time. For example, as shown Table 3, the production time of the well $i_2$ is 82 h, 22.1 h higher than in the base case, which can operate until five periods (Table 2). Thus, operating with the above-mentioned staggered cycles, not only the reservoir pressure is achieved more quickly, but also the wells produce crude oil even in small periods of time. In the optimal solution, the objective function is 15 000 barrels, a 10.2% higher than that of the case study. Finally, it is important to emphasize that, in this instance, the three intermediate tanks are at full capacity, resulting in a hard constraint to the model.

Figure 10 compares the results for the objective functions obtained in these two above-mentioned instances with the one found for the base case. Besides, the increment in the maximum number of periods considerably influences the resolution time of the model.

Due to the previous analysis, it is evident that both the number of periods and their lengths must be model variables to select the best scheme that optimize the crude oil production. If these variables are fixed, the effect of the staggered behavior of wellbore pressure for reaching the reservoir pressure would not be possible and the crude oil production would be considerably lower.

*5.3.2. Example 2: Capacity of Intermediate Tanks.* As already mentioned, capacity of intermediate tanks limits the objective function. Thus, the base case (NJ = 6), as well as the instances presented in example 1 (NJ = 3 and NJ = 9), are solved here by incrementing the size of the intermediate tanks up to 8000 barrels each. From Figure 10, it can be seen that the objective function rises for NJ = 6 and NJ = 9 whereas for NJ = 3 it remains the same. In fact, when wells can operate up to NJ = 9 periods, the accumulated production of final products OP is 16 000 barrels, 6.7% more barrels than the same scenario in the example 1, where the three available intermediate tanks are at full capacity.

As mentioned above, the objective function remains in 12 201 barrels when NJ = 3. This is explained by the production potential of the wells. For the posed case study, 12 201 barrels is the highest volume of crude oil that wells can produce in only three time periods, no matter which restriction is relaxed.

*5.3.3. Example 3: Limited Dispatching Capacity.* In this example, there is a limit on the volume of crude oil that can be


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**Table 3. Length and Number of the Time Periods for Each Well with NJ = 9**

| well | length of time periods [h] | | | | | | | | | no. of periods, $n_i$ | production time $(\sum_j t^o_{ij})$ [h] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $t^o_{i1}$ | $t^c_{i2}$ | $t^o_{i3}$ | $t^c_{i4}$ | $t^o_{i5}$ | $t^c_{i6}$ | $t^o_{i7}$ | $t^c_{i8}$ | $t^o_{i9}$ | | |
| $i_1$ | 8.1 | 33.9 | 24.0 | 7.9 | 0.3 | 7.9 | 7.4 | 30.4 | 24.0 | 9 | 63.8 |
| $i_2$ | 8.2 | 23.4 | 32.5 | 7.0 | 0.3 | 7.0 | 8.5 | 24.4 | 32.5 | 9 | 82.0 |
| $i_3$ | 12.5 | 8.7 | 14.1 | 61.8 | 46.8 | | | | | 5 | 73.4 |
| $i_4$ | 33.0 | 6.8 | 0.3 | 2.7 | 1.2 | 6.7 | 15.2 | 46.6 | 31.5 | 9 | 81.2 |
| $i_5$ | 33.0 | 74.8 | 00.8 | 2.4 | 33.0 | | | | | 5 | 66.8 |
| $i_6$ | 19.6 | 63.2 | 48.0 | 11.6 | 1.7 | | | | | 5 | 69.3 |

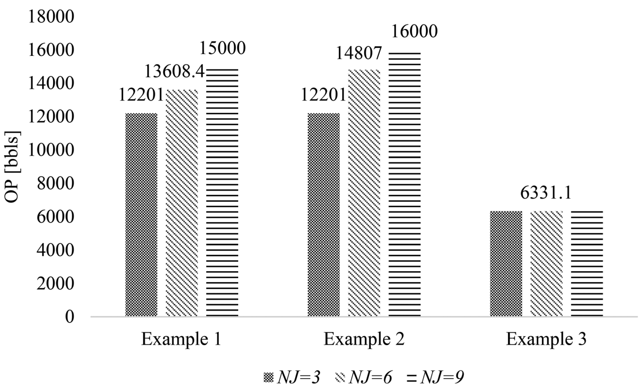


**Figure 10.** Model performance for the different examples.

dispatched from the intermediate tanks, either due to a limited pumping capacity or because of a design flaw. In fact, if this constraint is considered, the benefit of increasing the number of periods to obtain larger productions cannot be observed. Figure 10 compares this example with the examples 1 and 2, where there are not limitations on the crude oil dispatching capacity. As shown, a limited dispatching capacity of crude oils gives, as result, the same objective function (i.e., 6331.1 barrels) for NJ = 3, NJ = 6, and NJ = 9. Although the wells can produce more crude oil with a higher number of periods, the volume of end products at the end of the planning horizon is considerably lower than the base case study. Thus, as shown with this example, a limited pumping capacity in downstream facilities is a strong restriction that avoids finding better solutions in the production planning problem.

As it was shown previously, constraints about operation of wells, pressure drops in the interconnectivities, capacity of intermediate tanks, and sulfur concentration in blending operations can affect the model performance significantly.

## 6. CONCLUSIONS

A new disjunctive multiperiod model for the simultaneous production planning and crude oil blending in an on-shore oil field with known topology and a fixed interconnectivity scheme has been presented. In contrast to previous works, the number of periods in which each well can open/shut-in and the length of these periods are decision variables of the model. Furthermore, the nonlinearities are considered in the model both in the wellbore pressure behavior and in the bilinear terms present in the quality balances. When the well is not producing, a new expression to describe the recovery pressure of the wellbore is introduced, avoiding the use of additional binary variables. Besides, pressure drops in wells, pipelines, valves, and manifolds are modeled.

The proposed approach integrates three relevant aspects of the petroleum industry, oil production planning, interconnectivity among wells, and blending operations, providing a more realistic approximation of the problem. Thus, an important feature of the formulation is its capability for evaluating different decisions involved in the oil production system, from the reservoirs (upstream) to the downstream facilities, which are usually treated in a separate manner.

The proposed model was formulated with GDP and reformulated into an MINLP by using the big-M representation for its resolution. To demonstrate the scope of the formulation, a case study and several examples were presented where the parameters of the model were analyzed. Numerical results show that the production in oil fields is strongly dependent on the maximum allowed number of periods. Also, these results clearly demonstrate the importance of implementing an integral formulation due to restrictions such as production potential of wells, capacity of intermediate tanks and sulfur quality specification for the end products are active constraints of the model, significantly affecting the value of the objective function.

It is important to mention that the MINLP model presents nonconvex bilinear equations; therefore, the global optimality of the solutions cannot be guaranteed. The study of this model through techniques of global optimization will be the subject of a future work.

## ■ ASSOCIATED CONTENT

### Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.7b03526.

Big-M reformulation of the GDP model (PDF)

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**Notes**

The authors declare no competing financial interest.

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## ■ NOMENCLATURE

### Sets and Indices

$I$ wells ($i$ = 1, ..., NI)

$J$ periods ($j$ = 1, ..., NJ)



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$M$ manifolds ($m$, $r$ = 1, ..., NM)
$P$ intermediate tanks ($p$ = 1, ..., NP)
$K$ end products ($k$ = 1, ..., NK)
$O_m$ wells connected to manifold $m$ ($i$ = 1, ..., $\mathrm{NI}_m$)

**Parameters**

$c1_i$, $c2_i$ geological and physical properties to calculate pressure drop of well $i$
$c1_i^{\mathrm{rec}}$, $c2_i^{\mathrm{rec}}$ geological and physical properties to calculate recover pressure of well $i$
$P_i^{\mathrm{up}}$ reservoir pressure of well $i$
$P_i^{\mathrm{low}}$ lowest operative pressure of well $i$
$q_i^{\mathrm{up}}$ volumetric production flow rate of well $i$
$\rho_i$ crude oil density of well $i$
$L_i$ pipe length of well $i$
$D_i$ pipe diameter of well $i$
$f_i$ pipe friction factor of well $i$
$C_i^{\mathrm{v}}$ valve parameter of well $i$
$\rho_m$ crude oil density of manifold $m$
$L_m$ pipe length of manifold $m$
$D_m$ pipe diameter of manifold $m$
$f_m$ pipe friction factor of manifold $m$
$q_m^{\mathrm{up}}$ volumetric production flow rate of manifold $m$
$\mathrm{CP}_p$ capacity of intermediate tank $p$
$\mathrm{CV}_m^{\mathrm{max}}$ valve parameter of manifold $m$
$\mathrm{SQ}_m$ sulfur quality of manifold $m$
$H$ planning horizon
$\Delta$ scalar of accuracy
$g$ gravitational acceleration
$g_c$ unit conversion factor

**Continuous Variables**

$t_{ij}^{\mathrm{o}}$ operation time of well $i$ in period $j$
$t_{ij}^{\mathrm{c}}$ shut-in time of well $i$ in period $j$
$\mathrm{tp}_{ij}$ processing time of well $i$ in period $j$
$\mathrm{tr}_{ij}$ real time of well $i$ in period $j$
$P_{ij}^{\mathrm{in}}$ initial pressure of well $i$ in period $j$
$P_{ij}^{\mathrm{f}}$ final pressure of well $i$ in period $j$
$Q_{ij}$ oil volume of well $i$ in period $j$
$\mathrm{QT}_i$ total oil volume of well $i$
$n_i$ number of periods of well $i$
$P_{ij}^{\mathrm{t}}$ pressure at the top of the pipe of well $i$ in period $j$
$P_{mj}^{\mathrm{in}}$ pressure at the inlet point of manifold $m$ in period $j$
$P_{mj}^{\mathrm{d}}$ pressure at the outlet point of manifold $m$ in period $j$
$b_{mp}$ oil volume from manifold $m$ to intermediate tank $p$
$C_{mj}$ oil volume of manifold $m$ in period $j$
$\mathrm{CT}_m$ total oil volume of manifold $m$
$\mathrm{AP}_{mj}$ aperture of valve in manifold $m$ in period $j$
$C_m^{\mathrm{v}}$ valve parameter of manifold $m$
$x_{pk}$ oil volume from intermediate tank $p$ to end product $k$
$E_k$ total volume of end product $k$
$S_p$ sulfur quality of intermediate tank $p$
$Z_k$ sulfur quality of end product $k$
OP accumulated production of the end products

**Binary Variables**

$w_{ij}$ 1 if the period $j$ of well $i$ exists; 0 otherwise
$y_{ij}$ 1 if well $i$ is producing in period $j$; 0 otherwise
$v_p$ 1 if the intermediate tank $p$ is used; 0 otherwise
$u_{mp}$ 1 if crude oil from manifold $m$ goes to intermediate tank $p$; 0 otherwise

## REFERENCES


(1) Faried, M.; Samer, M.; Abdelsalam, E.; Yousef, R. S.; Attia, Y. A.; Ali, A. S. Biodiesel production from microalgae: Processes, technologies and recent advancements. *Renewable Sustainable Energy Rev.* **2017**, *79*, 893−913.

(2) Rand, J.; Hoen, B. Thirty years of North American wind energy acceptance research: What have we learned? *Energy Research & Social Science* **2017**, *29*, 135−148.

(3) Fthenakis, V.; Mason, J. E.; Zweibel, K. The technical, geographical, and economic feasibility for solar energy to supply the energy needs of the US. *Energy Policy* **2009**, *37* (2), 387−399.

(4) Goldemberg, J. The promise of clean energy. *Energy Policy* **2006**, *34* (15), 2185−2190.

(5) Chew, T. L.; Bhatia, S. Catalytic processes towards the production of biofuels in a palm oil and oil palm biomass-based biorefinery. *Bioresour. Technol.* **2008**, *99* (17), 7911−7922.

(6) Atadashi, I. M.; Aroua, M. K.; Aziz, A. A. Biodiesel separation and purification: a review. *Renewable Energy* **2011**, *36* (2), 437−443.

(7) Fukuda, H.; Kondo, A.; Noda, H. Biodiesel fuel production by transesterification of oils. *J. Biosci. Bioeng.* **2001**, *92* (5), 405−416.

(8) Shafiee, S.; Topal, E. When will fossil fuel reserves be diminished? *Energy Policy* **2009**, *37* (1), 181−189.

(9) Baumeister, C.; Kilian, L. Understanding the Decline in the Price of Oil since June 2014. *Journal of the Association of Environmental and Resource Economists* **2016**, *3* (1), 131−158.

(10) Khan, M. I. Falling oil prices: Causes, consequences and policy implications. *J. Pet. Sci. Eng.* **2017**, *149*, 409−427.

(11) Raman, R.; Grossmann, I. E. Modelling and computational techniques for logic based integer programming. *Comput. Chem. Eng.* **1994**, *18* (7), 563−578.

(12) Haverly, C. A. Studies of the behavior of recursion for the pooling problem. *Acm sigmap bulletin* **1978**, *25*, 19−28.

(13) Ben-Tal, A.; Eiger, G.; Gershovitz, V. Global minimization by reducing the duality gap. *Math. Program.* **1994**, *63* (1), 193−212.

(14) Foulds, L. R.; Haugland, D.; Jörnsten, K. A bilinear approach to the pooling problem. *Optimization* **1992**, *24* (1−2), 165−180.

(15) Adhya, N.; Tawarmalani, M.; Sahinidis, N. V. A Lagrangian approach to the pooling problem. *Ind. Eng. Chem. Res.* **1999**, *38* (5), 1956−1972.

(16) Audet, C.; Brimberg, J.; Hansen, P.; Digabel, S. L.; Mladenović, N. Pooling problem: Alternate formulations and solution methods. *Manag. Sci.* **2004**, *50* (6), 761−776.

(17) Meyer, C. A.; Floudas, C. A. Global optimization of a combinatorially complex generalized pooling problem. *AIChE J.* **2006**, *52* (3), 1027−1037.

(18) Kolodziej, S. P.; Grossmann, I. E.; Furman, K. C.; Sawaya, N. W. A discretization-based approach for the optimization of the multiperiod blend scheduling problem. *Comput. Chem. Eng.* **2013**, *53*, 122−142.

(19) Lotero, I.; Trespalacios, F.; Grossmann, I. E.; Papageorgiou, D. J.; Cheon, M. S. An MILP-MINLP decomposition method for the global optimization of a source based model of the multiperiod blending problem. *Comput. Chem. Eng.* **2016**, *87*, 13−35.

(20) Haugland, D.; Jørnsten, K.; Shayan, E. Modelling petroleum fields with movable platforms. *Applied Mathematical Modelling* **1991**, *15* (1), 33−39.

(21) Haugland, D.; Hallefjord, Å.; Asheim, H. Models for petroleum field exploitation. *Eur. J. Oper. Res.* **1988**, *37* (1), 58−72.

(22) Iyer, R. R.; Grossmann, I. E.; Vasantharajan, S.; Cullick, A. S. Optimal planning and scheduling of offshore oil field infrastructure investment and operations. *Ind. Eng. Chem. Res.* **1998**, *37* (4), 1380−1397.

(23) van den Heever, S. A.; Grossmann, I. E. An iterative aggregation/disaggregation approach for the solution of a mixed-integer nonlinear oilfield infrastructure planning model. *Ind. Eng. Chem. Res.* **2000**, *39* (6), 1955−1971.

(24) van den Heever, S. A.; Grossmann, I. E.; Vasantharajan, S.; Edwards, K. Integrating complex economic objectives with the design and planning of offshore oilfield infrastructures. *Comput. Chem. Eng.* **2000**, *24* (2), 1049−1055.




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*Ind. Eng. Chem. Res.* 2018, 57, 653−665

(25) Ortiz-Gómez, A.; Rico-Ramírez, V.; Hernández-Castro, S. Mixed-integer multiperiod model for the planning of oilfield production. *Comput.-Aided Chem. Eng.* **2001**, *9*, 907−912.
(26) Ortiz-Gómez, A.; Rico-Ramírez, V.; Hernández-Castro, S. Mixed-integer multiperiod model for the planning of oilfield production. *Comput. Chem. Eng.* **2002**, *26*, 703−714.
(27) Kosmidis, V. D.; Perkins, J. D.; Pistikopoulos, E. N. Optimization of Well Oil Rate Allocations in Petroleum Fields. *Ind. Eng. Chem. Res.* **2004**, *43* (14), 3513−3527.
(28) Kosmidis, V. D.; Perkins, J. D.; Pistikopoulos, E. N. A mixed integer optimization formulation for the well scheduling problem on petroleum fields. *Comput. Chem. Eng.* **2005**, *29* (7), 1523−1541.
(29) Barragán-Hernández, V.; Vázquez-Román, R.; Rosales-Marines, L.; García-Sánchez, F. A strategy for simulation and optimization of gas and oil production. *Comput. Chem. Eng.* **2005**, *30* (2), 215−227.
(30) Flores-Salazar, M. A.; Vâzquez-Román, R.; Grossmann, I. E.; Iglesias-Silva, G. A multiperiod planning model for gas production system. *J. Pet. Sci. Eng.* **2011**, *77* (2), 226−235.
(31) Lang, J.; Zhao, J. Modeling and optimization for oil well production scheduling. *Chin. J. Chem. Eng.* **2016**, *24* (10), 1423−1430.
(32) Horne, R. N. *Modern Well Test Analysis*; Petroway Inc.: Palo Alto, CA, USA, 1998.
(33) Brooke, A.; Kendrick, D.; Meeraus, A.; Raman, R. *GAMS, A User's Guide*; GAMS Development Corporation, Washington, DC; 2012; http://www.gams.com.
(34) Perry, R. H.; Green, D. W. *Perry's Chemical Engineers' Handbook*, 7th ed.; McGrawHill, 1997.
(35) Smith, C. A.; Corripio, A. B. *Principles and Practices of Automatic Process Control*, 3rd ed.; John Wiley & Sons, Inc., 2006.
(36) Vieira, L. V.; Rainha, K. P.; de Castro, E. V. R.; Filgueiras, P. R.; Carneiro, M. T. W.; Brandão, G. P. Exploratory data analysis using API gravity and V and Ni contents to determine the origins of crude oil samples from petroleum fields in the Espírito Santo Basin (Brazil). *Microchem. J.* **2016**, *124*, 26−30.
(37) Ji, C.; Yi, H.; Chen, Z.; Xia, G.; Mao, L.; Li, Q.; Hu, H. Crude oil types of the Well Qiang D-2 in Qiangtang Basin, Tibet and its exploration significance. *Acta Petrolei Sinica* **2013**, *34* (6), 1070−1076.
(38) Raman, R.; Grossmann, I. E. Relation between MILP modelling and logical inference for chemical process synthesis. *Comput. Chem. Eng.* **1991**, *15* (2), 73−84.
(39) Lee, S.; Grossmann, I. E. New algorithms for nonlinear generalized disjunctive programming. *Comput. Chem. Eng.* **2000**, *24* (9), 2125−2141.
(40) Grossmann, I. E.; Lee, S. Generalized convex disjunctive programming: Nonlinear convex hull relaxation. *Comput. Optimization Appl.* **2003**, *26* (1), 83−100.



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