REPORT ON "ASSOUAD DIMENSIONS OF COMPLEMENTARY SETS"

Let a be a positive decreasing sequence with finite sum $L = \sum a$. We can associate complimentary sets E to this sequence by removing open intervals of length a from the line $[0, L]$. These sets are called complimentary sets and the article under review investigates their Assouad and Lower dimension. The class of all possible complementary sets $\mathcal{C}$ contains the set D_a, constructed by removing intervals in decreasing order, making D_a countable. The class $\mathcal{C}$ also contains C_a, the Cantor set obtained by removing the lengths a in a similar way to the construction of the middle third Cantor set. The most interesting results of this manuscript are the dichotomy that the Assouad dimension of the decreasing set is either full or 0, i.e. $\dim_A D_a = 0$ or $\dim_A D_a = 1$. Further, for all $E \in \mathcal{C}$, the decreasing set is maximal: $\dim_A E \leq \dim_A D_a$. The Assouad dimension of C_a is proved to be a lower bound to $\dim_A E$ and the authors investigate the attainable values of the Assouad dimension. In the last section of the paper some analogous results for the Lower dimension are given.

I am of the opinion that the results in this article are interesting and tie in well with some results about specific fractal sets, e.g. Moran, self-similar , and self-conformal sets. The article is generally well written [I could not find a single grammatical error] and nicely structured. In a few places [particularly the proof of Theorem 3.7] I recommend changes. Subject to those, I recommend publication in Proceedings of the Royal Society of Edinburgh.

Comments.

- p.3 ll.11ff.: if F is self-similar, then the WSP (and thus the OSC) implies that $\dim_A F = \dim_H F$ (irrespective of ambient Euclidean dimension). Further, the authors might want to add that in the case $\dim_A F < 1$, the Assouad dimension coinciding with Hausdorff dimension is equivalent to the WSP, and equivalent to F being Ahlfors regular, see [FaFr15]. It might be worth pointing out that some of these results also hold in the self-conformal setting, see [KR16] and [AT16].
- p.3 ll.-17f.: for clarity "then the **class of** complementary sets are the same"
- p.3 l.-9: be more explicit why this uniquely determines a set. The placement of cut out intervals might at first appear to contradict this statement.
- p.5 Theorem 2.2. The statement looks similar to the results in [15]. Worth mentioning the similarities?
- p.7 ll.16f.: phrasing awkward. "and the RHS of (2.3) **is equal to** 0"
- p.10 Corollary 3.3: leave out proof, it is obvious.
- p.10 Corollary 3.4: I do not see any motivation for this corollary, is it worth keeping? If so, omit the proof, it follows easily.
- p.15 ll.16ff. Here the level is given the variable i, however i is used in many other places and a different letter should be used. It is not clear how d_k is chosen. Equations (3.7) and (3.8) do not depend on the level i.
- p.16 Eq. (3.11): combine into single inequality.
- p.23 Remark (1): This would also be an appropriate place to remark upon the link to self-similar sets.

References

[AT16] J. Angelevska and S. Troscheit. A dichotomy of self-conformal subsets of $\mathbb{R}$ with overlaps, *preprint*, (2016), arXiv:1602.05821.

[FaFr15] Á. Farkas and J. M. Fraser. On the equality of Hausdorff measure and Hausdorff content, *J. Fractal Geom.*, **2**, (2015), 403–429.

[KR16] A. Käenmäki and E. Rossi. Weak separation condition, Assouad dimension, and Furstenberg homogeneity, *Ann. Acad. Sci. Fenn. Math.*, **41**, (2016), 465–490.