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Gravitons emission during pre-inflation from unified spinor fields

Luis Santiago Ridao[2,a], Marcos R.A. Arcodía[2,b], and Mauricio Bellini[1,2,c]

[1] Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350, C.P. 7600, Mar del Plata, Argentina
[2] Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR), Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Mar del Plata, Argentina

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Abstract. We obtain the equation that describe the conditions of quantization for neutral massless bosons on an arbitrary curved space-time, obtained using a particular theoretical formalism developed in a previous work (M.R.A. Arcodía and M. Bellini, arXiv:1703.01355). In particular, we study the emission of neutral massless spin-$(1,2)\hbar$ bosons during pre-inflation using the recently introduced unified spinor field theory. We conclude that during pre-inflation (which is governed by vacuum equation of state), gravitational radiation is emitted, which could be detected in the future, as primordial gravitational radiation.

1 Introduction

It is well known that Heisenberg suggested unified quantum field theory of a fundamental spinor field describing all matter fields in their interactions [1,2]. In his theory the masses and interactions of particles are a consequence of a self-interaction term of the elementary spinor field. The fact that manifolds with no-Euclidean geometry can help uncover new features of quantum matter makes it desirable to create manifolds of controllable shape and to develop the capability to add in synthetic gauge fields [3].

On the other hand, in a previous work we have developed construct a pure geometric spinor field theory on an arbitrary curved background, which is considered a Riemannian manifold. In the theory the spinor field $\hat{\Psi}^{\alpha}$ is responsible for the displacement of the extended Weylian manifold [4] with respect to the Riemannian background and the covariant derivative of the metric tensor in the Riemannian background manifold is null[1]. However, the Weylian covariant derivative on the extended Weylian manifold[2], is nonzero: ${g^{\alpha\beta}}_{\|\gamma} \neq 0$. In this formalism they are considered the couplings of the spinor fields with the background and their self-interactions in a generic manner. The theory is worked in 8 dimensions, 4 of them related to the space-time coordinates (x^{μ}), and the other 4 related to the inner space (ϕ^{μ}), described by compact coordinates. The former have the spin components as canonical momentums: (s_{μ}).

This paper is organized as follows: In sect. 2 we have described the space-time structure from a quantum approach to recover a line element on a background Riemannian manifold. In sect. 3 we have exposed the spinor field formalism for bosons and the dynamic equations. In sect. 4 we have studied the particular case of the neutral bosons on arbitrary curved backgrounds. In sect. 5 we studied the example of neutral massless bosons in the pre-inflationary epoch. In particular, we consider the emission of massless spinor fields with spins $s=\hbar$ and $s=2\hbar$. Finally, in sect. 6 we develop some final comments.

[a] e-mail: santiagoridao@hotmail.com
[b] e-mail: marcodia@mdp.edu.ar
[c] e-mail: mbellini@mdp.edu.ar

[1] We denote with a ∇, the Riemannian-covariant derivative and with Δ the Riemannian variation of some arbitrary tensor: $\Delta g_{\alpha\beta} = \nabla_{\gamma} g_{\alpha\beta}\, dx^{\gamma} = 0$.
[2] We denote the covariant derivative on the extended Weylian manifold with a $\|$.

2 Einstein-Hilbert action and quantum structure of space-time

If we deal with an orthogonal basis, the curvature tensor will be written in terms of the connections ${R^\alpha}_{\beta\gamma\delta} = {\Gamma^\alpha}_{\beta\delta,\gamma} - {\Gamma^\alpha}_{\beta\gamma,\delta} + {\Gamma^\epsilon}_{\beta\delta}{\Gamma^\alpha}_{\epsilon\gamma} - {\Gamma^\epsilon}_{\beta\gamma}{\Gamma^\alpha}_{\epsilon\delta}$. The Einstein-Hilbert (EH) action for an arbitrary matter Lagrangian density $\mathcal{L}$

$$\mathcal{I} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R}{2\kappa} + \mathcal{L}\right], \tag{1}$$

after variation, is given by

$$\delta\mathcal{I} = \int \mathrm{d}^4x\sqrt{-g}\left[\delta g^{\alpha\beta}\left(G_{\alpha\beta} + \kappa T_{\alpha\beta}\right) + g^{\alpha\beta}\delta R_{\alpha\beta}\right], \tag{2}$$

where $\kappa = 8\pi G$, G is the gravitational constant, $g^{\alpha\beta}\delta R_{\alpha\beta} = \delta\Theta(x^\alpha)$, such that $\delta\Theta(x^\alpha)$ is an arbitrary scalar field, and $T_{\alpha\beta}$ is the energy-momentum tensor defined by

$$T_{\alpha\beta} = 2\frac{\delta\mathcal{L}}{\delta g^{\alpha\beta}} - g_{\alpha\beta}\mathcal{L}. \tag{3}$$

When the flux $\delta\Theta(x^\alpha)$ that cross the Gaussian-like hypersurface defined on an arbitrary region of the space-time, is zero, the resulting equations that minimize the EH action, are the background Einstein equations: $G_{\alpha\beta} + \kappa\, T_{\alpha\beta} = 0$. However, when this flux is nonzero, one obtains in the last term of eq. (2). This flux becomes zero when there are no sources within this hypersurface. Hence, in order to make $\delta\mathcal{I} = 0$ in eq. (2), we must consider the condition: $G_{\alpha\beta} + \kappa T_{\alpha\beta} = \Lambda\, g_{\alpha\beta}$, where Λ is the cosmological constant. On the other hand, we can make the transformation

$$\bar{G}_{\alpha\beta} = G_{\alpha\beta} - \Lambda\, g_{\alpha\beta}, \tag{4}$$

where the scalar field $\delta\Theta$ complies $\Box\delta\Theta = 0$ [5,6], and the transformed Einstein equations with the equation of motion for the transformed gravitational waves, hold

$$\bar{G}_{\alpha\beta} = -\kappa\, T_{\alpha\beta}. \tag{5}$$

Equation (5) give us the Einstein equations with cosmological constant included. Notice that the scalar field $\delta\Theta(x^\alpha)$ appears as a scalar flux of some 4-vector with components δW^α:

$$\left[\delta W^\alpha\right]_{\|\alpha} = \delta\Theta(x^\alpha), \tag{6}$$

through the closed hypersurface $\partial\mathcal{M}$, which is situated in any region of space-time. Here, $\delta W^\alpha = \delta\Gamma^\epsilon_{\beta\epsilon} g^{\beta\alpha} - \delta\Gamma^\alpha_{\beta\gamma} g^{\beta\gamma}$ (see footnote[3]). In this work we shall use a recently introduced extended Weylian manifold [7] to describe quantum geometric spinor fields $\hat{\Psi}^\alpha$, where the connections are

$$\hat{\Gamma}^\alpha_{\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta\,\gamma \end{array} \right\} + \hat{\Psi}^\alpha\, g_{\beta\gamma}. \tag{8}$$

Here

$$\delta\hat{\Gamma}^\alpha_{\beta\gamma} = \hat{\Psi}^\alpha\, g_{\beta\gamma} \tag{9}$$

describes the quantum displacement of the extended Weylian manifold with respect to the classical Riemannian background, which is described by the Levi-Civita symbols in (8), and the variation of the Ricci tensor is

$$\delta\hat{R}_{\beta\gamma} = \left(\delta\hat{\Gamma}^\alpha_{\beta\alpha}\right)_{\|\gamma} - \left(\delta\hat{\Gamma}^\alpha_{\beta\gamma}\right)_{\|\alpha}, \tag{10}$$

where $\delta\hat{\Gamma}^\alpha_{\beta\alpha} = \hat{\Psi}^\alpha\, g_{\beta\gamma}$.

[3] We define the covariant derivative of some vector field Υ^β: $[\Upsilon^\beta]_{\|\alpha}$

$$\left[\Upsilon^\beta\right]_{\|\alpha} = \nabla_\alpha \Upsilon^\beta + \xi^2\, \delta\Gamma^\beta_{\epsilon\alpha}\Upsilon^\epsilon, \tag{7}$$

where ξ is the self-interaction constant, $\nabla_\alpha\Upsilon^\beta$ is the covariant derivative on the Riemann manifold and $\delta\Gamma^\beta_{\epsilon\alpha}$ is the displacement of the manifold with respect to the Riemann one.

2.1 Quantum structure of space-time

In order to describe the quantum structure of space-time we consider a the variation $\delta\hat{X}^{\mu}$ of the quantum operator $\hat{X}^{\mu}$:

$$\hat{X}^{\alpha}(x^{\nu}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3k\, \hat{\gamma}^{\alpha} \left[b_k\, \hat{X}_k(x^{\nu}) + b^{\dagger}_k\, \hat{X}^{*}_k(x^{\nu}) \right],$$

where $b^{\dagger}_k$ and b_k are the creation and destruction operators of space-time, such that $\langle B| \left[b_k, b^{\dagger}_{k'}\right] |B\rangle = \delta^{(3)}(\vec{k}-\vec{k'})$ and $\hat{\gamma}^{\alpha}$ are 4×4-matrices that comply with the Clifford algebra. Moreover, we shall define in the analogous manner the variation $\delta\hat{\Phi}^{\mu}$ of the quantum operator $\hat{\Phi}^{\mu}$ that describes the quantum inner space:

$$\hat{\Phi}^{\alpha}(\phi^{\nu}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3s\, \hat{\gamma}^{\alpha} \left[c_s\, \hat{\Phi}_s(\phi^{\nu}) + c^{\dagger}_s\, \hat{\Phi}^{*}_s(\phi^{\nu}) \right],$$

where $c^{\dagger}_s$ and c_s are the creation and destruction operators of the inner space, such that $\langle B| \left[c_s, c^{\dagger}_{s'}\right] |B\rangle = \delta^{(3)}(\vec{s}-\vec{s'})$. In our case the background quantum state can be represented in a ordinary Fock space in contrast with LQG [8, 9], where operators are qualitatively different from the standard quantization of gauge fields. These operators can be applied to some background quantum state, and describes a Fock space on an arbitrary Riemannian curved space-time $|B\rangle$, such that they comply with

$$\delta\hat{X}^{\mu}\,|B\rangle = \mathrm{d}x^{\mu}\,|B\rangle, \qquad \delta\hat{\Phi}^{\mu}\,|B\rangle = \mathrm{d}\phi^{\mu}\,|B\rangle. \tag{11}$$

The states $|B\rangle$ do not evolves with time because we shall consider the Heisenberg representation, in which only the operators evolve with time so that the background expectation value of the manifold displacement is null: $\langle B|\, \delta\hat{\Gamma}^{\alpha}_{\beta\gamma} |B\rangle = 0$. In order to describe the effective background space-time, we shall consider the line element

$$\mathrm{d}l^2 \delta_{BB'} = \mathrm{d}x^2 \delta_{BB'} + \mathrm{d}\phi^2 \delta_{BB'} = \langle B|\, \delta\hat{X}_{\mu} \delta\hat{X}^{\mu}\, |B'\rangle + \langle B|\, \delta\hat{\Phi}_{\mu} \delta\hat{\Phi}^{\mu}\, |B'\rangle, \tag{12}$$

where ϕ^{α} are the four compact dimensions related to their canonical momentum components s^{α} that describe the spin. The variations and differentials of the operators $\hat{X}^{\mu}$ and $\hat{\Phi}^{\mu}$ on the extended Weylian manifold, are given respectively by

$$\delta\hat{X}^{\mu}\,|B\rangle = \left(\hat{X}^{\mu}\right)_{\|\alpha} \mathrm{d}x^{\alpha}\,|B\rangle, \qquad \delta\hat{\Phi}^{\mu}\,|B\rangle = \left(\hat{\Phi}^{\mu}\right)_{\|\alpha} \mathrm{d}\phi^{\alpha}\,|B\rangle, \tag{13}$$

$$\mathrm{d}\hat{X}^{\mu}\,|B\rangle = \left(\hat{X}^{\mu}\right)_{,\alpha} \mathrm{d}x^{\alpha}\,|B\rangle, \qquad \mathrm{d}\hat{\Phi}^{\mu}\,|B\rangle = \left(\hat{\Phi}^{\mu}\right)_{,\alpha} \mathrm{d}\phi^{\alpha}\,|B\rangle, \tag{14}$$

with covariant derivatives

$$\left(\hat{X}^{\mu}\right)_{\|\beta} |B\rangle = \left[\nabla_{\beta}\hat{X}^{\mu} + \hat{\Psi}^{\mu}\hat{X}_{\beta} - \hat{X}^{\mu}\hat{\Psi}_{\beta}\right] |B\rangle, \tag{15}$$

$$\left(\hat{\Phi}^{\mu}\right)_{\|\beta} |B\rangle = \left[\nabla_{\beta}\hat{\Phi}^{\mu} + \hat{\Psi}^{\mu}\hat{\Phi}_{\beta} - \hat{\Phi}^{\mu}\hat{\Psi}_{\beta}\right] |B\rangle. \tag{16}$$

2.2 Bi-vectorial structure of inner space

We shall consider the squared of the $\delta\hat{\Phi}$-norm on the bi-vectorial space, and the squared $\delta\hat{X}$-norm on the vectorial space, are

$$\underset{\longleftrightarrow}{\delta\Phi}\overleftrightarrow{\delta\Phi} \equiv \left(\delta\hat{\Phi}_{\mu}\delta\hat{\Phi}_{\nu}\right)(\bar{\gamma}^{\mu}\bar{\gamma}^{\nu}), \tag{17}$$

$$\underset{\longrightarrow}{\delta X}\overrightarrow{\delta X} \equiv \delta\hat{X}_{\alpha}\delta\hat{X}^{\alpha}, \tag{18}$$

such that $\hat{\Phi}^{\alpha} = \phi\,\bar{\gamma}^{\alpha}$ and $\hat{X}^{\alpha} = x\,\bar{\gamma}^{\alpha}$ are, respectively, the components of the inner and coordinate spaces. Furthermore, $\bar{\gamma}_{\mu}$ are the (4×4) Dirac matrices that generate the vectorial and bi-vectorial structure of the space-time:

$$\begin{aligned} \left\langle B \middle| \hat{X}_{\mu}\hat{X}^{\mu} \middle| B \right\rangle &= x^2\, \mathbb{I}_{4\times4}, \\ \left\langle B \middle| \left(\hat{\Phi}_{\mu}\hat{\Phi}_{\nu}\right)(\bar{\gamma}^{\mu}\bar{\gamma}^{\nu}) \middle| B \right\rangle &= \left\langle B \middle| \frac{1}{4}\left\{\hat{\Phi}_{\mu},\hat{\Phi}_{\nu}\right\}\{\bar{\gamma}^{\mu},\bar{\gamma}^{\nu}\} - \frac{1}{4}\left[\hat{\Phi}_{\mu},\hat{\Phi}_{\nu}\right][\bar{\gamma}^{\mu},\bar{\gamma}^{\nu}] \middle| B \right\rangle \\ &= \phi^2\, \mathbb{I}_{4\times4}. \end{aligned} \tag{19}$$

The $\bar{\gamma}^{\mu}$ matrices, comply with the Clifford algebra

$$\bar{\gamma}^{\mu} = \frac{\mathbf{I}}{3!}\left(\bar{\gamma}^{\mu}\right)^{2}\,\epsilon^{\mu}{}_{\alpha\beta\nu}\bar{\gamma}^{\alpha\beta}\bar{\gamma}^{\nu}, \qquad \{\bar{\gamma}^{\mu},\bar{\gamma}^{\nu}\} = 2g^{\mu\nu}\,\mathbb{I}_{4\times4},$$

where $\mathbf{I} = \gamma^0\gamma^1\gamma^2\gamma^3$, $\mathbb{I}_{4\times4}$ is the identity matrix, $\bar{\gamma}^{\alpha\beta} = \frac{1}{2}\left[\bar{\gamma}^{\alpha},\bar{\gamma}^{\beta}\right]$. In this paper we shall consider the Weyl basis on a Minkowsky space-time (in Cartesian coordinates): $\{\gamma^a,\gamma^b\} = 2\eta^{ab}\mathbb{I}_{4\times4}$

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & -\sigma^1 \\ \sigma^1 & 0 \end{pmatrix},$$
$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & -\sigma^3 \\ \sigma^3 & 0 \end{pmatrix},$$

such that the Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

3 Spinor field

The expressions (6) and (8), give us

$$\frac{\delta\hat{W}^{\alpha}}{\delta l} = 3\hat{\Psi}^{\alpha}, \tag{20}$$

l-being the Weylian 4-length. The self-interacting effects do not necessary preserve the Riemannian flux of matter fields along the Gaussian hypersurface, so that $(\delta W^{\alpha})_{\|\alpha} = \nabla_{\alpha}\delta W^{\alpha} + \xi^2\,\delta W^{\alpha}\hat{\Psi}_{\alpha}$. Notice that when the coupling constant is zero: $\xi = 0$, the Riemannian flux on the extended Weylian manifold is equal to the flux on the Riemannian one. The flux equation can be rewritten using (20), so that

$$\nabla_{\alpha}\hat{\Psi}^{\alpha} + \xi^2\,\hat{\Psi}^{\alpha}\hat{\Psi}_{\alpha} = \frac{1}{3}\frac{\delta\hat{\Theta}}{\delta l}\,. \tag{21}$$

However, when we describe matter fields, the coupling ξ is nonzero. In general, ξ depends on the theory under study (*i.e.* on the group representation of the spinor fields), and can be proportional to some physical property of the field (mass, charge, etc). Spinors with $\xi = 0$, do not describe matter fields, but geometric fields.

In this framework, we can define respectively the slash and vector quantum fields $\not{\Psi} = \hat{\Psi}_{\alpha}\bar{\gamma}^{\alpha}$, $\overleftrightarrow{\Psi} = \Psi_{\alpha}\bar{\gamma}^{\alpha}$. The 4-vector components are $\hat{\Psi}_{\alpha} = \frac{\delta\hat{\Theta}}{\delta\hat{\Phi}^{\alpha}}$, where the flux of $\hat{\Psi}^{\alpha}$-field through the Gaussian hypersurface in eq. (21): $\hat{\Theta}(x^{\beta}|\phi^{\nu})$, can be represented according to (12), as a Fourier expansion in the momentum-space:

$$\hat{\Theta}\left(x^{\beta}|\phi^{\nu}\right) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4k\int \mathrm{d}^4s\left[A_{s,k}\,e^{i\underleftrightarrow{K}\cdot\overleftrightarrow{X}}\,e^{\frac{i}{\hbar}\underleftrightarrow{S}\,\overleftrightarrow{\Phi}} + B^{\dagger}_{k,s}\,e^{-i\underleftrightarrow{K}\cdot\overleftrightarrow{X}}\,e^{-\frac{i}{\hbar}\underleftrightarrow{S}\,\overleftrightarrow{\Phi}}\right].$$

We can define the spinor (complex) components $\hat{\Psi}_{\alpha}\left(x^{\beta}|\phi^{\nu}\right)$

$$\hat{\Psi}_{\alpha}\left(x^{\beta}|\phi^{\nu}\right) = \frac{i}{\hbar(2\pi)^4}\int \mathrm{d}^4k\int \mathrm{d}^4s\frac{\delta\left(\underleftrightarrow{S}\,\overleftrightarrow{\Phi}\right)}{\delta\hat{\Phi}^{\alpha}}\left[A_{s,k}\,e^{i\underleftrightarrow{K}\cdot\overleftrightarrow{X}}\,e^{\frac{i}{\hbar}\underleftrightarrow{S}\,\overleftrightarrow{\Phi}} - B^{\dagger}_{k,s}\,e^{-i\underleftrightarrow{K}\cdot\overleftrightarrow{X}}\,e^{-\frac{i}{\hbar}\underleftrightarrow{S}\,\overleftrightarrow{\Phi}}\right],$$

where $\langle B|A_{ks}B^{\dagger}_{k's'}|B\rangle = (\frac{c^3M_p^3}{\hbar})^2\delta^{(4)}(k-k')\,\delta^{(4)}(s-s')$, and

$$\frac{\delta}{\delta\hat{\Phi}^{\alpha}}\left(\underleftrightarrow{S}\,\overleftrightarrow{\Phi}\right) = \left(2g_{\alpha\beta}\mathbb{I}_{4\times4} - \bar{\gamma}_{\alpha}\bar{\gamma}_{\beta}\right)\hat{S}^{\beta} = 2\hat{S}_{\alpha} - \bar{\gamma}_{\alpha}\,s = \hat{S}_{\alpha}, \tag{22}$$

where $s\,\mathbb{I}_{4\times4} = \frac{1}{4}\,\hat{S}_{\beta}\bar{\gamma}^{\beta}$. Here, c is the speed of light, M_p is the Planckian mass, $\hbar = h/(2\pi)$, h-being the Planck constant. Additionally, the squared bi-vectorial $\hat{S}$-norm, is

$$\left\|\hat{S}\right\|^2 = \left\langle B\left|\underleftrightarrow{S}\,\overleftrightarrow{S}\right|B\right\rangle = \left\langle B\left|\left(\hat{S}_{\mu}\hat{S}_{\nu}\right)\left(\bar{\gamma}^{\mu}\bar{\gamma}^{\nu}\right)\right|B\right\rangle = s^2\mathbb{I}_{4\times4}, \tag{23}$$

for $\hat{S}_{\mu} = s\,\bar{\gamma}_{\mu}$. In order to quantize the spin, we shall consider the universal invariant

$$\left\langle B\left|\underleftrightarrow{S}\,\overleftrightarrow{\Phi}\right|B\right\rangle = \left\langle B\left|\left(\hat{S}_{\mu}\hat{\Phi}_{\nu}\right)\left(\bar{\gamma}^{\mu}\bar{\gamma}^{\nu}\right)\right|B\right\rangle = s\phi\,\mathbb{I}_{4\times4} = \left(2\pi n\hbar\right)\mathbb{I}_{4\times4}, \tag{24}$$

with n-integer. For this reason, gravitons (which have $s = 2\hbar$), will be invariant under $\phi = n\,\pi$ rotations and vectorial bosons (with $s = \hbar$), will be invariant under $\phi = 2n\pi$ rotations.

4 Dynamics of neutral bosons

Explicitly written, the dynamics of massless neutral vector bosons is given by [7]

$$\begin{aligned}\Box\hat{\Psi}^{\alpha} &- \nabla_{\beta}\left(\nabla^{\alpha}\hat{\Psi}^{\beta}\right) + 2\left(\nabla_{\beta}\hat{\Psi}^{\alpha}\right)\hat{\Psi}^{\beta} - 2\left(\nabla_{\beta}\hat{\Psi}^{\beta}\right)\hat{\Psi}^{\alpha}\\ &- \left(\nabla^{\alpha}\hat{\Psi}^{\gamma}\right)\hat{\Psi}_{\gamma} + 2\hat{\Psi}^{\alpha}\left(\nabla_{\gamma}\hat{\Psi}^{\gamma}\right) + \left(\nabla^{\gamma}\hat{\Psi}^{\alpha}\right)\hat{\Psi}_{\gamma}\\ &- 2\hat{\Psi}^{\gamma}\left(\nabla_{\gamma}\hat{\Psi}^{\alpha}\right) = 2\left[\hat{\Psi}^{\mu},\hat{\Psi}^{\alpha}\right]\hat{\Psi}_{\mu},\end{aligned} \tag{25}$$

such that in the case of bosons, we obtain

$$\left\langle B\left|\left[\hat{\Psi}_{\mu}(\mathbf{x},\boldsymbol{\phi}),\hat{\Psi}_{\nu}(\mathbf{x}',\boldsymbol{\phi}')\right]\right|B\right\rangle = \frac{s^2}{2\hbar^2}\left[\bar{\gamma}_{\mu},\bar{\gamma}_{\nu}\right]\sqrt{\frac{\eta}{g}}\,\delta^{(4)}\left(\mathbf{x}-\mathbf{x}'\right)\delta^{(4)}\left(\boldsymbol{\phi}-\boldsymbol{\phi}'\right), \tag{26}$$

where L_p is the Planckian length and $\sqrt{\frac{\eta}{g}}$ is the squared root of the ratio between the determinant of the Minkowsky metric: $\eta_{\mu\nu}$ and the metric that describes the background: $g_{\mu\nu}$. This ratio describes the inverse of the relative volume of the background manifold with respect to the Minkowsky one. The Fourier expansion for the spinor field $\hat{\Psi}_{\alpha}$ is

$$\hat{\Psi}_{\alpha} = \frac{i}{\hbar(2\pi)^4}\int \mathrm{d}^4k\int \mathrm{d}^4s\,\frac{\delta\left(\underset{\leftrightarrow}{S}\,\overleftrightarrow{\Phi}\right)}{\delta\hat{\Phi}^{\alpha}}\left[A_{s,k}\,e^{i\underset{\leftrightarrow}{K}\cdot\overleftrightarrow{X}}e^{\frac{i}{\hbar}\underset{\leftrightarrow}{S}\,\overleftrightarrow{\Phi}} - B^{\dagger}_{k,s}\,e^{-i\underset{\leftrightarrow}{K}\cdot\overleftrightarrow{X}}e^{-\frac{i}{\hbar}\underset{\leftrightarrow}{S}\,\overleftrightarrow{\Phi}}\right], \tag{27}$$

where $\frac{\delta(\underset{\leftrightarrow}{S}\,\overleftrightarrow{\Phi})}{\delta\hat{\Phi}^{\alpha}} = \hat{S}_{\alpha}$. If we deal with bosons, creation and destruction operators must comply [7]

$$\frac{4s^2L_p^2}{\hbar^2}\left(|A_{k,s}|^2 - |B_{k,s}|^2\right) = 0,\,\pm\left(\frac{c^3M_p^3}{\hbar}\right)^2. \tag{28}$$

The conditions (28) are required for scalar bosons (the first equality) and vector, or tensor bosons (the second equality). On the other hand, in order for the expectation value of the energy to be positive: $\langle B|\mathcal{H}|B\rangle \geq 0$, we must choose the negative signature in the second equality of (28). The expectation value for the local particle-number operator for bosons with wave-number norm k and spin s, $\hat{N}_{k,s}$, is given by[4]

$$\left\langle B\left|\hat{N}_{k,s}\right|B\right\rangle = -n_{k,s}\left(\frac{\hbar}{c^3M_p^3}\right)^2\int \mathrm{d}^4x\sqrt{-g}\int \mathrm{d}^4\phi\left\langle B\left|\left[\hat{\not{\Psi}}(\mathbf{x},\boldsymbol{\phi}),\hat{\not{\Psi}}^{\dagger}(\mathbf{x},\boldsymbol{\phi})\right]\right|B\right\rangle = n_{k,s}\,\mathbb{I}_{4\times4}, \tag{29}$$

where the slashed spinor fields are: $\hat{\not{\Psi}} = \bar{\gamma}^{\mu}\hat{\Psi}_{\mu}$, $\hat{\not{\Psi}}^{\dagger} = (\bar{\gamma}^{\mu}\hat{\Psi}_{\mu})^{\dagger}$. Furthermore, these fields comply with the algebra

$$\left\langle B\left|\left[\hat{\not{\Psi}}(\mathbf{x},\boldsymbol{\phi}),\hat{\not{\Psi}}^{\dagger}(\mathbf{x}',\boldsymbol{\phi}')\right]\right|B\right\rangle = \frac{4s^2L_p^2}{\hbar^2}\left(|A_{k,s}|^2 - |B_{k,s}|^2\right)\sqrt{\frac{\eta}{g}}\,\delta^{(4)}\left(\mathbf{x}-\mathbf{x}'\right)\delta^{(4)}\left(\boldsymbol{\phi}-\boldsymbol{\phi}'\right), \tag{30}$$

which must be nonzero in order that particles can be created. Notice that this is the case for bosons with spin nonzero, but in the case of scalar bosons, which have zero spin, one obtains that $(|A_{k,s}|^2 - |B_{k,s}|^2) = 0$, and $\langle B|\,[\hat{\not{\Psi}}(\mathbf{x},\boldsymbol{\phi}),\hat{\not{\Psi}}^{\dagger}(\mathbf{x}',\boldsymbol{\phi}')]\,|B\rangle = 0$. This result is valid in any relativistic scenario.

If we take the expectation value for (25), and we take into account (26) and (27), we obtain the following equation for the wave-numbers of bosons:

$$\begin{aligned}\left[\bar{\gamma}^{\beta},\bar{\gamma}^{\theta}\right]_{,\theta} &- \frac{1}{2}g^{\beta\theta}\left(\bar{\gamma}^{\nu}\right)_{,\theta}\bar{\gamma}_{\nu} - 2i\,k^{\beta}\mathbb{I}_{4\times4} + \frac{1}{2}g^{\nu\theta}\left(\bar{\gamma}^{\beta}\right)_{,\theta}\bar{\gamma}_{\nu} + \frac{i}{2}\bar{\gamma}^{\beta}\underset{\leftrightarrow}{k}\\ &= \frac{s^2}{2\hbar^2}\left\{\begin{matrix}\nu\\ \theta\,\nu\end{matrix}\right\}\left[\bar{\gamma}^{\theta},\bar{\gamma}^{\beta}\right] + \frac{1}{2}g^{\beta\theta}\left\{\begin{matrix}\mu\\ \nu\,\theta\end{matrix}\right\}\bar{\gamma}^{\nu}\bar{\gamma}_{\mu} - \frac{1}{2}g^{\mu\theta}\left\{\begin{matrix}\beta\\ \nu\,\theta\end{matrix}\right\}\bar{\gamma}^{\nu}\bar{\gamma}_{\mu},\end{aligned} \tag{31}$$

[4] To connect the Fock-space theory and the ordinary quantum mechanics one can introduce the wave function in position space by using the definition of a kind of $n_{k,s}$-particle state vector that describes a system of $n_{k,s}$ particles that are localized in coordinate space at the points $\mathbf{x}_1;\boldsymbol{\phi}_1\ldots\mathbf{x}_n;\boldsymbol{\phi}_n$:

$$|\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_n;\boldsymbol{\phi}_1,\boldsymbol{\phi}_2,\ldots,\boldsymbol{\phi}_n\rangle = \frac{1}{\sqrt{n_{k,s}!}}\hat{\not{\Psi}}^{\dagger}(\mathbf{x}_1;\boldsymbol{\phi}_1)\ldots\hat{\not{\Psi}}^{\dagger}(\mathbf{x}_n;\boldsymbol{\phi}_n)\,|B\rangle,$$

where here $|B\rangle$ is our reference state. This state is not a vacuum state because it describes a curved background state, but describes the Riemannian (classical) reference with respect to which we describe the quantum system.

where $\underset{\leftrightarrow}{k} = k^{\alpha}\bar{\gamma}_{\alpha}$ and $\bar{\gamma}_{\alpha} = E^{\mu}_{\alpha}\gamma_{\mu}$ are the components of the basis on the background metric, which are related by the vielbein E^{μ}_{α} with the 4×4 matrices γ^{μ} on the Minkowsky space-time. In our case we shall use Cartesian coordinates to describe spacial coordinates. In this paper we shall use the Weyl representation of the γ-matrices to generate the hyperbolic space-time.

5 Pre-inflation

The idea of a pre-inflationary expansion of the universe in which the universe begins to expand through a (global) topological phase transition was proposed in [10]. In this model the birth of the universe was studied using a complex time $\tau(t) = \int e^{i\hat{\theta}(t)}\mathrm{d}t$, such that the phase transition from a pre-inflationary to inflationary epoch was examined using a dynamical rotation of the complex time, $\tau(t)$, on the complex plane. After a particular choice of coordinates, one can define a dynamical variable θ: $\pi/2 \geq \hat{\theta}(t) > 0$, such that it describes the dynamics of the system and it is related with the expansion of the universe

$$\hat{\theta}(t) = \frac{\pi}{2}e^{-H_0 t}. \tag{32}$$

We consider the line element introduced in [10] to describe pre-inflation

$$\mathrm{d}\hat{S}^2 = \left(\frac{\pi a_0}{2}\right)^2 \frac{1}{\hat{\theta}^2}\left[\mathrm{d}\hat{\theta}^2 - \delta_{ij}\mathrm{d}\hat{x}^i\mathrm{d}\hat{x}^j\right]. \tag{33}$$

If we desire to describe an initially Euclidean 4D universe, that thereafter evolves to an asymptotic value $\hat{\theta} \to 0$, we must require $\hat{\theta}$ to have an initial value $\hat{\theta}_0 = \frac{\pi}{2}$. Furthermore, the nonzero components of the Einstein tensor, are

$$G_{00} = -\frac{3}{\hat{\theta}^2}, \qquad G_{ij} = \frac{3}{\hat{\theta}^2}\,\delta ij, \tag{34}$$

so that the energy density and the pressure are, respectively, given by

$$\rho(\hat{\theta}) = \frac{1}{\pi G}\frac{3}{(\pi a_0)^2}, \qquad P(\hat{\theta}) = -\frac{1}{\pi G}\frac{3}{(\pi a_0)^2}. \tag{35}$$

The equation of state for the metric (33), is

$$\frac{P}{\rho} = -1. \tag{36}$$

We shall describe the case where the asymptotic evolution of the Universe is described by a vacuum expansion. In this case the asymptotic scale factor, Hubble parameter and the potential are, respectively, given by

$$a(t) = a_0\, e^{H_0 t}, \qquad \frac{\dot{a}}{a} = H_0 \qquad V = \frac{3}{8\pi G}H_0^2, \tag{37}$$

so that, due to the fact that $\frac{\delta V}{\delta\phi} = 0$, the background field background solution of the background dynamics

$$\phi'' - \frac{2}{\hat{\theta}}\phi' = 0, \tag{38}$$

is

$$\phi(t) = \phi_0. \tag{39}$$

This solution describes the background solution of the field that drives a phase transition of the global geometry from a 4D Euclidean space to a 4D hyperbolic space-time. The exact back-reaction effects were considered in [5,6]. In the present paper we shall consider the emission of gravitons (massless bosons of spin 2 and spin 1), using unified spinor fields [7].

5.1 Graviton's emission in pre-inflation

To study the graviton's emission during pre-inflation we shall use eq. (31), for spin $s = 2\hbar$ (see eq. (26)). In this case eq. (31) can be separated into two new equations

$$\left[\bar{\gamma}^i, \bar{\gamma}^0\right]_{,0} = \frac{s^2}{8\hbar^2} \left\{ \begin{array}{c} \nu \\ 0\,\nu \end{array} \right\} \left[\bar{\gamma}^0, \bar{\gamma}^i\right], \tag{40}$$

$$k^\beta \mathbb{I}_{4\times 4} = \frac{1}{6} k^\alpha \left[\gamma^\beta, \gamma_\alpha\right] + \frac{3is^2}{8\hbar^2} \left\{ \begin{array}{c} \nu \\ \theta\,\nu \end{array} \right\} \left[\bar{\gamma}^\theta, \bar{\gamma}^\beta\right]. \tag{41}$$

Equations (40) and (41) give us the conditions that must be fulfilled by the modes with wave number k and massless bosons with spin $s = 2\hbar$, in eq. (25). In particular, eq. (40) is fulfilled only by $s = 2\hbar$-spin massless bosons (gravitons). During pre-inflation $\bar{\gamma}^\beta = \frac{2\hat{\theta}}{\pi a_0} \gamma^\beta$ and $(\bar{\gamma}^\beta)_{,0} = \frac{1}{\hat{\theta}} \bar{\gamma}^\beta$. From (41) we obtain four vector equations that provide us the solutions for the k^β-components of the graviton's propagation during pre-inflation. Using the fact that (for $i, j = 1, 2, 3$)

$$\left\{ \begin{array}{c} 0 \\ 0\,0 \end{array} \right\} = \left\{ \begin{array}{c} 1 \\ 0\,1 \end{array} \right\} = \left\{ \begin{array}{c} 2 \\ 0\,2 \end{array} \right\} = \left\{ \begin{array}{c} 3 \\ 0\,3 \end{array} \right\} = -\frac{1}{\hat{\theta}}, \tag{42}$$

we obtain the resulting values for k^β

$$k^0 = -\left(\frac{18\,i}{\pi a_0}\right) \hat{\theta}, \qquad k^1 = k^2 = 0, \qquad k^3 = \pm \left(\frac{74\,i}{\pi a_0}\right) \hat{\theta}, \tag{43}$$

such that the physical wave number norm of gravitons that propagate in the $\hat{z}$-direction, is

$$\frac{|k|^2}{a^2(\hat{\theta})} = \frac{(k_\alpha k^\alpha)}{a^2(\hat{\theta})} = \left[\frac{1288}{(\pi a_0)^2}\right] \hat{\theta}^2 > 0. \tag{44}$$

Notice that it tends to zero with the expansion of the universe, due to the fact $\hat{\theta} \to 0$ with the increasing of the scale factor $a(\hat{\theta}) = \frac{\pi a_0}{2\hat{\theta}}$. However, due to the fact we are dealing with photons, the k-squared norm must be null on physical coordinates. Therefore, the effective frequency and the z-component of the wave number in physical coordinates should be altered in the following manner (we use natural units):

$$\omega^2 \equiv \left(\tilde{k}^0\right)^2 = \left[\Im\left(\frac{k^0}{a(\hat{\theta})}\right)\right]^2 + \frac{|k|^2}{a^2} = \left(\frac{74}{\pi a_0}\right)^2 \hat{\theta}^2, \qquad \left(\tilde{k}^3\right)^2 = \left[\Im\left(\frac{k^3}{a(\hat{\theta})}\right)\right]^2, \tag{45}$$

such that $\tilde{k}_\alpha \tilde{k}^\alpha = 0$. Therefore, the redefined physical values $\tilde{k}^\alpha$, should be the values experimentally measured. The physical wavelength results to be $\lambda_{ph} = \left(\frac{2\pi}{37}\right)\left(\frac{a(\hat{\theta})}{a_0}\right) H_0^{-1}$, such that $H_0 = a_0^{-1}$. In other words the physical wavelength of gravitons is something smaller than the physical Hubble radius during pre-inflation.

5.2 Massless s = $\hbar$-bosons emission in pre-inflation

In order to study the massless $s = \hbar$ bosons emitted during the pre-inflationary epoch, we shall use eq. (31), with (26), for $s = \hbar$. In this case eq. (31) can be splited into two equations:

$$\left[\bar{\gamma}^i, \bar{\gamma}^0\right]_{,0} = \frac{s^2}{2\hbar^2} \left\{ \begin{array}{c} \nu \\ 0\,\nu \end{array} \right\} \left[\bar{\gamma}^0, \bar{\gamma}^i\right], \tag{46}$$

$$k^\beta \mathbb{I}_{4\times 4} = \frac{1}{2} k^\alpha \left[\gamma^\beta, \gamma_\alpha\right]. \tag{47}$$

Equation (46) is needed to assure its validity for $s = \hbar$, so that eq. (47) is fulfilled in order to obtain the wave number components of the wave. Notice that if we sum eqs. (46) and (47), we obtain eq. (31) for the metric (12). The wave number solutions for the four eqs. (47), are

$$k^0 = -\left(\frac{i}{\pi^2 a_0^2}\right) \hat{\theta}, \qquad k^1 = k^2 = 0, \qquad k^3 = \mp \left(\frac{3\,i}{\pi a_0}\right) \hat{\theta}. \tag{48}$$

The physical wave number norm of $s = \hbar$-bosons that propagate in the $\hat{z}$-direction, is

$$\frac{|k|^2}{a^2(\hat{\theta})} = \frac{(k_\alpha k^\alpha)}{a^2(\hat{\theta})} = \left[\frac{8}{(\pi a_0)^2}\right]\hat{\theta}^2 > 0, \tag{49}$$

which, as in the case of gravitons, tends to zero with the expansion of the universe. In order for the k-squared norm to be null, the frequency and z-wave number must be altered in physical coordinates, $\tilde{k}_\alpha \tilde{k}^\alpha = 0$:

$$\omega^2 \equiv \left(\tilde{k}^0\right)^2 = \left[\Im\left(\frac{k^0}{a(\hat{\theta})}\right)\right]^2 + \frac{|k|^2}{a^2} = \left(\frac{3}{\pi a_0}\right)^2 \hat{\theta}^2, \qquad \left(\tilde{k}^3\right)^2 = \left[\Im\left(\frac{k^3}{a(\hat{\theta})}\right)\right]^2. \tag{50}$$

These should be the values measured in an experiment. In this case, the physical wavelength for massless $s = \hbar$-bosons, is $\lambda_{ph} = (\frac{4\pi}{3})(\frac{a(\hat{\theta})}{a_0})H_0^{-1}$, which is something bigger than both the physical graviton's wavelength and the Hubble horizon.

6 Final comments

Following the unified spinor field theory recently introduced, we have obtained the universal equation of motion for massless bosons with quantization included: eq. (31). This equation describes the dynamics of massless bosons with different spin on arbitrary Riemannian background. The dynamics of some particular spinor field is given when we consider eq. (26) in the background (Riemannian) expectation value of eq. (25). In particular, we have explored the case of a pre-inflationary scenario described in sect. 5. In this epoch the universe suffered a global topological phase transition that made possible the transition between a global 4D Euclidean universe and an hyperbolic one through an expansion governed by a vacuum equation of state: $P = -\rho$. A remarkable result here obtained is that during this epoch gravitons and $s = \hbar$-bosons take a positive relativistic squared norm in physical coordinates: $\frac{|k|^2}{a^2(\hat{\theta})}|_{s=(1,2)\hbar} > 0$, which tends to zero with the expansion of the universe. However, by redefining the physical coordinates in order to obtain $\tilde{k}_\alpha \tilde{k}^\alpha = 0$, we obtain that the wavelengths of both gravitons and photons is increased co-moving with the Hubble radius of the universe: $\lambda_{Ph} \sim a/H_0$. Our calculations show that gravitational radiation is emitted during the big bang. As was shown in (29), bosons with $s = (1,2)\hbar$ can be created in any relativistic scenario, and therefore can be created during pre-inflation. However, this is not the case of scalar bosons, which have zero spin. Because the wave-length of both, photons and gravitons are of the order of the Hubble horizon, the frequency should be very low, of the order of the inverse of the edge of the universe, which make it very difficult to be detected because of the diluting effects of the expansion of the universe. However, they should be responsible for very large-scale gravitational and electromagnetic primordial fundamental wavelengths, which are coherent, and could be detected in the future in the extreme (low-frequencies) range of the primordial electromagnetic and gravitational spectrum.

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