Systems & Control Letters 116 (2018) 71–77

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Distributed Kalman filter in a network of linear systems

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ARTICLE INFO

Article history:
Received 13 September 2017
Received in revised form 29 March 2018
Accepted 7 April 2018
Available online 8 May 2018

Keywords:
Estimation
Kalman filter
Distributed systems

ABSTRACT

This paper is concerned with the problem of distributed Kalman filtering in a network of interconnected subsystems with distributed control protocols. We consider networks, which can be either homogeneous or heterogeneous, of linear time-invariant subsystems, given in the state-space form. We propose a distributed Kalman filtering scheme for this setup. The proposed method provides, at each node, an estimation of the state parameter, only based on locally available measurements and those from the neighbor nodes. The special feature of this method is that it exploits the particular structure of the considered network to obtain an estimate using only one prediction/update step at each time step. We show that the estimate produced by the proposed method asymptotically approaches that of the centralized Kalman filter, i.e., the optimal one with global knowledge of all network parameters, and we are able to bound the convergence rate. Moreover, if the initial states of all subsystems are mutually uncorrelated, the estimates of these two schemes are identical at each time step.

1. Introduction

There has been an increasing activity in the study of distributed estimation in a network environment. This is due to its broad applications in many areas, including formation control Subbotin and Smith [1], Lin et al. [2], distributed sensor network Zhang et al. [3] and cyber security Teixeira et al. [4], Zamani et al. [5]. This paper examines the problem of distributed estimation in a network of subsystems represented by a finite dimensional state-space model. Our focus is on the scenario where each subsystem obtains some noisy measurements, and broadcasts them to its nearby subsystems, called *neighbors*. The neighbors exploit the received information, together with an estimate of their internal states, to make a decision about their future states. This sort of communication coupling arises in different applications. For example, in control system security problems Teixeira et al. [4], distributed state estimation is required to calculate certain estimation error residues for attack detection. Similarly, for formation control Lin et al. [6], Zheng et al. [7], Lin et al. [8], each subsystem integrates measurements from its nearby subsystems, and states of each subsystem need to be estimated for distributed control design purposes. The main objective of this paper is to collectively estimate the states of all subsystems within such a network. We will propose a novel distributed version of the celebrated Kalman filter.

The current paper, in broad sense, belongs to the large body of literature regarding distributed estimation. One can refer to Lopes and Ali [9], Kar et al. [10], Conejo et al. [11], Gómez-Expósito et al. [12], Marelli and Fu [13], Olfati-Saber [14], Ugrinovskii [15], Ugrinovskii [16], Zamani and Ugrinovskii [17], Khan and Moura [18], Olfati-Saber [19], He et al. [20] and the survey paper Ribeiro et al. [21], as well as references listed therein, for different variations of distributed estimation methods among a group of subsystems within a network. A consensus based Kalman filter was proposed in Olfati-Saber [14]. The author of Ugrinovskii [15] utilized a linear matrix inequality to minimize a H_∞ index associated with a consensus based estimator, which can be implemented locally. Some of the results there were then extended to the case of switching topology in Ugrinovskii [16]. The same problem was solved using the minimum energy filtering approach in Zamani and Ugrinovskii [17]. The reference [20] proposed an event-based distributed Kalman filter for estimating a common state in a sensor network. A common drawback of the state estimation methods described above is that, being based on consensus, they require, in theory, an infinite number of consensus iterations at each time step. This results in computational and communication overload. To avoid this, in this paper we exploit the network structure to

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https://doi.org/10.1016/j.sysconle.2018.04.005
0167-6911/

achieve a distributed Kalman filter method which requires only one prediction/update step at each time step.

Moreover, it is worthwhile noting that there is a major difference between the above-mentioned works and the problem formulation in the current paper. More precisely, in the former, the aim of each subsystem is to estimate the aggregated state which is common to all subsystems. In contrast, in the problem studied here, each subsystem is dedicated to the estimation of its own internal state, which in general is different from those of other subsystems. This allows the distributed estimation algorithm to be scalable to networked systems with a large number of subsystems where requiring each subsystem to estimate the aggregated state is both computationally infeasible and practically unnecessary.

To show the effectiveness of the proposed algorithm, we compare our method with the classical (centralized) Kalman filter, which is known to be optimal (in the minimum error covariance sense). The classical method requires the simultaneous knowledge of parameters and measurements from all subsystems within the network to carry out the estimation. In contrast, our proposed distributed estimation algorithm runs a local Kalman filter at each subsystem, which only requires the knowledge of local measurements and parameters, as well as measurements from neighbor subsystems. Hence, it can be implemented in a fully distributed fashion. We show that the state estimate, and its associated estimation error covariance matrix, produced by the proposed distributed method asymptotically converge to those produced by the centralized Kalman filter. We provide bounds for the convergence of both the estimate and the estimation error covariance matrix. A by-product of our result is that, if the initial states of all subsystems are uncoupled (i.e., they are mutually uncorrelated), the estimates produced by our method are identical to that of the centralized Kalman filter.

The remainder of the paper is structured as follows. In Section 2, we describe the network setup and its associated centralized Kalman filter. In Section 4, we describe the proposed distributed Kalman filter scheme. In Section 5, we demonstrate the asymptotic equivalence between the proposed distributed filter and the centralized one, and provide bounds for the convergence of the estimates and their associated estimation error covariances. Simulation results that support our theoretical claims are presented in Section 6. Finally, concluding remarks are given in Section 7.

2. System description

In this paper we study networks of N time-invariant subsystems. Subsystem i is represented by the following state-space model:

$$x_{k+1}^{(i)} = A^{(i)}x_k^{(i)} + z_k^{(i)} + w_k^{(i)}, \tag{1}$$

$$y_k^{(i)} = C^{(i)}x_k^{(i)} + v_k^{(i)}. \tag{2}$$

The subsystems are interconnected as follows:

$$z_k^{(i)} = \sum_{j\in\mathcal{N}_i} L^{(i,j)}y_k^{(j)}, \tag{3}$$

where $x_k^{(i)} \in \mathbb{R}^{n_i}$ is the state, $y_k^{(i)} \in \mathbb{R}^{p_i}$ the output, $w_k^{(i)}$ is an i.i.d Gaussian disturbance process with $w_k^{(i)} \sim \mathcal{N}(0, Q_i)$, and $v_k^{(i)}$ is an i.i.d. Gaussian measurement noise process with $v_k^{(i)} \sim \mathcal{N}(0, R_i)$. We further suppose that $\mathcal{E}\left(w_k^{(i)}w_k^{(j)\top}\right) = 0$ and $\mathcal{E}\left(v_k^{(i)}v_k^{(j)\top}\right) = 0$, $\forall i \neq j$ and $\mathcal{E}\left(x_k^{(i)}w_k^{(j)\top}\right) = 0$, $\mathcal{E}\left(x_k^{(i)}v_k^{(j)\top}\right) = 0$ $\forall i, j$. We also denote the neighbor set of the subsystem i by $\mathcal{N}_i = \left\{j : L^{(i,j)} \neq 0\right\}$.

Remark 1. We note in (1)–(2) that the coupling between neighboring subsystems is solely caused through the $z_k^{(i)}$ term in'(3). The main motivation for considering such coupling comes from distributed control, where'(1) represents the model of an autonomous subsystem (or agent) with $z_k^{(i)}$ being the control input, and (3) represents a distributed control protocol, which employs feedback only from neighboring measurements. This type of distributed control is not only common for control of multi-agent systems (see, for example, Lin et al. [2], Lin et al. [6], Lin et al. [8], Zheng et al. [7]), but also realistic for large networked systems, since only neighboring information is both easily accessible and most useful for each subsystem.

It is worthwhile noting that the dynamical descriptions (1)–(3) can be regarded as a very general setting for the well-known consensus algorithm [22], i.e., when it is run over a group of interconnected multi-input-multi-output linear subsystems expressed in state space form. Additionally, this model can capture interactions within linear dynamical networks. Interested readers can refer to Zamani et al. [23], Sanandaji et al. [24], Sanandaji et al. [25] and Dankers et al. [26], where the authors exploited a similar model for conducting system identification analysis in linear dynamical networks. Finally, this model turns out to be an effective one for studying properties of networked subsystems [5].

We emphasize that the distributed state estimation problem arises for the networked system (1)–(3) because of our allowance for measurement noises $v_k^{(i)}$ in (2). This consideration is very important for applications because measurement noises are unavoidable in practice. This also sharply distinguishes our distributed control formulation from most distributed control algorithms in the literature, where perfect state measurement is often implicitly assumed.

We define $\xi_k^\top = \left[\left(\xi_k^{(1)}\right)^\top, \ldots, \left(\xi_k^{(I)}\right)^\top\right]$ and $\Xi_k = \{\xi_1, \ldots, \xi_k\}$, where (ξ, Ξ) stands for either (x, X), (y, Y), (z, Z), (w, W) or (v, V); moreover, we denote $\Upsilon = \operatorname{diag}\left\{\Upsilon^{(1)}, \ldots, \Upsilon^{(I)}\right\}$, where Υ stands for either A, B, C, Q or R, and $L = \left[L^{(i,j)} : i, j = 1, \ldots, N\right]$.

Using the above notation, we let the initial state of all subsystems have the joint distribution $x_0 \sim \mathcal{N}(\mu, P)$. We can also write the aggregated model of the whole network as

$$\begin{aligned} x_{k+1} &= Ax_k + LCx_k + w_k + BLv_k \\ &= \tilde{A}x_k + e_k, \end{aligned} \tag{4}$$

$$y_k = Cx_k + v_k, \tag{5}$$

with

$$\tilde{A} = A + LC \quad \text{and} \quad e_k = w_k + Lv_k. \tag{6}$$

It then follows that

$$\operatorname{cov}\left(\begin{bmatrix} e_k \\ v_k \end{bmatrix}\begin{bmatrix} e_k^\top & v_k^\top \end{bmatrix}\right) = \begin{bmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^\top & R \end{bmatrix}, \tag{7}$$

where $\tilde{Q} = Q + LRL^\top$ and $\tilde{S} = LR$.

3. Centralized Kalman filter

Consider the standard (centralized) Kalman filter. For all $k, l \in \mathbb{N}$, let

$$\begin{aligned} \hat{x}_{k|l} &\triangleq \mathcal{E}\left(x_k | Y_l\right), \\ \Sigma_{k|l} &\triangleq \mathcal{E}\left(\left[x_k - \hat{x}_{k|l}\right]\left[x_k - \hat{x}_{k|l}\right]^\top\right). \end{aligned} \tag{8}$$

Our aim in this subsection is to compute $\hat{x}_{k|k}$ in a standard centralized way. Notice that Eq. (7) implies that, in the aggregated system formed by (1)–(2), the process noise e_k and the measurement noise v_k are mutually correlated. Taking this into account, it follows from [27, S 5.5] that the prediction and update steps of the (centralized) Kalman filter are initialized by $\hat{x}_{0|0} = \mu$ and $\Sigma_{0|0} = P$, and proceed as follows:

1. **Prediction:**

$$\hat{x}_{k+1|k} = \left(\tilde{A} - \tilde{S}R^{-1}C\right)\hat{x}_{k|k} + \tilde{S}R^{-1}y_k \\ = A\hat{x}_{k|k} + Ly_k, \tag{9}$$

and

$$\Sigma_{k+1|k} = \left(M - \tilde{S}R^{-1}C\right)\Sigma_{k|k}\left(M - \tilde{S}R^{-1}C\right)^\top \\ + \tilde{Q} - \tilde{S}R^{-1}\tilde{S} \\ = A\Sigma_{k|k}A^\top + Q. \tag{10}$$

2. **Update:**

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(y_k - C\hat{x}_{k|k-1}\right), \tag{11}$$

$$\Sigma_{k|k} = (I - K_kC)\,\Sigma_{k|k-1}, \tag{12}$$

with

$$K_k = \Sigma_{k|k-1}C^\top\left(C\Sigma_{k|k-1}C^\top + R\right)^{-1}. \tag{13}$$

4. Distributed Kalman filter

Consider the ith subsystem (1)–(2). Notice that, since the measurements $y_k^{(j)}$, $j \in \mathcal{N}_i$, are known by the ith subsystem, they can be treated as external inputs. This observation leads us to the following intuitive approach for a distributed Kalman filter scheme.

Let, for all $i = 1, \ldots, I$ and $k, l \in \mathbb{N}$,

$$\hat{x}_{k|l}^{(i)} \triangleq \mathcal{E}\left(x_k^{(i)} | y_m^{(j)}; j \in \mathcal{N}_i \cup \{i\}, m = 1, \ldots, l\right), \\ \Sigma_{k|l}^{(i)} \triangleq \mathcal{E}\left(\left[x_k^{(i)} - \hat{x}_{k|l}^{(i)}\right]\left[x_k^{(i)} - \hat{x}_{k|l}^{(i)}\right]^\top\right). \tag{14}$$

Then, the prediction and update steps for the proposed distributed Kalman filter are initialized by $\hat{x}_{0|0}^{(i)} = \mu^{(i)}$ and $\Sigma_{0|0}^{(i)} = P^{(i,i)}$, and proceed as follows:

1. **Prediction:**

$$\hat{x}_{k+1|k}^{(i)} = A^{(i)}\hat{x}_{k|k}^{(i)} + \sum_{j\in\mathcal{N}_i} L_{i,j}^{(i,j)} y_k^{(j)}, \tag{15}$$

$$\Sigma_{k+1|k}^{(i)} = A^{(i)}\Sigma_{k|k}^{(i)}A^{(i)\top} + Q^{(i)}. \tag{16}$$

2. **Update:**

$$\hat{x}_{k|k}^{(i)} = \hat{x}_{k|k-1}^{(i)} + K_k^{(i)}\left(y_k^{(i)} - C^{(i)}\hat{x}_{k|k-1}^{(i)}\right), \tag{17}$$

$$\Sigma_{k|k}^{(i)} = \left(I - K_k^{(i)}C^{(i)}\right)\Sigma_{k|k-1}^{(i)}, \tag{18}$$

with

$$K_k^{(i)} = \Sigma_{k|k-1}^{(i)}C^{(i)\top}\left(C^{(i)}\Sigma_{k|k-1}^{(i)}C^{(i)\top} + R^{(i)}\right)^{-1}. \tag{19}$$

5. Optimality analysis

Since the distributed Kalman filter approach given in Section 4 is motivated by intuition, the question naturally arises as to which extent it is optimal. In this section we address this question. To this end, we define $\left(\hat{x}_{k|l}^\star, \Sigma_{k|l}^\star\right)$, where $\hat{x}_{k|l}^{\star\top} = \left[\hat{x}_{k|l}^{(i)\top} : i = 1, \ldots, N\right]$ and $\Sigma_{k|l}^\star = \operatorname{diag}\left(\Sigma_{k|l}^{(i)} : i = 1, \ldots, N\right)$, to be the outcomes of distributed filter and $\left(\hat{x}_{k|l}, \Sigma_{k|l}\right)$ to be those of centralized one. In Section 5.1, we show that the estimation error covariance of the distributed filter $\Sigma_{k|k}^\star$ converges to that of the centralized one $\Sigma_{k|k}$, and provide a bound for this convergence. In Section 5.2, we do the same for the convergence of $\hat{x}_{k|k}^\star$ to $\hat{x}_{k|k}$.

5.1. Convergence of $\Sigma_{k|k}^\star$ to $\Sigma_{k|k}$

In this section, we show that the covariance matrices $\Sigma_{k|k}$ and $\Sigma_{k|k}^\star$ exponentially converge to each other, and introduce a bound on $\left\|\Sigma_{k|k} - \Sigma_{k|k}^\star\right\|$. To this end, we require the following definition from [28, Def 1.4].

Definition 2. For $n \times n$ matrices $P, Q > 0$, the Riemannian distance is defined by

$$\delta(P, Q) = \sqrt{\sum_{k=1}^n \log^2 \sigma_k\left(PQ^{-1}\right)},$$

where $\sigma_1(X) \geq \cdots \geq \sigma_n(X)$ denote the singular values of matrix X.

Several properties of the above definition, which we use to derive our results, are given in the following proposition.

Proposition 3 ([29, Proposition 5]). *For $n \times n$ matrices $P, Q > 0$, the following holds true:*

1. $\delta(P, P) = 0$.
2. $\delta\left(P^{-1}, Q^{-1}\right) = \delta(Q, P) = \delta(P, Q)$.
3. *For any $m \times m$ matrix $W > 0$ and $m \times n$ matrix B, we have*
$$\delta(W + BPB^\top, W + BQB^\top) \leq \frac{\alpha}{\alpha + \beta}\delta(P, Q),$$
where $\alpha = \max\{\|BPB^\top\|, \|BQB^\top\|\}$ and $\beta = \sigma_{\min}(W)$.
4. *If $P > Q$, then $\|P - Q\| \leq \left(e^{\delta(P,Q)} - 1\right)\|Q\|$.*

The main result of this section is given in Theorem 5. Its proof requires the following technical result.

Lemma 4. *Let $\Gamma_{k|l} = \Sigma_{k|l}^{-1}$ and $\Gamma_{k|l}^\star = \Sigma_{k|l}^{\star -1}$. Then*

$$\left\|\Sigma_{k|k}\right\|, \left\|\Sigma_{k|k}^\star\right\| \leq \sigma, \\ \left\|\Gamma_{k|k}\right\|, \left\|\Gamma_{k|k}^\star\right\| \leq \omega,$$

and

$$\delta\left(\Sigma_{k|k}, \Sigma_{k|k}^\star\right) \leq \upsilon^k \delta\left(P, P^\star\right), \tag{20}$$

$$\delta\left(\Gamma_{k|k}, \Gamma_{k|k}^\star\right) \leq \upsilon^k \delta\left(P, P^\star\right), \tag{21}$$

where

$$\sigma = \max\left\{\|P\|, \|P^\star\|, \left\|\bar{\Sigma}\right\|\right\}, \tag{22}$$

$$\omega = \max\left\{\left\|P^{-1}\right\|, \left\|P^{\star -1}\right\|, \left\|\bar{\Sigma}^{-1}\right\|\right\}, \tag{23}$$

with $P^\star$ denoting the diagonal matrix formed by the block diagonal entries of the matrix P,

$$\upsilon = \upsilon_1\upsilon_2, \quad \upsilon_1 = \frac{\sigma\|A\|^2}{\sigma\|A\|^2 + \left\|Q^{-1}\right\|^{-1}}, \\ \upsilon_2 = \frac{\omega}{\omega + \left\|U^{-1}\right\|^{-1}}, \quad U = C^\top R^{-1} C, \tag{24}$$

and $\bar{\Sigma} = \lim_{k\to\infty}\Sigma_{k|k}$.

Proof. Let $\bar{\Sigma}^\star = \lim_{k\to\infty}\Sigma_{k|k}^\star$ and

$$\tilde{\sigma} = \max\left\{\|P\|, \|P^\star\|, \left\|\bar{\Sigma}\right\|, \left\|\bar{\Sigma}^\star\right\|\right\}, \tag{25}$$

$$\tilde{\omega} = \max\left\{\left\|P^{-1}\right\|, \left\|P^{\star -1}\right\|, \left\|\bar{\Sigma}^{-1}\right\|, \left\|\bar{\Sigma}^{\star -1}\right\|\right\}. \tag{26}$$

We can then appeal to the fact that the Riccati equation is monotonic Bitmead et al. [30], to conclude that, for all $k \in \mathbb{N}$,

$$\left\| \Sigma_{k|k} \right\| \leq \max \left\{ \|P\|, \left\| \bar{\Sigma} \right\| \right\} \leq \tilde{\sigma}, \tag{27}$$

$$\left\| \Sigma^{\star}_{k|k} \right\| \leq \max \left\{ \left\|P^{\star}\right\|, \left\| \bar{\Sigma}^{\star} \right\| \right\} \leq \tilde{\sigma}, \tag{28}$$

$$\left\| \Gamma_{k|k} \right\| \leq \max \left\{ \left\|P^{-1}\right\|, \left\| \bar{\Sigma}^{-1} \right\| \right\} \leq \tilde{\omega}, \tag{29}$$

$$\left\| \Gamma^{\star}_{k|k} \right\| \leq \max \left\{ \left\|P^{\star^{-1}}\right\|, \left\| \bar{\Sigma}^{\star^{-1}} \right\| \right\} \leq \tilde{\omega}. \tag{30}$$

Recall that

$$\Sigma_{k+1|k} = A\Sigma_{k|k}A^{\top} + Q.$$

Also, from [27, p. 139], we have

$$\Gamma_{k|k} = \Gamma_{k|k-1} + U.$$

Clearly, similar relations hold for $\Sigma^{\star}_{k|l}$ and $\Gamma^{\star}_{k|l}$. Then, it follows from Proposition 3-3 that,

$$\begin{aligned}\delta\left(\Sigma_{k+1|k}, \Sigma^{\star}_{k+1|k}\right) &= \delta\left(A\Sigma_{k|k}A^{\top} + Q, A\Sigma^{\star}_{k|k}A^{\top} + Q\right) \\ &\leq \tilde{\upsilon}_1 \delta\left(\Sigma_{k|k}, \Sigma^{\star}_{k|k}\right),\end{aligned} \tag{31}$$

$$\begin{aligned}\delta\left(\Gamma_{k|k}, \Gamma^{\star}_{k|k}\right) &= \delta\left(\Gamma_{k|k-1} + U, \Gamma^{\star}_{k|k-1} + U\right) \\ &\leq \tilde{\upsilon}_2 \delta\left(\Gamma_{k|k-1}, \Gamma^{\star}_{k|k-1}\right),\end{aligned} \tag{32}$$

with

$$\tilde{\upsilon}_1 = \frac{\tilde{\sigma}\,\|A\|^2}{\tilde{\sigma}\,\|A\|^2 + \left\|Q^{-1}\right\|^{-1}} \quad \text{and} \quad \tilde{\upsilon}_2 = \frac{\tilde{\omega}}{\tilde{\omega} + \left\|U^{-1}\right\|^{-1}}.$$

It then follows from (31)–(32) and Proposition 3-2, that

$$\delta\left(\Sigma_{k|k}, \Sigma^{\star}_{k|k}\right) \leq \tilde{\upsilon}^k \delta\left(P, P^{\star}\right),$$
$$\delta\left(\Gamma_{k|k}, \Gamma^{\star}_{k|k}\right) \leq \tilde{\upsilon}^k \delta\left(P, P^{\star}\right).$$

with $\tilde{\upsilon} = \tilde{\upsilon}_1\tilde{\upsilon}_2$. Finally, the above implies that $\bar{\Sigma}^{\star} = \bar{\Sigma}$. Hence, the parameters $\tilde{\sigma}$ and $\tilde{\omega}$ given in (25)–(26) are equivalent to σ and ω in (22)–(23), respectively, and the result follows. □

We now introduce the main result of the section, stating a bound on $\left\|\Sigma_{k|k} - \Sigma^{\star}_{k|k}\right\|$.

Theorem 5. *Let $\tilde{\Sigma}_{k|l} = \Sigma_{k|l} - \Sigma^{\star}_{k|l}$ and $\tilde{\Gamma}_{k|l} = \Gamma_{k|l} - \Gamma^{\star}_{k|l}$. Then*

$$\left\|\tilde{\Sigma}_{k|k}\right\| \leq \kappa\sigma\upsilon^k \quad \text{and} \quad \left\|\tilde{\Gamma}_{k|k}\right\| \leq \kappa\omega\upsilon^k,$$

where

$$\kappa = e^{\delta(P,P^{\star})} - 1.$$

Proof. Using (22)–(23), together with (20)–(21), Proposition 3-4 and Lemma 11, we obtain

$$\begin{aligned}\left\|\tilde{\Sigma}_{k|k}\right\| &\leq \left(e^{\upsilon^k\delta(P,P^{\star})} - 1\right)\left\|\Sigma_{k|k}\right\| \leq \kappa\upsilon^k\left\|\Sigma_{k|k}\right\| \\ &\leq \kappa\sigma\upsilon^k, \\ \left\|\tilde{\Gamma}_{k|k}\right\| &\leq \kappa\upsilon^k\left\|\Gamma_{k|k}\right\| \leq \kappa\omega\upsilon^k. \quad \square\end{aligned}$$

5.2. Convergence of $\hat{x}^{\star}_{k|k}$ to $\hat{x}_{k|k}$

In this subsection, we study the convergence of state estimate $\hat{x}^{\star}_{k|k}$, obtained through the distributed method, and that of the centralized one $\hat{x}_{k|k}$. Moreover, we derive a bound on the error $\hat{x}_{k|k}$-$\hat{x}^{\star}_{k|k}$. We start by introducing a number of lemmas which are instrumental for establishing our main results.

Lemma 6. *Let $\tilde{x}_{k|l} = \hat{x}_{k|l} - \hat{x}^{\star}_{k|l}$. Then*

$$\tilde{x}_{k+1|k} = H_k\tilde{x}_{k|k-1} + \xi_k. \tag{33}$$

where

$$\begin{aligned}H_k &= A\left(I - \Sigma_{k|k}U\right), \\ \xi_k &= a_k + b_k, \\ a_k &= A\Sigma_{k|k}\tilde{\Gamma}_{k|k}\hat{x}^{\star}_{k|k-1}, \\ b_k &= A\tilde{\Sigma}_{k|k}\Gamma^{\star}_{k|k}\hat{x}^{\star}_{k|k}.\end{aligned}$$

Proof. Let $\gamma_{k|l} = \Gamma_{k|l}\hat{x}_{k|l}$, $\gamma^{\star}_{k|l} = \Gamma^{\star}_{k|l}\hat{x}^{\star}_{k|l}$ and $\tilde{\gamma}_{k|l} = \gamma_{k|l} - \gamma^{\star}_{k|l}$. We can easily obtain

$$\tilde{x}_{k+1|k} = A\tilde{x}_{k|k}.$$

Also, from [27, p. 140], we obtain

$$\tilde{\gamma}_{k|k} = \tilde{\gamma}_{k|k-1}.$$

Then it is easy to check that

$$\tilde{x}_{k|k} = \hat{x}_{k|k} - \hat{x}^{\star}_{k|k} = \Sigma_{k|k}\gamma_{k|k} - \Sigma^{\star}_{k|k}\gamma^{\star}_{k|k} = \Sigma_{k|k}\tilde{\gamma}_{k|k} + \tilde{\Sigma}_{k|k}\gamma^{\star}_{k|k},$$

and

$$\begin{aligned}\tilde{\gamma}_{k|k-1} &= \gamma_{k|k-1} - \gamma^{\star}_{k|k-1} \\ &= \Gamma_{k|k-1}\hat{x}_{k|k-1} - \Gamma^{\star}_{k|k-1}\hat{x}^{\star}_{k|k-1} = \Gamma_{k|k-1}\tilde{x}_{k|k-1} + \tilde{\Gamma}_{k|k-1}\hat{x}^{\star}_{k|k-1}.\end{aligned}$$

We then have

$$\begin{aligned}\tilde{x}_{k+1|k} &= A\Sigma_{k|k}\tilde{\gamma}_{k|k-1} + A\tilde{\Sigma}_{k|k}\gamma^{\star}_{k|k} = A\Sigma_{k|k}\Gamma_{k|k-1}\tilde{x}_{k|k-1} + \xi_k \\ &= A\Sigma_{k|k}\left(\Gamma_{k|k} - U\right)\tilde{x}_{k|k-1} + \xi_k = H_k\tilde{x}_{k|k-1} + \xi_k. \quad \square\end{aligned}$$

Lemma 7. *Let*

$$\Delta_k = \mathcal{E}\left(\tilde{x}_{k|k-1}\tilde{x}^{\top}_{k|k-1}\right). \tag{34}$$

Then

$$\Delta_k \leq H_k\Delta_{k-1}H_k^{\top} + \lambda\upsilon^k I, \tag{35}$$

where I is the identity matrix, υ is defined in (24), and

$$\lambda \triangleq \sup_{k\in\mathbb{N}}\left(\zeta + 2\sqrt{\zeta\,\|H_k\|^2\,\|\Delta_{k-1}\|}\right) < \infty, \tag{36}$$

with

$$\begin{aligned}\zeta &= (\alpha+\beta) + 2\sqrt{\alpha\beta}, \\ \alpha &= \kappa^2\omega^2\sigma^2\,\|A\|^2\left(\sigma\,\|A\|^2 + \|Q\|\right), \\ \beta &= \kappa^2\omega^2\sigma^3\,\|A\|^2.\end{aligned} \tag{37}$$

Proof. We split the argument in steps:

Step (1) From Lemmas 6 and 12

$$\begin{aligned}\left\|\mathcal{E}\left(\xi_k\xi_k^{\top}\right)\right\| &\leq \left\|\mathcal{E}\left(a_ka_k^{\top}\right)\right\| + \left\|\mathcal{E}\left(b_kb_k^{\top}\right)\right\| \\ &\quad + 2\sqrt{\left\|\mathcal{E}\left(a_ka_k^{\top}\right)\right\|\,\left\|\mathcal{E}\left(b_kb_k^{\top}\right)\right\|}.\end{aligned}$$

Now, using Lemma 4,

$$\begin{aligned}\left\|\mathcal{E}\left(a_ka_k^{\top}\right)\right\| &\leq \|A\|^2\left\|\Sigma_{k|k}\right\|^2\left\|\tilde{\Gamma}_{k|k}\right\|^2\left\|\mathcal{E}\left(\hat{x}^{\star}_{k|k-1}\hat{x}^{\star\top}_{k|k-1}\right)\right\| \\ &\leq \kappa^2\omega^2\sigma^2\,\|A\|^2\left\|\Sigma^{\star}_{k|k-1}\right\|\upsilon^{2k} \leq \alpha\upsilon^{2k},\end{aligned}$$

and

$$\begin{aligned}\left\|\mathcal{E}\left(b_kb_k^{\top}\right)\right\| &\leq \|A\|^2\left\|\Gamma^{\star}_{k|k}\right\|^2\left\|\tilde{\Sigma}_{k|k}\right\|^2\left\|\mathcal{E}\left(\hat{x}^{\star}_{k|k}\hat{x}^{\star\top}_{k|k}\right)\right\| \\ &\leq \kappa^2\omega^2\sigma^2\,\|A\|^2\left\|\Sigma^{\star}_{k|k}\right\|\upsilon^{2k} \leq \beta\upsilon^{2k}.\end{aligned}$$

Then

$$\left\|\mathcal{E}\left(\xi_k\xi_k^{\top}\right)\right\| \leq \zeta\upsilon^{2k}.$$

Step (2) From (33) and Lemma 12, we have

$$\begin{aligned}\Delta_k &\leq H_k\Delta_{k-1}H_k^{\top} + \mathcal{E}\left(\xi_k\xi_k^{\top}\right) \\ &\quad + 2\sqrt{\|H_k\Delta_{k-1}H_k\|\left\|\mathcal{E}\left(\xi_k^{\top}\xi_k\right)\right\|}I \leq F_k(\Delta_{k-1}),\end{aligned}$$

with

$$F_k(X) = H_k X H_k^\top + \left(\zeta \upsilon^k + 2\sqrt{\zeta \|H_k\|^2 \|X\|}\right) I \upsilon^k.$$

Clearly, if $A > B$ then $F_k(A) > F_k(B)$. Also, there clearly exists $\bar{k}$ and $\bar{\Delta}$ such that $F_k(\bar{\Delta}) < \bar{\Delta}$, for all $k \geq \bar{k}$. Hence, $\lim_{k\to\infty} \Delta(k) < \infty$, and the result follows. □

The following result states a family of upper bounds on the norm of the covariance matrix of $\tilde{x}_{k|l}$.

Theorem 8. *Consider Δ_k as defined in (34). Let $H_k = V_k J_k V_k^{-1}$ and $\bar{H} = \bar{V}\bar{J}\bar{V}^{-1}$ be the Jordan decompositions of H_k and $\bar{H}$, respectively. Then for every $\epsilon > 1$, there exists $k_\epsilon \in \mathbb{N}$ such that*

$$\|\Delta_k\| \leq A_\epsilon \psi_\epsilon^k + B_\epsilon \upsilon^k,$$

where

$$A_\epsilon = \frac{\lambda \psi_\epsilon \phi_\epsilon}{\psi_\epsilon - \upsilon}, \qquad B_\epsilon = \frac{\lambda \upsilon \phi_\epsilon}{\upsilon - \psi_\epsilon}.$$

and

$$\psi_\epsilon = \epsilon \rho\left(\bar{H}\right), \qquad \bar{H} = \lim_{k\to\infty} H_k, \tag{38}$$

$$\phi_\epsilon = \epsilon \left\|\bar{V}\right\|^2 \left\|\bar{V}^{-1}\right\|^2 \left(\frac{m_\epsilon}{\epsilon\rho\left(\bar{H}\right)}\right)^{2(k_\epsilon - 1)},$$

$$m_\epsilon = \max\left\{1, \|H_1\|, \ldots, \left\|H_{k_\epsilon - 1}\right\|\right\}.$$

Proof. We split the argument in steps:

Step (1) Let

$$D_k = H_k D_{k-1} H_k^\top + \lambda I \upsilon^k.$$

with $D_1 = 0$, From (35), and since $\Delta_1 = D_1 = 0$, it follows that

$$\Delta_k \leq D_k. \tag{39}$$

Step (2) Let

$$\begin{aligned}
\Pi_{l,k} &= H_{k-1} H_{k-2} \cdots H_l \\
&= V_{k-1} J_{k-1} V_{k-1}^{-1} \cdots V_l J_l V_l^{-1}.
\end{aligned}$$

From (38), there exists $k_\epsilon \in \mathbb{N}$ such that, for all $k \geq k_\epsilon$,

$$\begin{aligned}
\|V_k\| &\leq \sqrt{\epsilon}\left\|\bar{V}\right\|, \\
\left\|V_k^{-1}\right\| &\leq \sqrt{\epsilon}\left\|\bar{V}^{-1}\right\|, \\
\left\|V_{k+1}^{-1} V_k\right\| &\leq \sqrt{\epsilon}, \\
\|J_k\| &\leq \sqrt{\epsilon}\rho\left(\bar{H}\right).
\end{aligned}$$

Then, for all $k \geq l$,

$$\begin{aligned}
\left\|\Pi_{l,k}\right\| &\leq m_\epsilon^{k_\epsilon - l} \|V_{k-1}\| \|J_{k-1}\| \left\|V_k^{-1} V_{k-1}\right\| \times \\
&\quad \cdots \times \left\|V_{k_\epsilon+1}^{-1} V_{k_\epsilon}\right\| \left\|J_{k_\epsilon}\right\| \left\|V_{k_\epsilon}\right\| \\
&\leq \sqrt{\epsilon}\left\|\bar{V}\right\| \left\|\bar{V}^{-1}\right\| m_\epsilon^{k_\epsilon - l}\left(\epsilon\rho\left(\bar{H}\right)\right)^{k - k_\epsilon} \\
&= \sqrt{\epsilon}\left\|\bar{V}\right\| \left\|\bar{V}^{-1}\right\| \left(\frac{m_\epsilon}{\epsilon\rho\left(\bar{H}\right)}\right)^{k_\epsilon - l} \left(\epsilon\rho\left(\bar{H}\right)\right)^{k-l} \\
&\leq \sqrt{\phi_\epsilon}\psi_\epsilon^{\frac{k-l}{2}}.
\end{aligned}$$

Step (3) We have

$$D_{k+1} = \lambda \sum_{l=1}^{k} \upsilon^l \Pi_{l,k} \Pi_{l,k}^\top.$$

Let $d_k = \|D_k\|$. Then

$$d_k \leq \lambda \sum_{l=1}^{k} \upsilon^l \left\|\Pi_{l,k}\Pi_{l,k}^\top\right\| \leq \lambda\phi_\epsilon \sum_{l=1}^{k} \upsilon^l \psi_\epsilon^{k-l} = (h_k * u_k),$$

with

$$h_k = \phi_\epsilon \psi_\epsilon^k \qquad \text{and} \qquad u_k = \lambda \upsilon^k.$$

Taking z-transform we get

$$\begin{aligned}
d(z) = h(z)u(z) &= \frac{\lambda\phi_\epsilon}{\left(1 - \psi_\epsilon z^{-1}\right)\left(1 - \upsilon z^{-1}\right)} \\
&= \frac{A_\epsilon}{1 - \psi_\epsilon z^{-1}} + \frac{B_\epsilon}{1 - \upsilon z^{-1}}.
\end{aligned}$$

Hence,

$$d_k = A_\epsilon \psi_\epsilon^k + B_\epsilon \upsilon^k,$$

and the result follows from the definition of d_k and (39). □

Theorem 8 states that the covariance of the difference between $\hat{x}^\star_{k|k-1}$ and $\hat{x}_{k|k-1}$ is bounded by two exponential terms. The term $B_\epsilon \upsilon^k$ is due to the convergence of the Kalman gain $K_k^\star$ to K_k, while the term $A_\epsilon \psi_\epsilon^k$ is due to the convergence of the states given by the system dynamics. In order to use this result to show the asymptotic convergence of $\hat{x}^\star_{k|k-1}$ to $\hat{x}_{k|k-1}$, we need that $\upsilon < 1$ and $\psi_\epsilon < 1$, for some $\epsilon > 0$. While it is clear from (24) that the former is true, guaranteeing the latter is not that straightforward. The following proposition addresses this issue.

Proposition 9. *If the pair $[A, C]$ is completely detectable and the pair $\left[A, Q^{1/2}\right]$ is completely stabilizable, then $\rho\left(\bar{H}\right) < 1$, where $\rho(\bar{H})$ denotes the spectral radius of matrix $\bar{H}$.*

Proof. Let $K_k^\star = \operatorname{diag}\left(K_k^{(i)} : i = 1, \ldots, N\right)$. From Theorem 5,

$$\lim_{k\to\infty} K_k = \lim_{k\to\infty} K_k^\star \triangleq \bar{K}.$$

Now,

$$\begin{aligned}
\hat{x}_{k+1|k} &= A\left(I - K_k C\right)\hat{x}_{k|k-1} + \left(AK_k + L\right) y_k, \\
\hat{x}^\star_{k+1|k} &= A\left(I - K_k^\star C\right)\hat{x}^\star_{k|k-1} + \left(AK_k^\star + L\right) y_k.
\end{aligned}$$

Hence, if we had that $K_k = K_k^\star = \bar{K}$, for all $k \in \mathbb{N}$, then

$$\tilde{x}_{k+1|k} = A\left(I - \bar{K}C\right)\tilde{x}_{k|k-1}.$$

However, under the same assumption, according to Lemma 6, $\tilde{x}_{k+1|k} = \bar{H}\tilde{x}_{k|k-1}$. Hence,

$$\bar{H} = A\left(I - \bar{K}C\right).$$

i.e., $\bar{H}$ equals the matrix that determines the asymptotic dynamics of the centralized Kalman filter's estimation error. Then, in view of the model (4)–(5), the result follows from [27, S 4.4]. □

5.3. The case when the initial covariance is block diagonal

It turns out that, when the initial covariance matrix has a block diagonal structure both estimation methods are completely identical. This is summarized in the following corollary.

Corollary 10. *Consider the network of subsystems (1)–(2). If the matrix P is block diagonal, then the distributed Kalman filter scheme (15)–(19) produces, for each i, the same estimate as the centralized Kalman filter (9)–(13).*

Proof. Recall that matrices A, Q, C and R are all block diagonal. It then follows from (10) that, if $\Sigma_{k|k}$ is block diagonal, so is $\Sigma_{k+1|k}$. One can easily check from (12) and (13) that the same holds for K_k and $\Sigma_{k|k}$ if $\Sigma_{k|k-1}$ is block diagonal. Since $\Sigma_{1|0} = P$ is block diagonal, it follows that the matrices $\Sigma_{k|k-1}$ and $\Sigma_{k|k}$ remain block diagonal for all k. Now, it is straightforward to verify that (9)–(13)

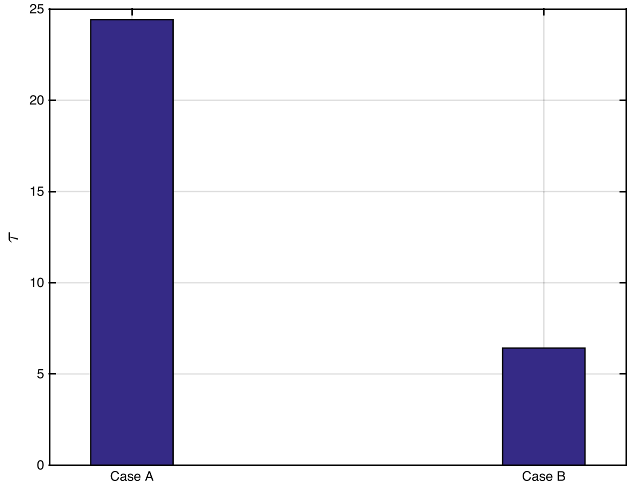

Fig. 1. Settling time vs. different choices of topologies.

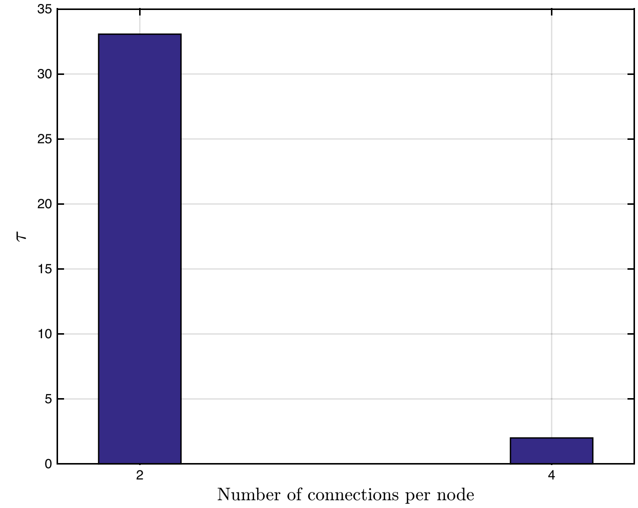

Fig. 2. Settling time vs. number of connections per node.

become equivalent to (15)–(19), when $\Sigma_{k|k}$ and $\Sigma_{k|k-1}$ are block diagonal. Hence, the distributed and centralized Kalman filters produce the same estimate and the result follows. □

6. Simulations

In this section, we present four numerical experiments to study the convergence of the proposed distributed Kalman filter to its centralized counterpart. In the first experiment, we compare the convergence on networks with different topologies. To this end, we consider a directed communication topology involving ten subsystems with first-order dynamics. The subsystems' initial conditions are drawn from the normal distribution $\mathcal{N}(0, P)$. The initial covariance matrix P is chosen by randomly generating a positive-definite matrix using $P = \mathcal{L}\mathcal{L}^\top + \epsilon_0 I_{10}$, with $\epsilon_0 = 0.1$, and the entries of $\mathcal{L} \in \mathbb{R}^{10\times 10}$ are drawn from the uniform distribution $\mathcal{U}(0, 1)$. Also, $v_k \sim \mathcal{N}(0, 0.1 I_{10})$ and $w_k \sim \mathcal{N}(0, 0.1 I_{10})$. The poles of these ten subsystems are randomly chosen from the uniform distribution $\mathcal{U}(0.4, 0.8)$. We consider two different topologies. The first one is a *path* topology, whose weights, i.e., the scalars $L^{(i,j)}$, are randomly selected from the distribution $\mathcal{U}(0, 1)$. The second one is a *random* topology, whose weights are randomly drawn while guaranteeing the stability of the overall system. We refer to these two topologies as Case A and Case B, respectively. We examine the convergence rate of the proposed filtering algorithm for these two cases. To measure this rate, we use the settling time, which we define as

$$\tau = \min_{\tilde{\tau}} \left\{ \max_{k:\tilde{\tau} \le k \le T} \|\tilde{x}_{x_{k|k-1}}\|_2 < 0.1 \|\tilde{x}_{0|-1}\|_2 \right\}, \tag{40}$$

where T is the running time. For each one of these two topologies, we plot the average value of τ obtained over 200 realizations of the process noise $w_k^{(i)}$, measurement noise $v_k^{(i)}$, initial condition and the random selection of the topology weights. The results are shown in Fig. 1. We see that in Case B the convergence is faster. The reason for this is that the random topology generated in Case B has a much larger number of edges compared to the path topology in Case A. Indeed, the matrix L associated with Case B is a very dense one, i.e., it contains very few zero entries.

In the second experiment we study the effect of number of connections per node on the convergence rate. To this end, we consider a network of five nodes with first-order dynamics, poles at 0.15 and topology weights $L^{(i,j)} = 0.1$. We perform 200 Monte Carlo simulations for two classes of topologies, namely, with two and four connections per node. One can observe that the former results in a *cycle* graph and the latter delivers a *complete* graph. The simulation results are depicted in Fig. 2. Again, we see how the settling time decreases with the number of connections.

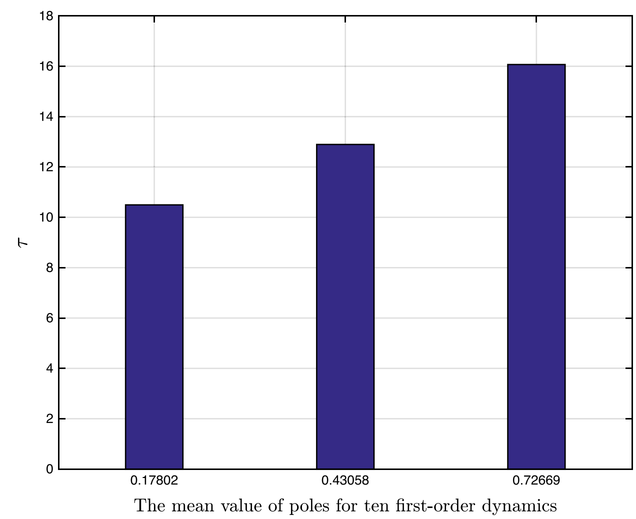

Fig. 3. Settling time vs. mean value of subsystems' poles, in a network with random topology.

In the third experiment we study the effect of subsystems' dynamics on the settling time of the distributed Kalman filter. We consider a network with ten nodes with first-order dynamics, interconnected with a random topology. We consider three cases, in which the subsystems' poles are drawn from the uniform distributions $\mathcal{U}(0.1, 0.3)$, $\mathcal{U}(0.2, 0.6)$ and $\mathcal{U}(0.6, 0.8)$, respectively. Fig. 3 shows the dependence of τ, obtained by averaging 200 Monte Carlo runs, on the mean value of the subsystems' poles. We see how the settling time increases with this value.

In our final experiment, we compare the convergence rate for networks with different sizes. We consider networks ranging from 80 to 600 nodes. For each case, we consider identical nodes with first-order dynamics, having poles at 0.3, and connected using a loop topology whose nonzero values are $L^{(i,j)} = 0.1$. Fig. 4 depicts the dependence of the convergence rate τ, again obtained by averaging 200 Monte Carlo runs, on the number of nodes. We see how this rate slightly decreases with the network size, and remains nearly constant for large networks.

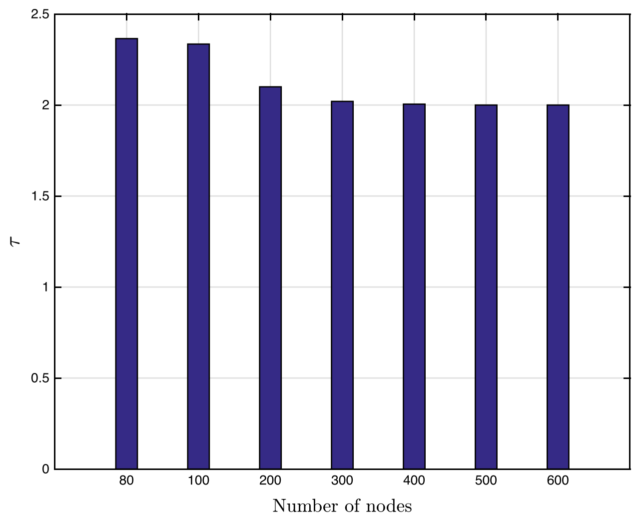

Fig. 4. Settling time vs. number of nodes, in a network with path topology.

7. Conclusion

We studied the distributed Kalman filter problem in a network of linear time-invariant subsystems. We proposed a distributed Kalman filter scheme, which uses only local measurements, and we studied the extent to which this scheme approximates the centralized (i.e., optimal) Kalman filter. It turned out that the covariance matrix associated with the initial value of the state vector plays an important role. We showed that if this matrix is block diagonal, the proposed distributed scheme is optimal. Moreover, if that condition is dropped, the estimation error covariances, and the associated estimates, obtained through these two approaches approximate each other exponentially fast. We also established proper bounds on error between estimates and its covariance matrix.

Appendix. Some lemmas

Lemma 11 ([29, Lemma 25]). *For every* $x \in \mathbb{R}$ *and* $0 \leq y \leq 1$, $e^{xy} - 1 \leq (e^x - 1)\, y$.

Lemma 12 ([29, Lemma 26]). *If* $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix} \geq 0$, *then* $\|B\| \leq \sqrt{\|A\|\,\|C\|}$.

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