

BRANCHING LAWS, SOME RESULTS AND NEW EXAMPLES

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ABSTRACT. For a connected, noncompact matrix simple Lie group G so that a maximal compact subgroup K has three dimensional simple ideal, in this note we analyze the admissibility of the restriction of irreducible square integrable representations for the ambient group when they are restricted to certain subgroups that contains the three dimensional ideal. In this setting we provide a formula for the multiplicity of the irreducible factors. Also, for general G such that G/K is an Hermitian G -manifold we give a necessary and sufficient condition so that a square integrable representations of the ambient group is admissible over the semisimple factor of K .

1. INTRODUCTION

Let G be a connected noncompact simple matrix Lie group. Henceforth, we fix a maximal compact subgroup K of G and we assume both groups have the same rank. We also fix $T \subset K$ a maximal torus. Thus, T is a compact Cartan subgroup of G . Under these hypotheses, Harish-Chandra showed there exists irreducible unitary representations of G so that its matrix coefficients are square integrable with respect to a Haar measure on G . One aim of this note is to write down explicit branching laws for the restriction of some irreducible square integrable representation to specific subgroups H of G . A second objective is to show that when G is simple, the symmetric space G/K has G -invariant quaternionic structure, and H is a specific subgroup locally isomorphic to the group $SU(2, 1)$, then an irreducible square integrable representations for G has an admissible restriction to H if and only if it is a quaternionic discrete series representation. The last objective is to present results on admissible restriction of square integrable representations to specific subgroups of G . To begin with, we recall a description of the irreducible square integrable representations for G . Harish-Chandra showed that the set of equivalence classes of irreducible square integrable representations is parameterized by a lattice contained in the dual of the Lie algebra of a compact Cartan subgroup. In order to state our results we need

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to explicit the parametrization and set up some notation. As usual, the Lie algebra of a Lie group is denoted by the corresponding lower case German letter. The complexification of a real vector space V is denoted by adding the subindex \mathbb{C} . However, the root space for a root is denoted by the real Lie algebra followed by a subindex equal to the root. V^* denotes the dual space to a vector space V . Let θ be the Cartan involution which corresponds to the subgroup K , the associated Cartan decomposition is denoted by $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$. Let $\Phi(\mathfrak{g}, \mathfrak{t})$ denote the root system attached to the Cartan subalgebra $\mathfrak{t}_{\mathbb{C}}$. Hence, $\Phi(\mathfrak{g}, \mathfrak{t}) = \Phi_c \cup \Phi_n = \Phi(\mathfrak{k}, \mathfrak{t}) \cup \Phi_n(\mathfrak{g}, \mathfrak{t})$ splits up as the disjoint union of the set of compact roots and the set of noncompact roots. From now on, we fix a system of positive roots Δ for Φ_c . For this note, either the highest weight or the infinitesimal character of an irreducible representation of K is dominant with respect to Δ . The Killing form on the Lie algebra \mathfrak{g} gives rise to an inner product $(\ , \)$ in $i\mathfrak{t}^*$. As usual, let $\rho = \rho_g$ denote half of the sum of the roots for some system of positive roots for $\Phi(\mathfrak{g}, \mathfrak{t})$.

A Harish-Chandra parameter for G is $\lambda \in i\mathfrak{t}^*$ such that $(\lambda, \alpha) \neq 0$ for every $\alpha \in \Phi(\mathfrak{g}, \mathfrak{t})$, and so that $\lambda + \rho$ is the differential of a character of T . To each Harish-Chandra parameter, λ , Harish-Chandra associated a unique irreducible square integrable representation $(\pi_{\lambda}^G, V_{\lambda})$ of G . Moreover, he showed the map $\lambda \mapsto \pi_{\lambda}^G$ is a bijection from the set of Harish-Chandra parameters dominant with respect to Δ onto the set of equivalence classes of irreducible square integrable representations for G . For a proof [W1].

In [GW], the authors have considered quaternionic real form G of a complex simple Lie group and constructed a specific subgroup H locally isomorphic to $SU(2, 1)$, their setting is as follows: a system of positive roots Ψ so that $\Delta \subset \Psi \subset \Phi(\mathfrak{g}, \mathfrak{t})$ is called *small* if the maximal root β for Ψ is compact, Ψ has at most two noncompact simple roots α_0, α_1 and after we write $\beta = n_0\alpha_0 + n_1\alpha_1 +$ a linear combination of compact simple roots, we have the inequality $n_0 + n_1 \leq 2$. A noncompact connected simple Lie group G is a *quaternionic real form* if \mathfrak{g} is an inner form of a complex simple Lie algebra and if a compactly imbedded Cartan subalgebra \mathfrak{t} has the property that $\Phi(\mathfrak{g}, \mathfrak{t})$ admits a small system of positive roots so that $n_0 + n_1 = 2$. In [GW], the list of the Lie algebras for the quaternionic real forms is presented, we reproduce the list in Section 3. It can be shown that the set of equivalence classes of the set of quaternionic real forms is equal to the set of equivalence classes of the set of noncompact simple Lie groups G so that the associated global symmetric space admits a G -invariant quaternionic structure.

In order to state Theorem 1, we fix a quaternionic real form G , a small system of positive roots $\Psi \supset \Delta$ and a noncompact simple root α for Ψ . An irreducible square integrable representation (π_{λ}^G, V) is called *quaternionic discrete series representation* if the Harish-Chandra parameter λ is dominant with respect to Ψ .

For the quaternionic real form G , a particular copy \mathfrak{h} of $\mathfrak{su}(2, 1)$ contained in \mathfrak{g} is constructed in [GW]. For this, they verify the equality

$$\frac{2(\beta, \alpha)}{(\alpha, \alpha)} = 1.$$

Thus, the Lie subalgebra $\mathfrak{h}_{\mathbb{C}}$ of $\mathfrak{g}_{\mathbb{C}}$ spanned by the root vectors corresponding to the roots $\{\pm\alpha, \pm\beta\}$ is isomorphic to $\mathfrak{sl}(3, \mathbb{C})$ and invariant under the conjugation of $\mathfrak{g}_{\mathbb{C}}$ with respect to \mathfrak{g} . Hence, $\mathfrak{h} := \mathfrak{g} \cap \mathfrak{h}_{\mathbb{C}}$ is a real form for $\mathfrak{h}_{\mathbb{C}}$. This real form has a compactly embedded Cartan subalgebra, namely, $\mathfrak{u} := \mathfrak{t} \cap \mathfrak{h}$. Thus, \mathfrak{h} is isomorphic to $\mathfrak{su}(2, 1)$. Henceforth, we identify the set $\Phi(\mathfrak{h}, \mathfrak{u})$ with the subset $\{\pm\alpha, \pm\beta, \pm(\beta - \alpha)\}$ of $\Phi(\mathfrak{g}, \mathfrak{t})$.

(1.0) Let H denote the analytic subgroup of G with Lie algebra \mathfrak{h} . Then, $L := K \cap H$ is a maximal compact subgroup for H . The system $\Phi(\mathfrak{h}, \mathfrak{u})$ has three systems of positive roots to which the root β belongs to. The one of our interest is the non-holomorphic system

$$\Psi_q := \Psi \cap \Phi(\mathfrak{h}, \mathfrak{u}) = \{\beta - \alpha, \alpha, \beta\}.$$

The simple roots for Ψ_q are $\beta - \alpha, \alpha$. For a root $\gamma \in \Phi(\mathfrak{g}, \mathfrak{t})$, we denote its coroot by $\check{\gamma} \in i\mathfrak{t}$. Let Λ_1, Λ_2 denote the fundamental weights for Ψ_q , labeled so that $\Lambda_1(\check{\alpha}) = 0$.

(1.1) Owing to results in [DV], [W2], [Kb2], which we will review in section 2, it follows that for a Harish-Chandra parameter λ dominant with respect to the small system Ψ the irreducible representation $(\pi_{\lambda}^G, V_{\lambda})$ restricted to H is an admissible representation. That is, there exists a sequence of Harish-Chandra parameters for H , dominant with respect to β ,

$$\mu_1, \mu_2, \dots, \mu_j, \dots \text{ in } i\mathfrak{u}^*$$

and there exists positive integers

$$n^{G,H}(\lambda, \mu_j), j = 1, 2, \dots$$

so that the restriction of $(\pi_{\lambda}^G, V_{\lambda})$ to H is unitarily equivalent to the discrete Hilbert sum

$$\sum_{j=1}^{\infty} n^{G,H}(\lambda, \mu_j) (\pi_{\mu_j}^H, V_{\mu_j}).$$

In [GW] it is shown $\Psi_n := \Psi \cap \Phi_n$ has $2d$ elements. Our hypothesis that G is a quaternionic real form, forces the root spaces for the roots $\pm\beta$ span a three dimensional simple ideal $\mathfrak{su}_2(\beta)$ in \mathfrak{k} . We denote by \mathfrak{k}_2 the complementary ideal to $\mathfrak{su}_2(\beta)$ in \mathfrak{k} . Hence, we have the decompositions

$$\mathfrak{t} = \mathbb{R}i\check{\beta} + (\mathfrak{t} \cap \mathfrak{k}_2) \text{ and } \Delta = \{\beta\} \cup \Phi(\mathfrak{k}_2, \mathfrak{t} \cap \mathfrak{k}_2) \cap \Psi.$$

For each $\lambda \in \mathfrak{t}_{\mathbb{C}}$, we write $\lambda = \lambda_1 + \lambda_2$ with $\lambda_1 \in \mathbb{C}\check{\beta}, \lambda_2 \in \mathfrak{t}_{2\mathbb{C}} := \mathfrak{t}_{\mathbb{C}} \cap \mathfrak{k}_{2\mathbb{C}}$. Let $q_{\mathfrak{u}} : \mathfrak{t}^* \rightarrow \mathfrak{u}^*$ denote the restriction map.

(1.3) We will verify (in 2.9) that for a Harish-Chandra parameter λ dominant for the small system Ψ , we have λ_2 is a Harish-Chandra parameter for K_2 or perhaps for a two fold cover of K_2 . From now on, $\pi_{\lambda_2}^{K_2}$ denotes the irreducible

representation for \mathfrak{k}_2 of infinitesimal character λ_2 . As usual, $\Delta_{T \cap K_2}(\pi_{\lambda_2}^{K_2})$ denotes the set of $T \cap K_2$ -weights for the representation $\pi_{\lambda_2}^{K_2}$ and $M(\lambda_2, \nu)$ stands for the multiplicity of the weight $\nu \in \Delta_{T \cap K_2}(\pi_{\lambda_2}^{K_2})$.

In (2.9) we verify that for λ dominant with respect to small system Ψ the weight $\lambda_1 + \nu + a\Lambda_1 + bq_u(\lambda_2)$ is dominant with respect to the system Ψ_q for every $a, b \in \mathbb{Z}_{\geq 0}$, and for every $U \cap K_2$ -weight ν of $\pi_{\lambda_2}^{K_2}$. One result of this note, is:

Theorem 1. *Let G be a quaternionic real form, H as in (1.0) and $(\pi_\lambda^G, V_\lambda)$ a quaternionic discrete series representation for G . Then, $n^{G,H}(\lambda, \mu) \neq 0$ if and only if $\mu = (n+d)\Lambda_1 + (m+d)\Lambda_2 + q_u(\lambda_2) + q_u(\nu)$ with arbitrary $m, n \in \mathbb{Z}_{\geq 0}$ and $T \cap K_2$ -weight ν for $\pi_{\lambda_2}^{K_2}$. Moreover*

$$n^{G,H}(\lambda, \mu) = \sum_{\substack{\nu \in \Delta_{T \cap K_2}(\pi_{\lambda_2}^{K_2}), \\ m, n \in \mathbb{Z}_{\geq 0}, \\ \mu = (n+d)\Lambda_1 + (m+d)\Lambda_2 + q_u(\lambda_2) + q_u(\nu)}} M(\lambda_2, \nu) \binom{m+d-2}{d-2} \binom{n+d-2}{d-2}.$$

A question that naturally arises is: What are the Harish-Chandra parameters for G , dominant with respect to Δ , so that π_λ^G has an admissible restriction to H ? The answer to this question is given in Proposition 1.

A group G locally isomorphic to either $SO(3, n)$ shares with the quaternionic real forms that a suitable copy of the algebra \mathfrak{su}_2 is an ideal in a maximal compactly embedded subalgebra for \mathfrak{g} . A group locally isomorphic to $SO(3, 2p+1)$ has no square integrable representations. For a group locally isomorphic to $SO(3, 2n)$ and $n \geq 2$, in Proposition 2 we show that no irreducible square integrable representation of G has an admissible restriction to the usual copy of "SO(3)" contained in G . For the quaternionic group $Sp(1, p)$ the usual factor "Sp(1)" of a maximal compact subgroup is contained in certain image H_0 of $Sp(1, 1)$. In Proposition 3, for a quaternionic discrete series for $Sp(1, p)$, we show it has admissible restriction to H_0 , we compute the Harish-Chandra parameters of the irreducible H_0 -factors and their respective multiplicities.

The group $SU(2, 1)$ can be mapped into a simple Lie group G in perhaps several ways by maps $\phi : SU(2, 1) \rightarrow G$, a question is: What are the triple (G, π_λ^G, ϕ) such that π_λ restricted to the image of ϕ is an admissible representation. In [Va1] we find that for the analytic subgroup H_1 that corresponds to the image of $\mathfrak{su}(2, 1)$ in the rank one real form of a complex group type F_4 no square integrable representation of the ambient group has an admissible restriction to H_1 .

We would like to comment that this note grew up from results in the respective Ph. D. thesis of Sebastian Simondi and Oscar Marquez successfully defended at the Faculty of Mathematics Astronomy and Physics at the Universidad Nacional de Córdoba, Argentine, in 2007 and 2011 respectively.

2. PROOF OF THEOREM 1

As in the hypothesis G is a connected, quaternionic simple Lie group and H is the subgroup locally isomorphic to $SU(2, 1)$. To begin with, we sketch a proof for the statement: *For λ dominant with respect to the small system Ψ , the representation $(\pi_\lambda^G, V_\lambda)$ restricted to H is admissible.* In fact, for a system of positive roots $\Sigma \subset \Phi(\mathfrak{g}, \mathfrak{t})$ in [DV] is attached an ideal $\mathfrak{k}_1(\Sigma)$ for the Lie algebra \mathfrak{k} . The ideal is equal to the real form of the ideal of $\mathfrak{k}_\mathbb{C}$ spanned by $\{[Y_\gamma, Y_\phi] : \gamma, \phi \in \Sigma \cap \Phi_n, Y_\gamma \in \mathfrak{g}_\gamma\}$ together with a subspace of the center $\mathfrak{z}_\mathfrak{k}$ of \mathfrak{k} .

(2.0) For the system Ψ , cf. [GW] Prop. 1.3, Table 2.5, we have that any root in $\Phi_c \cap \Psi$ not equal to β is a linear combination of compact simple roots for Ψ . Thus, for two noncompact roots in Ψ , its sum, is a root only when the sum is equal to β . Thus, $\mathfrak{k}_1(\Psi)$ is equal to $\mathfrak{su}_2(\beta)$ plus the contribution of the center. Now, from the list of the quaternionic real forms, we read that $\mathfrak{z}_\mathfrak{k}$ is nonzero only for G locally isomorphic to $SU(2, p)$. For $\mathfrak{su}(2, p)$, in [DV], it is shown that for Ψ the contribution of $\mathfrak{z}_\mathfrak{k}$ to $\mathfrak{k}_1(\Psi)$ is just the zero subspace. Hence, for a quaternionic system Ψ we have

$$\mathfrak{k}_1(\Psi) = \mathfrak{su}_2(\beta).$$

Because of the definition of H we have $K_1(\Psi)$ is contained in H , hence, Theorem 1 in [DV] yields that for λ dominant with respect to Ψ the representation $(\pi_\lambda^G, V_\lambda)$ has an admissible restriction to $H \cap K$ as well as to the subgroup H . In [Kb3] we find a different proof of the admissibility. Therefore, there exists a sequence of Harish-Chandra parameters for H , $\mu_1, \dots, \mu_n, \dots \in i\mathfrak{u}^*$, for which, we may assume for every j , $(\mu_j, \beta) > 0$, and positive integers $n^{G,H}(\lambda, \mu_j)$ so that the restriction of $(\pi_\lambda^G, V_\lambda)$ restricted to H is equivalent to the Hilbert sum

$$\sum_j n^{G,H}(\lambda, \mu_j) \pi_{\mu_j}^H.$$

We are left to compute μ_j , to show each μ_j is dominant for Ψ_q and to compute the integers $n^{G,H}(\lambda, \mu_j)$.

For this we recall results in [DV], [He]. For $\gamma \in i\mathfrak{t}^*$. (resp in $i\mathfrak{u}^*$.) we consider the Dirac distribution δ_γ and the the discrete Heaviside distribution defined by the series

$$y_\gamma := \sum_{n \geq 0} \delta_{\frac{\gamma}{2} + n\gamma} = \delta_{\frac{\gamma}{2}} + \delta_{\frac{\gamma}{2} + \gamma} + \delta_{\frac{\gamma}{2} + 2\gamma} + \dots$$

For any strict multiset $\gamma_1, \dots, \gamma_r$ the convolution $y_{\gamma_1} \star \dots \star y_{\gamma_r}$ is a well defined distribution. In particular, we have

$$\underbrace{y_\gamma \star \dots \star y_\gamma}_r := y_\gamma^r = \sum_{n \geq 0} \binom{n+r-1}{r-1} \delta_{(\frac{r}{2}+n)\gamma}$$

We have

$$\mathfrak{u}_\mathbb{C} = \mathbb{C}\check{\alpha} + \mathbb{C}\check{\beta}, \mathfrak{u}_\mathbb{C} \cap \mathfrak{k}_{2_\mathbb{C}} = \mathfrak{u}_\mathbb{C} \cap \mathfrak{t}_{2_\mathbb{C}} = \mathfrak{h}_\mathbb{C} \cap \mathfrak{k}_{2_\mathbb{C}} = \mathbb{C}(\check{\beta} - 2\check{\alpha}) = \mathbb{C}\Lambda_2.$$

$q_u : \mathfrak{k}_\mathbb{C}^* \rightarrow \mathfrak{u}_\mathbb{C}^*$ denotes restriction map.

Next, we recall the sub-root system $\Phi_3 := \{\gamma \in \Phi(\mathfrak{k}, \mathfrak{t}) : q_u(\gamma) = 0\}$. Because of (2.1) and (2.3)

$$\Phi_3 = \{\gamma : (\gamma, \alpha) = (\gamma, \beta) = 0\} = \{\gamma \in \Phi(\mathfrak{k}_2, \mathfrak{t}_2) : q_{u \cap \mathfrak{t}_2}(\gamma) = 0\}.$$

The Weyl group for the system Φ_3 is denoted by W_3 . Because of (1.2) the Weyl group W for the pair $(\mathfrak{k}, \mathfrak{t})$ is equal to the product $\langle S_\beta \rangle \times W(\mathfrak{k}_2, \mathfrak{t})$. Thus,

$$W_3 \backslash W = \langle S_\beta \rangle \times W_3 \backslash W(\mathfrak{k}_2, \mathfrak{t}).$$

Let

$$\begin{aligned} \Delta(\mathfrak{k}/\mathfrak{l}) &:= q_u[\Psi \cap \Phi(\mathfrak{k}, \mathfrak{t}) \setminus \Phi_3] \setminus \Phi(\mathfrak{l}, \mathfrak{u}) = q_u[\{\beta\} \cup \Psi(\mathfrak{k}_2, \mathfrak{t}) \setminus \Phi_3] \setminus \{\beta\} \\ &= q_{u \cap \mathfrak{t}_2}(\Psi \cap \Phi(\mathfrak{k}_2, \mathfrak{t} \cap \mathfrak{k}_2) \setminus \Phi_3) =: \Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2). \end{aligned}$$

We set $\rho_3 = \frac{1}{2} \sum_{\gamma \in \Psi \cap \Phi_3} \gamma$ and for $\sigma \in i\mathfrak{t}^*$, the Weyl polynomial is defined to

be $\varpi(\sigma) := \frac{\prod_{\gamma \in \Psi \cap \Phi_3} (\sigma, \gamma)}{\prod_{\gamma \in \Psi \cap \Phi_3} (\rho_3, \gamma)}$. As before, we write $\lambda = \lambda_1 + \lambda_2$ with $\lambda_1 \in \mathbb{R}i\check{\beta}$

and $\lambda_2 \in \mathfrak{t}_2$. Then, owing to (1.2) for $\gamma \in \Phi(\mathfrak{k}_2, \mathfrak{t}) \cap \Psi$ we have the equality $\lambda(\check{\gamma}) = \lambda_2(\check{\gamma})$. Thus, λ_2 is a Harish-Chandra parameter for K_2 or perhaps for a two-fold cover of K_2 . Actually, it readily follows that λ_2 is a Harish-Chandra parameter for K_2 if and only if $\frac{\beta}{2}$ lifts to a character of T . Therefore, if necessary replacing G by a two-fold cover, we have λ_2 is a Harish-Chandra parameter for K_2 .

We now state according to [He] the branching law for the restriction of the irreducible representation $\pi_{\lambda_2}^{K_2}$ of infinitesimal character λ_2 to the one dimensional torus $H \cap K_2 = U \cap K_2$. The restriction of $\pi_{\lambda_2}^{K_2}$ to $H \cap K_2$ is the sum of one-dimensional representations $\sigma_1, \dots, \sigma_r$ with multiplicity $M(\lambda_2, \sigma_j)$ for $j = 1, \dots, r$. The formula of Heckman for this particular case reads

$$\begin{aligned} \sum_{\mu \in \Delta_{U \cap K_2}} M(\lambda_2, \mu) \delta_\mu \\ \left(\pi_{\lambda_2}^{K_2} \right) \\ &= \sum_{s \in W_3 \backslash W(\mathfrak{k}_2, \mathfrak{t})} \epsilon(s) \varpi(s\lambda_2) \delta_{q_{u \cap \mathfrak{t}_2}(s\lambda_2)} \star y_{\Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2)}. \end{aligned}$$

Another fact necessary for the proof is a formula in [DV] for the restriction of π_λ^G to the subgroup H . The hypothesis for the truth of the formula is $K_1(\Psi)$ being a subgroup of H which in our case holds because of our choice of Ψ and H . The hypothesis on G and on the system Ψ yields for each $w \in W$ the multiset

$$S_w^H := [\Delta(\mathfrak{k}/\mathfrak{l}) \cup q_{\mathfrak{u}}(w\Psi_n)] \setminus \Phi(\mathfrak{h}, \mathfrak{u}).$$

is strict. This, also follows from an explicit computation of S_w^H , which, we will carry out later on. The formula that encodes the parameters μ_j and the multiplicities $n^{G,H}(\lambda, \mu_j)$ is:

$$\sum_{\mu \in i\mathfrak{u}^*: (\mu, \beta) > 0} n^{G,H}(\lambda, \mu) (\delta_\mu - \delta_{S_\beta \mu}) = \sum_{w \in W_3 \setminus W} \epsilon(w) \varpi(w\lambda) \delta_{q_{\mathfrak{u}}(w\lambda)} \star y_{S_w^H}.$$

To elaborate on (2.4) and on (2.6) we recall a few known results. It is convenient to think of $(\mathfrak{u} \cap \mathfrak{su}_2(\beta))^*$ (resp. \mathfrak{t}_2^*) as the linear functionals on \mathfrak{t} so that vanishes on \mathfrak{t}_2 (resp. on $\mathfrak{u} \cap \mathfrak{su}_2(\beta)$), hence, for $\lambda_2 \in \mathfrak{t}_2$, we have the equality $q_{\mathfrak{u}}(\lambda_2) = q_{\mathfrak{u} \cap \mathfrak{t}_2}(\lambda_2)$. For $w \in W(\mathfrak{k}_2, \mathfrak{t}_2)$ we have the equalities

$$\begin{aligned} q_{\mathfrak{u}}(w\lambda) &= \lambda_1 + q_{\mathfrak{u} \cap \mathfrak{t}_2}(w\lambda_2) \\ q_{\mathfrak{u}}(wS_\beta \lambda) &= S_\beta(\lambda_1) + q_{\mathfrak{u} \cap \mathfrak{t}_2}(w\lambda_2) \\ \varpi(w\lambda) &= \varpi(w\lambda_2) \\ \varpi(wS_\beta \lambda) &= \varpi(w\lambda_2). \end{aligned}$$

From table 2.5 in [GW] it follows that any root in $\Phi(\mathfrak{k}_2, \mathfrak{t})$ is linear combination of compact simple roots in Ψ . Thus, lemma 3.3 in [HS] yields

$$w\Psi_n = \Psi_n \text{ for } w \in W(\mathfrak{k}_2, \mathfrak{t}).$$

In [GW] Proposition 1.3 it is shown that $\Psi_n = \left\{ \gamma \in \Psi : \frac{2(\beta, \gamma)}{(\beta, \beta)} = 1 \right\}$, and that the map $\gamma \mapsto \beta - \gamma$ is an involution in Ψ_n . Thus, the number of elements of Ψ_n is an even number $2d$ and we may write

$$\Psi_n = \{\gamma_2, \dots, \gamma_d, \beta - \gamma_2, \dots, \beta - \gamma_d, \alpha, \beta - \alpha\}.$$

Hence, we have $S_\beta(\Psi_n) = -\Psi_n$. Also in [GW] Proposition 2 it is shown that

$$q_{\mathfrak{u}}(\gamma_j) = \Lambda_1 \text{ for } j = 2, \dots, d.$$

The equality $\Lambda_1 + \Lambda_2 = \beta$ yields $q_{\mathfrak{u}}(\beta - \gamma_j) = \Lambda_2$. From these and (2.8) we conclude for $w \in W(\mathfrak{k}_2, \mathfrak{t})$

$$q_{\mathfrak{u}}(w\Psi_n) = \underbrace{\{\Lambda_1, \dots, \Lambda_1\}}_{d-1}, \underbrace{\{\Lambda_2, \dots, \Lambda_2\}}_{d-1}, \alpha, \beta - \alpha\}.$$

$$q_{\mathfrak{u}}(wS_\beta \Psi_n) = S_\beta(q_{\mathfrak{u}}(w\Psi_n)) = \underbrace{\{\Lambda_1, \dots, \Lambda_1\}}_{d-1}, \underbrace{\{\Lambda_2, \dots, \Lambda_2\}}_{d-1}, \alpha, \alpha - \beta\}.$$

The previous calculations let us conclude.

For $w \in W(\mathfrak{k}_2, \mathfrak{t}_2)$,

$$S_w^H = \underbrace{\{\Lambda_1, \dots, \Lambda_1\}}_{d-1} \underbrace{\{\Lambda_2, \dots, \Lambda_2\}}_{d-1} \cup \Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2).$$

$$S_{S_\beta w}^H = \underbrace{\{-\Lambda_1, \dots, -\Lambda_1\}}_{d-1} \underbrace{\{-\Lambda_2, \dots, -\Lambda_2\}}_{d-1} \cup \Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2).$$

The right hand side of (2.6), after we apply the previous calculations, becomes equal to

$$\begin{aligned} & \sum_{s \in W_{\mathfrak{J}} \setminus W(\mathfrak{k}_2)} \epsilon(s) \varpi(s\lambda_2) \delta_{\lambda_1} \star \delta_{q_{\mathfrak{u}} \cap \mathfrak{k}_2}(s\lambda_2) \star y_{\Lambda_1}^{d-1} \star y_{\Lambda_2}^{d-1} \star y_{\Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2)} \\ & - \sum_{s \in W_{\mathfrak{J}} \setminus W(\mathfrak{k}_2)} \epsilon(s) \varpi(s\lambda_2) \delta_{S_\beta \lambda_1} \star \delta_{q_{\mathfrak{u}} \cap \mathfrak{k}_2}(s\lambda_2) \star y_{S_\beta \Lambda_1}^{d-1} \star y_{S_\beta \Lambda_2}^{d-1} \star y_{\Delta(\mathfrak{k}_2/\mathfrak{u} \cap \mathfrak{k}_2)} \\ & = \sum_{\sigma \in \Delta_{U \cap K_2}(\pi_{\lambda_2}^{K_2})} M(\lambda_2, \sigma) \delta_\sigma \star [\delta_{\lambda_1} \star y_{\Lambda_1}^{d-1} \star y_{\Lambda_2}^{d-1} \star \delta_{S_\beta \lambda_1} \star y_{S_\beta \Lambda_1}^{d-1} \star y_{S_\beta \Lambda_2}^{d-1}] \\ & = \sum_{\sigma, p, q \in \mathbb{Z}_{\geq 0}} M(\lambda_2, \sigma) \binom{p+d-2}{d-2} \binom{q+d-2}{d-2} \delta_{\lambda_1 + \sigma} \star \delta_{p\Lambda_1} \star \delta_{q\Lambda_2} \\ & + \sum_{\sigma, p, q \in \mathbb{Z}_{\geq 0}} M(\lambda_2, \sigma) \binom{p+d-2}{d-2} \binom{q+d-2}{d-2} \delta_{S_\beta(\lambda_1 + \sigma)} \star \delta_{pS_\beta \Lambda_1} \star \delta_{qS_\beta \Lambda_2}. \end{aligned}$$

(2.9) We now show: For every $p, q \in \mathbb{Z}_{\geq 0}$, and for every $U \cap K_2$ -weight σ of $\pi_{\lambda_2}^{K_2}$ the weight $\lambda_1 + \sigma + p\Lambda_1 + q\mathfrak{u}(\lambda_2)$ is dominant with respect to the system $\Psi_q = \{\alpha, \beta - \alpha, \beta\}$.

In fact, because of a Theorem of Kostant, every T_2 -weight of $\pi_{\lambda_2}^{K_2}$ lies in the convex hull of $\{s(\lambda_2), s \in W(\mathfrak{k}_2, \mathfrak{t})\}$. Thus, there exists non negative real numbers c_t so that $\sigma = \sum_{t \in W(\mathfrak{k}_2, \mathfrak{t})} c_t q_{\mathfrak{u}}(t\lambda_2)$ and $\sum_t c_t = 1$. The hypothesis λ is regular and dominant with respect to Ψ yields, $\lambda_1(\check{\beta}) = \lambda(\check{\beta}) > 0$.

We write $(\lambda_1 + \sigma + p\Lambda_1 + q\lambda_2, \alpha) = \lambda(\check{\beta})\alpha(\check{\beta}) + \sum_t (q_{\mathfrak{u}}(t\lambda_2), \alpha) + p(\Lambda_1, \alpha)$ and $q_{\mathfrak{u}}(t\lambda_2) = (t\lambda_2, \beta - 2\alpha)(\beta - 2\alpha)$. Now, since $\alpha \in i\mathfrak{u}^*$, we have,

$$(q_{\mathfrak{u}}(t\lambda_2), \alpha) = (t\lambda_2, \alpha) = (\lambda_2, t^{-1}\alpha) = (\lambda, t^{-1}\alpha) > 0$$

because t is a product of reflections about compact simple roots for Ψ , $\alpha \in \Psi_n$ and (2.7).

For

$$\begin{aligned}
& (\lambda_1 + \sigma + p\Lambda_1 + q\lambda_2, \beta - \alpha) \\
&= \sum_t c_t(t\lambda_1, \beta - \alpha) + \sum_t c_t(t\lambda_2, \beta - \alpha) + q(\lambda_2, \beta - \alpha) \\
&= \sum_t c_t(q_u(t\lambda_1 + t\lambda_2), \beta - \alpha) + q(\lambda_2, \beta - \alpha) \\
&= \sum_t c_t(t\lambda, \beta - \alpha) > 0
\end{aligned}$$

because of $\beta - \alpha \in \Psi_n$, $t \in W(\mathfrak{k}_2, \mathfrak{t})$ and λ is regular dominant for Ψ . We have concluded the proof of Theorem 1, because we have shown that the left hand side of (2.6) is expressed as claims the statement of Theorem 1. This finishes the proof of Theorem 1.

Note Wallach in [W2] considered the case the lowest K -type for π_λ is equal to a representation of $\mathfrak{su}_2(\beta)$ times the trivial representation of K_2 .

3. ADMISSIBLE RESTRICTIONS TO "SU(2, 1)" OF DISCRETE SERIES FOR QUATERNIONIC REAL FORMS

To begin with we list the the Lie algebra of the Lie groups where Theorem 1 applies. Up to equivalence, the list of the Lie algebras for quaternionic real forms is: $\mathfrak{su}(2, n)$, $\mathfrak{so}(4, n)$, $EII = \mathfrak{e}_{6(2)}$, $EVI = \mathfrak{e}_{7(-5)}$, $EIX = \mathfrak{e}_{8(-24)}$, $FI = \mathfrak{f}_{4(4)}$ and $G = \mathfrak{g}_{2(2)}$.

For the corresponding groups, we show that a square integrable irreducible representation for G has an admissible restriction to $H = "SU(2, 1)"$ if and only if the Harish-Chandra parameter is dominant with respect to the small system Ψ .

Proposition 1. *Let G be a quaternionic real form, a small system of positive roots Ψ , $\mathfrak{su}_2(\beta)$, \mathfrak{k}_2 , H as in the previous section. Let Σ be a system of positive roots in $\Phi(\mathfrak{g}, \mathfrak{t})$ so that $\Delta \subset \Sigma$. Then, a square integrable irreducible representation with Harish-Chandra parameter dominant with respect to Σ has an admissible restriction to H if and only if $\Sigma = \Psi$.*

Proof: From the list of Vogan's diagram, we notice there exists a subgroup of H_1 of G so that (G, H_1) is a symmetric pair and $H \subset H_1$ and $T \subset H_1$. Hence, if $\Sigma \supset \Delta$ is a system of positive roots for $\Phi(\mathfrak{g}, \mathfrak{t})$ so that some irreducible square integrable representation π_μ^G with μ dominant with respect to σ has admissible restriction to H , then, [Kb3] Theorem 2.8 implies π_μ^G has admissible restriction H_1 . Owing to [DV] Prop. 2, we have $\mathfrak{k}_1(\Sigma)$ is a subalgebra of \mathfrak{h}_1 , except for some G locally isomorphic to $SO(4, 2n)$ the Lie algebra \mathfrak{k} is the sum of two simple ideals, hence $\mathfrak{k}_1(\Sigma)$ is equal to $\mathfrak{su}_2(\beta)$. A case by case computation forces $\Sigma = \Psi$. For a group G locally isomorphic to $SO(4, 2n)$, $n \geq 2$ we select two different choices of H_1 which forces once again $\mathfrak{k}_1(\Sigma)$ to be equal to a copy of $\mathfrak{su}_2(\beta)$ and $\Sigma = \Psi$. \square

4. OTHER GROUPS

A group G locally isomorphic to either $SO(3, n)$ or $Sp(1, n)$ share with the quaternionic real forms that a copy \mathfrak{su}_2 is an ideal in any maximal compactly embedded subalgebra of \mathfrak{g} . Next, we analyze admissible restriction of square integrable representations to the subgroup corresponding to the copy of \mathfrak{su}_2 mentioned in the previous sentence.

We recall that from a criterium of Harish-Chandra it follows that a group locally isomorphic to $SO(3, 2n + 1)$ has no irreducible square integrable representation, whereas, a group locally isomorphic to $SO(3, 2n)$ does have a non empty discrete series. For a group locally G isomorphic to $SO(3, p)$ a maximally compactly imbedded subalgebra is isomorphic to the direct sum of the ideals $\mathfrak{so}(3), \mathfrak{so}(p)$. For the next statement we denote the analytic subgroup of G corresponding to $\mathfrak{so}(3)$ by K_1 .

Proposition 2. *For a group G locally isomorphic to $SO(3, 2n)$ no irreducible square integrable representation has an admissible restriction to K_1 .*

Proof: Because, $n \geq 1$ we have that K_1 is contained in a subgroup H_1 of G locally isomorphic to $SO(3, 1)$. Next, we recall Theorem 1.2 in [Kb1] which gives us: if a unitary representation of G has an admissible restriction to K_1 then it has an admissible restriction to H_1 . Hence, if irreducible square integrable representation of G had admissible restriction to K_1 we would have that H_1 has a nonempty discrete series, which is not true since H_1 is locally isomorphic to $SO(3, 1)$. Another proof follows from [DV] and the fact that $K_1(\Psi)$ never is equal to K_1 . \square

For a group G locally isomorphic to $Sp(1, q)$ we fix as maximal compact subgroup K and a compact Cartan subgroup T . Therefore, there exists an orthogonal basis $\{\epsilon_1, \delta_1, \dots, \delta_q\}$ for it^* and a system of positive roots Σ so that

$$\Sigma \cap \Phi_c = \{2\epsilon_1, \delta_i \pm \delta_j, 1 \leq i < j \leq q, 2\delta_j, j = 1, \dots, q\}$$

and $\Sigma \cap \Phi_n = \{\epsilon_1 \pm \delta_j, j = 1, \dots, q\}$. The simple roots are $\epsilon_1 - \delta_1, \delta_j - \delta_{j+1}, j = 1, \dots, q, 2\delta_q$. The maximal root is $\beta = 2\epsilon_1$. It readily follows that $\mathfrak{k}_1(\Sigma) = \mathfrak{su}_2(2\epsilon_1)$. Let \mathfrak{h}_0 denote the real form of the Lie subalgebra spanned by the root vectors corresponding to the roots

$$\Phi(\mathfrak{h}_0, \mathfrak{u}) := \{\pm 2\epsilon_1, \pm 2\delta_1, \pm (\epsilon_1 \pm \delta_1)\}.$$

Then, \mathfrak{h}_0 is isomorphic to $\mathfrak{sp}(1, 1)$. As for the quaternionic case, let H_0 denote the analytic subgroup of G associated to \mathfrak{h}_0 . Owing to [DV] Theorem 1, we have that for λ dominant with respect to Σ the representation π_λ^G restricted to H_0 is admissible when the Harish-Chandra. Let $\mu_j, n^{G, H_0}(\lambda, \mu_j)$ be as in (1.1). Let $\Sigma_q := \Sigma \cap \Phi(\mathfrak{h}_0, \mathfrak{u})$. Let $HC_{L \cap K_2}(\pi_{\lambda_2}^{K_2})$ denotes the set of Harish-Chandra parameters for the $L \cap K_2$ -irreducible factors of the restriction of $\pi_{\lambda_2}^{K_2}$ to the subgroup $L \cap K_2$. We have,

Proposition 3. *Assume λ is dominant with respect to Σ . Then, for $j = 1, \dots$ the parameters $\mu_j := \lambda_1 + \sigma + j\epsilon_1$ are dominant with respect to Σ_q . Besides,*

$n^{G,H_0}(\lambda, \mu) \neq 0$ if and only if $\mu = \mu_j$ for some j . Moreover

$$n^{G,H_0}(\lambda, \mu) = \sum_{\substack{\sigma \in HC_{L \cap K_2}(\pi_{\lambda_2}^{K_2}), \\ p \in \mathbb{Z}_{\geq 0} \\ \mu = \lambda_1 + \sigma + p\epsilon_1}} M(\lambda_2, \sigma) \binom{p + 2q - 3}{2q - 3}.$$

Proof: We begin writing the equalities (2.4) and (2.6) for the setting of the Proposition. For this particular case (2.4) reads

$$\begin{aligned} \sum_{\mu \in HC_{L \cap K_2}(\pi_{\lambda_2}^{K_2})} M(\lambda_2, \mu) &= \sum_{r \in W(L \cap K_2, U \cap K_2)} \epsilon(r) \delta_{r\mu} \\ &= \sum_{s \in W_3 \setminus W(\mathfrak{k}_2, \mathfrak{t})} \epsilon(s) \varpi(s\lambda_2) \delta_{q_{\mathfrak{u} \cap \mathfrak{k}_2}(s\lambda_2)} \star y_{\Delta(\mathfrak{k}_2 / \mathfrak{l} \cap \mathfrak{k}_2)}. \end{aligned}$$

The multiset

$$S_w^{H_0} := [\Delta(\mathfrak{k} / \mathfrak{l}) \cup q_{\mathfrak{u}}(w\Psi_n)] \setminus \Phi(\mathfrak{h}, \mathfrak{u})$$

is strict. This, follows from an explicit computation of $S_w^{H_0}$, which, we will carry out after the next formula. The formula (2.6) becomes:

$$\begin{aligned} \sum_{\substack{\mu \in i\mathfrak{u}^* \\ (\mu, \epsilon_1) > 0, (\mu, \delta_1) > 0}} n^{G,H}(\lambda, \mu) &\left(\sum_{t \in W(L, U)} \epsilon(t) \delta_{t\mu} \right) \\ &= \sum_{w \in W_3 \setminus W} \epsilon_1(w) \varpi(w\lambda) \delta_{q_{\mathfrak{u}}(w\lambda)} \star y_{S_w^H}. \end{aligned}$$

In this case $\mathfrak{u} \cap \mathfrak{k}_1 = \mathbb{R}i\epsilon_1$, $\mathfrak{u} \cap \mathfrak{k}_2 = \mathbb{R}i\delta_1$, $\mathfrak{l} \cap \mathfrak{k}_2 = \mathfrak{su}_2(2\delta_1)$. Furthermore

$$\Psi \cap \Phi_3 = \{\delta_i \pm \delta_j, 2 \leq i < j \leq q\},$$

$W = \langle S_{2\epsilon_1} \rangle \times W(\mathfrak{k}_2, \mathfrak{t}_2)$ and $W_3 \setminus W = \langle S_{2\epsilon_1} \rangle \times W_3 \setminus W(\mathfrak{k}_2, \mathfrak{t}_2)$.

For $w \in W(\mathfrak{k}_2, \mathfrak{t}_2)$, $w\Psi_n = \Psi_n$, $wS_{2\epsilon_1}\Psi_n = -\Psi_n$. and

$$\begin{aligned} q_{\mathfrak{u}}(w\Psi_n) \setminus \Phi(\mathfrak{h}, \mathfrak{u}) &= \underbrace{\{\epsilon_1, \dots, \epsilon_1\}}_{2(q-1)}. \\ q_{\mathfrak{u}}(wS_{2\epsilon_1}\Psi_n) \setminus \Phi(\mathfrak{h}, \mathfrak{u}) &= \underbrace{\{-\epsilon_1, \dots, -\epsilon_1\}}_{2(q-1)}. \end{aligned}$$

$$\Delta(\mathfrak{k}, \mathfrak{l}) = q_{\mathfrak{u}}(\Psi_c \setminus \Phi_3) \setminus \Phi(\mathfrak{h}, \mathfrak{u}) = \underbrace{\{2\epsilon_1, 2\delta_1, \delta_1, \dots, \delta_1\}}_{2(q-1)} \setminus \Phi(\mathfrak{h}, \mathfrak{u}) = \Delta(\mathfrak{k}_2, \mathfrak{l} \cap \mathfrak{k}_2).$$

Therefore, for $w \in W(\mathfrak{k}_2, \mathfrak{t})$ we have,

$$S_w^{H_0} = \underbrace{\{\epsilon_1, \dots, \epsilon_1\}}_{2(q-1)} \cup \Delta(\mathfrak{k}_2, \mathfrak{l} \cap \mathfrak{k}_2).$$

$$S_{S_2\epsilon_1 w}^{H_0} = \underbrace{\{\epsilon_1, \dots, \epsilon_1\}}_{2(q-1)} \cup \Delta(\mathfrak{k}_2, \mathfrak{l} \cap \mathfrak{k}_2).$$

After replacing $S_w^{H_0}$ by the result obtained in the previous line, the right hand side of the formula similar to the one in (2.6) becomes

$$\begin{aligned} & \sum_{\substack{t \in \{1, S_2\epsilon_1\} \\ s \in W_{\mathfrak{g}} \setminus W(\mathfrak{k}_2, \mathfrak{l}_2)}} \epsilon(t)\epsilon(s)\varpi(s\lambda_2)\delta_{t\lambda_1} \star \delta_{q_{\mathfrak{u}}(s\lambda_2)} \star y_{\Delta(\mathfrak{k}_2/\mathfrak{l} \cap \mathfrak{k}_2)} \star y_{t\epsilon_1}^{2(q-1)} \\ &= \sum_{t \in \{1, S_2\epsilon_1\}} \epsilon(t) \sum_{\substack{r \in W(\mathfrak{k}_2, \mathfrak{l}_2) \\ \sigma \in HC}} \epsilon(r)M(\lambda_2, \sigma)\delta_{t\lambda_1+r\sigma} \star y_{t\epsilon_1}^{2(q-1)} \\ &= \sum_{\substack{t, r, \sigma \\ p \in \mathbb{Z}_{\geq 0}}} \epsilon(t)\epsilon(s)M(\lambda_2, \sigma) \binom{p+2(q-1)-1}{2(q-1)-1} \delta_{tr(\lambda_1+\sigma+p\epsilon_1)} \\ &= \sum_{p \geq 0} \sum_{\sigma} M(\lambda_2, \sigma) \binom{p+2q-3}{2q-3} \sum_{w \in W(L, U)} \epsilon(w)\delta_{w(\lambda_1+\sigma+p\epsilon_1)}. \end{aligned}$$

By a reasoning similar to (2.9) we obtain that $\lambda_1 + \sigma + p\epsilon_1$ is dominant with respect to Ψ_q and we have concluded the proof of Proposition 3. \square

5. SIMONDI'S THESIS

The Ph. D. thesis of Sebastian Simondi was defended by March 2007. Most of his results were verified in a case by case checking. By now, some of his results are a consequence of work of Toshi Kobayashi [Kb2], [Kb3] and [DV]. This section does not include proofs of the stated results, we will point out those results that follows from the work of T. Kobayashi, Kobayashi-Oshima, DV and those results we believe are still not in the literature. A complete version of Simondi's thesis is on:

www2.famaf.unc.edu.ar/publicaciones/documents/seried/DMat49.pdf.

(5.1) We now describe the results. For this, we fix a noncompact connected matrix simple Lie group G a maximal compact subgroup K for G . Henceforth, H is a closed reductive subgroup of G so that $L := H \cap K$ is a maximal compact subgroup for H and that (G, H) is a symmetric pair. Hence, (K, L) is Riemannian symmetric pair.

As in the previous sections we assume G admits irreducible square integrable representations, we would like to point out that in the course of the computation was made an extensive use of the description for the set of

equivalence classes of square integrable irreducible representations given by Harish-Chandra in terms of Harish-Chandra parameters.

In [DV], [Kb1], [Kb2] and [Kb3] we find criteria for checking whether or not the restriction of an square integrable representation for G is an admissible representation for a subgroup H . By mean of these criteria, the classification of the symmetric pairs given by Berger and a case by case checking, we have,

Theorem 2. *Assume (G, H) is a symmetric pair and (π, V) is an irreducible square integrable representation for G . If \mathfrak{k} is a simple Lie algebra, then the restriction of π to H is not an admissible representation.*

Nowadays, this result follows from [DV] or from the work of [KO].

For the next result we fix a maximal compact connected subgroup L' for K so that the rank of K is equal to the rank of L' .

Theorem 3. *Let (π, V) be an irreducible square integrable representation for G . We assume \mathfrak{k} is a simple Lie algebra. Then, π restricted to L' is not an admissible representation.*

When L' is a maximal compact subgroup of a reductive subgroup H of G so that (G, H) is symmetric pair Theorem 3 follows from Theorem 2 and results in [DV]. For the other subgroups L' the proof has been done in a case by case checking based on the classification of the equal rank maximal subgroups of K obtained by Borel-de Siebenthal and work of Toshi Kobayashi on criteria on admissibility of restriction of representations.

Under the hypothesis \mathfrak{k} is not a simple Lie algebra, (G, H) a symmetric pair and the subgroups L, K are of the same rank, we obtain a complete list, in the language of Harish-Chandra parameters, of the square integrable representations for G which do not have an admissible restriction to H . Nowadays this results are included in [KO] [Va2].

For the last result of this note we further assume (G, K) is an Hermitian symmetric pair. Then, the center of K is a one dimensional torus. Let K_{ss} denote the semisimple factor of K . We fix a maximal torus T for K . The hypothesis on (G, K) allows us to choose, once for all, a holomorphic system of positive roots Ψ_h in $\Phi(\mathfrak{g}, \mathfrak{t})$. In [Kb1] it is shown that either a holomorphic or a antiholomorphic discrete series for G has an admissible restriction to K_{ss} if and only if G/K is not a tube domain. The next result gives a criteria which allows to determine when an arbitrary irreducible square integrable representation has admissible restriction to K_{ss} . For this we recall set of equivalence classes for irreducible square integrable representations is parameterized by the set of Harish-Chandra parameters λ dominant with respect to $\Psi_h \cap \Phi_c$. The regularity of λ determines a system of positive roots $\Psi_\lambda := \{\alpha \in \Phi(\mathfrak{g}, \mathfrak{t}) : \lambda(\check{\alpha}) > 0\}$ which satisfies $\Psi_\lambda \cap \Phi_c = \Psi_h \cap \Phi_c$. In Table 1 we list for each Hermitian symmetric pair (G, K) subsets I, \tilde{I} of Ψ_h .

Theorem 4. *Assume (G, K) is an Hermitian symmetric pair and fix an irreducible square integrable representation $(\pi_\lambda^G, V_\lambda)$ for G of Harish-Chandra*

parameter λ dominant with respect $\Psi_h \cap \Phi_c$. Then, π_λ^G restricted to K_{ss} is an admissible representation if and only if either I or \tilde{I} is a subset of Ψ_λ .

The proof of the last Theorem is carried out in basis of classification of Hermitian symmetric pairs and criteria due to Kobayashi [Kb2], [Kb3].

\mathfrak{g}	\mathfrak{k}	
$\mathfrak{sp}(n, \mathbb{R})$	$\mathfrak{u}(n)$	If $n = 2l$ $I = \{(e_k + e_{n-k+1})\}_{k=1}^l$, $\tilde{I} = -I$ If $n = 2l + 1$ $I = \{2e_{l+1}\} \cup \{(e_k + e_{n-k+1})\}_{k=1}^l$, $\tilde{I} = -I$
$\mathfrak{so}^*(2n)$	$\mathfrak{u}(n)$	If $n = 2l$ $I = \{(e_k + e_{n-k+1})\}_{k=1}^l$, $\tilde{I} = -I$ If $n = 2l + 1$ $I = \{(e_k + e_{n-k+1})\}_{k=1}^l \cup \{e_{l+1} + e_{l+2}\}$, $\tilde{I} = \{(-e_k - e_{n-k+1})\}_{k=1}^l \cup \{-e_l - e_{l+1}\}$
$\mathfrak{su}(p, q)$	$\mathfrak{su}(p) \oplus \mathfrak{u}(q)$	$I = \{e_i - e_{\gamma_i}\}_{i=1}^p$ $\tilde{I} = \{-e_i + e_{b_i}\}_{i=1}^p$
$\mathfrak{e}_{6(-14)}$	$\mathfrak{so}(10) \oplus \mathfrak{so}(2)$	$I = \{\varepsilon_1, \varepsilon_2, e_1 + e_5, e_2 + e_5\}$ $\tilde{I} = -I$
$\mathfrak{e}_{7(-25)}$	$\mathfrak{e}_6 \oplus \mathfrak{so}(2)$	$I = \{\eta_1, \eta_2, e_1 + e_6\}$ $\tilde{I} = -I$

Table 1

Here, in notation of Bourbaki:

$$\gamma_i = i + \left[\frac{(i-1)(q-p)}{p} \right] + p, \text{ for } 1 \leq i \leq p,$$

$$b_i = \begin{cases} i + 1 + \left[\frac{i(q-p)}{p} \right] + p & \text{if } \frac{i(q-p)}{p} \notin \mathbb{Z}, \\ i + \frac{i(q-p)}{p} + p & \text{if } \frac{i(q-p)}{p} \in \mathbb{Z}, \end{cases} \text{ for } 1 \leq i \leq p,$$

$$\varepsilon_1 = \frac{1}{2}(-e_1 - e_2 - e_3 - e_4 + e_5 - e_6 - e_7 + e_8),$$

$$\varepsilon_2 = \frac{1}{2}(-e_1 - e_2 + e_3 + e_4 + e_5 - e_6 - e_7 + e_8),$$

$$\eta_1 = \frac{1}{2}(-e_1 + e_2 - e_3 - e_4 + e_5 + e_6 - e_7 + e_8)$$

$$\eta_2 = \frac{1}{2}(-e_1 - e_2 + e_3 + e_4 - e_5 + e_6 - e_7 + e_8)$$

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