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Stochastic Unit Commitment & Optimal Allocation of Reserves: A Hybrid Decomposition Approach

Carlos J. López-Salgado, Osvaldo Añó, and Diego M. Ojeda-Esteybar

***Abstract*—The Unit Commitment is still a widely studied problem especially when more renewable energy of stochastic character is being added to power systems. This paper proposes a model for weekly planning of systems involving hydro, wind and thermal energy under a stochastic perspective. The proposed model follows the endogenous reserve determination criterion to achieve a simultaneous optimization of energy and reserves considering wind uncertainty, forced outages of equipment, a DC Flow model of the network and cascaded head sensitive hydro systems, among others. The paper also develops a practical solution methodology based on Outer Approximation and Benders decomposition. Testing has been conducted over four systems, and computational results demonstrate that the proposed model and solution are useful and effective.**

***Index Terms*—reserve allocation, spinning reserves, supplementary reserves, unit commitment, uncertainty, Benders decomposition, outer approximation**

NOMENCLATURE

Indexes/Parameters

t	Index of stages running from 1 to T
i	Index of thermal plants running from 1 to $\mathcal{G}$
j	Index of hydroelectric plant with an associated reservoir running from 1 to $\mathcal{J}$ for short term reservoirs and from 1 to $\mathcal{J}^{\mathrm{S}}$ for seasonal reservoirs
k	Index of wind farms running from 1 to $\mathcal{W}$
n	Index of buses running from 1 to $\mathcal{N}$
m	Index of transmission lines running from 1 to $\mathcal{M}$
c	Index of contingencies running from 0 to $\mathcal{C}$
e	Index of wind realizations running from 1 to $\mathcal{E}$

Sub-sets

$\boldsymbol{\Omega}_n$	Sub-set of generation units (thermal, hydro and wind) connected to bus n

Parameters

TU_i,TD_i	Minimum up and down times of thermal plant i [h]
SU_i,SD_i	Start-up and shut down costs of thermal plant i [\$]
a_i, b_i	Coefficients of linear cost function of thermal plant i
WV_j	Water value of stored energy in seasonal reservoir j valid throughout the short term horizon [\$/MWh] or [\$/hm^3]
c_i^{P}	Cost of spinning reserve of thermal plant i for primary and secondary regulation [\$/MWh]
c_i^{U}, c_i^{D}	Cost of spinning up and spinning down reserve of thermal plant i for tertiary regulation [\$/MWh]
c_i^{S}	Cost of non-spinning reserve of thermal plant i [\$/MWh]
VOLL	Value of lost load [\$/MWh]
L_{nt}	Forecast load at bus n during period t [MW]
RREQ_t	Required fast spinning reserve for primary and secondary regulation during period t [MW]
$\mathrm{F}_m^{\max}$	Power flow limit of transmission line m [MW]
$\mathrm{P}_i^{\min}$, $\mathrm{P}_i^{\max}$	Min and max power of thermal plant i [MW]
$\mathrm{R}_i^{\mathrm{up}}$, $\mathrm{R}_i^{\mathrm{dn}}$	Ramp-up and ramp-down limits of thermal plant i [MW/h]
$\mathrm{Q}_j^{\max}$	Maximum flow rate of hydroelectric plant j [m^3/s]
$\mathrm{V}_j^{\min}$, $\mathrm{V}_j^{\max}$	Min and max volume of reservoir j $\left[\mathrm{hm}^3\right]$
I_{jt}	Inflow to reservoir j during period t [m^3/s]
$\mathrm{W}_{kt}^{\mathrm{offer}}$	Forecast wind power of farm k for period t, for scheduling purposes [MW]
WR_{kt}^{e}	Wind power realization of farm k in scenario e during stage t [MW]
U_i^c	Status of thermal plant i for contingency c ($1 =$ available, $0 =$ unavailable)
U_m^c	Status of transmission line m for contingency c ($1 =$ available, $0 =$ unavailable)
σ^e	Probability of wind power realization e
π^c	Probability of contingency c
$\mathrm{p}^{e,c}$	Probability of stochastic realization $\{e,c\}$ where $\mathrm{p}^{e,c} = \sigma^e \times \pi^c$

Variables

u_{it}	Commitment of thermal plant i for period t
a_{it}	Start-up of thermal plant i at beginning of period t
z_{it}	Shut-down of thermal plant i at beginning of period t
us_{it}	Commitment of thermal plant i for provision of supplementary reserve during period t
p_{it}	Scheduled power of plant i for period t [MW]
h_{jt}	Scheduled power of hydroelectric plant j for period t [MW]
w_{kt}	Scheduled wind power of farm k for period t [MW]
rp_{it}	Scheduled fast (primary & secondary) regulation reserve for thermal plant i for period t [MW]
ru_{it}	Scheduled tertiary regulation up reserve for thermal plant i for period t [MW]
rd_{it}	Scheduled tertiary regulation down reserve for thermal plant i for period t [MW]

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rs_{it}	Scheduled supplementary reserve for thermal plant i for period t [MW]
v_{jt}	Volume of reservoir j at the end of period t $\left[\mathrm{hm}^3\right]$
q_{jt}	Discharge flow of reservoir j during period t $\left[\mathrm{hm}^3/\mathrm{h}\right]$
s_{jt}	Spilled flow of reservoir j during period t $\left[\mathrm{hm}^3/\mathrm{h}\right]$
f_{mt}	Power flow through line m during period t [MW]
$p_{it}^{e,c}$	Output power of plant i for stochastic realization $\{e,c\}$ during period t [MW]
$f_{mt}^{e,c}$	Power flow through line m for stochastic realization $\{e,c\}$ during period t [MW]
$ws_{kt}^{e,c}$	Spilled wind power of farm k for stochastic realization $\{e,c\}$ during period t [MW]
$pd_{nt}^{e,c}$	Energy deficit at bus n for stochastic realization $\{e,c\}$ during period t [MW]

I. Introduction

A. Background and Litarature Review

THIS paper analyzes the Weekly Unit Commitment (WUC) problem of systems involving a mix of thermal, hydro and wind power. This is today a very important scheduling phase, especially when power systems go through the necessity of increasing their reception of renewable resources of uncertain character; and is a relevant task for Independent System Operators (ISO) or agents seeking to maximize their economic benefit. Hydrothermal systems are susceptible to load interruptions due to the forced outage of equipment or adverse wind power realizations; therefore, the system operator must ensure optimal levels of reserves while procuring the minimal economic harm to users. Nevertheless, since most works addressing the problem within the weekly horizon follow deterministic standpoints, the WUC results become ineffective when the system is later exposed to contingencies. Documented works in literature are mainly centered on strategies to deal effectively with the integer variables needed to model the commitment status of plants, the head-sensitive hydro plants sharing a hydrology connection between them, the time linked constraints associated to large scale problems and the consideration of the transmission network to account for transmission losses or to comply with security constraints. The optimization problem is therefore described as a large scale MINLP problem, not easily handled by existing commercial solvers. Accordingly, [1]-[2] define MILP models after linearizing the production function of head dependent reservoirs using integer variables, [3] proposes an elaborated nonlinear approach, while [4] applies the convex hull algorithm to exogenously deal with the head effect of the hydro units. As an example of application to a real case, the work of [5] describes an MIP model applied to the Portuguese system including wind and pumped-hydro units. The model is implemented in the UC and Economic Dispatch (ED) phases and solved using a commercial solver. The Genetic Algorithm (GA) is employed in [6] and an improved Particle Swarm Optimization (PSO) in [7], but it is well known that these strategies are still limited by the problem size [8]. Generalized Benders Decomposition (GBD) is applied by [9] to solve the UC problem considering a full AC power flow model, while [10] improves the convergence profile of the aforementioned work by proposing the use of strong cuts, which results in a more effective solution of the master problem. Multi-objective optimization is proposed in [11] considering non smooth fuel cost functions of thermal plants and nonlinear hydro production functions; the problem is solved applying lexicographic optimization. Among the works adopting a stochastic perspective, [12] deals simultaneously with load, hydrology and wind uncertainty within a daily context and proposes a hybrid scheme driven by the Benders decomposition (BD) algorithm, where the sub-problem is further decomposed and solved using Outer Approximation (OA). In [13], a network constrained UC is presented, adopting the form of a two-stage stochastic programming model; the resulting Mixed Integer Nonlinear Programming (MINLP) problem is tackled with BD, after employing a heuristic technique to relax the inter-temporal constraints imposed by the ramping capabilities of units. A Robust Optimization (RO) model is given in [14] considering load uncertainty, handling the stochastic variable by application of Information Gap Decision Theory (IGDT); while in [15] the short term strategic bidding problem of a wind-hydro producer is addressed employing interval optimization within a daily context.

An additional body of related literature involves the energy and reserve co-optimization problem, most of which is restricted to purely thermal systems within an hourly or at most daily time frame and to a single type of reserve. For instance [16] presents a deterministic model which is solved applying LP. A second group of works employs the endogenous reserve determination criterion, and hence a reserve deployment stage is defined within the formulation. In [17], a LP model is combined with simulation tools, while [18]-[20] describe the problems as stochastic MILP models. A two-stage decision framework is presented in [21]-[22], which extended the above models to include wind uncertainty (additionally to equipment failures) and also ended with stochastic LP or NLP models that are solved directly or by special Lagrangian relaxation.

Finally, two-stage robust optimization models have gain considerable attention due to the fact that these models require moderate information about the underlying uncertainty, especially when it is challenging to construct a stochastic model to capture randomness. For instance [23] considers demand response and wind uncertainty, and applies a cutting-plane method to solve the problem. In [24], the net power injected to nodes is considered as stochastic variable, and a hybrid decomposition approach based on BD and OA is developed; while [25] applies BD in the solution of a weekly model, where water inflow is the only uncertain variable involved. Since convexity of the second stage problem is a requirement of robust models, none of the works above consider nonlinear constraints in their primal formulations.

B. Motivation and Contributions

Regardless of research efforts in these subjects, it has not yet been found a proposal for the weekly horizon that involves simultaneously the following elements: i) deployment of the different types of reserves due to uncertain wind power realizations along with equipment failures, and ii) an operation policy

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for head-sensitive short-term reservoirs in cascaded hydro systems. The reservoirs addressed here are small compared to seasonal reservoirs, i.e., small in terms of the length of the regulation period, but their MW provision is significant to the power system. Hydro energy from this type of plants is very important in Latin America as well as in many other countries worldwide; however, as seen above, the impact of the conformation of reserves over these plants is not perceptible in models with a daily scope. Moreover, although the weekly schedule will not be used for the daily dispatch, an in advance consideration of operating reserves and uncertainties will yield a much more tight management policy for the reservoirs than one that ignores such factors.

Therefore, this paper proposes a stochastic WUC model where deployment of reserves, cost of interruptions and uncertain wind scenarios are quantified so that water in reservoirs is optimally pre-allocated in the light of expected energy and reliability costs.

Consistent with the previous discussion, the main contributions of this paper are listed below:

1) A stochastic WUC model is introduced, including four types of operating reserves (fast spinning for primary & secondary regulation; up, down and supplementary for tertiary regulation), involving head-sensitive hydro plants linked to short term reservoirs, considering wind power uncertainty and modeling failure of system components. To the best of the authors' knowledge, for the weekly horizon, a model assembling the elements above has not been reported in literature.
2) Development and application of a useful hybrid decomposition method based on OA and BD to cope with the complex optimization problem that cannot be solved by directly employing commercial MINLP solvers. Particularly, the solution of MILP model provides feasible commitment decisions, while a NLP model provides energy & reserve allocation among thermal plants and water release decisions for hydro stations. The nonlinear and non-convex NLP model is tackled through BD. When convergence of the BD process is reached the resulting master and sub problems are LP models, a fact that satisfies the convergence's condition of the OA algorithm. According to the authors' research, there has not been a coordinated application of the OA and BD algorithms affording the conformation presented in this paper. Two hybrid approaches employing BD and OA are [12] and [24], but their methods are both driven by BD instead of OA, as in the present contribution.
3) The UC solution preserves the accuracy of the nonlinear nature of hydroelectric power; therefore, the obtained water allocation policy is not left exposed to future infeasibilities due to exogenous linearization of the hydro power functions

The rest of the paper is organized as follows. In Section II the stochastic-WHS model is formulated. In Section III the solution approach is introduced and the problems that form the solution architecture are defined. In Section IV the model is tested in four case studies and the future research is identified. The paper conclusions are summarized in Section V.

II. Problem Formulation

A. Stochastic Framework and Modeling Assumptions

The proposed formulation follows a two-stage stochastic decision framework as portrayed in Fig. 1, recognizing that the two stages refer to the same temporal instance.

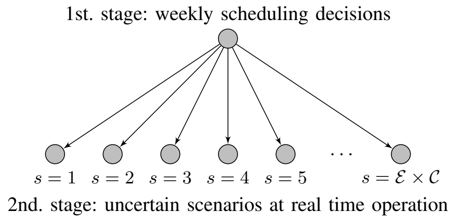

Fig. 1: The stochastic framework of the proposed formulation

The first stage is identified as the weekly scheduling decisions, which are adjusted to wind power realizations and equipment failures at the second stage, where deployment of operating reserves are represented. In this sense, prior to uncertainty realizations in the second stage, the system is optimally prepositioned by means of the scheduling decisions in the first stage [13].

Three assumptions are needed before presenting the proposed model:

1) The start-up and shut-down costs of units providing supplementary reserves are assumed to be embedded in the reserve provision cost
2) Suppliers of non spinning reserves are able to handle by themselves the technical aspects of the minimum on and off times and ramping limits of their plants, so that these constraints are not binding at the real operation phase
3) The stochastic processes that model wind uncertainty and forced outages of equipment are exogenous to the optimization problem, and statistically independent among them.

B. Problem Formulation

Objective Function. The objective function of the proposed model is defined as follows:

$$\text{Min} \quad \sum_{t=1}^{T} \left(c_t^{\text{UC}} + c_t^{\text{OPER}} + c_t^{\text{BAL}} \right) + c_T^{\text{F}} \tag{1}$$

where:

$$c_t^{\text{UC}} = \sum_i \left(a_i u_{it} + \text{SU}_i a_{it} + \text{SD}_i z_{it} \right) \tag{2}$$

$$c_t^{\text{OP}} = \sum_i \left(b_i p_{it} + c_i^{\text{P}} rp_{it} + c_i^{\text{U}} ru_{it} + c_i^{\text{D}} rd_{it} + c_i^{\text{S}} rs_{it} \right) \tag{3}$$

$$c_t^{\text{BAL}} = \sum_{\{e,c\}}^{\mathcal{E}\times\mathcal{C}} \mathrm{p}^{e,c} \Big(\sum_i \big[b_i \left(p_{it}^{ec} - p_{it} \right) \big] + \sum_n \text{VOLL} pd_{nt}^{ec} \Big) \tag{4}$$

The objective function penalizes the costs pertaining to a scheduling phase (the UC costs c_t^{UC}; the operating and reserve provision costs, c_t^{OP}; and the future cost of stored water in seasonal reservoirs, c_T^{F}), and the expected correction costs incurred in real time operation, c_t^{BAL}. The future cost c_T^{F} of seasonal reservoirs is a known function obtained from a long term model, and is associated to the expected water storage level at the end of the week (period T).

I) Constraints at scheduling stage.

$$\left\{ \begin{array}{ll} u_{it} - u_{i(t-1)} = a_{it} - z_{it} & (5) \\ a_{it} + z_{it} \leq 1 & (6) \\ u_{it} + us_{it} \leq 1 & (7) \\ u_{it}, a_{it}, z_{it}, us_{it} \in \{0,1\} \end{array} \right\} \quad \forall\, i, \forall\, t \quad (8)$$

$$\sum_{l=0}^{\text{TU}_i-1} u_{i(t+l)} \geq \text{TU}_i\big(u_{it} - u_{i(t-1)}\big) \quad , \forall\, i \quad (9)$$

$$\sum_{l=0}^{\text{TD}_i-1} \big(1 - u_{i(t+l)}\big) \geq \text{TD}_i\big(u_{i(t-1)} - u_{it}\big) \quad , \forall\, i \quad (10)$$

$$\left\{ \begin{array}{ll} \text{P}_i^{\text{min}} \cdot u_{it} \leq p_{it} \leq \text{P}_i^{\text{max}} \cdot u_{it} & (11) \\ 0 \leq rp_{it} + ru_{it} \leq \big(\text{P}_i^{\text{max}} - \text{P}_i^{\text{min}}\big) u_{it} & (12) \\ p_{it} + rp_{it} + ru_{it} \leq \text{P}_i^{\text{max}} & (13) \\ p_{it} - rd_{it} \geq \text{P}_i^{\text{min}} \cdot u_{it} \end{array} \right\} \quad \forall\, i, \forall\, t \quad (14)$$

$$\sum_i rp_{it} \geq \text{RREQ}_t \quad , \forall t \quad (15)$$

$$\text{P}_i^{\text{min}} \cdot us_{it} \leq rs_{it} \leq \text{P}_i^{\text{max}} \cdot us_{it} \quad , \forall i, \forall t \quad (16)$$

$$\left\{ \begin{array}{ll} p_{it} + ru_{it} - \big(p_{i(t-1)} - rd_{i(t-1)}\big) \leq \text{R}_i^{\text{up}} & (17) \\ p_{i(t-1)} + ru_{i(t-1)} - \big(p_{it} - rd_{it}\big) \leq \text{R}_i^{\text{dn}} & (18) \\ p_{it}, rp_{it}, ru_{it}, rd_{it}, rs_{it} \geq 0 \end{array} \right\} \quad \forall\, i, \forall t \quad (19)$$

$$-\text{F}_m^{\text{max}} \leq \sum_n \psi_{nm} \cdot \big(g_{nt} - \text{L}_{nt}\big) \leq \text{F}_m^{\text{max}} \quad , \forall\, m, \forall t \quad (20)$$

$$g_{nt} = \sum_{i \in \mathbf{\Omega}_n} p_{it} + \sum_{j \in \mathbf{\Omega}_n} h_{it} + \sum_{k \in \mathbf{\Omega}_n} w_{kt} \quad , \forall n, \forall t \quad (21)$$

$$\sum_i p_{it} + \sum_j h_{jt} + \sum_k w_{kt} = \sum_n \text{L}_{nt} \quad , \forall t \quad (22)$$

$$0 \leq w_{kt} \leq \text{W}_{kt}^{\text{offer}} \quad , \forall\, k, \forall t \quad (23)$$

$$\left\{ \begin{array}{ll} v_{jt} = v_{j(t-1)} + \text{CF}\Big(\text{I}_{jt} - \sum_{j'=1}^{\mathcal{J}+\mathcal{J}^s} \mathcal{T}_{jj'} \big[q_{j't} + s_{j't}\big]\Big) & (24) \\ \text{Q}_j^{\text{min}} \leq q_{jt} \leq \text{Q}_j^{\text{max}} & (25) \\ \text{V}_j^{\text{min}} \leq v_{jt} \leq \text{V}_j^{\text{max}} & (26) \\ h_{jt} = \text{c}_{1j} q_{jt} + \text{c}_{2j} q_{jt} v_{jt} + \text{c}_{3j} q_{jt} v_{jt}^2 + \text{c}_{4j} q_{jt}^2 & (27) \\ h_{jt}, s_{jt} \geq 0 \end{array} \right\} \quad \forall\, j \in \mathcal{J}, \forall t \quad (28)$$

$$v_{jT} \geq \text{V}_j^{\text{end}} \quad , \forall\, j \in \mathcal{J} \quad (29)$$

II) Constraints at reserve deployment stage:

$$\left\{ \begin{array}{ll} -\text{F}_m^{\text{max}} \text{U}_m^c \leq \sum_n \psi_{nm}^c \big(g_{nt}^{ec} - \text{L}_{nt}\big) \leq \text{F}_m^{\text{max}} \text{U}_m^c \quad , \forall\, m & (30) \\ g_{nt}^{ec} = \sum_{i \in \mathbf{\Omega}_n} p_{it}^{ec} + \sum_{j \in \mathbf{\Omega}_n} h_{it} + \sum_{k \in \mathbf{\Omega}_n} \big(\text{W}_{kt}^e - ws_{kt}^{ec}\big) \quad , \forall\, n & (31) \\ \sum_i p_{it}^{ec} + \sum_j h_{jt} + \sum_k \big(\text{W}_{kt}^e - ws_{kt}^{ec}\big) = \sum_n \big(\text{L}_{nt} - pd_{nt}^{ec}\big) & (32) \\ p_{it}^{ec} \leq \big(p_{it} + ru_{it} + rs_{it}\big) \text{U}_i^c \quad , \forall i & (33) \\ p_{it}^{ec} \geq \big(p_{it} - rd_{it}\big) \text{U}_i^c \quad , \forall\, i & (34) \\ \text{WR}_{kt}^e - ws_{kt}^{ec} \geq 0 \quad , \forall\, k & (35) \\ ws_{kt}^{ec} \geq 0 \quad , \forall\, k \end{array} \right\} \quad \forall\, e, \forall c, \forall t \quad (36)$$

While constraints (5),(6) and (9)-(10) are common to most UC models, constraint (7), assures that if a thermal plant is committed for generation and spinning reserve ($u_{it} = 1$), it cannot provide non-spinning reserve ($us_{it} = 0$). Constraints (11)-(16) are the limits on active power and the different type of reserves involved, while (17)-(18) are the ramping constraints, which are also related to the provision of reserves [26]. The need for fast spinning reserve is estabished exogenously as a result of a system security analysis, as expressed by (15), where if hydroelectric plants are to provide fast regulation, these plants are known beforehand and therefore their contribution to fast regulation is not a decision variable in the present model. Equations (20)-(22) correspond to a DC power flow representation, where ψ_{nm} is the power transfer distribution factor of bus n to line m, which gives the ratio between the change of flow on the line and the change of power injection at the bus, once a reference bus has been established [27]. The term g_{nt} contains the power injection at bus n, and (22) is the total power balance at period t, which is required to complement the system of equations given by (20).

Expression (24) is the water balance equation, where CF is a scalar used to convert m^3/s to hm^3, and $\mathcal{T}_{jj'}$ denotes the hydrology coupling between reservoirs. $\mathcal{T}_{jj'} = 1$ for all $j = j'$ and $\mathcal{T}_{jj'} = -1$ if output flow from reservoir j' goes directly to reservoir j; otherwise $\mathcal{T}_{jj'} = 0$. The nonlinear production function of hydro plants is given in (27), where $\text{c}_{1j}, \text{c}_{2j}, \text{c}_{3j}, \text{c}_{4j}$ are scalars. This equation expresses the head effects on the efficiency of the plant. Although variants to this expression can be found in literature, the decomposition scheme with which this function is dealt with can be naturally extended to consider alternative forms. Expression (29) establishes the minimum volume that must be available at the end of the week, and this is usually set as a percentage of the initial volume. This constraint is only applicable to short term reservoirs, since the use of water from seasonal reservoirs is determined by their water value for the week under study. Finally, constraints (33)-(36) bind the scheduling decisions to real operation. Equation (33) is of key importance in the co-optimization model, as it expresses the limitation on the total power that can be used at a particular uncertain realization.

III. Proposed Solution

A. Outer Approximation Background

Outer Approximation [29]-[31] is a basic approach for solving MINLP models based on decomposition, outer-approximation and relaxation. The strategy consists of solving an alternating finite sequence of a NLP model (Primal problem) and a relaxed version of a MILP model (Relaxed Master problem) as

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shown in Figure 2. The fundamental idea of OA is to develop linear representations of the nonlinear constraints and apply relaxation.

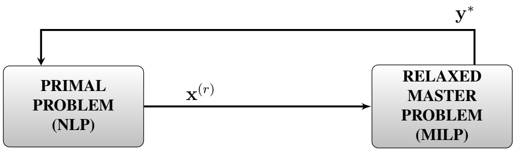

Fig. 2: Basic functioning of the OA algorithm

Consider the problem:

$$\text{Min } \left\{\mathbf{c}^{\mathrm{T}}\mathbf{x} + \mathbf{d}^{\mathrm{T}}\mathbf{y}\right\} \tag{37}$$

$$\text{S.t:} \quad g\left(\mathbf{x}\right) \leq 0 \tag{38}$$

$$\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} \leq \mathbf{d} \tag{39}$$

$$\underline{\mathbf{x}} \leq \mathbf{x} \leq \overline{\mathbf{x}}, \qquad \mathbf{x} \in \mathbb{R} \tag{40}$$

$$\underline{\mathbf{y}} \leq \mathbf{y} \leq \overline{\mathbf{y}}, \qquad \mathbf{y} \in \{0,1\} \tag{41}$$

where $g\left(\mathbf{x}\right)$ is assumed to be a convex function. The algorithm consists in solving (37)-(41) removing constraint (38) and replacing it by its linearized function in the current solution:

$$g\left(\mathbf{x}^{(r)}\right) + \nabla g\left(\mathbf{x}^{(r)}\right)^{\mathrm{T}}\left[\ \mathbf{x} - \mathbf{x}^{(r)}\ \right] \leq 0 \tag{42}$$

where $\mathbf{x}^{(r)}$ is the optimal solution of the Primal problem, which is problem (37)-(41) when $\mathbf{y}$ is fixed to a feasible solution, i.e., the previos optimal solution of the Relaxed Master (RM) problem. The convergence proof of the outer approximation, as well as other variants of the algorithm can be found in [30]-[31]. The algorithm either converges to a locally optimal solution or never finds a feasible solution if the problem is infeasible.

B. A Hybrid Outer Approximation (OA)/Benders Decomposition (BD) with Parallelization Approach

The formulation in Section II renders a complex stochastic MINLP problem. This problem is solved in a sequence of iterations driven by the OA algorithm, shifting between the Primal problem, and the RM problem, as explained above. Although this dynamic provides for an effective solution of most deterministic MINLP models, uncertainty modeling still causes the Primal and RM problems to remain characterized as large scale optimization problems. Therefore, the proposed solution further decomposes the Primal problem applying Benders decomposition, as seen in Fig. 3. Finally, a proper selection of constraints defined within Master and the Subproblem of the BD algorithm provides for the parallelization of the Subproblem in T subproblems, allowing the solution of smaller stochastic NLP models.

C. The Primal problem

If the binary variables are set to a feasible solution, the Primal is a stochastic NLP problem with objective function: (1) and constraints: (11)-(36). The Primal is decomposed in a Master and a Subproblem (which at the same time is decomposed in T subproblems), as seen in Fig. 3.

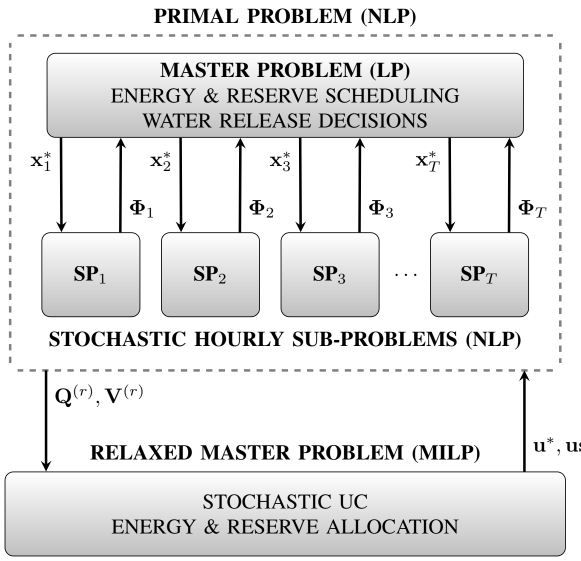

Fig. 3: The decomposition & parallelization scheme to solve the stochastic MINLP problem

1) The BD Master problem within the OA PRIMAL problem: The Master problem for this part is a LP problem, with the objective function:

$$\boldsymbol{\mu}_{\mathrm{M}}^{\mathrm{P}} = \text{Min} \quad \sum_{t}\left(c_t^{\mathrm{UC}*} + c_t^{\mathrm{OPER}} + \boldsymbol{\beta}_t\right) + c_T^{\mathrm{F}} \tag{43}$$

with $c_t^{\mathrm{UC}*}$ already known and c_t^{OPER} as in (3). The term $\boldsymbol{\beta}_t$ is the approximation of the cost of subproblem t that is iteratively improved by means of the Benders cut. The objective function (43) is subject to:

- Constraints (11)-(19)
- Constraints (24)-(26) and (28)
- The Benders cuts for each period:

$$\begin{aligned}\boldsymbol{\beta}_t \geq\ & \boldsymbol{\mu}_{St}^{\mathrm{P}} + \sum_i \left[\lambda_{it}^{\mathrm{P}}\left(p_{it} - p_{it}^*\right) + \lambda_{it}^{\mathrm{RU}}\left(ru_{it} - ru_{it}^*\right)\right.\\ & \left. + \lambda_{it}^{\mathrm{RD}}\left(rd_{it} - rd_{it}^*\right) + \lambda_{it}^{\mathrm{RS}}\left(rs_{it} - rs_{it}^*\right)\right]\\ & + \sum_j \left[\lambda_{jt}^{\mathrm{Q}}\left(q_{jt} - q_{jt}^*\right) + \lambda_{jt}^{\mathrm{V}}\left(v_{jt} - v_{jt}^*\right)\right]\end{aligned} \tag{44}$$

where $\boldsymbol{\mu}_{St}^{\mathrm{P}}$ is the optimal cost of hourly subproblem t; $\lambda_{it}^{\mathrm{P}}, \lambda_{it}^{\mathrm{RU}}, \lambda_{it}^{\mathrm{RD}}, \lambda_{it}^{\mathrm{RS}}, \lambda_{jt}^{\mathrm{Q}}$ and $\lambda_{jt}^{\mathrm{V}}$, are Lagrange multipliers obtained from the solution of subproblems (the elements of vector $\mathbf{\Phi}_t$ in Fig. 3); and $p_{it}^*, ru_{it}^*, rd_{it}^*, rs_{it}^*, q_{jt}^*$ and v_{jt}^* are the optimal values from the previous solution of the Master problem (the elements of $\mathbf{x}_t^*$ in Fig. 3). It can be noticed that this work uses a Benders cut for each hour as done in [10], which is quite effective when combined with the parallelization feature.

2) The BD hourly sub problems within the OA Primal problem: The T hourly sub problems for this part are NLP models, and they have the objective function:

$$\boldsymbol{\mu}_{St}^{\mathrm{P}} = \text{Min} \quad c_t^{\mathrm{BAL}} \tag{45}$$

where c_t^{BAL} is given in (4). The objective function above is subject to:

- Constraints (20)-(23)
- Equation (27), the hydroelectric power function

■ Constraints (30)-(36)
■ Constraints related to the decision variables determined at the Master problem:

$$p_{it} = p^*_{it} \qquad : \lambda^{\mathrm{P}}_{it} \tag{46}$$
$$ru_{it} = ru^*_{it} \qquad : \lambda^{\mathrm{RU}}_{it} \tag{47}$$
$$rd_{it} = rd^*_{it} \qquad : \lambda^{\mathrm{RD}}_{it} \tag{48}$$
$$rs_{it} = rs^*_{it} \qquad : \lambda^{\mathrm{RS}}_{it} \tag{49}$$
$$q_{jt} = q^*_{jt} \qquad : \lambda^{\mathrm{Q}}_{jt} \tag{50}$$
$$v_{jt} = v^*_{jt} \qquad : \lambda^{\mathrm{V}}_{jt} \tag{51}$$

The lower bound $Z^{\mathrm{L}}_{\mathrm{B}}$ and upper bound $Z^{\mathrm{U}}_{\mathrm{B}}$ are as follows:

$$Z^{\mathrm{L}} = \boldsymbol{\mu}^{\mathrm{P}}_{\mathrm{M}} \tag{52}$$
$$Z^{\mathrm{U}} = \sum_t \boldsymbol{\mu}^{\mathrm{P}}_{St} + \left(\boldsymbol{\mu}^{\mathrm{P}}_{\mathrm{M}} - \sum_t \boldsymbol{\beta}_t\right) \tag{53}$$

The BD algorithm within the OA Primal problem is summarized next:

1) Initialization. Set counter $p = 0$.
2) Solve the BD Master problem (first Master problem solution with no Benders cuts). The solution provides a feasible solution $\mathbf{x}^*$ and a lower bound $Z^{\mathrm{L}}_{\mathrm{B}}$
3) Using the solution values in $\mathbf{x}^*$, solve the hourly sub problems. The optimal cost of subproblems allows for the calculation of the upper bound $Z^{\mathrm{U}}_{\mathrm{B}}$
4) Test for convergence: If $Z^{\mathrm{U}} - Z^{\mathrm{L}} \leq \epsilon$, then STOP, as a solution has been reached; else, $p = p + 1$ and GOTO next step.
5) Using the Lagrange multipliers of constraints (46)-(51) and the solution vector of the previous iteration $\mathbf{x}^{*(p-1)}$, solve the Master problem adding new Benders cuts (44) and obtain an improved lower bound Z^{L}
6) Go to step 3.

Solving the Primal problem provides an upper bound UB of the process, and feasible solution values for q_{jt} and v_{jt}. These values are used to linearize the nonlinear constraint (27) which must be removed from the RM problem and replaced by linearized cuts.

D. The Relaxed Master problem

The RM is a MILP model which encompass the objective function (1) and linear constraints (5)-(26) and (28)-(36); that is, the hydroelectric power function is removed from the model, and replaced by the linearized cuts:

$$h_{jt} + \mathrm{kq}^{(r)}_{jt} q_{jt} + \mathrm{kv}^{(r)}_{jt} v_{jt} + \mathrm{k0}^{(r)}_{jt} \leq 0 \tag{54}$$

which are added at each iteration r of the OA algorithm. It can be shown that the coefficients $\mathrm{kq}^{(r)}_{jt}$, $\mathrm{kv}^{(r)}_{jt}$ and $\mathrm{k0}^{(r)}_{jt}$ are given by:

$$\mathrm{kq}^{(r)}_{jt} = -\left(\mathrm{c}_1 + \mathrm{c}_2 v^*_{jt} + \mathrm{c}_3 \left(v^*_{jt}\right)^2 + 2\mathrm{c}_4 q^*_{jt}\right) \tag{55}$$
$$\mathrm{kv}^{(r)}_{jt} = -\left(\mathrm{c}_2 q^*_{jt} + 2\mathrm{c}_3 q^*_{jt} v^*_{jt}\right) \tag{56}$$
$$\mathrm{k0}^*_{jt} = \mathrm{c}_4 \left(q^*_{jt}\right)^2 + \mathrm{c}_2 q^*_{jt} v^*_{jt} + 2\mathrm{c}_3 q^*_{jt} \left(v^*_{jt}\right)^2 \tag{57}$$

where it must be noticed that q^*_{jt} and v^*_{jt} are taken from the previous execution of the BD algorithm, once it has converged, i.e., the previous solution of the Primal problem. Notice that constraint (27) is the only non-linearity considered here. If the system involves thermal plants with quadratic cost functions, these can be adequately handled by piecewise linearization, with a negligible effect. Finally, transmission line losses could also be included in the model, which would imply defining a linearized loss cut as in [28].

To solve the RM problem, this work proposes the use of a large-scale commercial MIP optimizer, taking advantage of the growing development experienced by this kind of solvers in recent years. The solution of the RM provides an improved solution vector of the binary variables u_{it} and us_{it}, which are included as fixed variables in the next execution of the BD algorithm that solves the Primal problem.

E. The hybrid OA/BD algorithm and its convergence

The proposed hybrid OA/BD algorithm is depicted in Figure 4. The convergence of the algorithm represents the solution of overall problem, aiming to solve the UC problem under uncertainty, and its final product is a set of optimal values for the on and off status of thermal plants, the configuration of energy and reserves among these units, an optimal water release policy for the entire week, and a set of linearized cuts (54) delimiting the output power of hydro plants.

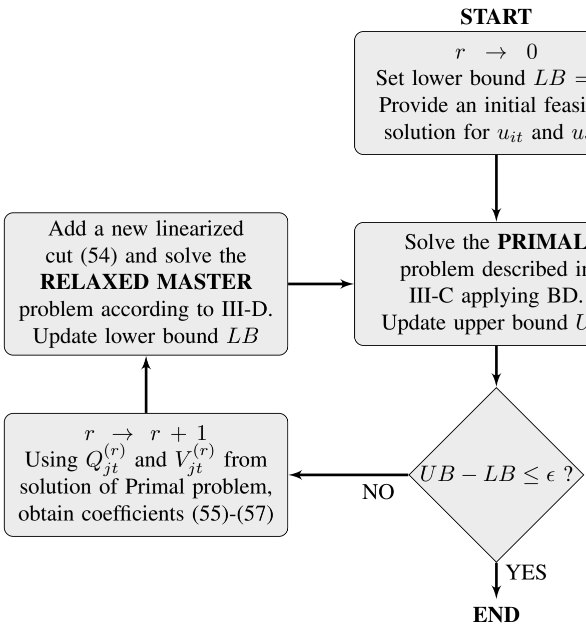

Fig. 4: The proposed Hybrid OA/BD solution algorithm

In principle, the convergence of the algorithm relies on the convexity in $\mathbf{x}$ of (38). This constraint is the compact matrix representation of (27) in the formulation. The hydropower function is clearly nonlinear and non convex. The OA algorithm requires the continuous Primal problem (resulting from fixing the binary variables to any feasible values) to be convex; otherwise, convergence cannot be guaranteed. However, as stated in [22], local convexity in the neighbourhood of the optimal solution is sufficient to guarantee convergence in most practical applications. The proposed strategy deals with this complication in the Primal problem (NLP model) applying BD.

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The release and storage variables q_{jt} and v_{jt} are placed in the Benders master, which is a linear multi period problem, but the hydropower function is left at the Benders sub problem. In this way, after solving the master and fixing q_{jt} and v_{jt} to their optimal solution values q^*_{jt} and v^*_{jt} respectively, the hydropower function can be expressed as:

$$h_{jt} \leq \mathrm{c}_{1j} q^*_{jt} + \mathrm{c}_{2j} q^*_{jt} v^*_{jt} + \mathrm{c}_{3j} q^*_{jt} \left(v^*_{jt}\right)^2 + \mathrm{c}_{4j} \left(q^*_{jt}\right)^2 \quad (58)$$

As it can be seen, it becomes a linear constraint, and hence the Benders sub problems can be addressed employing an LP solver. When the BD converges, the Benders Subproblem is expressed as the sum of T linear subproblems, which are linear and convex in the neighborhood of the optimal solution. Due to the tolerances established for both the main (OA) and the BD algorithm, the algorithm's convergence is to a local optima, but the quality of the solution is evaluated in terms of the system overall cost.

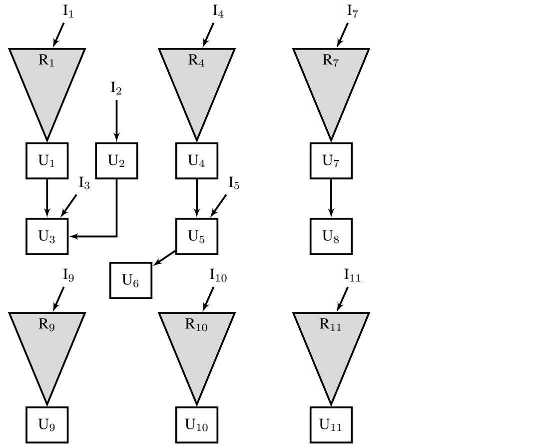

Fig. 5: The cascaded hydro system coupled to the IEEE-14 bus network

IV. Case Studies

The proposed model is illustrated using a modified IEEE-14 bus and further testing is done over three other systems. The optimization problems have been developed using the AMPL algebraic modelling language [33], and the parallelization enhancement relies on the use of the MATLAB's *parfor* function and the AMPL API interface. The cases have been run using XPRESS and CPLEX for the MILP and LP problems, and MINOS for the NLP problems, on a 64-bit windows based server with 64 GB of RAM and 28 Intel Xeon processor at 2.0 GHz. For all cases studies, data is available upon request, as well as the AMPL models and Matlab scripts.

A. Extended IEEE-14 bus

1) System description: The system presented in [6], corresponding to the Chilean Central Interconnected System, has been inserted into the the IEEE-14 bus network. It consists of 10 thermal plants, and 11 hydroelectric units from which 6 have an associated water reservoir. Figure 5 illustrates the hydraulic coupling of the reservoirs and the hydroelectric units involved. Additionally to the thermal and hydro plants, two wind farms are incorporated to the system. To model wind power uncertainty, three wind scenarios were considered and to model equipment failures, a credible set of contingencies was defined. The total number of stochastic realizations is of 3 (wind) $\times$ 7 (failures) = 21 scenarios per hour.

2) Extended IEEE 14-bus results: Table I summarizes the performance of the proposed solution when using a tolerance of 0.5% for both the main and the BD algorithms. It can be seen that the process converged in 2 iterations with an error of 0.31%. According to the time frame considered, results indicate that it is feasible to attain a solution at a reasonable time. To provide an initial solution for the binary variables when solving the RM problem, the solution from the previous iteration was used, and although the problem involved more constraints (an additional set of linear cuts), this artifice improved the solution time in the last iteration, as seen in Table I.

Figure 6 portrays the total allocation of thermal and hydro power and the total system's load, reflecting the expected behaviour in the sense that the cheaper thermal generation is placed at the base of the load curve, and deployment of hydro power is moderate during off-peak periods while intensive during on-peak periods. Observe that for the weekend, hydro generation levels are higher than in working days. This fact reflects a difference with respect to deterministic models (where hydroelectric generation follows the load curve in a more uniform way), which is induced by the incorporation of operating reserves and uncertain realizations. Since hydro generation is more conservative during peak periods, greater accumulation of water may occur in the final stages of the planning period.

Additionally, Figure 7 displays the total allocation of the three types of reserves considered in the example, for each period. The total load has been divided by 4 so that the load profile can be easily compared to that of the reserves. Notice that in contrast to most deterministic models, where reserve needs are established exogenously as a percentage of the total load, the up reserve allocation ($\mathrm{R^u}$ in Fig. 7) does not exhibit this sort of relationship. Indeed, this association is only observed between the load and the primary reserve, which is an exogenous variable in the present model. Another fact worth noting is that for the whole weekend, the up reserve needs are lower than the reserves for primary regulation, which for this example was set at 5% of the total load. This particular result suggests that following deterministic rules may lead to an over estimation of spare capacity for the tertiary regulation interval.

TABLE I: Results for extended IEEE-14 bus system

	Costs [$]		Solution Time [min]		Primal
Main iters	LB	UB	Primal	RM	iters
0	0	48530373	1.11	-	7
1	3165537	3182213	2.16	52.18	16
2	3171440	3180730	2.08	44.17	16

The energy and reserves allocation among thermal plants is presented in Table II. It is seen that thermal plants $\mathrm{GT_1}$ and

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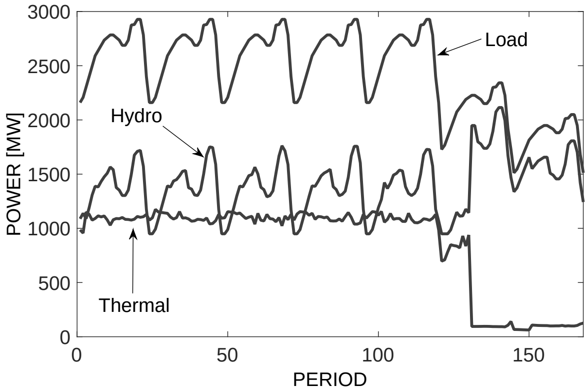

Fig. 6: Energy allocation in extended IEEE14-bus case

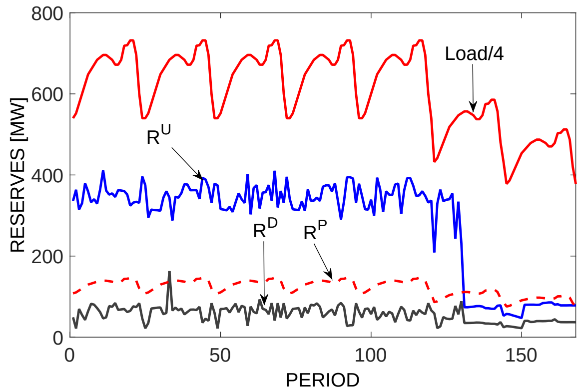

Fig. 7: Reserves allocation in extended IEEE14-bus case

GT_2 are only allocated with energy and primary reserve. This is in part because they both have the lowest production cost and the highest reserve cost. On the other hand, this units are among the plants prone to fail in the COPT table, which will disable them to operate at the post-contingency phase.

TABLE II: Energy & reserve allocation among plants

Plant	Energy [MWh]	RP [MW · h]	RU [MW · h]	RD [MW · h]
GT_1	45194	18	0	0
GT_2	44714	629	0	0
GT_3	27800	392	10771	3446
GT_4	9701	2311	21588	2719
GT_5	7280	7841	2879	2480
GT_6	5260	34	12255	580
GT_7	3450	9287	223	210

In tables III and IV the performance of the proposed hybrid method is assessed in relation to other existing strategies. Table III records the results for a deterministic version of the formulation applied to the 14-bus case, where deployment of reserves and the transmission network are neither modeled. The advantage of the OA driven algorithm is clearly seen, especially when comparing its results with that of commercial MINLP solvers, reflecting a cost difference of less than 0.01% and a significant reduction in computation time.

TABLE III: Asssesment of alternative methods to solve a deterministic-WUC

Method	Solution time	Iters	Total Cost [$]
Bonmin 1.8.0	154 h	-	1,628,587
Knitro 10.0.0	85 h	-	1,628,587
BD	03:54 min	36	1,628,681
OA	00:49 min	4	1,628,649

Table IV shows the results when trying to solve the full stochastic-WUC, proving the difficulty encountered when including the transmission network and modelling reserves deployment along with the inherent nonlinear nature of the system. These results also confirm the effectiveness of the proposed hybrid method.

TABLE IV: Asssesment of alternative methods to solve the proposed stochastic-WUC

Method	Solution time	Iters	Remark
Bonmin 1.8.10	> 72 h	-	No solution after 72 h.
Knitro 10.0.0	> 72 h	-	No solution after 72 h.
OA	-	-	Unable to solve Primal (large scale NLP problem)
BD	> 36 h	-	Unable to reach convergence after 36 h
Hybrid OA/BD	01:42 h	4	

To have a rather general idea on the model sensitivity to wind power realizations, 100 simulations have been run. The experiment based on the random generation of percentage values from a uniform distribution. At each simulation, all values within the initial set of wind scenarios (assumed to accurately reflect the stochastic process) were allowed to vary within the range [0.7 1.3]; that is, between -30 % and +30% of the values used to obtain the results in Table I. To simplify the simulations, the optimal solution values for the commitment variables where kept fix, so the simulations where performed solving the Primal problem only. This simplification implies that all costs obtained are higher than those obtained if the complete algorithm were applied.
A histogram for the total cost distribution has been constructed, and is shown in Figure 8, and the basic statistic parameters of the process are shown on Table V.

TABLE V: Results for 100 simulations of wind power realizations in the 14-bus case

Mean	3,188,171.64 [$]
Standard Error	421.92 [$]
Median	3,188,011.41 [$]
Standard Deviation	4,219.20 [$]
Range	23,438.36 [$]
Minimum	3,176,916.90 [$]
Maximum	3,200,355.26 [$]

Although these numbers should be assessed in the light of the proportion of wind power contribution to the power system,

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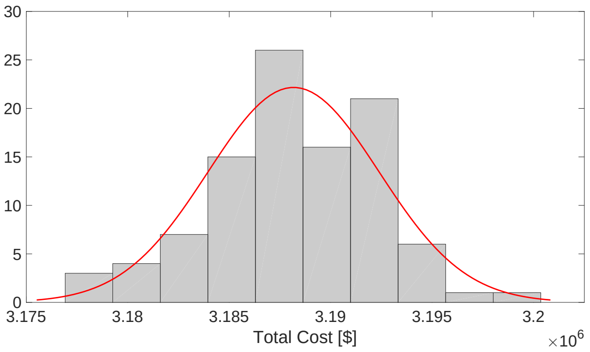

Fig. 8: Cost distribution of simulations in the 14-bus case

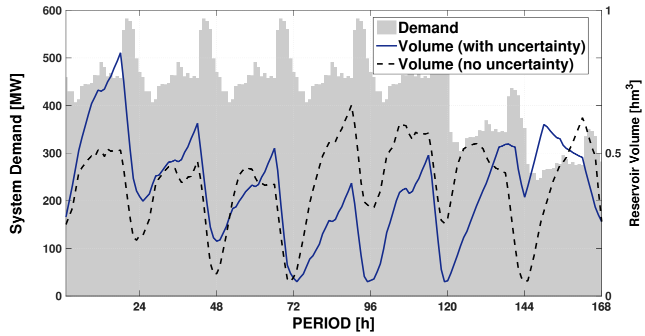

Fig. 9: Total demand and volume trajectories for deterministic and proposed models in 9-bus case

they seem to suggest that even with sub-optimal values for the on and off status of plants, a solid response from the model could be expected. This is expressed by the general symmetry of the distribution, the closeness between the Mean and the Median, and the value of the Range, which is less than 1% of the Mean.

B. Additional tests

Results in Table I clearly indicate that the RM problem is the one demanding the major computational effort. The intention pursued in the following set of tests was to assess the solution times required when solving linear versions of the proposed formulation, that is, simulating the solution of the RM problem. The tests were performed over three systems of different size: a 9-bus system and modified IEEE 57-bus and 118-bus systems. For the three cases above, non-spinning reserve was considered in the problems. Table VI summarizes the characterization of each problem and the simulation results, suggesting that for small, medium and large-scale hydrothermal systems, the MILP model is feasible to be solved.

TABLE VI: Assesment of model solution for the three cases

Problem Parameters	9-bus	IEEE 57-bus	IEEE 118-bus
Thermal plants	3	7	54
Hydro plants	1	8	8
Wind farms	2	0	0
Transmission lines	9	78	179
Contingencies	83	18	44
Wind scenarios	5	0	0
Problem Size			
Binary variables	2,016	4,701	36,288
Continuous Variables	1,722,644	418,299	2,910,264
Constraints	1,510,346	299,491	2,214,373
Solution time [hh:mm]	00:10	00:02	12:55

Although the aim of these simulations was to show that solving RM problem is realizable in real systems, some useful insights are derived from the results. For instance, for the 9-bus system, Figure 9, compares the volumes trajectories from a deterministic model (no reserves for tertiary regulation) with that of the proposed model.

The trajectories suggest that from the first day the storage levels depart from each other. This is clearer in day 1, where about 333000 additional cubic meters of water are stored before facing the peak period (occurring in the sixth sub-interval of the day) and the rest of the week. This difference in storage levels preceding the peak periods is also evidenced in days 2 and 3. It is also seen that with the proposed model, at the peak periods the hydro generation is more intensive, as seen in days 1-3 and 5. In other words, when uncertainty and reserves deployment are involved in the model, more water is expected to be released from the plant during the peak period, and hence the reservoir could reach a lower volume level than that obtained in a deterministic dispatch. Again, the causal reason is that having thermal plants out of service or wind farms not able to provide the forecast wind power are both probable events.

Additionally, Figure 10 displays the allocation of energy for each hour of the week, and Figure 11 depicts the allocation of the different types of reserves. As seen in Table VI, this case included two wind farms, so downward reserves must be scheduled to accommodate excess wind power production at the real operation. It is also observed that provision of supplementary reserve is incremented during the weekend, when the load is lower than in workdays. This is explained by observing the energy allocation of Figure 10, where thermal power is reduced at that period (thermal units providing active power and spinning reserve are shut-down); and because of this an additional unit is committed for provision supplementary reserve, whose startup cost at the real operation phase is not quantified, but is rather assumed to be embedded in the reserve provision cost.

Figure 12 shows the energy & reserve allocation for the 57-bus case, which didn't consider wind power, but included the 8 cascaded reservoirs in [34]. An interesting result of the output relates to the levels of upward spinning reserves. Notice that the provision of spinning reserves basically oscillates between two levels, in some proportion to the provision of active power from thermal units, instead of the load, as any exogenous rule for reserve determination would suggest.

C. Identification of future research

In relation to deployment of hydropower when facing adverse scenarios in the actual operation, it is observed from (32) that

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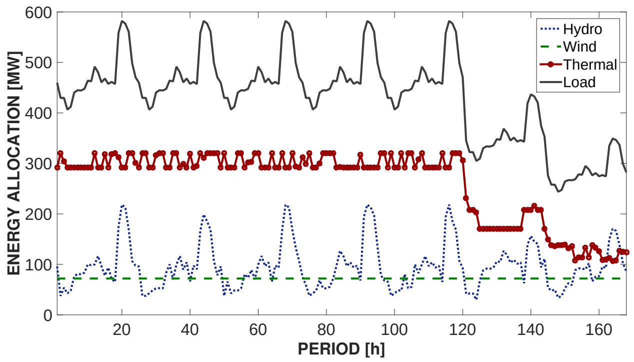

Fig. 10: Energy allocation in 9-bus case

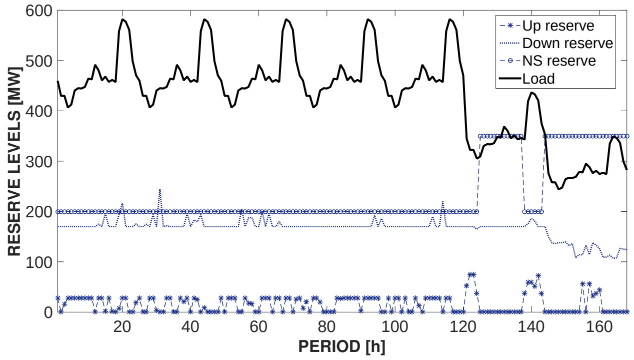

Fig. 11: Allocation of reserves in the 9-bus case

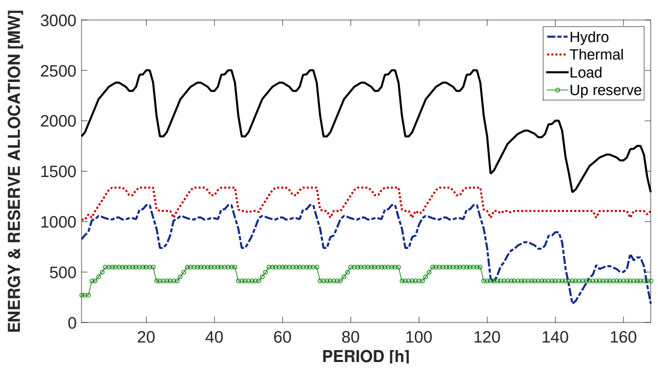

Fig. 12: Energy & reserve allocation in the 57-bus case

water release decisions remain fix during the actual operation, i.e. , for tertiary regulation purposes, no more (or less) hydro power would be used than the amount previously allocated at the weekly scheduling. In this sense, it is also necessary to analyze the problem using the dual approach, and to attain future cost functions that are effective for each sub-period (hour) within the weekly horizon. This would provide a mechanism for adjusting the release levels of hydro stations based on their volume levels.

V. Conclusions

This work addresses the weekly hydrothermal scheduling problem in the presence of wind uncertainty and random failures of plants and transmission lines, laying attention on the procurement of reserves needed at the real operation phase (spinning and supplementary). A new hybrid scheme that relies on both the Outer Approximation and the Benders decomposition algorithm has been proposed. At the expense of computational cost, the proposed strategy allows for an iterative build of linear approximations of the production function of hydro stations, allowing a comparison of the accuracy of this estimation in terms of the total system cost. By means of simulations performed over an extended 14-bus, a 9-bus and two large scale systems, it has been shown that the model is feasible to be implemented in real size problems and that it provides reasonable energy and reserves allocation guidelines. The proposed solution has proven to be effective in solving the optimization problem.

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