Computers and Chemical Engineering 57 (2013) 78–94

Contents lists available at SciVerse ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Toward integrated production and distribution management in multi-echelon supply chains

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ARTICLE INFO

Article history:
Received 17 September 2012
Received in revised form
19 December 2012
Accepted 8 January 2013
Available online 31 January 2013

Keywords:
MILP-based approach
Production and distribution scheduling
Multi-site system
Logistics

ABSTRACT

The effective management of multi-site systems involves the proper coordination of activities performed in multiple factories, distribution centers (DCs), retailers and end-users located in many different cities, countries and/or continents. To optimally manage numerous production and transportation decisions, a novel monolithic continuous-time MILP-based framework is developed to determine the best short-term operational planning to meet all customer requests at minimum total cost. The formulation lies on the unit-specific general precedence concept for the production scheduling problem whereas the immediate precedence notion is used for transportation decisions. To illustrate the applicability and potential benefits of the model, a challenging example corresponding to a supply chain comprising several locations geographically spread in six European countries has been solved to optimality with modest CPU times. Several scenarios with different logistics features were addressed in order to remark the significant advantages of using the integrated approach.

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1. Introduction

In the current context of a global and very competitive economy, multiple production and distribution activities must be properly coordinated in order to satisfy strict market requirements at minimum cost. Many big companies are not only manufacturing the demanded products but also distributing them to the customer location at a predefined due date. This implies a proper consideration of complex temporal and capacity interdependencies arising between production processes and transportation activities. From the operational perspective, both problems have been traditionally treated separately and independently from any supply chain (SC) environment. This decoupled approach works acceptably well if there is sufficient finished goods inventory to buffer the production and distribution operations from each other (Chandra & Fisher, 1994). However, higher inventory costs and the trend to operate in a just in time manner are putting pressure on firms to reduce stocks in their distribution chains. Consequently, the efficient synchronization of production and distribution activities remains as an open and challenging area for research. Only few contributions have been reported so far in this direction, and most of them are mainly focused on the integration at the strategic and tactical level of supply chain networks.

On the one hand, there are been tremendous research efforts over the last decade in the field of short-term production scheduling problem in the chemical industry, and several extended reviewer papers can be found in Floudas and Lin (2004), Méndez, Cerdá, Grossmann, Harjunkoski, and Fahl (2006), Li and Ierapetritou (2008), Verderame, Elia, Li, and Floudas (2010), and Phanden, Jain, and Verma (2011). In particular, some approaches were developed to represent sequence-dependent setup operations appearing in a large number of industrial applications. Cerdá, Henning, and Grossmann (1997) introduced an MILP mathematical formulation for the scheduling of a given set of production orders in a multiproduct batch plant based on the linear structure of the processing sequence at every unit. More recently, Kopanos, Laínez, and Puigjaner (2009) proposed a new unit-specific general precedence formulation, based on a continuous time-domain representation, as a means of tackling scheduling problems with sequence-dependent setup time and/or cost issues. In addition simultaneous batching and scheduling in single-stage facilities was studied by Castro, Erdirik-Dogan, and Grossmann (2008) and Castro and Grossmann (2012) and in multi-stage facilities by Liu and Karimi (2008), Prasad and Maravelias (2008), and Sundaramoorthy and Maravelias (2008a, 2008b).

On the transportation side, a wide variety of vehicle routing problems has been extensively analyzed and solved by the communities of Operations Research and Process Systems Engineering. Widely known as a NP-hard problem (Laporte & Semet, 2002; Prins, 2004), the basic VRP has been studied for decades. Different variants of this problem, usually referred to the pickup and delivery

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http://dx.doi.org/10.1016/j.compchemeng.2013.01.004

Nomenclature

Subscripts

i, i', i''	nodes
b, b'	batches
p, p'	products
u	processing units
v, v'	vehicles

Sets

B	set of batches that can be processed
BP_p	set of batches assigned to product p
D_i	set of vehicles housed at operational base i
F	set of factories
FU_i	processing units that can perform tasks at factory i
I	set of nodes (factories, warehouses, distribution centers, customers)
P	set of products
S	set of customers
U	set of processing units
V	set of vehicles
W	set of warehouses

Parameters

$capV_v$	volume capacity of vehicle v
$capW_v$	weight capacity of vehicle v
$cmax_u$	maximum batch size in processing unit u
$cmin_u$	maximum batch size in processing unit u
$dem_{i,p}$	amount of product p demanded by node i
$dist_{i,i'}$	*km* distance between nodes i and i'
$inv_{i,p}$	initial inventory of product p at node i
$pc_{p,u}$	processing cost of product unit p in unit u
$pt_{p,u}$	processing time of product unit p in unit u
$setupC_{p,p'}$	sequence-dependent setup cost between p and p'
$setupT_{p,p'}$	sequence-dependent setup time between p and p'
sp_v	average travel speed of vehicle v
vfc_v	fixed cost of using vehicle v
vft_v	fixed stop time of vehicle v
$volume_p$	unit volume for product p
vvc_v	unit distance cost for vehicle v
vvt_v	unit load/unload time per unit of product for vehicle v
$weight_p$	unit weight for product p
$M_z, M_t, M_v, M_{tv}, M_{cb}$	upper bounds for big-M constraints

Binary variables

H_v	variable denoting that vehicle v is used
$FI_{i,v}$	variable determining that node i is the last one visited in the route of vehicle v
$IN_{i,v}$	variable determining that node i is the first one visited in the route of vehicle v
$PR_{i,i',v}$	variable determines that node i is visited right before node i' in the route of vehicle v
$VA_{i,v}$	variable denoting that node i is visited by vehicle v
$VVAA_{v,v'}$	variable computing if vehicle v' loads goods delivered by vehicle v
$X_{b,u}$	variable denoting that batch b is allocated to unit u
$Y_{b,b',u}$	variable determining if batch b is processed before b' in unit u
$Z_{b,v}$	variable denoting that batch b is loaded on vehicle v

Continuous variables

$A_{b,v}$	quantity of product of *batch* b allocated to vehicle v
$AI_{p,v}$	amount of product p at stock loaded on vehicle v
$AA_{p,v,v'}$	amount of product p transshipments between vehicle v and v'
$FT_{b,u}$	completion time for batch b in unit u
$LOAD_{p,v}$	total amount of product p loaded on vehicle v
$POS_{b,b',u}$	position difference between batch b and b' in unit u
$SEQ_{b,b',u}$	variable determining that batch b is processed right before batch b' in processing unit u
$ST_{b,u}$	starting time for batch b in processing u
STV_v	departure time of vehicle v
$TB_{b,u}$	size of batch b allocated to unit u
TTV_v	overall traveling time for route of vehicle v
$TV_{i,v}$	travel time up to node i for route of vehicle v
$UNLOAD_{i,p,v}$	total amount of product p unloaded at node i from vehicle v

activities (PDP), have been explored as well. In particular, the multiple vehicle time-window-constrained pickup and delivery problem (MVPDPTW) faced in Dondo, Méndez, and Cerdá (2008) is capable of handling transport requests with multiple origins and/or destinations, heterogeneous vehicles and multiple depots.

In the last years, the development of mathematical models to represent transportation activities involved on N-echelon distribution systems has attracted an increasing attention from researchers worldwide due to its potential applications in real-life scenarios. For the short-term operational planning of multi-echelon, multi-product distribution systems, Dondo, Méndez, and Cerdá (2009) introduced the so-called vehicle routing problem in supply chain management (VRP-SCM), which is a generalization of the classical multi-echelon vehicle routing problem that handles multiple items and allows direct shipping of products from manufacturing sites to customers. In the same year, Wen, Larsen, Clausen, Cordeau, and Laporte (2009) proposed a new variant of the VRP called VRP with cross docking (VRPCD), where a set of homogeneous vehicles are used to transport orders from the suppliers to the corresponding customer via a cross-docking platform. The objective of the VRPCD is to minimize the total traveling time while respecting time window constraints at the nodes and a time horizon for the whole transportation operation. The consolidation of goods from multiple suppliers to a single destination has also been taken into account by Tsiakis, Shah, and Pantelides (2001) and Dondo, Méndez, and Cerdá (2011) by developing mathematical formulations considering warehousing operations and cross docking strategies.One of the first detailed analysis of an integrated production and distribution system was presented by Chandra and Fisher (1994). This paper evaluates the savings that can be obtained by properly coordinating production scheduling with transportation decisions. The motivating example was a two-echelon system using direct shipments from factory or inventory in the first echelon and a set of geographically dispersed customers in the second echelon. These authors found that the effective coordination of production and distribution may produce savings in operating costs ranging from 3 to 20%. Méndez, Bonfill, et al. (2006) proposed a MILP formulation focused on the operational level of supply chain management to efficiently coordinate the short-term production and transport scheduling. Such model aims toward a multipurpose batch plant producing a number of products over time and maintaining and inventory of finished goods that have to be distributed to a number of delivery centers. However, transportation decisions were treated in a simplified way and intermediate facilities like warehouses or DCs were not considered. Later, other approaches have been also developed to optimally coordinate short-term production and distribution activities, assuming a given supply chain structure. Verderame, Shaik, and Floudas (2007) studied the operational planning of a multisite network involving several batch production facilities and distribution centers. Based on customer demands, products are shipped from factories to DCs, where consumers pick up the orders. The problem was modeled through a MILP discrete-time formulation that provides the daily production mix at each factory and the product flows from manufacturing sites to DCs. Finally, Gupta et al. (2012) applied a multi-period MILP model for optimal operational planning of industrial gases with the objective of minimizing the total production and distribution cost. Although the decisions take into account multiple plants, product and shared customers, such model is quite limited in terms of routing activities.

This paper introduces a novel MILP framework to integrate production and transportation activities in a multi-site system. The proposed formulation lies on a continuous time representation which allows making operational decisions on a daily basis. The production problem is focused on the short-term production scheduling for multiproduct batch plants involving a single processing stage and multiple in-phase parallel units with sequence-dependent setup times and/or costs. Such type of operations appears in several application industrial areas such as textile, printing, container and bottle, plastic and paper, etc. (Kopanos et al., 2009). On the distribution side, a three-echelon network involving manufacturing centers, warehouses, DCs, and end-customers is supported by the model. The distribution activities are performed by a fleet of heterogeneous vehicles and three main transportation strategies for delivering several types of products from supplier (plants or DCs) to clients can be used: direct shipment, warehousing or cross docking. If a direct shipment strategy is considered, goods are shipped directly from the manufacturer to the end user. Direct shipments eliminate the expenses of operating a DC and reduce lead times. Warehousing is a traditional approach in which goods are received from manufacturers and stored. When a customer order arrives, items are retrieved, packed and shipped to the customer. Finally, cross docking (also referred to a just-in-time distribution) is a relatively new logistics technique that has been successfully applied by several retail chains. A cross dock is a transshipment facility in which incoming shipments (possibly originated from several manufacturers) are sorted, consolidated with other products and transferred directly to outgoing retailers without intermediate storage or order picking. As a result, shipments spend just a few hours at the facility (Ghiani et al., 2004). Successful stories on cross docking that resulted in considerable competitive advantages have been reported by several industries with high distribution costs like food and beverage producers, pharmaceutical companies, automobile manufacturers and retail chains (Sharma, 2010).

In the proposed approach, the optimal operation of logistics system is determined to achieve a proper customer service level at minimum cost. The main objective is to minimize the total operational cost associated with production and transportation activities. Hard and soft time windows for deliveries can be easily handled through the MILP mathematical framework.

2. Problem description

Production and distribution activities commonly arising in chemical supply chains involve the production and shipping of a number of commodities from multiple factories to customers directly and/or via distribution centers or regional warehouses. Since such operations are performed on a known network infrastructure, the integrated problem can be defined as a set of nodes $i \in I$ representing facilities situated at fixed locations. The nodes stand for factories, warehouses, distribution centers, and customer zones, which can be categorized into three subsets (i) $F \subset I$, (ii) $W \subset I$, and (iii) $S \subset I$, where F is the set of factories, W is the set of intermediate nodes, and S is the set of end-users. All together constitutes a three-echelon supply chain (see Fig. 1). The routes used for product shipping are determined by a set of minimum-cost arcs $a \in A$, which interconnects nodes in the network. Such arcs correspond to road segments characterized by a distance-based transportation cost and travel time.

In order to manage the manufacturing and shipment of multiple items, the production problem is based on single-stage parallel-line multiproduct batch plants. The configuration of each facility, i.e. its production capacity, inventory level, and operational costs are used for making production decisions. The set of processing units that can accomplish tasks at each factory i is defined through FU_i. In turn, each task performed may comprise an integer number of batches of the same product. Set P defines the range of products requested by customers and BP_p denotes the set of batches assigned to product p. In addition, sequence-dependent setup operations are considered to represent the activities arising in several industrial applications.

Intermediate facilities may keep finite stocks of fast moving products (warehousing) and/or act as cross-dock platforms for slow-moving and high value items. Warehouses are often used not only to maintain inventories but also to sort or consolidate goods. Since a replenishment order from a warehouse usually includes multiple products often available at different production sites, more than one vehicle coming from a factory can stop at the same DC to accomplish delivery operations. Products received may be later shipped to customer zones or stocked as inventory. Thus, vehicles can stop at intermediate facilities to perform loading or unloading activities. In multi-echelon problems, both inbound and outbound commodities are relevant. This is the case, for example, when DCs have to be located taking in account both the transportation cost from plants to DCs and the transportation costs from DCs to customers. Consequently, the integrated problem must consider constraints aiming at balancing inbound and outbound flows.

Thereby, factories and intermediate facilities act as supply points with product stocks stored at both facilities. Shipments can go directly from manufacturers and/or via warehouses to customers. A fleet of trucks is utilized for moving finished products to end-users. The vehicle fleet V involves a fixed number of heterogeneous vehicles with different properties (travel costs, travel times, and capacity). Each vehicle has an operational base situated at either a plant or a warehouse, where its trip must be started and ended. Set of trucks available at node i is defined through D_i. Vehicles only carry out pickup operations at the facility where are housed.

Since a customer order may include several products often available at different production sites, the consolidation of shipments from multiple factories to warehouses should be made before transporting products to a single destination (see Fig. 2). As partial shipments are not allowed, the service of a customer must be performed by a single vehicle. Consequently, vehicle routes must include only one stop at the same site and several products must be transported on the same truck; however, each vehicle has a finite load capacity in weight $capW_v$ and volume $capV_v$ that cannot be exceeded. Since the total shipment size must never exceed the maximum volume/weight capacity of the truck, two important product properties for truck loading are the weight ($weight_p$) and volume ($volume_p$) of a single unit of product p. If a typical customer shipment size is small, and customers are dispersed over a wide geographic area, a large fleet of small trucks may be required. As a result, direct shipment is common when fully loaded trucks are required by customers or when perishable goods have to be delivered timely.

The proposed integrated model for coordinating production and distribution activities in a N-echelon supply chain is able to consider: (i) single-stage parallel-line multiproduct batch plants, (ii) product sequence-dependent setup, (iii) initial inventories in

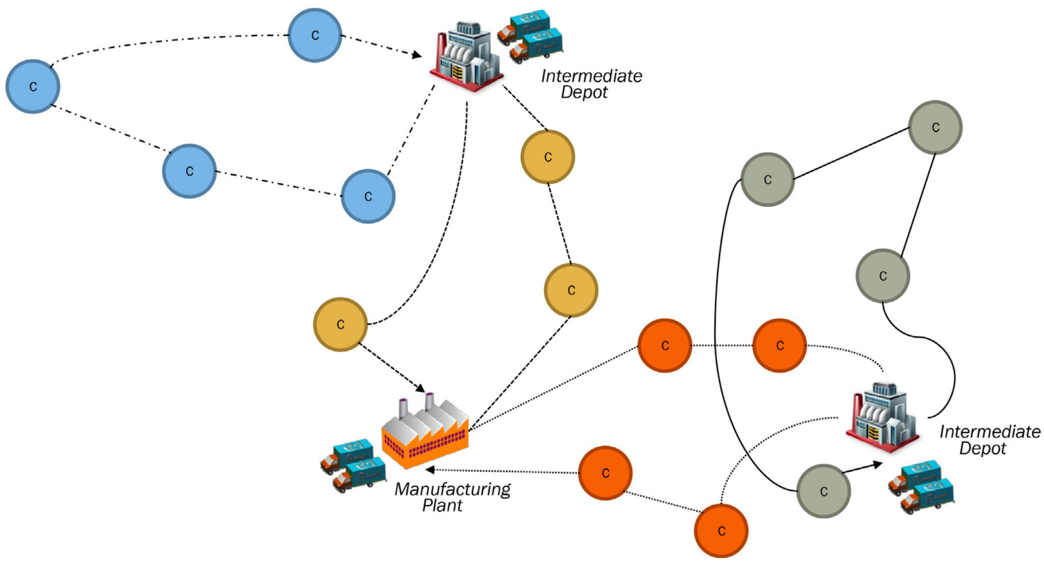

Fig. 1. A three-echelon network.

plants and intermediate facilities, (iv) synchronization between production activities and routing decisions, (v) heterogeneous vehicles, (vi) customer time windows, and (vii) cross-docking at intermediate facilities. The problem consists of determining the detailed production (batches to be produced, assignment of batches to processing units, sequencing and timing) and transport schedules (loads, unloads, routing and timing) while minimizing the total operational cost.

In Sections 3 and 4, a novel MILP-based continuous time formulation for the problem described above is introduced.

3. Model assumptions

The model assumptions are as follows:

1. Problem data are known with certainty and remain invariant over time.
2. There are several multiproduct batch plants. Each one has multiple processing units working in parallel.
3. Every processing unit cannot process more than one batch at a time.
4. The quantity of batches that can be processed at every factory is known a priori. Batches of the same product can be grouped in a single lot.
5. Setup times and costs are product sequence-dependent.
6. Processing times, product sequence-dependent setup times and/or cost are deterministic.
7. Batch sizes are not known a priori, but each equipment unit has a minimum and maximum production capacity that cannot be exceeded.
8. Initial product inventories are usually available at source nodes, and product demands are only specified for customer locations.
9. Shipments to factories nodes are not expected during the planning horizon.

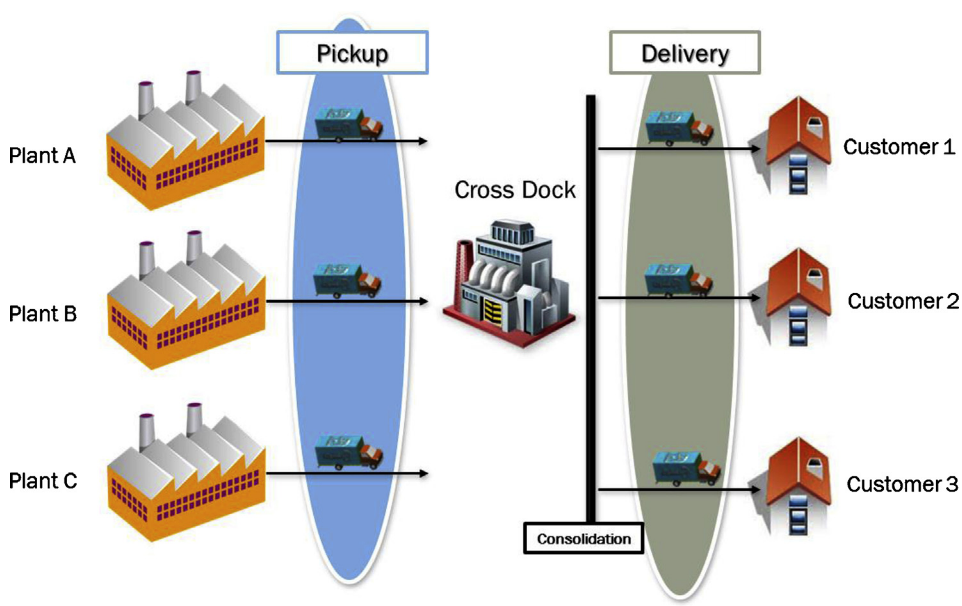

Fig. 2. Transshipment operations.

10. DCs and warehouses can receive lots of products from multiple plants.
11. Cross docking operations at warehouses are allowed.
12. Pickup and delivery services, if required, can both be performed by vehicles at intermediate nodes.
13. Each source facility, like factories or warehouses, has a fleet of vehicle available.
14. A vehicle only can load lots of product from the base (factory o warehouse) where is situated and can provide delivery services to multiple warehouses or customer locations.
15. Several products can be transported on the same vehicle, but its weight/volume capacity must never be exceeded. The weight and volume of a single unit of product are problem data.
16. A customer request can include more than one type of product.
17. Every customer location can be visited at most by one vehicle. Partial shipments to end-users are not allowed.
18. Each vehicle load/unload length time hast two components: a fixed time and a variable time that is proportional to the amounts of products to be picked-up and/or delivered.
19. If lots of products received at DCs should not be immediately loaded into outbound trucks, they can be temporarily stored until the time of shipping them to the assigned destinations.

4. The MILP mathematical model

The MILP mathematical model described in this section applies the unit-specific general precedence concept for production scheduling problem whereas the immediate precedence notion is used for distribution scheduling problem. The three different precedence-based mathematical formulations that have been reported in literature are: (i) unit-specific immediate precedence, (ii) immediate precedence, and (iii) general precedence. A complete survey about these alternatives can be found in Méndez, Cerdá, et al. (2006). Although the first two approaches require a larger number of binary variables in comparison to the last one, it will be showed that, for this specific problem, the use of immediate precedence variables in the objective function reduces significantly the computational effort needed to find the optimal solution.

The proposed mathematical model, which relies on a continuous time representation, includes three constraint categories: (a) *production constraints* determining the best scheduling of multiproduct batch plants, (b) *transportation constraints* computing the set of optimal vehicle routes to ensure that customer requests are satisfied, and (c) *integration constraints* monitoring the synchronization between production activities and routing decisions.

4.1. Production constraints

For every multiproduct batch plant performing activities in the supply chain, the short-term scheduling problem consists of determining: (a) batches to be processed, (b) batch sizes, (c) batch allocation to units, and (d) batch sequencing in each equipment item. As product sequence-dependent changeovers are considered, the production representation is based on the unit-specific general precedence concept introduced by Kopanos et al. (2009). However, it is worth to remark that based on different plant features, other existing formulations may be used for production scheduling.

In order to reduce the model size and consequently, the computational effort, batch allocation and sequencing decisions have been decoupled in three different sets of variables. The assignment variable $X_{b,u}$ denotes that batch b has been allocated to processing unit u. On the other hand, the 0–1 sequencing variable $Y_{b,b',u}$ indicates that batch b is processed before batch b', when both are assigned to the same unit u. Finally, the continuous positive variable $SEQ_{b,b',u}$ is defined to denote that the processing of batch b takes places in unit u immediately before batch b'. This continuous variable is then used in the objective function to determine associated costs to changeover operations. Note that a set of additional constraints must be defined to link sequencing assignment variable $SEQ_{b,b',u}$ with the sequencing assignment binary variable $Y_{b,b',u}$.

4.1.1. Unit allocation constraint

Eq. (1) restrains that each batch b can be assigned at most to one processing unit u in each factory i. $X_{b,u}$ is equal to 1 if batch b is allocated to unit u; otherwise, it is set to zero. FU_i denotes the set of units working in parallel in factory i.

$$\sum_{u \in FU_i} X_{b,u} \le 1 \quad \forall b \in B,\ i \in F \tag{1}$$

4.1.2. Unit capacity constraints

Minimum and maximum batch sizes are enforced by the pair of Eq. (2). $TB_{b,u}$ is a continuous positive variable defining the size of batch b processed in unit u. It value is greater than zero only if b is allocated to u; parameters $cmin_u$ and $cmax_u$ represent the minimum and maximum production capacity of every unit u respectively.

$$\left.\begin{aligned} &TB_{b,u} \le \mathrm{cmax}_u X_{b,u} \\ &TB_{b,u} \ge \mathrm{cmin}_u X_{b,u} \end{aligned}\right\} \quad \forall b \in B,\ u \in U \tag{2}$$

4.1.3. Sequencing-timing constraints

In order to represent the batch sequencing constraint, Eq. (3) is defined for every pair of batches (b,b') allocated to the same unit u. To compute the earliest start time to process batch b', the equation takes into account the final processing time of preceding batch b as well as the product setup time between b and b'. $Y_{b,b',u}$ is the global sequencing-allocation binary variable described above. If (b,b') are allocated to the same processing unit u and b is processed before b', $Y_{b,b',u}$ is equal to 1 and the constraint is activated by the model. $SEQ_{b,b',u}$ is the positive variable that defines the immediate precedence of two batches (b,b'). M_t is an upper bound for the corresponding time variable.

$$\begin{aligned} ST_{b',u} \ge\ & FT_{b,u} + setupT_{p,p'} SEQ_{b,b',u} - M_t(1 - Y_{b,b',u}) \\ & \times\ \forall (p,p') \in P,\ b \in BP_p,\ b' \in BP_{p'} : b \ne b' \end{aligned} \tag{3}$$

Eq. (4) computes the completion time of batch b in the assigned unit u. It is determined from the starting and processing time in u. Parameter $pt_{p,u}$ denotes the time required to process one unit of product p in unit u.

$$FT_{b,u} \ge ST_{b,u} + TB_{b,u} pt_{p,u} \quad \forall p \in P,\ b \in BP_p,\ u \in U \tag{4}$$

4.1.4. Sequencing-allocation constraints

$Y_{b,b',u}$ only can value 1 if both batches b and b' are allocated to the same equipment u. This restriction is computed by Eq. (5):

$$\left.\begin{aligned} &Y_{b,b',u} + Y_{b',b,u} \ge X_{b,u} + X_{b',u} - 1 \\ &Y_{b,b',u} \le X_{b,u} \\ &Y_{b,b',u} \le X_{b',u} \end{aligned}\right\} \quad \forall (b,b') \in B : b \ne b',\ u \in U \tag{5}$$

To tackle sequence-dependent changeovers and manage them in the objective function through a 0–1 variable, the continuous positive variables $POS_{b,b',u}$ and $SEQ_{b,b',u}$ are needed. The first one will be set to zero only if batch b is processed right before batch b' and they are allocated to the same equipment unit u (see Fig. 3). It is represented by Eq. (6):

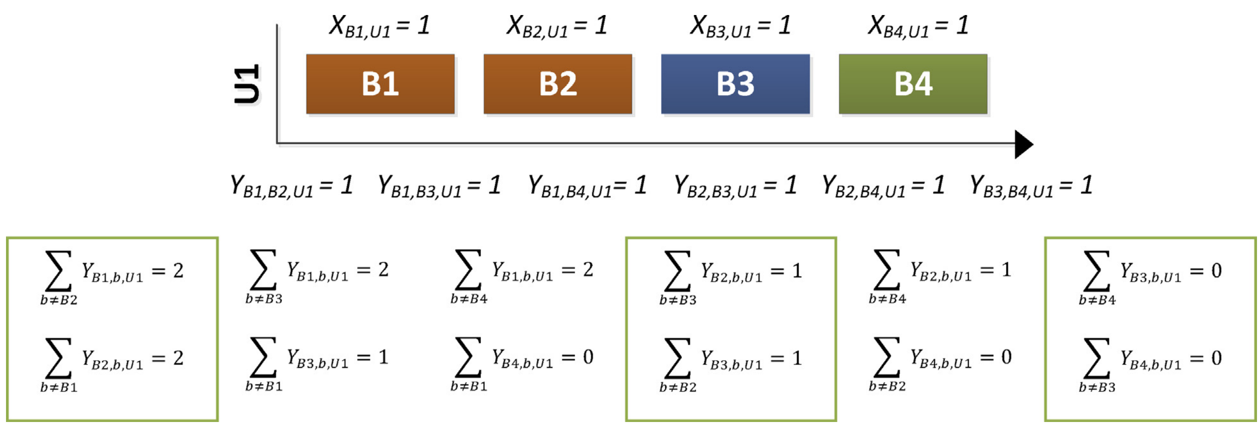

Fig. 3. Graphical representation of variable $POS_{b,b',u}$.

$$POS_{b,b',u} = \sum_{b'' \in B:(b,b') \neq b''} (Y_{b,b'',u} - Y_{b',b'',u}) + M_t(1 - Y_{b,b',u}) \times \forall (b, b') \in B : b \neq b',\ u \in U \quad (6)$$

If the position difference variable $POS_{b,b',u}$ is equal to zero, that is, when batch b has been processed exactly before batch b', Eq. (7) set the value of $SEQ_{b,b',u}$ to 1 because this variable is included in the objective function as a positive term.

$$POS_{b,b',u} + SEQ_{b,b',u} \geq 1 \quad \forall (b, b') \in B : b \neq b',\ u \in U \quad (7)$$

4.2. Constraints for integrating production and distribution decisions

The work developed by Méndez, Bonfill, et al. (2006) is adopted as reference in order to define the integration constraints determining the link between production and distribution decisions. In turn, in this section, new equations are presented to handle cross-docking operations at intermediate depots.

4.2.1. Product availability constraints

Both factories and warehouses may have inventory of products at beginning of the planning horizon. For each of these facilities, Eq. (8) states that the total amount of every product p taken from inventory for delivery must not be greater than the initial stock available at i ($inv_{i,p}$). Variable $AI_{p,v}$ computes the quantity of initial stock of product p allocated to vehicle v.

$$inv_{i,p} \geq \sum_{v \in D_i} AI_{p,v} \quad \forall i \in (F \cup W),\ p \in P \quad (8)$$

In case that the initial inventory is not enough to satisfy the overall demand, production tasks must be performed in factories. The batch production can be split to be loaded on different trucks housed in the factory where the batch is produced. Variable $A_{b,v}$ determines the quantity of product of batch b assigned to vehicle v. As shows Eq. (9), the value of this variable is always bounded by the batch size. In turn, considering that intermediate nodes only act as storage facilities with no production, Eq. (10) restrains the cargo of vehicles based at DCs.

$$\sum_{u \in FU_i} TB_{b,u} \geq \sum_{v \in D_i} A_{b,v} \quad \forall b \in B,\ i \in F \quad (9)$$

$$A_{b,v} \leq 0 \quad \forall b \in B,\ i \in W,\ v \in D_i \quad (10)$$

To determine to which trip is assigned every batch, a binary variable $Z_{b,v}$ is used. It is equal to 1 only if production of batch b is loaded (partially or totally) on vehicle v; M_z is a scalar whose value is equal to the maximum batch size.

$$M_z Z_{b,v} \geq A_{b,v} \quad \forall b \in B,\ v \in V \quad (11)$$

4.2.2. Cross-docking constraints

Eq. (12) states that the cargo of product p delivered by vehicle v to warehouse i ($UNLOAD_{i,p,v}$) can be cross-docked before being sent to their destinations. Variable $AA_{p,v,v'}$ determines the quantity of product p received at facility i from vehicle v that then is loaded on vehicle v' to be delivered to end-users.

$$UNLOAD_{i,p,v} \geq \sum_{v' \in D_i} AA_{p,v,v'} \quad \forall i \in (F \cup W),\ p \in P,\ v \in V \quad (12)$$

Note that if Eq. (12) is forced to be an equality by the model, so the amount of product p received from vehicle v is fully cross-docked; otherwise, a positive inventory of p will remain at node i at the end of the planning horizon.

Moreover, since more than one vehicle from factory can accomplish delivery tasks at the same cross-docking facility, a binary variable $VVAA_{v,v'}$ is used in Eq. (13) to determine if vehicle v' loads products delivered by v; M_v is an scalar whose value is equal to maximum vehicle load capacity.

$$M_v VVAA_{v,v'} \geq AA_{p,v,v'} \quad \forall p \in P,\ (v, v') \in V : v \neq v' \quad (13)$$

4.2.3. Vehicle loading constraints

The continuous positive variables $AI_{p,v}$, $A_{b,v}$ and $AA_{p,v,v'}$, which were defined above, allow to compute through Eq. (14) the total amount of product p transported on vehicle v ($LOAD_{p,v}$).

$$LOAD_{p,v} = \sum_{b \in BP_p} A_{b,v} + AI_{p,v} + \sum_{v' \in V : v \neq v'} AA_{p,v',v} \quad \forall v \in V,\ p \in P \quad (14)$$

However, the pair of Eq. (15) enforces the condition that the total cargo transported by each truck must never be greater than the maximum volumetric and weight vehicle capacity, defined by parameters $capV_v$ and $capW_v$, respectively. Parameters $weight_p$ and $volume_p$ define the weight and volume per unit of product p respectively. If vehicle v is used, the binary variable H_v is set to 1; otherwise, its value is set to zero.

$$\left. \begin{array}{l} \sum_{p \in P} (LOAD_{p,v} weight_p) \leq capW_v * H_v \\ \sum_{p \in P} (LOAD_{p,v} volume_p) \leq capV_v * H_v \end{array} \right\} \quad \forall v \in V \quad (15)$$

Traveling start time constraints

The earliest start time of each vehicle route (STV_v) is determined by Eqs. (16), (17) or (18). If a vehicle only loads products from inventory, Eq. (16) is used by the model. Parameter vvt_v defines the loading time per unit of product of vehicle v.

$$STV_v \geq \left(\sum_{p\in P} LOAD_{p,v}\right) vvt_v \quad \forall v\in V \tag{16}$$

Moreover, Eq. (17) enforces the condition that if batch b is allocated to vehicle v, i.e. $Z_{b,v}=1$, the ending time for batch b should be added to Eq. (16) to compute the departure time of v. Parameter M_{tv} represents an upper bound for the corresponding time variable.

$$STV_v \geq FT_{b,u} + \left(\sum_{p\in P} LOAD_{p,v}\right) vvt_v - M_{tv}(1-Z_{b,v}) \times \forall b\in B,\ i\in F,\ u\in FU_i,\ v\in D_i \tag{17}$$

Furthermore and for cross docking operations, Eq. (18) states that if $VVAA_{v,v'}$ is equal to 1, the travel time of vehicle v up to stop at facility i plus fixed and variable stop times to carry out loading/unloading tasks are considered to compute the earliest start time of vehicle route v'. Parameter vft_v represents the fixed time required for delivery operations of vehicle v.

$$STV_{v'} \geq TV_{i,v} + \sum_{p\in P}(LOAD_{p,v'} vvt_{v'} + UNLOAD_{i,p,v} vvt_v) + vft_v - M_{tv}(1-VVAA_{v,v'}) \quad \forall i\in W,\ v\in V,\ v'\in D_i : v\neq v' \tag{18}$$

4.3. Transportation constraints

The immediate precedence notion is used to define vehicle routes, which can be regarded as a sequence of vehicle stops at different locations. In this new formulation presented for the distribution problem, transportation decisions are decoupled in the following set of binary variables: (i) allocation variable $VA_{i,v}$ computes if vehicle v visits node i to accomplish delivery operations, (ii) sequencing variable $PR_{i,i',v}$ denotes that node i is visited right before node i' by vehicle v, (iii) $IN_{i,v}$ denotes that node i is the first visited in the route of vehicle v, (iv) $FI_{i,v}$ defines that node i is the last to be visited in the trip of vehicle v before it return to the base node, and (v) H_v values 1 if vehicle v is used; otherwise, it is set to zero.

Route building constraints

Eq. (19) indicates that every customer location $i\in S$ can at most be visited by a single vehicle during the planning horizon. If $VA_{i,v}=1$, then vehicle v will be visiting node i to perform a set of delivery tasks.

$$\sum_{v\in V} VA_{i,v} \leq 1 \quad \forall i\in S \tag{19}$$

Eq. (20) states that each manufacturer plant is a pure pickup node that delivers products to DCs or warehouses and customer locations. Thus, no vehicle will be visiting these locations to provide delivery services.

$$VA_{i,v} \leq 0 \quad \forall i\in F,\ v\in V \tag{20}$$

On the other hand, distribution centers or regional warehouses only receive and store product from manufactures and deliver them to customers. In other words, no vehicle loading lots of product from its based situated at intermediate depot can provide delivery services to other warehouse. Eq. (21) enforces such condition.

$$\sum_{i'\in W}\sum_{v\in D_{i'}} VA_{i,v} \leq 0 \quad \forall i\in W \tag{21}$$

Product demand constraints

Eq. (22) states that the total amount of product p delivered to each customer site i must always satisfy its demand ($dem_{i,p}$).

$$\sum_{v\in V} UNLOAD_{i,p,v} \geq dem_{i,p} \quad \forall i\in S,\ p\in P \tag{22}$$

For intermediate nodes, Eq. (23) restrains the gap between inbound and outbound commodities to ensure that customer requests of these locations will be satisfied.

$$inv_{i,p} + \sum_{v\in V} UNLOAD_{i,p,v} - \sum_{v\in D_i} LOAD_{p,v} \geq dem_{i,p} \quad \forall i\in W,\ p\in P \tag{23}$$

Since we have defined the variable $UNLOAD_{i,p,v}$ as the amount of product p delivered by vehicle v to location i, Eq. (24) enforces the condition that a delivery operation performed by vehicle v at customer node i can only take place if vehicle v has been assigned to i. Parameter M_{cb} is used as an upper bound.

$$M_{cb} VA_{i,v} \geq UNLOAD_{i,p,v} \quad \forall i\in I,\ v\in V,\ p\in P \tag{24}$$

In addition, Eq. (25) states that the quantity of every product p supplied by a vehicle v to the assigned destinations can never exceed the initial load of product p available on v.

$$\sum_{i\in I} UNLOAD_{i,p,v} \leq LOAD_{p,v} \quad \forall p\in P,\ v\in V \tag{25}$$

Note that if the above constraint is fulfilled as equality, every product unit picked up by vehicle is delivered to a demanding location before the end of the vehicle trip.

Traveling time constraints

Traveling time from the base i to the first location i' visited by vehicle v ($IN_{i',v}=1$) is given by Eq. (26). $TV_{i',v}$ must never be lower than the sum of the travel time along the arc (i,i') plus the start time of trip of vehicle v. Parameter $dist_{i,i'}$ defines the km distance between node i and node i', whereas the value of sp_v represents the average speed of vehicle v, respectively. Scalar M_t is an upper bounds for the corresponding time variable.

$$TV_{i',v} \geq STV_v + \frac{dist_{i,i'}}{sp_v} - M_t(1-IN_{i',v}) \quad \forall (i,i')\in I : i\neq i',\ v\in D_i \tag{26}$$

If the immediate precedence variable $PR_{i,i',v}$ is equal to 1, vehicle v will visit node i right before node i'. The accumulated time of vehicle v up to stop node i' should be greater that the corresponding value up to preceding visit to node i. The difference ($TV_{i',v} - TV_{i,v}$) must never be lower than the sum of the travel time along the route from i to i' plus the time required to perform delivery operations at node i.

$$TV_{i',v} \geq TV_{i,v} + \frac{dist_{i,i'}}{sp_v} + vft_v + \sum_{p\in P}(UNLOAD_{i,p,v} vvt_v) - M_t(1-PR_{i,i',v}) \quad \forall (i,i')\in I : i\neq i',\ v\in V \tag{27}$$

Finally, the overall traveling time for the route assigned to vehicle v is computed by Eq. (28). If $FI_{i',v}$ is equal to 1, the node i' is the last to be visited by v before it return to its base i. The duration of each trip is determined by adding both the duration of discharge activities carried out at node i' and the traveling time to return to

Table 1
Production features.

	Unit	Batch size (units)		Processing cost ($/units)			Processing time (h/units)		
		Min	Max	P1	P2	P3	P1	P3	P3
Barcelona	E1	200	400	6.5	4	5.5	0.1	0.1	0.1
	E2	200	400	6.5	4	5.5	0.1	0.1	0.1
Bratislava	E1	200	400	5.5	3	4.5	0.1	0.1	0.1
	E2	200	400	5.5	3	4.5	0.1	0.1	0.1

Table 2
Vehicle characteristics.

	Location	Capacity		Traveling cost ($/km)
		Weight (kg)	Volume (m^3)	
V1 (all Scenarios)	Barcelona	20,000	25	3
V2 (all Scenarios)	Bratislava	20,000	25	3
V3 (all Scenarios)	Madrid	12,000	18	2.5
V4 (all Scenarios)	Stuttgart	12,000	18	2.5
V5 (Scenario 1)	Bratislava	15,000	20	3

the base node i to the time required for reaching the last visited node i'.

$$TTV_v \geq TV_{i',v} + \frac{dist_{i',i}}{sp_v} + vft_v + \sum_{p \in P}(UNLOAD_{i',p,v}vvt_v) - M_t(1 - FI_{i',v}) \quad \forall (i, i') \in I : i \neq i',\ v \in D_i \tag{28}$$

Route sequencing constraints

If vehicle v is used, exactly one location must be first visited and exactly one location is the last to be visited in the route of v. These constraints are enforced by the pair of Eq. (29).

$$\left.\begin{array}{l} \sum_{i \in I} IN_{i,v} = H_v \\ \sum_{i \in I} FI_{i,v} = H_v \end{array}\right\} \quad \forall v \in V \tag{29}$$

A single location i can be the first/last to be visited by vehicle v, only if this node was assigned to v. It represented by the set of Eq. (30).

$$\left.\begin{array}{l} IN_{i,v} \leq VA_{i,v} \\ FI_{i,v} \leq VA_{i,v} \end{array}\right\} \quad \forall i \in I,\ v \in V \tag{30}$$

Whenever a pair of nodes i, i' are related through the immediate precedence relationship, i.e. $PR_{i,i',v} = 1$, both locations must be visited by the same vehicle v. This condition is imposed through Eq. (31).

$$\left.\begin{array}{l} PR_{i,i',v} \leq VA_{i,v} \\ PR_{i,i',v} \leq VA_{i',v} \end{array}\right\} \quad \forall (i, i') \in I,\ v \in V \tag{31}$$

A node i can be visited by vehicle v either in the first place ($IN_{i,v} = 1$) or right after another location i' ($PR_{i',i,v} = 1$), called its immediate predecessor. Moreover, every node i can be either allocated to the last position in the route of vehicle v ($FI_{i,v} = 1$), or right before another node i' ($PR_{i,i',v} = 1$), called its immediate successor. These constraints are represented by the pair of Eq. (32).

$$\left.\begin{array}{l} IN_{i,v} + \sum_{i' \in I: i \neq i'} PR_{i',i,v} = VA_{i,v} \\ FI_{i,v} + \sum_{i' \in I: i \neq i'} PR_{i,i',v} = VA_{i,v} \end{array}\right\} \quad \forall i \in I,\ v \in V \tag{32}$$

4.4. Objective function

The objective function is to minimize the total cost involving sum of the production and distribution costs over the whole planning horizon. The production activities comprise unit production and changeovers costs while transportation operations involve distance-based travel costs which generally decrease with the number of intermediate facilities because of shipment consolidation and shorter outbound distances. To achieve a better performance in the search of the optimal solution, the immediate precedence variables $PR_{i,i',v}$ are directly used in the objective function formulation. Thus, a considerably reduction in the computational effort needed to solve the problem to global optimality is obtained.

$$Min \left[\sum_{p \in P} \sum_{b \in BP_p} \sum_{u \in U} \left(TB_{b,u} pc_{p,u} + \sum_{b' \in BP_{p'}} setupC_{p,p'} SEQ_{b,b',u} \right) + \sum_{i \in (F \cup W)} \sum_{v \in D_i} \sum_{i' \in I} (IN_{i',v} + FI_{i',v}) dist_{i,i'} vvc_v + \sum_{v \in V} \sum_{i \in I} \sum_{i' \in I} PR_{i,i',v} dist_{i,i'} vvc_v \right] \tag{33}$$

The minimum total travel time has been chosen as an alternative objective. In the other words, the optimal distribution activities must be completed as early as possible. After solving the MILP model, the assignment variables $X_{b,u}$ and $VA_{i,v}$ and the sequencing variables $SEQ_{b,b',u}$, $IN_{i,v}$, $FI_{i,v}$, and $PR_{i,i',v}$ are fixed at their optimal values. The resulting model is solved once more but now the total travel time becomes the new problem objective function.

$$Min \left[\sum_{v \in V} TTV_v \right] \tag{34}$$

Table 3
Product specific weights and volumes.

	P1	P2	P3
Weight (kg/unit)	3	6	5
Volume (m^3/unit)	0.005	0.015	0.010

Table 4
Initial inventories in supplier nodes and product demands at customer locations.

	P1	P2	P3
Product inventories			
Barcelona	0	0	0
Bratislava	0	0	0
Madrid	400	200	200
Stuttgart	400	200	200
Product demands			
Berlin	50	200	0
Bilbao	300	100	100
Calais	150	50	50
Frankfurt	50	150	100
Hamburg	200	0	200
Lisbon	200	150	200
Lyon	170	100	150
Madrid	0	250	0
Milan	50	150	50
Nantes	50	50	0
Paris	200	350	200
Stuttgart	0	0	240
Torino	50	75	75
Valencia	180	180	180

4.5. Solution strategies for large problems

In order to assess the value of coordinating production and distribution scheduling, alternative sequential methodologies can be applied to integrate production and distribution decisions. Specifically, we implemented three strategies to solve the problem addressed: two of them in which the production scheduling and vehicle routing problems are solved separately and one in which both problems are integrated within a single model. Such strategies are described below:

1. *Production cost-based approach:* The production schedule is first optimized assuming both an instantaneous delivery of goods and a limited number of available vehicles for delivering finished products. Detailed routing decisions are ignored in this first step. The assignment variable $VA_{i,v}$ is then fixed to satisfy to each customer demand from vehicles located at the lowest cost factory. After solving the production model, variables $X_{b,u}$ and $VA_{i,v}$ are fixed at their optimal values and detailed routing decisions are made subject to inventory availability defined by the production schedule. This is performed by solving again the entire MILP model.

Table 5
Detailed production-transport scheduling corresponding to Scenario 1.

		Batch	Product type	Size	Starting time	Completion time	Processing time
Barcelona	E1	B7	P3	280	0.0	28.0	28.0
		B1	P1	280	29.0	57.0	28.0
	E2	B4	P2	280	0.0	28.0	28.0
		B5	P2	200	28.5	48.5	20.0
Bratislava	E1	B1	P1	200	0.0	20.0	20.0
		B2	P1	370	20.5	57.5	37.0
	E2	B4	P2	400	0.0	40.0	40.0
		B5	P2	200	40.5	60.5	20.0
		B6	P2	325	61	93.5	32.5
		B7	P3	200	94.5	114.5	20.0
		B8	P3	400	115.0	155.0	40.0
		B9	P3	265	155.5	182.0	26.5
Production Cost							$15132.5

	Site	Arrival time	Departure time	P1	P2	P3	Used capacity %w	Used capacity %v
V1	Barcelona	–	61.2	+280	+480	+280	25.60	45.60
	Madrid	70.1	73.1	−100	−300	−100		
	Valencia	78.2	81.4	−180	−180	−180		
	Barcelona	86.4	–	–	–	–		
V2	Bratislava	–	190.4	+520	+725	+865	51.18	88.5
	Stuttgart	201.2	207.2	−250	−400	−590		
	Lyon	216.2	218.9	−170	−100	−150		
	Torino	223.3	225.1	−50	−75	−75		
	Milan	227.2	229.2	−50	−150	−50		
	Bratislava	242.5	–	–	–	–		
V3	Madrid	–	77.3	+500	+250	+300	45.00	51.39
	Lisbon	84.3	87.5	−200	−150	−200		
	Bilbao	97.1	100.1	−300	−100	−100		
	Madrid	104.5	–	–	–	–		
V4	Stuttgart	–	214.4	+650	+600	+550	83.00	98.61
	Frankfurt	216.6	218.8	−50	−150	−100		
	Hamburg	224.3	226.9	−200	0	−200		
	Calais	235.3	237.3	−150	−50	−50		
	Nantes	243.9	245.4	−50	−50	0		
	Paris	249.6	253.6	−200	−350	−200		
	Stuttgart	260.5	–	–	–	–		
V5	Bratislava	–	41.0	+50	+200	0	16.25	9.00
	Berlin	50.7	52.7	−50	−200	0		
	Bratislava	62.4	–	–	–	–		
Routing cost								$28770.5

Table 6
Detailed production-transport scheduling for Scenario 2.

		Batch	Product type	Size	Starting time	Completion time	Processing time
Barcelona	E1	B7	P3	330	0.0	33.0	33.0
		B8	P3	200	33.5	53.5	20.0
		B4	P2	400	54.5	94.5	40.0
		B5	P2	330	95.0	128.0	33.0
		B6	P2	200	128.5	148.5	20.0
	E2	B1	P1	200	0.0	20.0	20.0
		B2	P1	280	20.5	48.5	28.0
		B3	P1	400	49.0	89.0	40.0
Bratislava	E1	B1	P1	370	0.0	37.0	37.0
		B1	P1	200	37.5	57.5	20.0
		B4	P2	400	58.5	98.5	40.0
		B5	P2	275	99.0	126.5	27.5
	E2	B7	P3	215	0.0	21.5	21.5
		B8	P3	400	22.0	62.0	40.0
Production cost							$20342.50

	Site	Arrival time	Departure time	P1	P2	P3	Used capacity	
							%w	%v
V1	Barcelona	–	157.9	+880	+930	+530	54.35	94.60
	Paris	172.7	176.7	−200	−350	−200		
	Calais	180.8	182.8	−150	−50	−50		
	Nantes	191.4	192.8	−50	−50	0		
	Bilbao	202.6	205.6	−300	−100	−100		
	Madrid	211.3	213.1	0	−200	0		
	Valencia	218.2	221.4	−180	−180	−180		
	Barcelona	226.3	–	–	–	–		
V2	Bratislava	–	133.9	+570	+675	+615	44.18	76.50
	Berlin	143.6	145.6	−50	−200	0		
	Hamburg	149.8	152.4	−200	0	−200		
	Frankfurt	159.4	161.6	−50	−150	−100		
	Stuttgart	164.5	165.7	0	0	−40		
	Lyon	174.7	177.4	−170	−100	−150		
	Torino	181.8	183.6	−50	−75	−75		
	Milan	185.7	187.7	−50	−150	−50		
	Bratislava	201.0	–	–	–	–		
V3	Madrid	–	2.2	+200	+150	+200	25.00	29.17
	Lisbon	9.2	12.4	−200	−150	−200		
	Madrid	19.4	–	–	–	–		
Routing cost								$25334.00

2. *Distribution cost-based methodology:* First, the transportation problem is just considered to determine the best product flows from factories and warehouses to customer zones. Production decisions are ignored and both the manufacturing plants and intermediate facilities are considered as full storage locations with products stocks available at the start of the planning horizon. The assignment variable $X_{b,u}$ is fixed to 0 and the parameter $inv_{i,p}$ is fixed to the maximum production capacity of each plant. After solving the distribution model, the variable $X_{b,u}$ is released and parameter $inv_{i,p}$ is returned to its original value. In a second step, the production activities are explicitly defined according to the best distribution scheduling generated (optimal value of binary variable $VA_{i,v}$).
3. *Fully integrated approach*: in contrast to previous strategies, a single monolithic MILP model is solved. No decision variable is predefined by this approach and the management of interdependencies between plant operations and vehicles activities are all decisions made by the model. In the result section it is demonstrated that this approach shows significant potential savings with respect to the previous sequential approaches.

5. Case study and computational results

The applicability and effectiveness of the proposed MILP formulation are illustrated by effectively coping with an integrated production and transportation scheduling problem usually arising in the daily operations of chemical supply chains. A case study that involves the management of a multi-site system comprising 16 locations (2 factories, 2 DCs, and 16 customers) geographically spread in six European countries has been solved through the proposed approach. Such an example is a modified version of case studies previously tackled by Bonfill, Espuña, and Puigjaner (2008) and Dondo et al. (2011). The example involves two single-stage multiproduct batch plants, which are located in Barcelona and Bratislava. These facilities have two different processing units in parallel that are able to process batches of three different products (P1, P2 or P3). In Table 1, the minimum and maximum product batch sizes for each unit as well as processing times and costs per unit of product are given. From this table, it is important to remark that Bratislava-based plant takes advantage of lower manufacturing costs in comparison to Barcelona's facility because of the lower cost of raw materials, manpower and taxation available in Slovakia.

Two warehouses are located in Madrid and Stuttgart in order to serve the customers located in the neighborhood of these facilities. The DC placed in Madrid is serviced from Barcelona, while that Stuttgart has to Bratislava's plant as the pre-assigned supplier. Four vehicles V1-V4 are available to fulfill the required distribution activities of three types of products P1–P3 from factories and warehouses to delivery nodes. Information related to vehicle

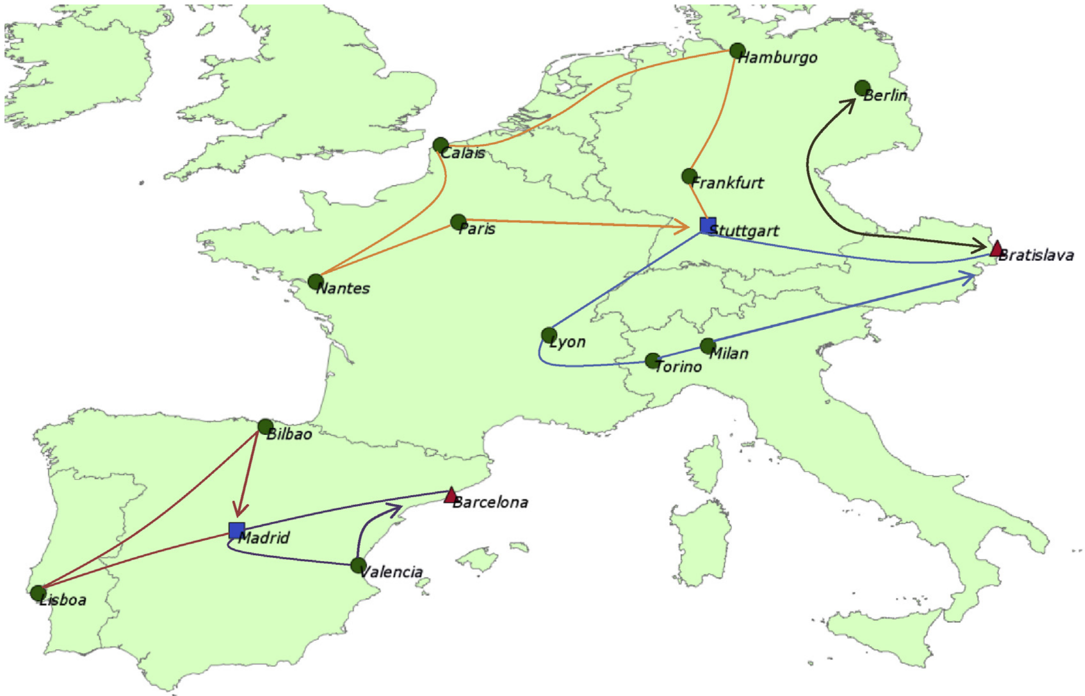

Fig. 4. Optimal vehicle routes for Scenario 1.

characteristics is given in Table 2, while Table 3 provides the weight and volume per unit of each product. The available vehicles can perform pickup operations at a rate of 250 units/h, while the stop time needed for carry out delivery operations in each site comprises a fixed time of 1 h plus a variable time period that directly increases with the total cargo to be discharged at a rate equal to the previously one mentioned. Moreover, the available stocks in supplier facilities at the beginning of planning horizon and the demands of products at customer sites are reported in Table 4.

The inherent advantages of coordinating production and distribution activities in the supply chain will be highlighted by optimally solving several scenarios with different logistics features. In particular, we have considered three approaches in order to compare operational costs. It is worth to remark that due to the close interaction between the production and distribution decisions, any small change in the problem configuration may significantly impact on the solution generated. This situation can be easily observed in the fourth scenario that considers time windows in some customer

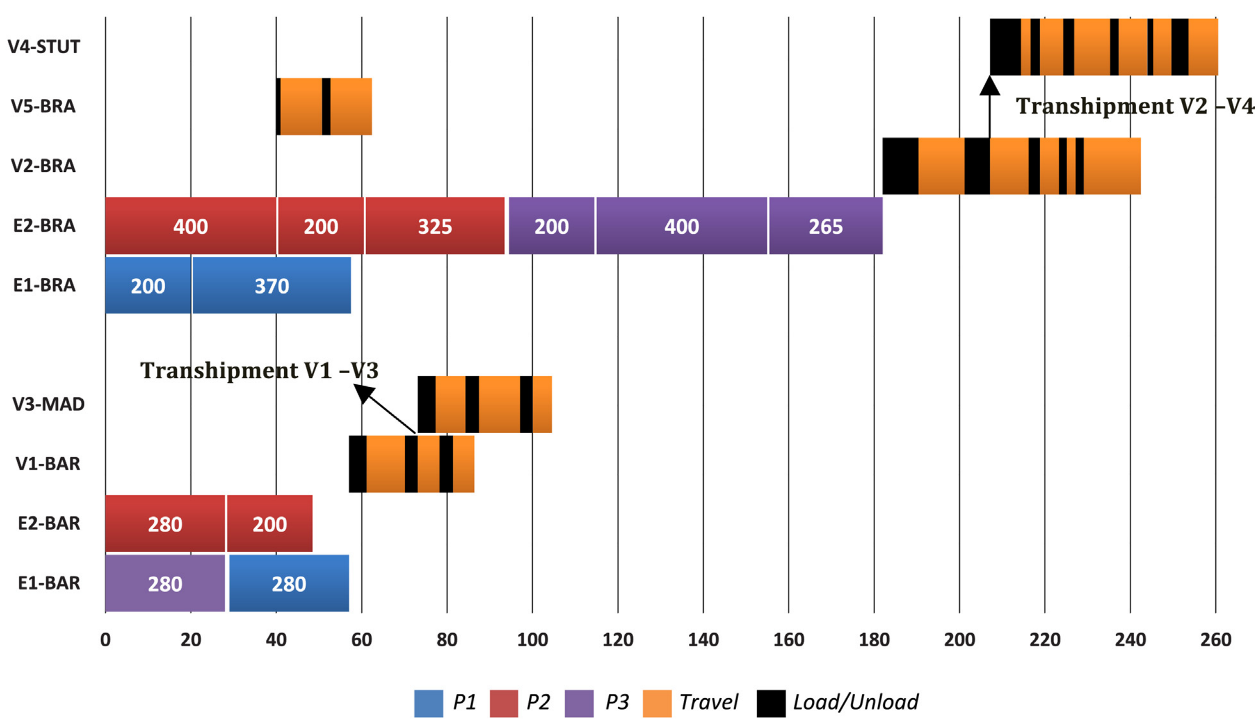

Fig. 5. Production-transport schedule for Scenario 1.

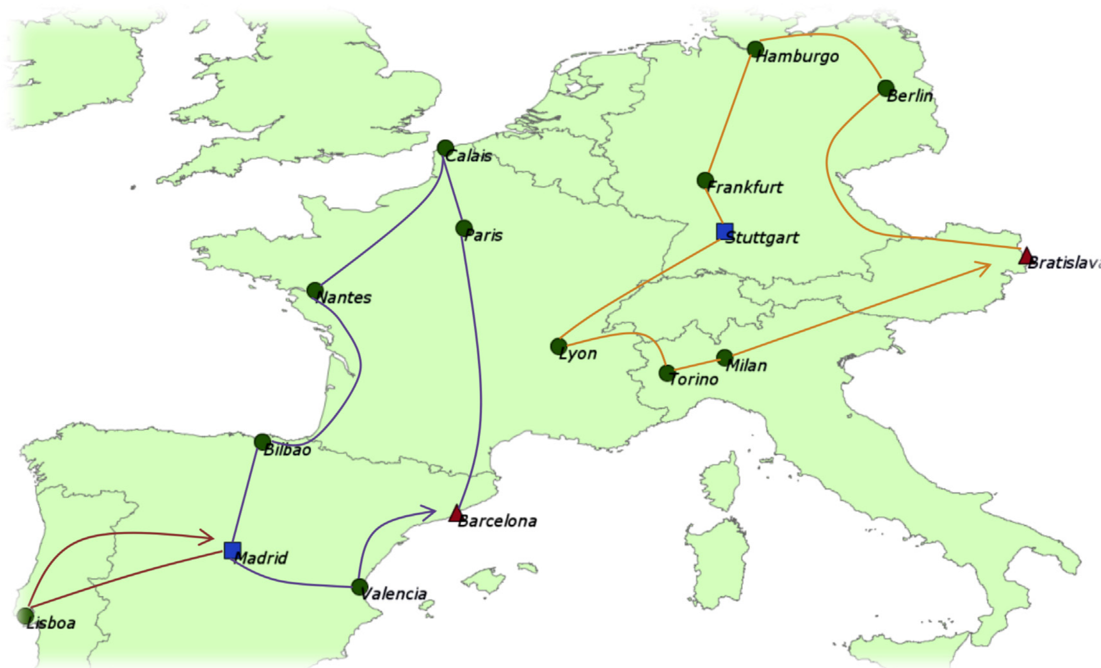

Fig. 6. Optimal vehicle routes for Scenario 2.

locations. All alternatives were solved to optimality with a modest computational effort by using a DELL PRECISION T5500 Workstation with six-core Intel Xeon Processor (2.67 GHz) and the modeling language GAMS and CPLEX 12.2 as the MILP solver. A relative optimality tolerance of 0.001 is adopted. Model sizes, computational time and objective values are summarized in Table 9.

5.1. Scenario 1

In the first scenario, the logistics problem previously described was solved by prioritizing a production cost-based criterion. Following this direction, most customer orders are to be satisfied from the lowest-cost factory, situated at Bratislava, or by its associated warehouse placed at Stuttgart. Thus, the cities of Milan, Torino, Hamburg, Berlin, Paris, Lyon, Nantes, Calais, and Frankfurt were pre-assigned to either supplier node in Bratislava or Stuttgart while the remaining demanding points situated in the sphere of influence of Barcelona and Madrid can be visited by either V1 or V3.

If Scenario 1 is solved by using the vehicle fleet adopted in the original problem (V1–V4), the resulting mathematical model has no feasible solution. This is because the vehicle capacities are not sufficient to service all the demanding cities associated to Bratislava

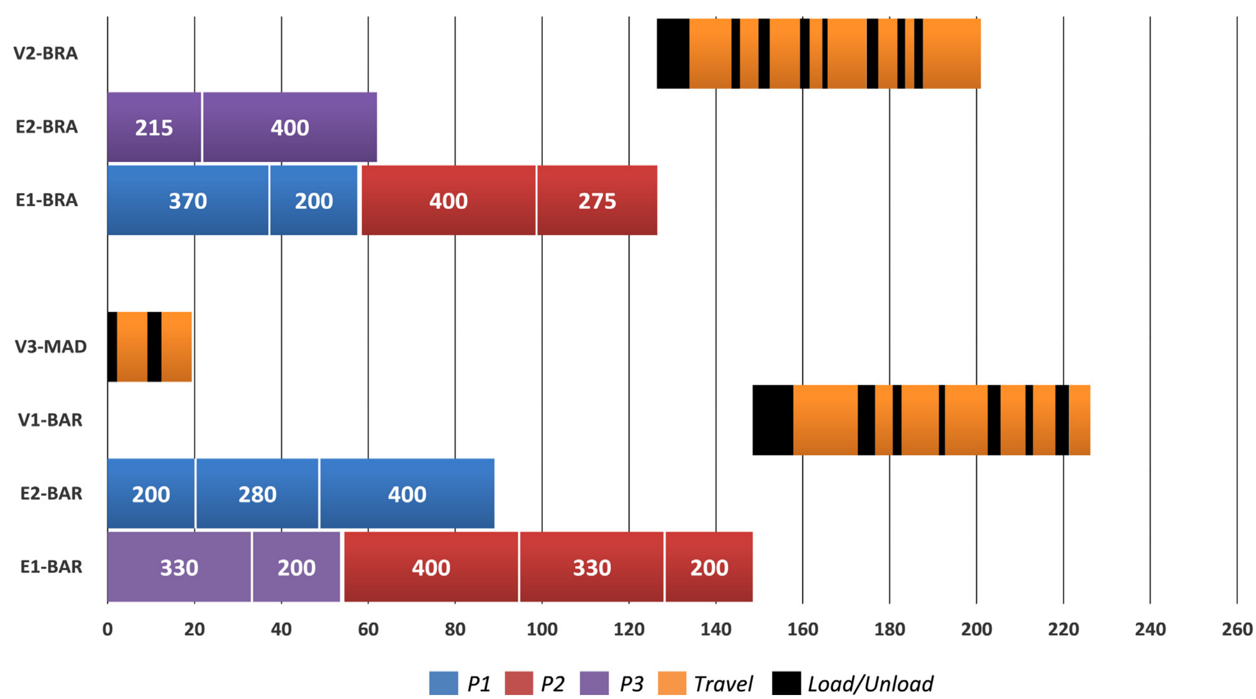

Fig. 7. Production-transport schedule for Scenario 2.

Table 7
Detailed production-transport scheduling for Scenario 3.

		Batch	Product type	Size	Starting time	Completion time	Processing time
Barcelona	E1	B1	P1	330	0.0	33.0	33.0
	E2	B4	P2	200	0.0	20.0	20.0
		B5	P2	330	20.5	53.5	33.0
		B7	P3	280	54.5	82.5	28.0
Bratislava	E1	B1	P1	320	0.0	32.0	32.0
		B2	P1	200	32.5	52.5	20.0
	E2	B4	P2	200	0.0	20.0	20.0
		B5	P2	275	20.5	48.0	27.5
		B6	P2	400	48.5	88.5	40.0
		B7	P3	265	89.5	116.0	26.5
		B8	P3	400	116.5	156.5	40.0
		B9	P3	200	157.0	177.0	20.0
Production cost							$15232.5

	Site	Arrival time	Departure time	P1	P2	P3	Used capacity	
							%w	%v
V1	Barcelona	–	87.0	+330	+530	+280	27.85	49.60
	Madrid	96.0	99.4	−150	−350	−100		
	Valencia	104.5	107.7	−180	−180	−180		
	Barcelona	112.6	–	–	–	–		
V2	Bratislava	–	186.0	+520	+875	+865	55.68	97.50
	Stuttgart	196.8	202.0	−70	−475	−515		
	Frankfurt	204.9	207.1	−50	−150	−100		
	Calais	215.7	217.7	−150	−50	−50		
	Hamburg	228.4	231.0	−200	0	−200		
	Berlin	235.2	237.2	−50	−200	0		
	Bratislava	246.9	–	–	–	–		
V3	Madrid	–	104.0	+550	+300	+300	49.50	56.94
	Lisbon	111.0	114.2	−200	−150	−200		
	Nantes	130.9	132.3	−50	−50	0		
	Bilbao	139.9	142.5	−300	−100	0		
	Madrid	147.3	–	–	–	–		
V4	Stuttgart	–	208.5	+470	+675	+475	78.35	95.69
	Paris	215.5	219.5	−200	−350	−200		
	Lyon	224.6	227.3	−170	−100	−150		
	Torino	230.8	232.6	−50	−75	−75		
	Milan	234.2	236.2	−50	−150	−50		
	Stuttgart	241.8	–	–	–	–		
Routing cost								$26974.5

and Stuttgart. In fact, most production scheduling models found in literature always assume either an instantaneous delivery of goods or an unlimited number of available vehicles for delivering finished products.

In order to overcome the infeasibility mentioned before, the vehicle fleet has incorporated another vehicle V5 based on Bratislava. The scenario was solved to optimally in 66.4 s of CPU time (see Table 9). Fig. 5 shows the best batch sequence in each processing unit as well as the times at which vehicles start loading operations and leave their bases, together with arrival times and delivery operations performed at each visited node. For vehicle schedules, loading/unloading activities are represented with black rectangles while traveling operations are showed in orange color. More details about optimal vehicle routes are given in Table 5 and illustrated in Fig. 4. Deliveries to DCs and customer locations are reported with negative numbers, while pickups at supplier nodes are represented by positive numbers. The optimal solution costs are given in Table 5.

From the solution shown in Fig. 5 it can be observed that the Barcelona-based plant processes one batch of P3 (280 units) and then one batch of P1 (280 units) in unit 1, while two batches of P2 (480 units) are sequenced in unit 2. On the other hand, Bratislava's factory manufactures 570 units of P1, 925 units of P2, and 865 units of P3 by using the batch sequencing depicted in Fig. 5. The total amount of finished goods in both factories is later loaded on plant-based vehicles to be delivered to the assigned destinations. In addition cross-docking operations are performed at intermediate facilities. The pickup operations by vehicle V3 begin immediately after delivery of product P1–P3 by V1 to Madrid has been completed. The amounts of products unloaded from V1 at Madrid are loaded into vehicle V3 together with the initial stock and sent to the customer zones, including Madrid. These synchronized activities also can be observed during the visit of V2 to the facility situated in Stuttgart. In this location, delivery and pickup operations are sequentially performed by vehicle V2 and V4, respectively. The amounts of products received from Bratislava at warehouse Stuttgart are fully cross-dock and sent to their destinations. As a result, no product inventories remain at the DCs when the planning horizon ends.

Another important feature of the integrated model, properly addressed in the optimal solution of Scenario 1, can be shown in Fig. 5. As the cargo of vehicle V5 includes only 50 units of P1 and 200 units of P2, the pickup operations of this truck can start immediately after that the first two batches of products P1–P2 have been completed at Bratislava-base plant.

5.2. Scenario 2

In this scenario, we applied a sequential methodology where the transportation problem is considered at the first to determine

Table 8
Detailed production-transport scheduling for Scenario 4.

		Batch	Product type	Size	Starting time	Completion time	Processing time
Barcelona	E1	B7	P3	280	0.0	28.0	28.0
		B1	P1	330	29.0	62.0	33.0
	E2	B4	P2	200	0.0	20.0	20.0
		B5	P2	330	20.5	53.5	33.0
Bratislava	E1	B7	P3	265	0.0	26.5	26.5
		B8	P3	200	27.0	47.0	20.0
		B9	P3	400	47.5	87.5	40.0
		B4	P2	275	88.5	116.0	27.5
	E2	B5	P2	400	0.0	40.0	40.0
		B6	P2	200	40.5	60.5	20.0
		B1	P1	200	61.5	81.5	20.0
		B2	P1	320	82.0	114.0	32.0
Production cost							$15237.5

	Site	Arrival time	Departure time	P1	P2	P3	Used capacity	
							%w	%v
V1	Barcelona	–	66.5	+330	+530	+280	27.85	49.60
	Madrid	75.5	78.9	−150	−350	−100		
	Valencia	84.0	87.2	−180	−180	−180		
	Barcelona	92.1	–	–	–	–		
V2	Bratislava	–	125.0	+520	+875	+865	55.68	97.50
	Stuttgart	135.8	141.0	−70	−475	−515		
	Frankfurt	143.9	146.1	−50	−150	−100		
	Calais	154.7	156.7	−150	−50	−50		
	Hamburg	167.5	170.1	−200	0	−200		
	Berlin	174.2	176.2	−50	−200	0		
	Bratislava	185.9	–	–	–	–		
V3	Madrid	–	83.5	+550	+300	+300	49.50	56.94
	Lisbon	90.5	93.7	−200	−150	−200		
	Nantes	110.4	111.8	−50	−50	0		
	Bilbao	119.4	122.0	−300	−100	0		
	Madrid	126.8		–	–	–		
V4	Stuttgart	–	147.5	+470	+675	+475	78.35	95.69
	Paris	154.4	158.4	−200	−350	−200		
	Lyon	163.6	166.3	−170	−100	−150		
	Torino	169.8	171.6	−50	−75	−75		
	Milan	173.2	175.2	−50	−150	−50		
	Stuttgart	180.8	–	–	–	–		
Routing cost								$26974.5

the best product flows from factories and warehouses to customer zones. Then, the production activities are made according to the best distribution scheduling generated.

Because production decisions are ignored, both the manufacturing plants and intermediate facilities are considered as full storage locations with products stocks available at the start of the planning horizon. Thus, and in order to find the set of optimal vehicle routes, two major changes have been introduced in the model formulation. On one hand, initial stocks of P1–P3 at both Barcelona-based plant and Stuttgart-based plant are fully available with 1200 units of each type of product. On the other hand, production operations cannot be performed in these manufacturing facilities. Consequently, the routing decisions will be mainly determined by vehicle capacities and distances between cities. The best solution for the transportation problem is illustrated in Fig. 6. As a result, Barcelona satisfies demands from Madrid, Bilbao, Valencia, Nantes, Calais, and Paris, which are served by vehicle V1, while Stuttgart, Frankfurt, Lyon, Torino, Milan, Hamburg, and Berlin have been assigned to Bratislava and visited by V2. The remaining city, Lisbon, is served from the distribution center situated in Madrid through vehicle V3. Pickup and delivery operations are performed by V1 and V3 in Madrid respectively whereas V4 is not use and only delivery tasks are fulfilled by V2 at Stuttgart's warehouse to satisfy the product demands of customers situated within this city. After solving the distribution problem, the assignment variable $VA_{i,v}$ was fixed at their optimal values and the resulting MILP model was solved again but now to find the optimal scheduling production. As expected, this approach produced

Table 9
Computational results for all examples.

	CPU BS[a]	CPU gap[b]	Objective function	Binary variables	Continuous variables	Linear constraints
Scenario 1	7.8	66.4	43903.0	1885	1320	8540
Scenario 2	20.5	616.4	45676.50	1580	1177	7277
Scenario 3	22.9	172.3	42207.00	1592	1165	7277
Scenario 4	179.4	443.4	42212.00	1592	1165	7289

[a] Seconds to find the best solution.
[b] Seconds to achieve a 0.001 relative gap.

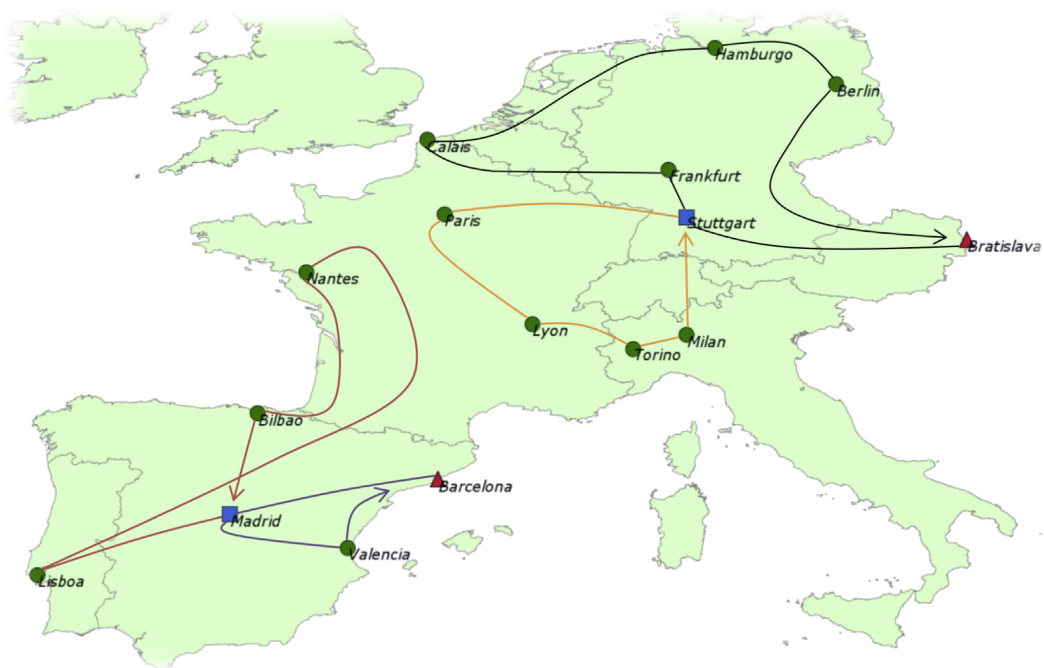

Fig. 8. Optimal vehicle routes for Scenario 3.

significant changes on the production activities performed at the manufacturing facilities regarding to above scenario. The best production and transportation schedule is shown in Fig. 7 and detailed in Table 6. It was found in 20.5 s of CPU time (see Table 9). The total production cost grows 34.43% with regards to Scenario 1 due to the additional customer demands served from Barcelona, which is the factory more expensive. However, the routing cost decreases from $28,770 to $25,334. Finally, the optimal integrated cost has increased from $43,903 to $45,676. By analyzing Fig. 7, it follows that vehicle V3 departs from Madrid at time 2.2 h after it completes the loading activities because cross-docking operations are not performed in the optimal solution. In addition, from such a picture, it is easy to conclude that although direct shipments eliminate the expenses of operating a DC, the non-use of intermediate depots increase the average length of the individual tours, and vehicles routes become longer with regards to Scenario 1.

5.3. Scenario 3

To highlight the benefits associated to the coordination of all activities performed in the supply chain under study, Scenario 3

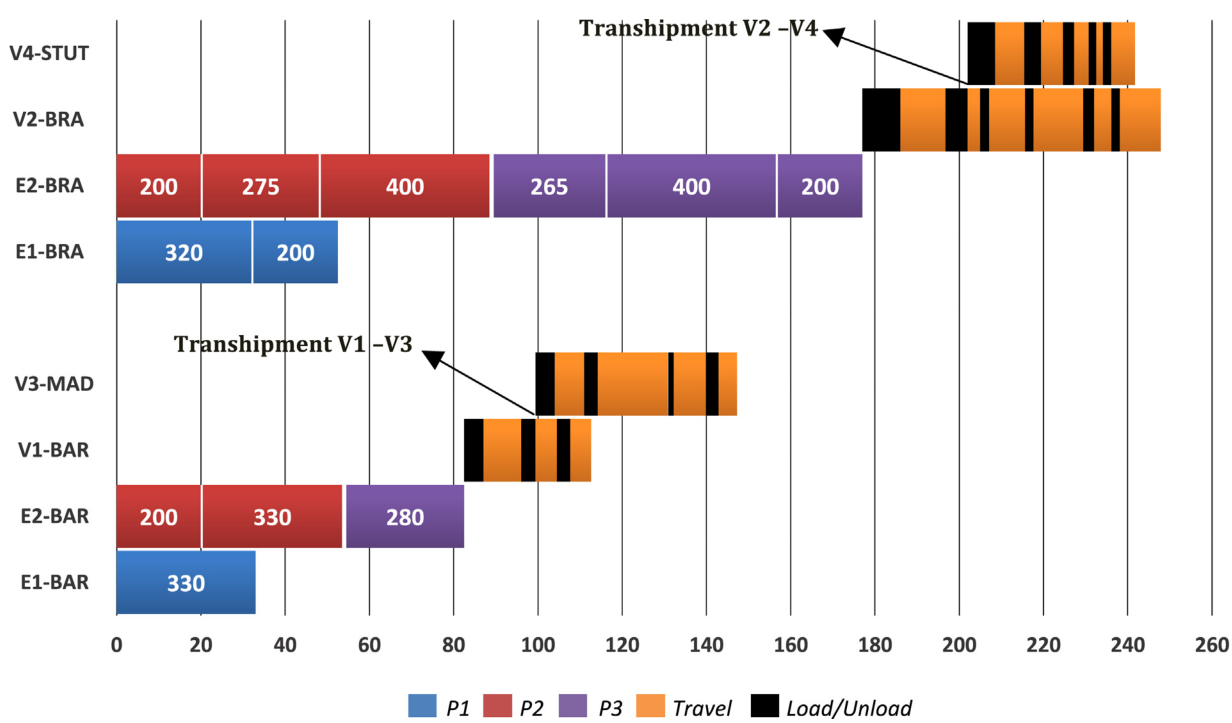

Fig. 9. Coordinated schedule for Scenario 3.

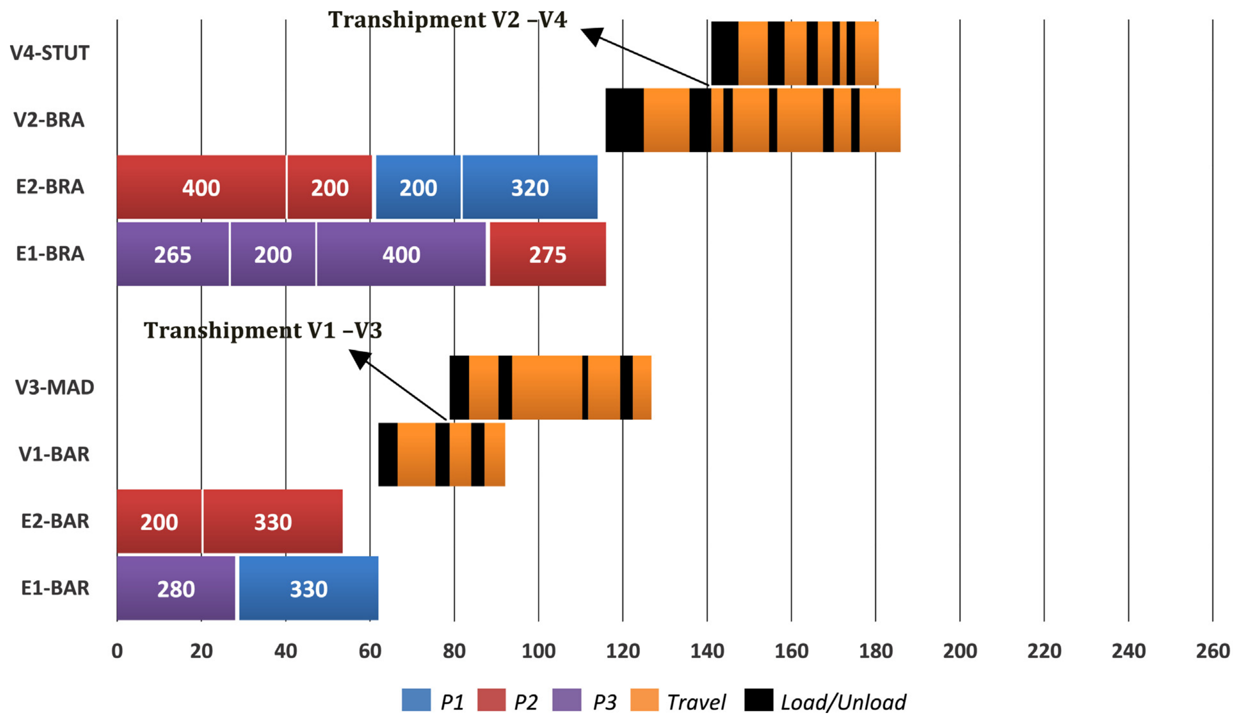

Fig. 10. Coordinated schedule for Scenario 4.

shows the best solution for the fully integrated approach. In contrast to previous scenarios, no decision variable is predefined by this approach and the management of interdependencies between plant operations and vehicles activities are all decisions left to the model. Despite the inherent higher problem complexity and model size (see Table 9), the best solution was found in just 22.9 s. The Gantt chart representation of the coordinated schedule is shown in Fig. 9. The set of optimal routes is presented in Table 7 and illustrated in Fig. 8. As shown in Fig. 9, since only one vehicle is available in each plant, pickup activities performed by vehicle V1 and V2 cannot begin before the batch processing has been completed in Barcelona and Bratislava, respectively. The amounts of products P1 (520 units), P2 (875 units), and P3 (865 units) manufactured at Bratislava-based plant are loaded into V2 and supplied to the assigned destinations whereas V1 departs from Barcelona with the loading of 330 units of P1, 530 units of P2, and 280 units of P3.

It is worth to remark that the optimal solution of the proposed fully integrated approach shows significantly savings with regards to previous scenarios. While compared with Scenario 1 the production cost is higher, the optimal total cost is decreased by almost 4.02%, from $43903.0 to $42207.0. Moreover, significant savings are obtained with respect to Scenario 2. Even though the distribution cost grows by 6.48%, the optimal total cost is decreased by almost 8.22%, from $ 45676.5 to $ 42207.0. Notice that for this particular example, an increase in distribution cost can implies a decrease in the production cost that results in a lower overall cost.

5.4. Scenario 4

The previous scenario is revisited but this time maximum service times for three locations are given. The cities of Paris and Frankfurt must be served before 168 h, while Lisboan's demand has to be satisfied within four days (96 h) counting from the beginning of the planning horizon.

Due to hard time windows are to be satisfied, the best solution, shown in Fig. 10 and detailed in Table 8, was found in 179.4 s. The vehicles routes are the same that Scenario 3 (see Fig. 8) and, consequently, the total transportation cost remain unchanged. However, the production time horizon decreases from 82.5 h to 62.0 h at Barcelona's factory, while Bratislava-based plant reduce the completion time of its production activities in 63 h. The solution obtained demonstrates that the distributing of products to the customer location at a given due date implies a proper consideration of complex temporal and capacity interdependencies arising between production processes and transportations activities. Though no major changes are observed in the optimal value of the objective function, the plants, and consequently the vehicles, complete their tasks much earlier than Scenario 3.

6. Conclusions

This paper has presented a novel optimization approach to the integrated operational planning of multi-echelon multiproduct production and transportation networks. In multi-site systems, products are usually manufactured in one or more factories, moved to warehouses for intermediate storage, and subsequently shipped to retailers or final consumers. To optimally manage such complex networks, an integrated MILP-based framework for production and distribution scheduling in supply chains has been proposed. The model addresses the problem of managing single-stage parallel-line multiproduct batch plants together with multi-echelon distribution networks transporting multiple products from factories to customers through direct shipping and/or via intermediate depots using warehousing and cross docking strategies.

The proposed approach can be applied to solve complex logistics problems in a reasonable computational time, providing a very detailed set of coordinated production and distribution schedules to meet all products demands at minimum total production and transportation cost. To illustrate the applicability and importance of the proposed method, the MILP formulation has been used to solve different scenarios of an illustrative case study involving the management of a supply chain comprising 16 locations geographically spread in six European countries. All variants were solved to optimality in a reasonable CPU time. Numerical solutions show significant potential savings (>8%) that can be obtained by solving the fully deterministic integrated approach. Due to the original decision-making process addressed in this paper involves many exogenous parameters that can vary quickly in the environment, uncertainty management will be carefully studied in a future work.

Acknowledgements

Financial support received from AECID under Grant PCI-A1/044876/11, from UNL under Grant PI-66-337 and ANPCyT under grant PICT-2010-1073 is acknowledged.

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