

Complexity-based discrepancy measures applied to detection of apnea-hypopnea events

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Abstract

In recent years an increasing interest in the development of discriminative methods based on sparse representations with discrete dictionaries for signal classification has been observed. It is still unclear, however, what is the most appropriate way for introducing discriminative information into the sparse representation problem. It is also unknown which is the best discrepancy measure for classification purposes. In the context of feature selection problems, several complexity-based measures have been proposed. The main objective of this work is to explore a method that uses such measures for constructing discriminative sub-dictionaries for detecting apnea-hypopnea events using pulse oximetry signals. Besides traditional discrepancy measures, we study a simple one called difference of conditional activation frequency (DCAF). We additionally explore the combined effect of over-completeness and redundancy of the dictionary as well as the sparsity level of the representation. Results show that complexity-based measures are capable of adequately pointing out discriminative atoms. Particularly DCAF yields competitive averaged detection accuracy rates of 72.57% at low computational cost. Additionally, ROC curve analyses show averaged diagnostic sensitivity and specificity of 81.88% and 87.32%, respectively. This shows that discriminative sub-dictionary construction methods for sparse representations of pulse oximetry signals constitute a valuable tool for apnea-hypopnea screening.

1 Keywords: Discriminative information, discrepancy measures, sparse representation, apnea-hypopnea
2 events, pulse oximetry signal.

3 1 Introduction

4 Although it is widely used and accepted, the notion of complexity has very often avoided a rigorous
5 formalization. It is therefore not surprising that no universally accepted measure exists yet for quantifying
6 such a concept. In particular, within information theory, the complexity of any element of a code, or
7 of any feature of a signal representation in the context of signal processing, is known to be strongly
8 related to the information it carries or, more precisely, to the value of its entropy. It is important to
9 point out however that, in the context of signal classification, the more informative features (in terms
10 of classification) are not necessarily the ones with larger entropy. Hence more “ad-hoc” measures are
11 needed. In fact, any appropriate complexity measure corresponding to a given feature should be instead,
12 strongly related to the amount of information about class membership provided by such a feature. One
13 could then think of using as measure of complexity the conditional entropy of the class given the feature.
14 However, features providing the most discriminative information regarding a class are almost always those
15 with lower conditional entropy values, and hence the best features for classification purposes will be the
16 least complex ones.

17 Information theory was originally based on the engineering of noisy communication channels, and
18 it is closely associated to a large number of disciplines such as signal processing, artificial intelligence,
19 complex systems and pattern recognition, to name only a few. We are particularly interested in the latter.
20 Pattern recognition is a discipline which is mainly oriented to the generation of algorithms or methods
21 that can decide an action based upon certain recognized similarities (patterns) in the input data. Within
22 signal classification, which is perhaps one of the most important subfields of pattern recognition, several
23 discrepancy measures have been used in problems coming from a wide variety of areas such as machine

learning [1], image and speech processing [2], neural networks [3] and biomedical signal processing [4, 5], among others. Among them the most commonly used is probably the Kullback-Leibler (KL) divergence [6, 7]. This divergence, also known as relative entropy, was used as a discriminative measure for selecting, from a large collection of orthonormal bases, the one attaining maximum information [1]. A more recent approach was introduced by Gupta *et.al.* [8] who used this divergence as a discrepancy measure in the traditional k-nearest neighbor (k-NN) algorithm, yielding competitive classification performances in the context of raw electroencephalographic signal classification. Although it provides certain computational and theoretical advantages, the lack of symmetry of the KL divergence has motivated the development of several symmetric versions such as the so called J-divergence [9] and the well known and widely used Jensen-Shannon divergence [10].

Sparse representation of signals constitute a useful technique which has drawn wide interest in recent years due to its success in many applications such as signal and image processing [11]. This technique allows the analysis of the signals by means of only a few well-defined basic waveforms. Due to its advantages, such as robustness to noise and dimension reduction, among others, sparse representation has acquired a large popularity in the area of biomedical signal processing. For example, this technique has been successfully applied to several problems including the estimation of the human respiratory rate [12] and electrocardiographic signal processing, both for signal enhancement and QRS complex detection, for improving heart disease analysis and diagnosis [13]. It is timely to point out however that, up to our knowledge, no applications of discrepancy measures to sparse representation for signals classification are known yet.

All reconstructive methods, such as principal components analysis (PCA), independent components analysis (ICA) and the previously mentioned sparse representations [14], produce particular types of signal representations minimizing a given cost functional which usually involves both fidelity and regularization terms. These methods have been successfully applied in a wide variety of problems such as signal denoising, missing data and outliers, among many others. On the other hand, discriminative methods such as linear discriminant analysis (LDA) are oriented to find optimal decision boundaries to be used for classification tasks. It is well known that for signal classification, which is our main interest in this work, discriminative methods generally outperform reconstructive methods. It is mainly for this reason that several authors have recently developed supervised approaches based on sparse representation which are simultaneously reconstructive and discriminative [15, 16].

The obstructive sleep apnea-hypopnea (OSAH) syndrome [17] is one of the most common sleep disorders and more often that not it remains undiagnosed and therefore not treated. This syndrome is caused by repeated events of partial or total blockage of the upper airway during sleeping, which correspond to events of hypopnea and apnea, respectively. To evaluate the severity degree of the OSAH syndrome, medical physicians have created the so called apnea-hypopnea index (AHI), which is defined as the average number of apnea-hypopnea events per hour of sleep. In terms of this index OSAH is classified as normal, mild, moderate or severe depending on whether such an index falls in the interval $[0, 5)$, $[5, 15)$, $[15, 30)$, or $[30, \infty)$, respectively. The gold standard test for OSAH diagnosis is a study called polysomnography (PSG). However, PSG is both costly and lengthy and the accessibility to this type of study is limited. Additionally, PSG studies require of information coming from a variety of physiological signals such as electroencephalography (EEG), airflow, pulse oximetry (SaO_2), etcetera. It is known however that cessation of breathing associated with apnea-hypopnea events are always accompanied by a drop in the oxygen saturation level in the SaO_2 signal record, although quite often such a drop is very small and almost impossible to detect by a human observer.

The main objective of this work is precisely to develop a technique based on sparse representations and the use of appropriate discriminative information that be able to accurately and efficiently detect apnea-hypopnea events by using only the SaO_2 signal. Several ways exists for combining discriminative information and sparse representations within the context of signal classification. We shall follow one consisting of using the discriminative information for detecting those atoms having the most frequent activations in order to provide them as input for a classifier. This approach was initially introduced in [4] where two methods using the absolute value of the activation differences of the atoms as a measure of the discriminative information for the detection of OSAH were presented. In this work a rigorous formalization of such a measure is introduced and compared with several other discrepancy measures for classifying apnea-hypopnea events. Also, the combined effect of using different sizes of non-redundant dictionaries and different sparsity degree is explored in detail. Results show clearly that the proposed measure is capable of adequately pointing out discriminative atoms in a full dictionary, yielding competitive accuracy rates in the detection of individual apnea-hypopnea events. Additionally, this new approach is computationally very cheap. In fact, it has proved to be at least twice faster than those associated to all other discrepancy measures.

83 The rest of this article is organized as follows: in Section 2 the obstructive sleep apnea-hypopnea
84 syndrome is explained. Sparse representation of signals is introduced in Section 3. In Section 4 sev-
85 eral discriminative information measures are presented. Section 6 contains a detailed description about
86 the performed experiments. Results and discussions are introduced in Section 7 while conclusions are
87 presented in Section 8.

88 2 Sleep apnea-hypopnea

89 Apnea-hypopnea events occur as a consequence of a functional-anatomic disturbance of the upper airway
90 producing its partial or total blockage. At the end of an apnea-hypopnea event, a pronounced desaturation
91 of the blood hemoglobin commonly occurs. These desaturations generate characteristic patterns in the
92 pulse oximetry record known as intermittent hypoxemias. The hypoxemia-reoxygenation cycles promote
93 oxidative stress, angiogenesis and tumor growth, favor the sympathetic activation with increment of
94 blood pressure and systemic and vascular inflammation with endothelial dysfunction which contributes
95 to multi-organic chronic morbidity, metabolic abnormalities and cognitive impairment [18]. Additionally,
96 strong correlations between neoplastic diseases and the OSAH syndrome have been described in [19].
97 Also, a recent study among male mice suggests that OSAH's intermittent hypoxia can be associated to
98 fertility reduction [20]. Currently this pathology affects more than 4% of the human population around
99 the world [21]. Additionally, it was found that aging, male gender, snoring and obesity are all risk factors
100 for OSAH syndrome [22].

101 Although very limited in many countries, overnight polysomnography (PSG) is currently the gold
102 standard tool for diagnosing OSAH syndrome. As previously mentioned, a full PSG consists of the
103 simultaneous measurement of several physiological signals such as EEG, electrocardiography (ECG),
104 respiratory effort, airflow, SaO₂ and electrical activity produced by skeletal muscles (EMG), etc. Mainly
105 due to its ease of acquisition, we are particularly interested in the SaO₂ signal. Figure 1 shows a typical
106 temporal plot of just a few physiological signals coming from a full PSG. This figure also depicts a
107 portion of an original raw airflow signal as well as the corresponding portion of the SaO₂ signal. The
108 corresponding labels of apnea-hypopnea events (dashed lines) are also shown. Finally, at the bottom of
109 this figure, the electrical activity of the heart as well as the sleep stages are shown. In a typical PSG
110 study, after a normal period of sleep the recorded signals are provided to medical experts who analyze the
111 whole record and mark the apnea-hypopnea events and sleep stages, needed for the posterior evaluation
112 the AHI index. Due to its complexity and cost, a few alternatives to PSG have been adopted. One
113 of the most popular ones is the so called home respiratory polygraphy (HRP) [23] which requires no
114 neurophysiological signals. Although studies have shown that there exists a high correlation between
115 AHI values generated by HRP and PSG studies [24], HRP still needs of several physiological signals,
116 whose acquisition strongly affects the normal sleeping of the person. It is therefore highly desirable to
117 develop a reliable OSAH screening system which makes use of as few as possible physiological signals. In
118 this regard, pulse oximetry, being a cheap and non-invasive technique, has become a suitable alternative
119 for screening purposes [25].

120 In this work we shall develop a method for the detection of apnea-hypopnea events that uses only the
121 SaO₂ signals. Our approach leads to a binary classification problem whose main purpose is the detection
122 of the presence (or not) of events of apnea and hypopnea. It is timely to point out that although our
123 method does take into consideration an appropriate fidelity term, we are by no means interested in
124 achieving accurate signal representation.

125 3 Sparse representations

126 As previously mentioned, one of the most popular reconstructive methods is based on sparse represen-
127 tations of the signals involved. Sparsity can be enforced by including upper bounds for the number of
128 non-zero coefficients in the representation of the given signals in terms of atoms in a dictionary.

Formally, the problem of sparse representations of signals can be separated into two sub-problems, the
so-called sparse coding problem and the dictionary learning problem. We shall now proceed to describe
in detail each one of these sub-problems. To be more precise, let $\mathbf{x} \in \mathbb{R}^N$ be a discrete signal and let
 $\Phi \in \mathbb{R}^{N \times M}$ (generally with $M \geq N$) be a dictionary whose columns $\phi_j \in \mathbb{R}^N$ are atoms that we want to
use for obtaining a representation of \mathbf{x} of the form $\mathbf{x} = \Phi \mathbf{a}$. Here, and in the sequel, we shall refer to the
vector $\mathbf{a} = [a_1 \ a_2 \ \dots \ a_M]^T \in \mathbb{R}^M$ as a “representation” of \mathbf{x} . Sparsity consists essentially of obtaining
a representation with as few non-zero elements as possible. A way of obtaining such a representation

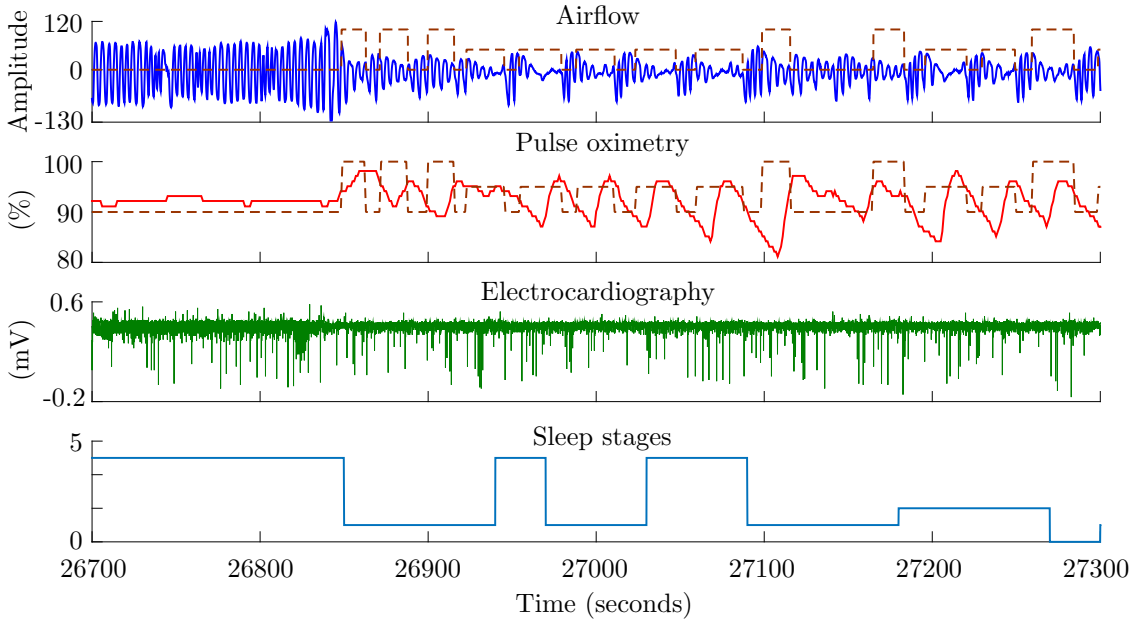


Figure 1: A portion of a few number of physiological signals coming from a full PSG. Dashed lines (brown) are apnea-hypopnea labels introduced by the medical expert.

consists of solving the following problem:

$$(P_0) : \min_{\mathbf{a}} \|\mathbf{a}\|_0 \text{ subject to } \mathbf{x} = \Phi\mathbf{a},$$

129 where $\|\mathbf{a}\|_0$ denotes the l_0 pseudo-norm, defined as the number of non-zero elements of \mathbf{a} .

130 Several questions regarding problem (P_0) immediately arise. Among them: *i*) does there exist an
 131 exact representation $\mathbf{x} = \Phi\mathbf{a}$?, *ii*) if an exact representation exists, is it unique?, *iii*) in the case of non-
 132 uniqueness, how do we find the “sparsest” representation?, *iv*) how difficult is it, from the computational
 133 point of view, to solve problem (P_0) ?. Although it is not an objective of this article to get into details
 134 about the answers to these questions, it turns out that imposing exact representation is most often a too
 135 restrictive and therefore inappropriate constrain and, on the other hand, solving (P_0) is generally an NP
 136 hard problem yielding this approach highly unsuitable for most applications. For more details we refer
 137 the reader to [26, §1.8].

In order to overcome some of the difficulties which entail solving problem (P_0) , several relaxed versions
 of it have been considered. One of them consists of allowing a small representation error while imposing
 an upper bound on the l_0 pseudo-norm of the representation:

$$(P_0^q) : \min_{\mathbf{a}} \|\mathbf{x} - \Phi\mathbf{a}\|_2 \text{ subject to } \|\mathbf{a}\|_0 \leq q,$$

138 where q is a prescribed integer parameter. This formulation takes into account the existence of possible
 139 additive noise terms; in other words it assumes that $\mathbf{x} = \Phi\mathbf{a} + \mathbf{e}$ where $\mathbf{e} \in \mathbb{R}^N$ is a small energy noise
 140 term. Thus, this approach is particularly suitable in most real applications (such as biomedical signal
 141 processing) where measured signals are always contaminated by noise. Several greedy strategies have
 142 been proposed for solving problem (P_0^q) [27, 28]. Among them, orthogonal matching pursuit (OMP)
 143 [28] is perhaps the most commonly used strategy. This greedy algorithm guarantees convergence to the
 144 projection of \mathbf{x} into the span of the dictionary atoms, in no more than q iterations. Figure 2 shows an
 145 example of the values of a particular coefficient a_{j^*} associated to the atom ϕ_{j^*} obtained by applying
 146 the OMP algorithm for a large number (almost half a million) of segments of SaO₂ signals and its
 147 corresponding activation histogram.

148 Although pre-constructed dictionaries, such as the well known wavelet packets [29], typically lead to
 149 fast sparse coding, they are almost always restricted to certain classes of signals. Is is mainly for this
 150 reason that new approaches introducing data-driven dictionary techniques emerged. A dictionary
 151 learning (*DL*) problem consists of simultaneously finding a dictionary Φ and representations of n signals
 152 \mathbf{x}_i , $1 \leq i \leq n$, (in terms of atoms of such a dictionary) complying with a sparsity constraint for each one
 153 of the n signals, while minimizing the total representation error. The (*DL*) problem associated to the
 154 data: $q, M, N \in \mathbb{N}$, $M \geq N$ and n signals in \mathbb{R}^N , $\mathbf{x}_1, \dots, \mathbf{x}_n$, can be formally written as:

$$(DL) : \min_{\substack{\Phi \in \mathbb{R}^{N \times M} \\ \mathbf{a}_i \in \mathbb{R}^M, \|\mathbf{a}_i\|_0 \leq q, 1 \leq i \leq n.}} \sum_{i=1}^n \|\mathbf{x}_i - \Phi \mathbf{a}_i\|_2$$

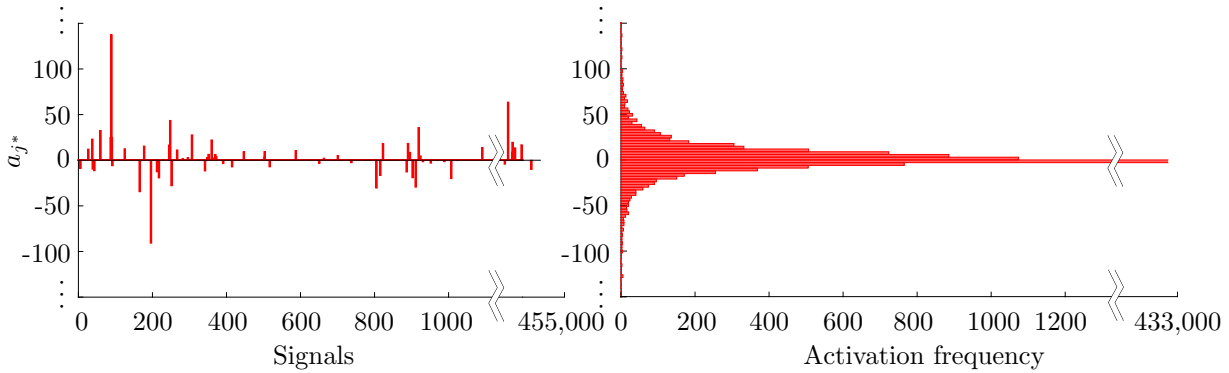


Figure 2: The values of the activations of a particular atom for each signal (left) and the corresponding histogram of activations (right).

155 The first data-based dictionary learning algorithms were originally developed almost three decades
 156 ago [30, 31, 32]. Some of them have their roots in probabilistic frameworks by considering the observed
 157 data as realizations of certain random variables [30, 31]. In [31] for example, the authors developed an
 158 algorithm for finding a redundant dictionary that maximizes the likelihood function of the probability
 159 distribution of the data. In that work, an analytic expression for the likelihood function was derived
 160 by approximating the posterior distribution by Gaussian functions. An iterative approach for dictionary
 161 learning, known as the “method for optimal directions” (MOD), was presented in [32]. The sparse coding
 162 stage of this method makes use of the OMP algorithm followed by a simple dictionary updating rule.
 163 A new iterative algorithm was recently proposed by Aharon *et.al.* in [14]. This new approach, called
 164 “K singular value decompositions” (KSVD), consists mainly of two stages: a sparse coding stage and a
 165 dictionary learning stage. The OMP algorithm is used for the sparse coding stage, which is followed by a
 166 dictionary updating step where the atoms are updated one at a time and the representation coefficients
 167 are allowed to change in order to minimize the total representation error.

168 4 Discriminative sub-dictionary construction

169 Although data-driven dictionary learning algorithms produce sparse representations of signals which are
 170 robust against noise and missing data, such representations turn out to be unsuitable if the final objective
 171 is signal classification. This is mainly so because those algorithms do not take into account any a-priori or
 172 available information concerning class membership. In order to overcome this difficulty, some strategies
 173 which incorporate appropriate class information have been proposed [4, 33, 16]. In [33], for instance, the
 174 authors developed a discriminative dictionary learning method by efficiently integrating a single predictive
 175 linear classifier into the cost function of the KSVD algorithm. A method incorporating a discriminative
 176 term into the cost function of the standard KSVD algorithm was presented in [16]. This method finds an
 177 optimal dictionary which is simultaneously representative and discriminative for face recognition tasks.
 178 In this work, we make use of a simple approach for detecting discriminative atoms from a previously
 179 learned dictionary and using them to build a new sub-dictionary. This approach, which was originally
 180 presented in [4], consists of solving two problems, namely: *i*) the above mentioned full (DL) problem
 181 and *ii*) a discriminative sub-dictionary (DSD) construction problem. We shall now proceed to describe
 182 problem *ii*). One way to obtain discriminative sub-dictionaries consists of maximizing an appropriate
 183 discriminative value functional $G(\cdot)$. Given a data matrix $\mathbf{X} \in \mathbb{R}^{N \times n}$, a class label vector $\mathbf{c} \in \mathcal{C}^n$ (where
 184 \mathcal{C} is the set of all classes; in the binary case $\mathcal{C} = \{c_1, c_2\}$), a dictionary $\Phi \in \mathbb{R}^{N \times M}$ and $p \in \mathbb{N}$ (with
 185 $p < M$), the most discriminative sub-dictionary $\hat{\Phi}^d \in \mathbb{R}^{N \times p}$, according to an appropriate prescribed
 186 discriminative value functional $G_{\mathbf{X}, \mathbf{c}, \Phi} : \mathbb{R}^{N \times p} \rightarrow \mathbb{R}_0^+$ is defined as:

$$(DSD) : \hat{\Phi}^d = \underset{\substack{\mathbf{d} \doteq [i_1 \ i_2 \ \dots \ i_p] \\ i_j \in \{1, 2, \dots, M\} \\ i_j \neq i_k \forall j \neq k,}}{\operatorname{argmax}} G_{\mathbf{X}, \mathbf{c}, \Phi}(\Phi^{\mathbf{d}})$$

187 where for $\mathbf{d} \doteq [i_1 \ i_2 \ \dots \ i_p]$, $\Phi^{\mathbf{d}}$ denotes the $N \times p$ matrix whose j^{th} -column is the i_j^{th} -column of Φ .

188 The function G , which must be provided, quantifies the discriminative power of each sub-dictionary $\Phi^{\mathbf{d}}$.
 189 Thus, large values of G correspond to highly discriminative sub-dictionaries while small values of G are
 190 associated to sub-dictionaries with low discriminability.

191 Several questions concerning problem (DSD) clearly emerge. Among them: i) how do we find an
 192 appropriate discriminative value function G ?, ii) given the functional G , does problem (DSD) have a
 193 solution?, iii) if it does, is it unique?, iv) in the case of non-uniqueness, how do we decide which sub-
 194 dictionary, among the optimizers, is the best for our classification purposes?, v) how difficult is it, in
 195 terms of computational cost, to solve problem (DSD)?. Although this problem has not been extensively
 196 studied, is it known that solving (DSD) is computationally very challenging for $p > 1$, mainly due to the
 197 combinatorial explosion problem. A way to overcome the computational complexities entailed by problem
 198 (DSD) consists of defining an appropriate discriminative value functional G for $p = 1$. In that way G is
 199 independently evaluated at each one of the atoms (columns) of Φ and the discriminative sub-dictionary
 200 $\Phi^{\mathbf{d}} \in \mathbb{R}^{N \times p^*}$ is constructed by stacking side-by-side the first p^* ranked columns of Φ with largest G
 201 values. This simplification is based on the assumption that each atom in the dictionary is used to model
 202 specific characteristics that are not completely modeled by the other atoms. Thus, the discriminative
 203 information provided by a particular atom will be different from the information contributed by other
 204 atoms.

205 5 Discriminative value functions for atom selection

206 Several ways for appropriately constructing discriminative value functions G exists. In this section we
 207 present two different approaches to define such a function. Namely *i*) using traditional discrepancy mea-
 208 sures and *ii*) using a new discriminative measure to which we shall refer as the “difference of conditional
 209 activation frequency” (DCAF). We shall previously need to introduce an appropriate setting and termi-
 210 nology regarding probability density functions (PDFs) in the context of sparse representations for signal
 211 classification.

212 Here, and in the sequel, we shall consider the vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ as realizations of a particular
 213 random vector \mathcal{X} . Any sparse representation of those vectors will result in the PDFs of each coefficient
 214 a_j (associated to the atom ϕ_j) showing a very concentrated peak at zero with heavy tails (as depicted in
 215 Figure 2). In the context of binary signal classification it is reasonable to think that if a given atom ϕ_{j^*}
 216 is highly discriminative, then the conditional PDFs $\pi(a_{j^*}|c_1)$ and $\pi(a_{j^*}|c_2)$ will be significantly different.
 217 Thus, if a dictionary Φ is poorly discriminative, then one should expect $\pi(a_j|c_1) \approx \pi(a_j|c_2)$ for all j .

218 Although the elements a_j of the representation vector \mathbf{a} are in general real numbers, for practical
 219 reasons it is appropriate to discretize them. That can be done in the usual way by partitioning the
 220 real line \mathbb{R} into intervals $I_k \doteq ((k - \frac{1}{2}) \Delta, (k + \frac{1}{2}) \Delta]$, $k \in \mathbb{Z}$, of length Δ and the associated discretized
 221 random variable $\mathcal{K}_j \doteq \sum_{k \in \mathbb{Z}} k \chi_{I_k}(a_j)$. The corresponding probability mass function (PMF) is $p_{\mathcal{K}_j}(k) =$
 222 $P(a_j \in I_k) = \int_{I_k} \pi(a_j) da_j$, $k \in \mathbb{Z}$. Figure 3 shows the estimated PMF and the corresponding conditional
 223 PMFs (given each one of the two classes), both for a non-discriminative and a discriminative atom using
 SaO₂ signals.

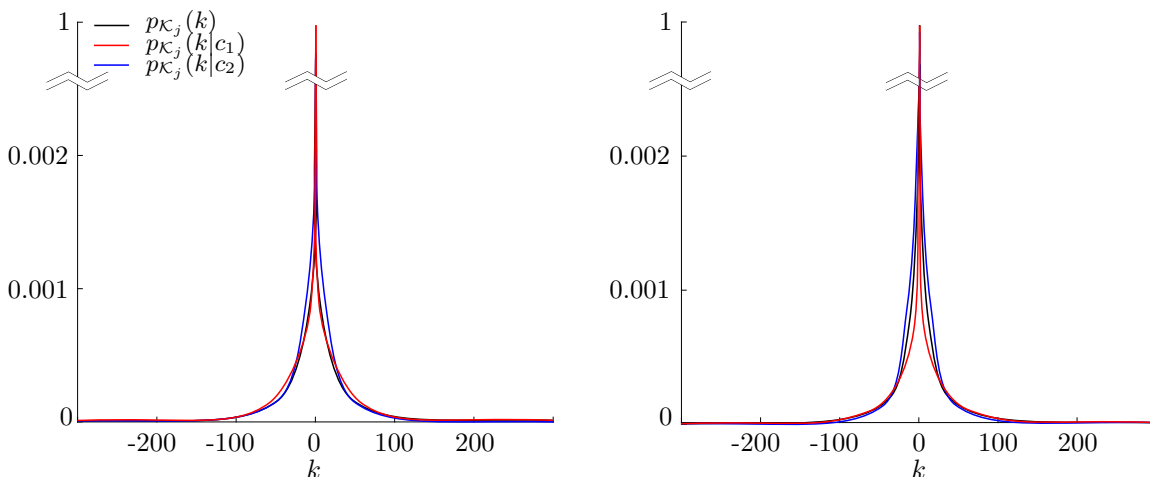


Figure 3: Estimated probability mass functions for a non-discriminative atom ϕ_j (left) and a discrimina-
 tive one (right).

225 We shall now proceed to define how we compute the discriminative value function G . Given the data
 226 matrix $\mathbf{X} \in \mathbb{R}^{N \times n}$, the corresponding class label vector $\mathbf{c} \in \mathcal{C}^n$ and a full dictionary $\Phi \in \mathbb{R}^{N \times M}$, the first
 227 step consists of obtaining the sparse matrix $\mathbf{A} \doteq [\mathbf{a}_1 \ \mathbf{a}_2 \ \cdots \ \mathbf{a}_n] \in \mathbb{R}^{M \times n}$ by applying the OMP algorithm.
 228 The j^{th} -row of this sparse matrix is then used for estimating the conditional PMFs $p_{\mathcal{K}_j}(\cdot|c_1)$ and $p_{\mathcal{K}_j}(\cdot|c_2)$.
 229 Finally, the value of G at the atom ϕ_j is computed as the discrepancy (as quantified by an appropriate
 230 discrepancy measure) between these two PMFs. In what follows, we introduce the discrepancy measures
 231 that we shall use in this work.

232 5.1 Traditional discrepancy measures

233 A great diversity of measures whose purpose is performing comparisons between probability distributions
 234 exists [34]. In this work the best known and more commonly used ones are compared in terms of their
 235 performance for selecting the most discriminative atoms in a dictionary. The KL, J and JS divergence
 236 measures were utilized, along with the Fisher score (F).

237 The KL divergence [7] is probably the most widely used information “distance” measure from a
 238 theoretical framework and it was successfully applied in numerous problems for signal classification [1,
 239 35, 36]. To compare the two conditional PMFs associated with the activation of the j^{th} -atom the KL
 240 distance was used as follows:

$$\text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) \doteq \sum_{k \in \mathbb{Z}} p_{\mathcal{K}_j}(k|c_1) \log \left(\frac{p_{\mathcal{K}_j}(k|c_1)}{p_{\mathcal{K}_j}(k|c_2)} \right), \quad (1)$$

241 assuming that $0 \log(0) \doteq 0$.

242 Despite the computational and theoretical properties provided by KL distance, what usually becomes
 243 a trouble in many problems of signal classification is its lack of symmetry. It can be easily seen that
 244 altering the order of the arguments in (1) can change the output value. To solve this issue a symmetric
 245 version of the KL distance can be used such as the J-divergence [9], which, even though was not initially
 246 created as a symmetric version of the KL distance, is the sum of the two possible KL distances between
 247 probability distributions. In this article the J-divergence is defined as follows:

$$\text{J}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) \doteq \text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) + \text{KL}(p_{\mathcal{K}_j}(\cdot|c_2), p_{\mathcal{K}_j}(\cdot|c_1)). \quad (2)$$

248 Another symmetric smoothed version of the KL distance is the JS divergence [10]. For the problem
 249 of comparing the two conditional probabilities associated to each class it is defined as:

$$\text{JS}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) \doteq w_1 \text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), q_{\mathcal{K}_j}(\cdot)) + w_2 \text{KL}(p_{\mathcal{K}_j}(\cdot|c_2), q_{\mathcal{K}_j}(\cdot)), \quad (3)$$

250 where $q_{\mathcal{K}_j}(\cdot) = w_1 p_{\mathcal{K}_j}(\cdot|c_1) + w_2 p_{\mathcal{K}_j}(\cdot|c_2)$ and w_1 and w_2 are the weights associated to each of the conditional
 251 PMFs, with $w_1, w_2 \geq 0$ and $w_1 + w_2 = 1$. An interesting feature of the JS-distance is the fact that
 252 different values of weights (w_1 and w_2) can be assigned to the probability distributions according to their
 253 importance. In this work $w_1 = P(c_1)$ and $w_2 = P(c_2)$ i.e. the weights are associated with the a-priori
 254 probabilities of the classes. Note that computing the JS-distance as defined here is the same as computing
 255 the mutual information between the class and the activations, i.e. $\text{JS}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) = \text{MI}(\mathcal{K}_j, \mathcal{C})$.

256 Within signal classification problems, F is a measure which has been extensively used. Unlike the other
 257 measures presented here, that require estimations of the conditional PMFs, F uses just two parameters
 258 of the distributions (the means and standard deviations). This makes this measure much less expensive
 259 computationally speaking, but implicitly assumes certain characteristics of the distribution under study
 260 (i.e. second order characteristics). In the case of univariate binary problem at hand the F can be defined
 261 as:

$$\text{F}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) \doteq \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}, \quad (4)$$

262 where μ_ℓ and σ_ℓ^2 are the mean and standard deviation of $p_{\mathcal{K}_j}(\cdot|c_\ell)$ [37].

263 Although the above mentioned discrepancy measures provide, in a certain sense, “measures” of dis-
 264 tance between two probability distribution functions, most of them (such as the KL divergence and those
 265 symmetric variants) are not strictly a metric. For instance, the KL divergence is a non-symmetric dis-
 266 crepancy measure where the triangular inequality is not satisfied. Nevertheless, $\text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2))$
 267 is a non-negative measure, i.e. $\text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) \geq 0$ and $\text{KL}(p_{\mathcal{K}_j}(\cdot|c_1), p_{\mathcal{K}_j}(\cdot|c_2)) = 0$ if and only if
 268 $p_{\mathcal{K}_j}(\cdot|c_1) = p_{\mathcal{K}_j}(\cdot|c_2)$.

5.2 Difference of conditional activation frequency

In a previous work a method called most discriminative column selection (MDCS) for the construction of a discriminative sub-dictionary was originally presented [4]. The sparse representations of the signals in terms of sub-dictionaries constructed using MDCS provided good performance in the detection of apnea-hypopnea events. In the mentioned work, the most discriminative atoms were identified by comparing the difference of conditional activation frequency (DCAF).

The candidates to be considered as “most discriminative” according to [4] are those atoms with higher absolute difference between conditional activation probabilities given the class. That is, an atom is considered as highly discriminative if it is active, in proportion, more times for one of the classes. The use of this approach as a measure of discriminative power follows from the idea that one of the most expressive parameters regarding the importance of a given atom is its activation probability. Moreover, if certain atoms are active mostly for a given class, then it is assumed they represent features of importance in the description of that particular class.

Following this idea, DCAF is defined as:

$$\text{DCAF}(\eta_1^j, \eta_2^j) \doteq |\eta_1^j - \eta_2^j|, \quad (5)$$

where:

$$\eta_\ell^j \doteq \frac{\text{number of activations of the } j^{\text{th}}\text{-atom for } c_\ell}{\text{number of } c_\ell \text{ samples}}. \quad (6)$$

The measure defined in (5) is symmetric, its value is always ≥ 0 , and is inexpensive in terms of computing¹.

It can easily be seen that the definition of η_ℓ^j in (6) is equal to the maximum likelihood estimation of the conditional probability of activation, i.e.:

$$p_{\mathcal{K}_j}(k \neq 0|c_\ell) \approx \eta_\ell^j. \quad (7)$$

Replacing this expression in (5) we can write,

$$\begin{aligned} \text{DCAF}(\eta_1^j, \eta_2^j) &\approx |p_{\mathcal{K}_j}(k \neq 0|c_1) - p_{\mathcal{K}_j}(k \neq 0|c_2)|, \\ &\approx |(1 - p_{\mathcal{K}_j}(k = 0|c_1)) - (1 - p_{\mathcal{K}_j}(k = 0|c_2))|, \\ &\approx |p_{\mathcal{K}_j}(k = 0|c_2) - p_{\mathcal{K}_j}(k = 0|c_1)|, \end{aligned} \quad (8)$$

finally expressing the DCAF in terms of the complementary conditional probabilities that the atoms will not be activated. With the exception of the F, all the measures presented in Section 5.1 can be expressed as summations, where only one of the terms is computed using the probabilities that $k = 0$. However, due to the high sparsity of the representations the terms associated with $k = 0$ are particularly important. This fact allows us to expect some correlation between the results obtained with the different discrepancy measures and the DCAF.

Figure 4 shows a representation of the conditional PMFs associated to the activations of two different atoms (left side) as well as an illustration of such functions where the peaks centered at zero ($k = 0$) were discarded (middle). It is important to note that, when excluding the zero-centered peak from the graphic, a significant reduction in the magnitude of the y -axis scale is produced which highlights the importance of the activation probability of sparse representations. However, the discrepancy between the distributions is not only due to the atoms activation probability, since slight differences between the probability values for all $k \neq 0$ exist (zoom-in region). Additionally, the absolute values of these differences are represented by the gray regions. It is also important to point out that, these area values shown in gray ($\sum_{k \neq 0} |p_{\mathcal{K}_\ell}(k|c_1) - p_{\mathcal{K}_\ell}(k|c_2)|$) are not necessarily equal to those corresponding to the DCAF values. Nevertheless, for symmetric PMFs with high kurtosis and heavy tails (such is the case of the PMFs used in this work), the conditional and a-priori distributions are similar and therefore both area values are close to each other.

6 Experimental setup

This section presents the proposed system and its configuration settings, aimed at detecting patients suspected of suffering from moderate-severe OSAH syndrome. It also describes the database used for training and testing the method along with the measures selected for assessing its performance.

The main objective of our research is to explore the effect of using discrepancy measures to rank the atoms according to their discriminative power. Also, the experiments are designed to determine the

¹If the classes are balanced the DCAF can be replaced just by simply counting, without the necessity of dividing with the number of samples.

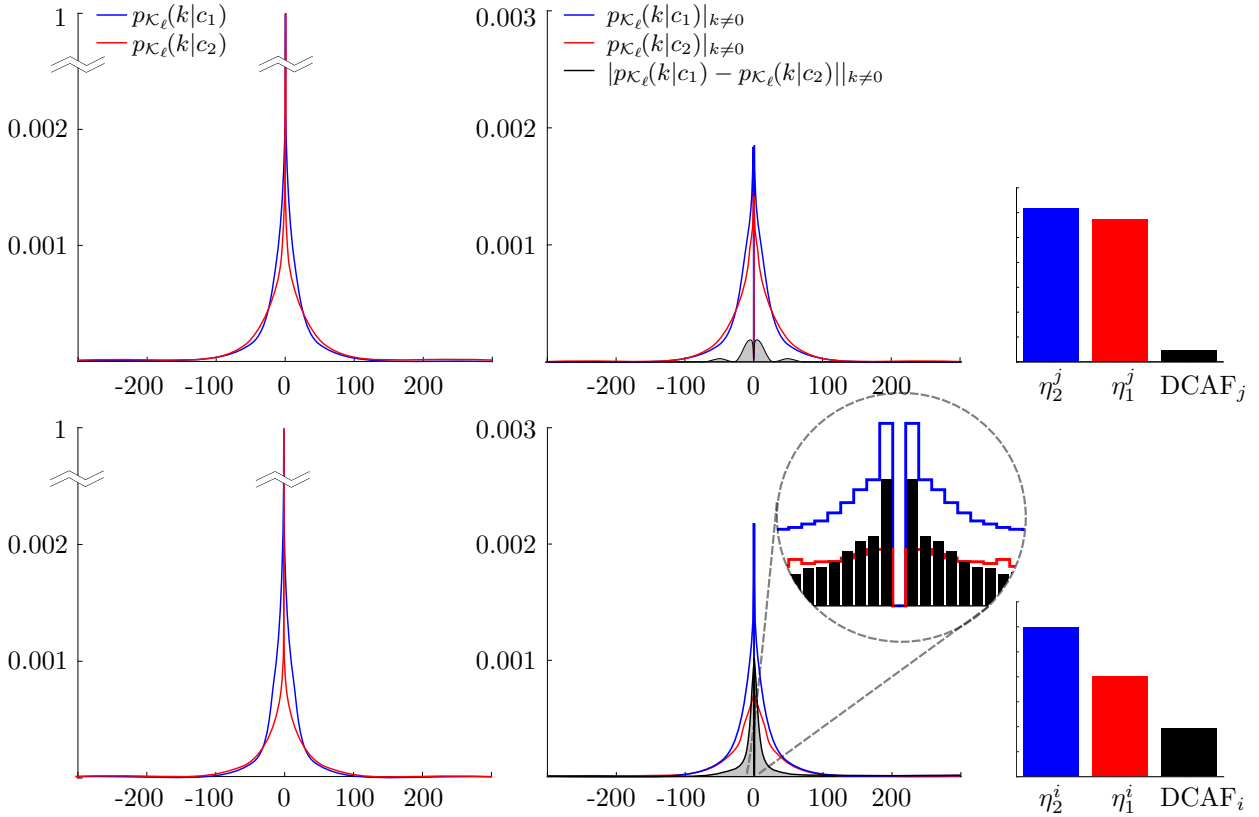


Figure 4: A representation of the conditional PMFs corresponding to the activations of two different atoms (left side), the same functions excluding the peaks centered at zero ($k = 0$) and the absolute value of their differences (middle) and a graphical interpretation of the DCAF (right side). The top row corresponds to a non-discriminative atom (ϕ_j) while the bottom row corresponds to a discriminative one (ϕ_i).

312 effect of using dictionaries with different degree of over-completeness (redundant dictionaries) for the
 313 detection of apnea-hypopnea events. Additionally, the performance of the system for different sizes of
 314 sub-dictionaries and sparsity degrees is analyzed.

315 Figure 5 shows a simplified block diagram of the presented system. It can be observed that our system
 316 comprises a training phase (above) and a testing phase (below). To clarify the system's description, we
 317 divided it into three different stages, namely Stage I, Stage II and Stage III. It can be seen that stages I
 318 and II are included into training and testing phases while Stage III is only used during testing. Stage I
 319 is composed by a pre-processing block whose inputs are the raw SaO₂ signals and its outputs are filtered
 320 segments of such signals, as described in Section 6.1. At the training phase, Stage II receives segmented
 321 signals and finds an optimal discriminative sub-dictionary. During the testing phase, Stage II obtains a
 322 sparse matrix in terms of the previously found sub-dictionary. These processes are thoroughly described
 323 in Section 6.2. Finally, the obtained sparse codes are used as input of Stage III. This stage detects
 324 apnea-hypopnea events and estimates the AHI value, as described in Section 6.3.

325 6.1 Database and signal's pre-processing

326 The sleep heart health study (SHHS) dataset [38, 39] was originally designed to study correlations between
 327 sleep-disordered breathing and cardiovascular diseases. This dataset includes a large number of PSG
 328 studies, each of them containing several physiological signals such as EEG, ECG, nasal airflow, SaO₂,
 329 among others. Medical expert annotations of sleep stages, arousals and apnea-hypopnea events are also
 330 provided. In this work, only the SaO₂ signal (sampled at 1Hz) and its corresponding apnea-hypopnea
 331 labels are considered for performing the experiments. In this article, the first online version of such a
 332 database (SHHS-2) is used. This version of the database contains a total of 995 freely available PSG

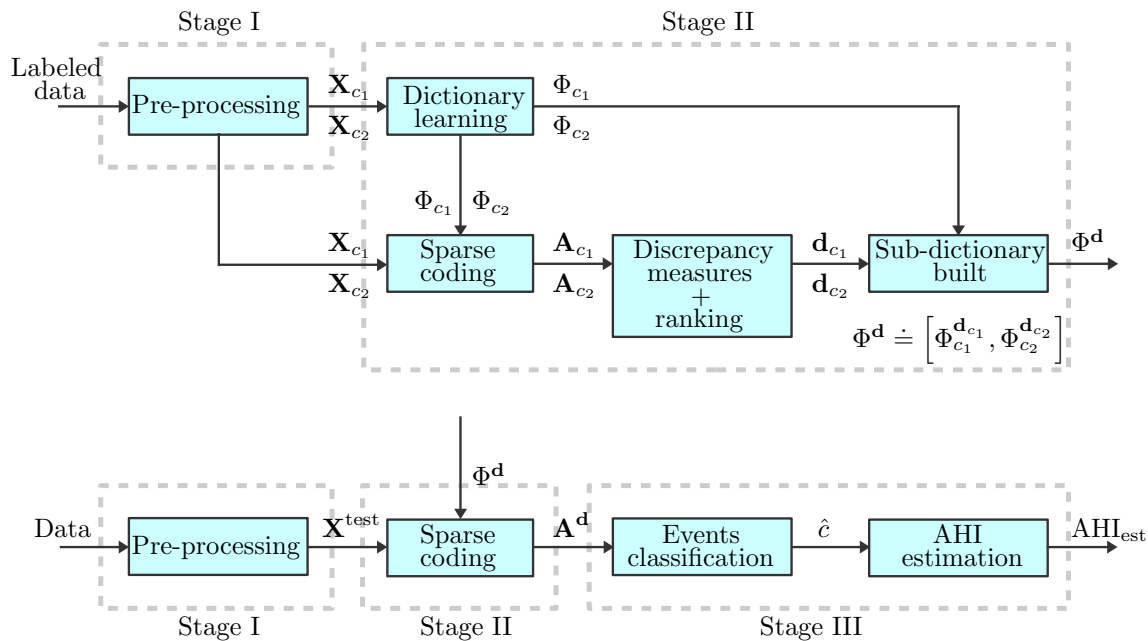


Figure 5: Block diagram of the proposed system during training (top) and testing (bottom).

333 studies².

334 The SaO₂ signals are mainly degraded by patient movements, baseline wander, disconnections and the
 335 limited resolution of pulse oximeters, among others factors. When a disconnection occurs, the recording
 336 during the time interval where the sensor signal is blocked is lost. In order to overcome this inconvenient,
 337 the values of blood oxygen saturation during such an interval are linearly interpolated. To denoise the
 338 signals, a wavelet processing technique [40] is used. The denoising process is performed by zeroing the
 339 approximation coefficients at level 8, as well as the coefficients of the first three detail levels of the discrete
 340 dyadic wavelet transform with mother wavelet Daubechies 2. The signals are then synthesized using the
 341 modified wavelet coefficients by inverse discrete dyadic wavelet transform. The application of this wavelet
 342 decomposition technique has the effect of a band-pass filter where the baseline wander and both the low
 343 frequency noise and the high frequency noise, as well as the quantization noise are eliminated. Figure 6
 344 shows a small fragment of the original raw SaO₂ signal (top) and its wavelet-filtered version (bottom).
 345 Labels of apnea-hypopnea events (dashed lines) introduced by the medical experts are also added. These
 346 labels were generated by medical experts using the airflow information and thus are not aligned to the
 347 desaturations, i.e. there is a variable delay between the start time of an event and the corresponding
 348 desaturation.

349 The application of the sparse representation technique requires an appropriate segmentation of the
 350 signals. Segments of length $N = 128$ (corresponding to 128 seconds of the signal recording) with a 75%
 351 overlapping between two consecutive segments are taken. It is appropriate to point out that although
 352 several overlapping percentages were tested, the best system performances were yielded by a 75% over-
 353 lapping. This redundancy prevents apnea-hypopnea events from being undetected. In this segmentation
 354 process, the time intervals where a disconnection occurs are discarded. The segments of pulse oximetry
 355 signals are then simultaneously arranged as column vectors $\mathbf{x}_i \in \mathbb{R}^N$ and labeled with ones (c_1) and
 356 minus ones (c_2), where a one corresponds to apnea-hypopnea events, and a minus one to the lack of it.
 357 Finally a signal matrix \mathbf{X} is built by stacking side-by-side the column vectors \mathbf{x}_i , i.e. the signal matrix
 358 is defined as $\mathbf{X} \doteq [\mathbf{x}_1 \ \mathbf{x}_2 \ \cdots \ \mathbf{x}_n]$.

359 As mentioned above, the entire dataset used in this work contains 995 complete studies, 41 of which
 360 were not taken into account for performing the experiments since the size of the signal vectors differs from
 361 the corresponding vector of class labels. Among the remaining 954 studies, a subset of 667 (70%) studies
 362 were randomly selected and fixed for learning the dictionary and training the classifier. The remaining
 363 287 (30%) studies were left out for the final test. The SaO₂ signals were filtered using wavelet filters and
 364 segmented as explained previously into column vectors of size 128. After performing the filtering and
 365 segmentation process, a signal matrix $\mathbf{X}^{\text{train}}$ of size 128×455515 is assembled by joining two previously
 366 constructed signal matrices, one for each class, $\mathbf{X}^{\text{train}} \doteq [\mathbf{X}_{c_1}^{\text{train}} \ \mathbf{X}_{c_2}^{\text{train}}]$, which contain 183163 and 272352

²<https://physionet.org/physiobank/>

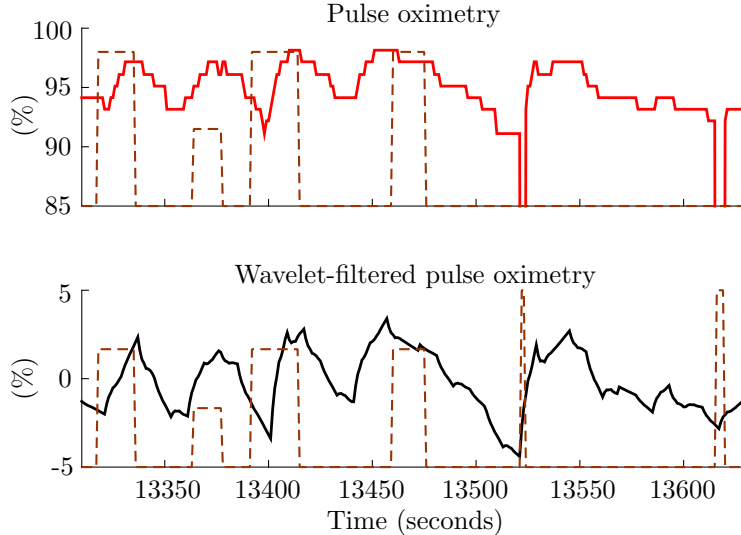


Figure 6: A small fragment of a pulse oximetry signal (top) and its wavelet-filtered version (bottom). Dashed lines represent labels of apnea-hypopnea events established by the medical expert.

367 segments, respectively. On the other hand, for each study included into the testing dataset, a testing
 368 matrix \mathbf{X}^{test} is built.

369 6.2 Sparse coding and sub-dictionary construction

370 In our experiments, the learning of the dictionaries is performed by using the traditional KSVD method
 371 [14]. Optimized MATLAB codes for dictionary learning using KSVD as well as for sparse coding using the
 372 OMP algorithm are freely available for academic and personal use at the Ron Rubinstein’s personal web
 373 page³. At the beginning, the atoms assigned to conform the initial dictionary are randomly selected from
 374 the input signal matrix for training without tacking into account any information about the classes. If
 375 the signal’s space dimension is fixed, which should be the effect of constructing dictionaries with different
 376 over-completeness degree?. To answer this question, three types of dictionaries denoted by $\Phi 1$ of size
 377 128×128 , $\Phi 2$ of size 128×256 and $\Phi 4$ of size 128×512 , corresponding to redundancy factors of 1,
 378 2 and 4, respectively, were built. First the dictionary $\Phi 1$ was constructed by joining two sub-complete
 379 dictionaries of sizes 128×64 denoted by $\Phi 1_{c_1}$ and $\Phi 1_{c_2}$ learned using a large number of training segments
 380 (a total of 100,000 segments for each of the classes) belonging to the classes c_1 and c_2 , respectively.
 381 Following the same idea, redundant dictionaries denoted by $\Phi 2$ (256 atoms) and $\Phi 4$ (512 atoms) were
 382 appropriately built. At the dictionary learning stage the number of non-zero elements was selected and
 383 fixed as a percentage value of 12.5 of the atoms conforming the dictionary. Also a total of 30 iterations
 384 of the KSVD algorithm were performed.

385 Once the dictionary has already been trained, the sparse representation vectors $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_n$ cor-
 386 responding to the input signals $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ are obtained by applying the OMP algorithm. In such a
 387 procedure, the nearest integer number to a percentage value of 12.5 of M is selected and fixed. The reason
 388 for having chosen this percentage value is because it presented the best trade-off between representativity
 389 and discriminability of the segments. Thus, sparsity values of $q = 16$, $q = 32$, and $q = 64$ are selected to
 390 represent the input signals for training in terms of the full dictionaries $\Phi 1$, $\Phi 2$, and $\Phi 4$, respectively.

391 Histograms are typically used to approximate data distributions. In this work we make use of his-
 392 tograms of the atom’s activations to approximate the PDFs. The discretization process was performed
 393 by using a Δ value of 0.5. The detection of the most discriminative atoms is obtained by maximizing the
 394 discrepancy between the conditional PMFs of the atom’s activations given the classes. This objective is
 395 achieved using the proposed DCAF measure as well as those denoted by KL, J, JS and F. The application
 396 of different discrepancy measures to the sparse vectors allows for the selection of different “discriminative
 397 atoms”, which implies the construction of discriminative sub-dictionaries which are essentially different.
 398 The construction of sub-dictionaries, here denoted by $\Phi 1^{\text{d}}$, $\Phi 2^{\text{d}}$, and $\Phi 4^{\text{d}}$, is performed by selecting atoms
 399 from $\Phi 1$, $\Phi 2$, and $\Phi 4$, respectively. Once the most discriminative atoms are detected, the sub-dictionary
 400 is built and consequently the feature vectors are obtained by applying the OMP algorithm. Finally each
 401 feature vector is assigned to be the input of the ELM classifier.

³<http://www.cs.technion.ac.il/~ronrubin/software.html>

6.3 Events detection and AHI estimation

Multilayer perceptron (MLP) neural networks trained for signal classification have proved to be a tool which provides quite good performances for OSAH syndrome detection [4], however, the process of training this class of neural network becomes very costly mainly in terms of time. For this reason, in this work we propose the use of extreme learning machine (ELM) [41] which is a type of single-hidden layer feed-forward neural networks (SLFNs), instead of using MLP neural networks. Theoretically, this algorithm (ELM) results in providing a good generalization performance at extremely fast learning speed. The experimental results based on a few artificial and real benchmark function approximation and classification problems including large complex applications show that ELM can produce good generalization performance in most cases and can learn thousands times faster than conventional popular learning algorithms for feedforward neural networks [42].

Basic ELM classifier’s MATLAB codes are available for download on the Guang-Bin Huang’s web page⁴. To train such a classifier, the main parameters to be fixed are the number of neurons in the hidden layer as well as the activation function of the neurons. In our experiments, the number of neurons in the hidden layer of the ELM corresponds to four times the feature vector dimension. Also the well know sigmoid activation function, which is the most common activation function in the nodes of the hidden and/or output layer, is chosen.

In order to evaluate the performance of the proposed classifier in the detection of individual apnea-hypopnea events (a local approach), or more specifically, in the identification of persons suspected of suffering from moderate-severe OSAH syndrome (a global approach), three performance measures are used. For the identification of single segments containing apnea-hypopnea events, the sensitivity (SE_{AH}) represents the total number of correctly classified segments of signals for which any apnea-hypopnea event occurred. Following the same idea, for the detection of individual segments of signals “not containing” any apnea-hypopnea event, the specificity (SP_{AH}) is defined as the total number of correctly classified segments for which any apnea-hypopnea is not present. The accuracy (AC_{AH}) is finally defined as follows:

$$AC_{AH} \doteq \frac{1}{n} \sum_{i=1}^n \delta(c_i, \hat{c}_i), \quad (9)$$

where n represents the total number of segments, c_i and \hat{c}_i denote the corresponding class label of the i^{th} -segment and the corresponding prediction of the classifier, respectively, and $\delta(x, y)$ represents the delta function whose output is true (one) if the condition $x = y$ is satisfied and false (zero) otherwise.

The differences in performance obtained for the event detection between each discrepancy measure were evaluated in order to test whether or not they are statistically significant. The test was performed assuming statistical independence of the classification errors for the different studies and approximating the error’s Binomial distribution by means of a normal distribution. This assumptions are reasonable due to the large number of SaO_2 signal segments available for each study (about 1100 segments per study, totaling 301306 segments).

The estimated AHI index (AHI_{est}) is defined as the average number of predicted events per hour of study. This new index is used for OSAH syndrome detection. In this case, the sensitivity (SE_{OSAH}) is defined as the ratio of persons with OSAH syndrome for whom the final test is positive, and the specificity (SP_{OSAH}) is defined as the ratio of health patients for whom the final test is negative. Also the area under the ROC curve (AUC) derived from a receiver operating characteristic (ROC) analysis [43] is used. A ROC analysis consists of computing the values of the sensitivity and specificity across all the possible detection threshold (DT) values. Then, the ROC curve is built by performing a plot of 1-specificity versus sensitivity values. This curve has been widely used by medical physicians for evaluating diagnostic tests [44]. A comparison between two different methods can be effectively done by finding the “optimal” (in certain sense) cut-off point of the curve and evaluating their corresponding performances. Finally, the accuracy AC_{OSAH} is defined as follows:

$$AC_{\text{OSAH}} \doteq \frac{1}{m} \sum_{i=1}^m \delta(AHI_{\text{est}}^{(i)} > DT, AHI^{(i)} > 15), \quad (10)$$

where m corresponds to the total number of studies coming from the testing dataset and “DT” is the detection threshold value which adjusts over-estimation of the events produced in the segmentation process. The value of DT results in the best cut-off point of the ROC curve. This point, which maximizes simultaneously sensitivity and specificity, corresponds to the minimum euclidean distance (d_{min}) to the point (0;1) of the ROC curve.

⁴http://www.ntu.edu.sg/home/egbhuang/elm_codes.html

7 Results and discussion

In this section results of the performed experiments are presented and discussed. This section is mainly separated into two sub-sections, namely *i*) the performance tuning section and *ii*) the optimal system performance section.

7.1 Performance tuning

This section presents results of the exploratory experiments performed to find optimal configurations of the proposed system. As explained in Section 6.2, three different full dictionaries called Φ_1 , Φ_2 and Φ_4 were learned by applying the standard KSVD algorithm. In this process, it is expected that most dictionary atoms would capture high frequency oscillations and normal respiration cycles in SaO₂ signals. It is important to point out however that, typical desaturations in signals associated to apnea-hypopnea events should be encoded by some atoms. Secondly, the sparse matrices \mathbf{A}_1 , \mathbf{A}_2 and \mathbf{A}_4 were obtained by applying the OMP algorithm. As described in Section 6.2 several measures were used to quantify the discriminative degree of individual atoms of each one of the studied dictionaries. Finally, the dictionary atoms were ranked in decreasing order of magnitude according to their discriminative power. Figure 7 shows the waveforms of the first seven ranked atoms of the dictionary Φ_1 according to our measure (first row) as well as the first seven ranked atoms of such a dictionary according to all other discrepancy measures (rows from two to five). It can be seen that the most discriminative atom selected by DCAF (dashed waveform) provides information about two well-defined desaturations in the signal. It is also important to point out that, this atom corresponds to the most discriminative one when using J divergence, or eventually when using the JS divergence. Moreover, one can clearly note that no highly discriminative atoms were taken when using Fisher score.

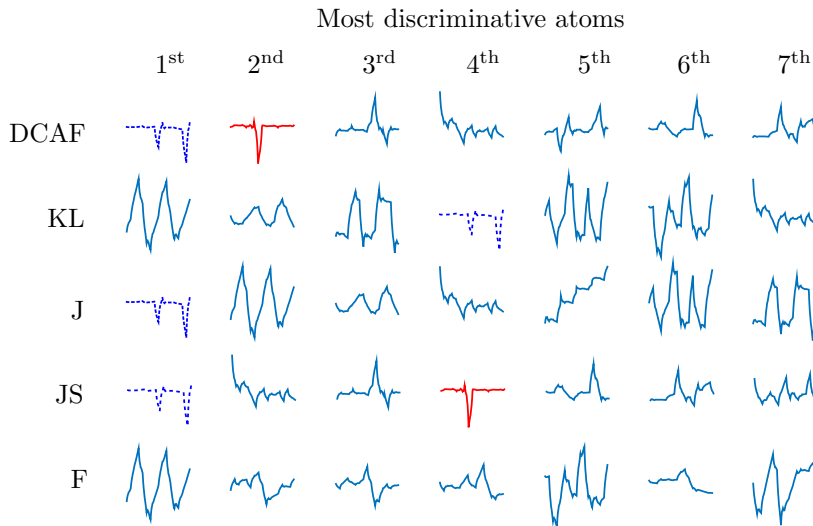


Figure 7: Waveforms corresponding to the first seven ranked atoms according to each one of the evaluated measures.

Discriminative sub-dictionaries called Φ_1^d , Φ_2^d and Φ_4^d were built by stacking side-by-side the first p ranked atoms from Φ_1 , Φ_2 and Φ_4 , respectively, according to their discriminative degree. It is appropriate to mention that, the evaluation of several discrepancy measures leads to the construction of different discriminative sub-dictionaries. However, optimal values of p (sub-dictionary size) and q (sparsity level) are parameters that need to be tuned. In order to find optimal values of such hyper-parameters, a grid search was performed.

The performance of our system was first tested by performing a “random selection” of the dictionary atoms. The involved results were fixed and appropriately used as reference. The random selection of the atoms was performed ten times. Additionally, for each one of the atoms random selection, 60 iterations of the grid search were performed. Thus, the accuracy rate’s variations introduced by the classifier were minimized. Figure 8 shows three images corresponding to averaged accuracy rates for each one of the evaluated dictionaries. Averaged accuracy rates (reference values) obtained by using the dictionary Φ_1 for the detection of apnea-hypopnea events are shown on the left of this figure. It can be seen that sparse representations in terms of Φ_1 , using the smallest sub-dictionary size and the highest sparsity

487 degree, result in better performance than the ones obtained by using all other configurations of Φ_1 and
 488 the over-complete dictionaries Φ_2 and Φ_4 . In this way, two regions can be distinguished corresponding to
 489 a high performance region and a low performance one. The first one, which is of our interest, is yielded
 490 by simultaneously employing a small sub-dictionary size (10%) and a high sparsity degree (5%).

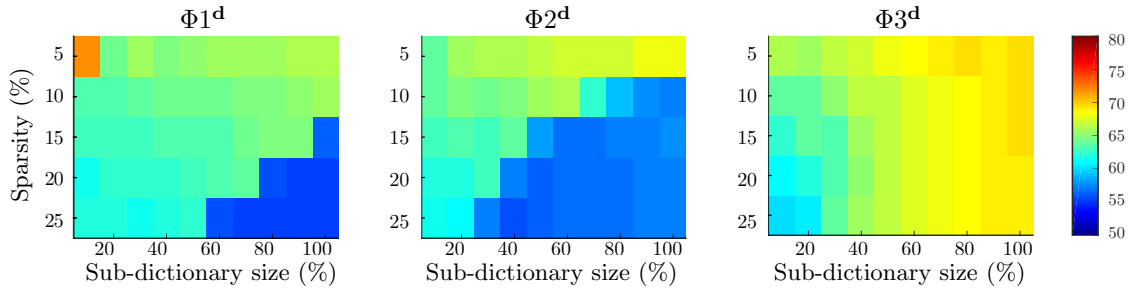


Figure 8: Averaged accuracy rates obtained by varying the percentages of the sub-dictionary size and the sparsity level according to a random ranking of the atoms.

491 Next, DCAF and four other discrepancy measures were used for appropriately constructing discrim-
 492 inative sub-dictionaries. Then, a grid search of hyper-parameters was performed by analyzing the per-
 493 formance that yields our system when using each one of the sub-dictionaries. Figure 9 shows five images
 494 corresponding to DCAF (upper-left) and the other four discrepancy measures. These images represent
 495 the differences between accuracy rates obtained by using discriminative measures and the reference one
 496 (random selection) for Φ_1 . Also, each pixel of these images correspond to particular percentages of sub-
 497 dictionary size and sparsity level. It can be observed that, independently of the discriminative measure,
 498 small percentages of sub-dictionary size yield good performances. It is appropriate to point out however
 499 that, the effect of the dimension (sub-dictionary size) in the performance of the system is more important
 than the one induced by using discriminative measures.

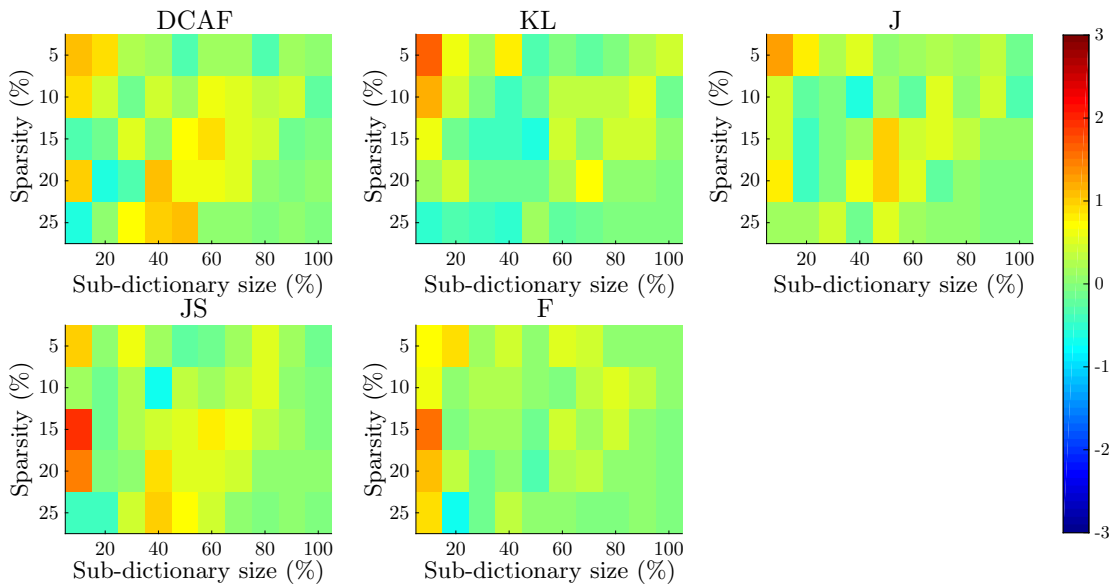


Figure 9: Five images representing differences between accuracy rates yielded by DCAF and all other discrepancy measures and random selection for Φ_1 .

500 Analogously, Figures 10 and 11 show five images which correspond to DCAF (upper-left) and all
 501 other discrepancy measures. The images depicted in Figures 10 and 11 represent the differences between
 502 accuracy rates obtained by using these measures and the reference one for dictionaries Φ_2 and Φ_4 ,
 503 respectively.
 504

505 If we compare the results shown in Figures 9, 10, and 11, then it can be conclude that the proposed
 506 system presents the best performance, in terms of accuracy rate in the detection of apnea-hypopnea
 507 events, when using the full dictionary Φ_1 . Although similar results were obtained applying the proposed
 508 DCAF measure and those traditional ones (see Figure 9), it is important to point out that the use of

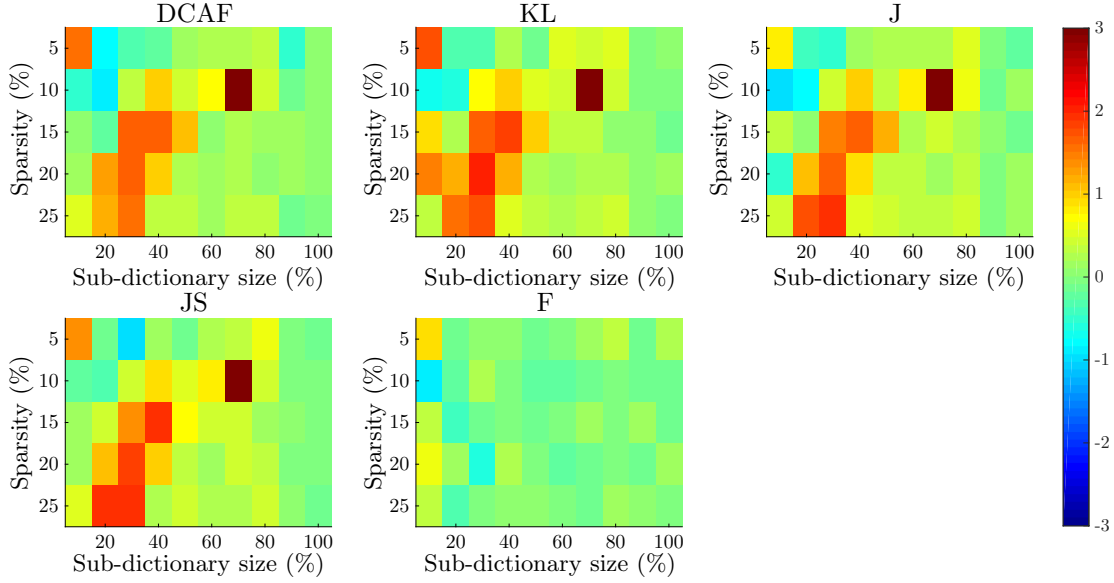


Figure 10: Five images representing differences between accuracy rates yielded by DCAF and all other discrepancy measures and random selection for Φ_2 .

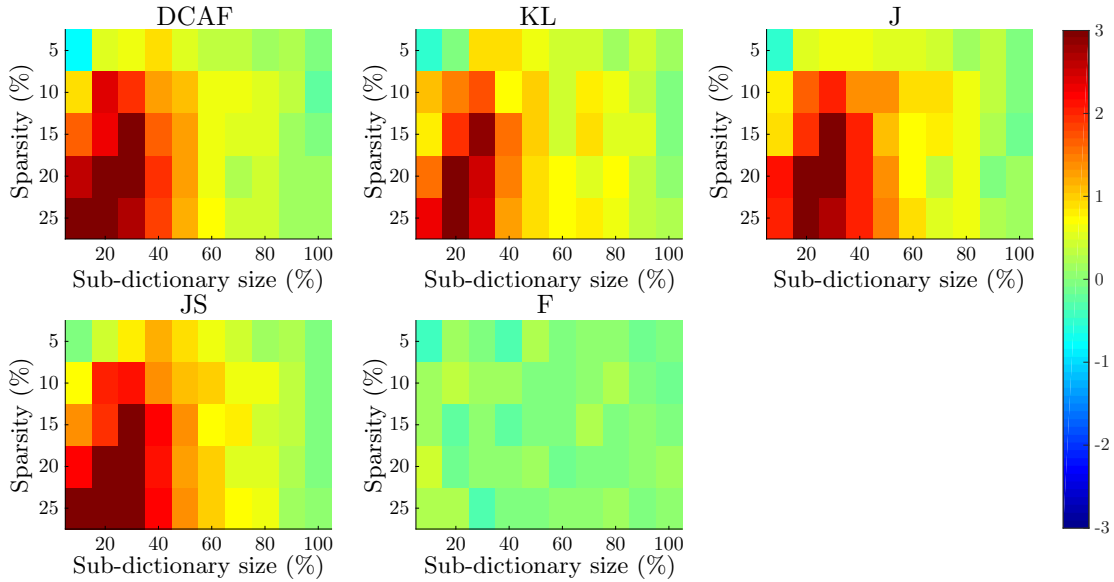


Figure 11: Five images representing differences between accuracy rates yielded by DCAF and all other discrepancy measures and random selection for Φ_4 .

509 discrepancy measures resulted in a significantly high improvement with respect to a “random” selection
 510 of the atoms. As discussed above, the dimension reduction in the sub-dictionary size as well as high
 511 sparse levels yielded high accuracy rates. This is the reason for which a small sub-dictionary size (10%)
 512 and high sparse level (5%) were chosen to perform the final test.

513 System performance changes were analyzed by performing a comparison between averaged accuracy
 514 rates obtained by using discriminative sub-dictionaries and the ones obtained by using full dictionaries.
 515 Table 1 shows averaged accuracy percentages obtained by taken into account fixed discriminative sub-
 516 dictionary sizes (10%) while allowing the sparsity level to change (rows from 3 to 7). The last row of
 517 this table presents averaged accuracy percentages yielded by using full dictionaries for different sparsity
 518 levels. It can be observed that, in all of cases, discriminative sub-dictionaries outperform full dictionaries
 519 in the detection of apnea-hypopnea events.

520 The impact of sparsity degree in the performance of our system is illustrated in Table 2. These results

Table 1: Averaged accuracy rates for sub-dictionary sizes of 10% regarding to each one of the evaluated full dictionaries.

Measure	$\Phi 1^d(128 \times 12)$		$\Phi 2^d(128 \times 24)$		$\Phi 4^d(128 \times 50)$	
	Max	Avg	Max	Avg	Max	Avg
DCAF	72.62	64.68	65.20	63.15	65.19	64.21
KL	73.20	64.91	65.44	63.53	65.42	63.66
J	72.82	64.88	64.50	62.82	65.39	63.68
JS	72.55	64.10	65.02	63.18	65.87	64.01
F	72.23	65.21	64.57	63.04	65.64	62.71
Full dictionary	66.39	59.77	68.13	59.57	69.28	69.21

521 were yielded by averaging accuracy rates obtained for a sparsity level of 5% and considering all possible
522 sub-dictionary sizes (from 10% to 90%). For example, the second row shows averaged accuracy rates
523 obtained by means of discriminative sub-dictionaries whose atoms were taken from $\Phi 1$, $\Phi 2$ and $\Phi 4$ by
using DCAF measure.

Table 2: Averaged accuracy rates by considering a sparsity level of 5% regarding to all possible sub-dictionary sizes.

Measure	$\Phi 1$	$\Phi 2$	$\Phi 4$
DCAF	66.41	66.51	67.95
KL	66.49	66.72	67.98
J	66.60	66.56	67.98
JS	66.41	66.57	68.15
F	66.53	66.54	67.58

524

525 7.2 Optimal system performance

526 Optimal system configurations were selected and fixed to perform the final test. In the previous section
527 it was found that discriminative sub-dictionaries constructed by taken atoms from the dictionary $\Phi 1$
528 yields better performances than the ones constructed by selecting atoms from the dictionaries $\Phi 2$ and
529 $\Phi 4$. Additionally, it was found that a discriminative sub-dictionary composed by only 12 atoms (10%)
530 and a sparsity level of one (5%) yield in the best accuracy rate of our system.

531 In order to overcome the variance introduced by ELM predictors, 60 repetitions of the testing process
532 were performed. Table 3 shows percentage values of minimum (Min), maximum (Max), average (μ)
533 and standard deviation (σ) corresponding to obtained accuracy rates in the detection of apnea-hypopnea
534 events. Although, DCAF perform similarly to the four other discrepancy measures, its performance is
535 achieved with a relatively low computational cost. Additionally, results show that performances obtained
536 by using discriminative measures for constructing sub-dictionaries always outperform the ones yielded by
537 making use of randomly constructed sub-dictionaries.

Table 3: Averaged accuracy rates for a sub-dictionary percentage of 10 for the detection of apnea-hypopnea events.

Measure	Min	Max	μ	σ
DCAF	71.72	73.14	72.57	0.345
Kullback-Leibler	72.06	73.78	73.26	0.390
Jeffrey	71.77	73.31	72.66	0.319
Jensen-Shannon	71.79	73.11	72.55	0.295
Fisher	71.01	72.77	72.18	0.325
Random Selection	70.01	71.51	70.91	0.372

538 We have also evaluated the statistical significance of the results presented in Table 3 by computing
539 the probability that using each one of the evaluated measures, including random selection (RS), yields in
540 better classification performances than the others. In order to perform this test, we assumed the statistical
541 independence of the classification errors for each study. Also it was possible to approximate the error's
542 binomial probability distribution by a normal distribution due to a wide availability of signals (301,306).
543 Table 4 summarizes the results of the performed statistical significance tests by considering a p-value

544 of 0.01. It can be seen that DCAF and three other discrepancy measures (KL, J and JS divergences)
 545 are significant different respect to random selection. Also, no significant difference was found between F
 546 score and random selection. Additionally is was found that DCAF does not perform significantly better
 that the KL, J and JS divergences.

Table 4: A summary of the performed statistical significance tests.

	RS	DCAF	KL	J	JS	F
RS	-	✓	✓	✓	✓	✗
DCAF	-	-	✗	✗	✗	✗
KL	-	-	-	✗	✗	✗
J	-	-	-	-	✗	✗
JS	-	-	-	-	-	✗
F	-	-	-	-	-	-

547 To determine the severity degree of OSAH syndrome, a ROC curve analysis was successfully performed
 548 by considering a detection AHI of 15. This index was selected in order to identify patients suspected
 549 of suffering from moderate-severe OSAH syndrome. Table Table 5 shows the minimum operating (cut-
 550 off) point of the ROC curves and maximum percentages of sensitivity, specificity and accuracy as well as
 551 maximum values of area under the ROC curve for AHI diagnostic threshold values of 15 (Figure 12 (left)).
 552 It can be seen that DCAF resulted in a maximum area under the ROC curve of 0.9250 and sensitivity and
 553 specificity percentages of 81.88 and 87.32, respectively. These are the maximum performance measures at
 554 which the minimum cut-off point of the ROC curve is attained. If we compare the performances attained
 555 between all of the evaluated measures, then the maximum SE and AUC value is yielded by J divergence.
 556 Also, JS divergence outperformed all the others in terms of ACC and DCAF resulted in the minimum
 557 cut-off point of the ROC curve.

Table 5: Maximum cut-off points for testing accuracy for a sub-dictionary percentage of 10 for the
 detection of apnea-hypopnea events.

Measure	d_{min}	SE	SP	ACC	AUC
DCAF	0.2211	81.88	87.32	84.60	0.9250
Kullback-Leibler	0.2242	81.46	87.39	84.43	0.9271
Jeffrey	0.2311	80.86	87.04	83.95	0.9283
Jensen-Shannon	0.2267	80.75	88.03	84.39	0.9244
Fisher	0.2280	80.66	87.91	84.29	0.9252

558 We additionally performed a ROC curve analysis of the averaged performances of DCAF and all
 559 the other discrepancy measures (Figure 12 (right)). A random selection was additionally included in our
 560 results in order to be able to compare performance changes. Table 6 the averaged minimum operating (cut-
 561 off) point of the ROC curves and averaged maximum percentages of sensitivity, specificity and accuracy
 562 as well as averaged maximum values of AUC values for the same OSAH syndrome diagnostic threshold.
 563 The result show that DCAF outperforms all the other discrepancy measures in terms of minimum optimal
 564 operating cut-off point of the ROC curve as well as in terms of sensitivity and accuracy rate. Also KL
 565 divergence resulted in the best averaged area under the curve ROC and the maximum averaged specificity
 566 was yielded by JS divergence. A significant performance improvement was observed when using DCAF
 567 or any of the other discrepancy measures compared to random selection.

569 Several applications exist where it is desirable to maximize the sensitivity. For instance, if the primary
 570 purpose of the test is “screening”, i.e. detection of early disease in a large numbers of apparently healthy
 571 persons, then a high sensitivity is generally desired. With this in mind, if a sensitivity of 98% is chosen
 572 in the ROC curves in Figure 12, for all used measures, the method achieves a specificity close to 45%.
 573 This fact shows that the analysis of pulse oximetry signals by means of the proposed method could be
 574 potentially applied as an efficient diagnostic screening tool in clinical practice.

575 In a previous work [4] it was shown that the MDCS method using DCAF to select discriminative
 576 atoms in a given dictionary, provides good accuracy rates in the detection of apnea-hypopnea events. In
 577 that work, a comparative analysis of the performances yielded by MDCS and other methods [45, 46, 47]
 578 has shown that MDCS outperforms all the others. It was also observed that the computational cost of
 579 MDCS is slightly higher than those required by the other three methods. On the other hand, in this
 580 work we show that MDCS using DCAF for selecting discriminative atoms performs similarly than MDCS

581 using several other traditional discrepancy measures. It is important to highlight that DCAF is very easy
 582 to compute and yields competitive performance rates in the detection of apnea-hypopnea events at a low
 583 computational cost.

Table 6: Averaged cut-off points for testing accuracy for a sub-dictionary percentage of 10 for the detection of apnea-hypopnea events.

Measure	d_{min}	SE	SP	ACC	AUC
DCAF	0.2211	81.88	87.32	84.60	0.9250
Kullback-Leibler	0.2242	81.46	87.39	84.43	0.9271
Jeffrey	0.2311	80.86	87.04	83.95	0.9283
Jensen-Shannon	0.2267	80.75	88.03	84.39	0.9244
Fisher	0.2280	80.66	87.91	84.29	0.9252
Random Selection	0.2396	80.85	85.60	83.23	0.9222

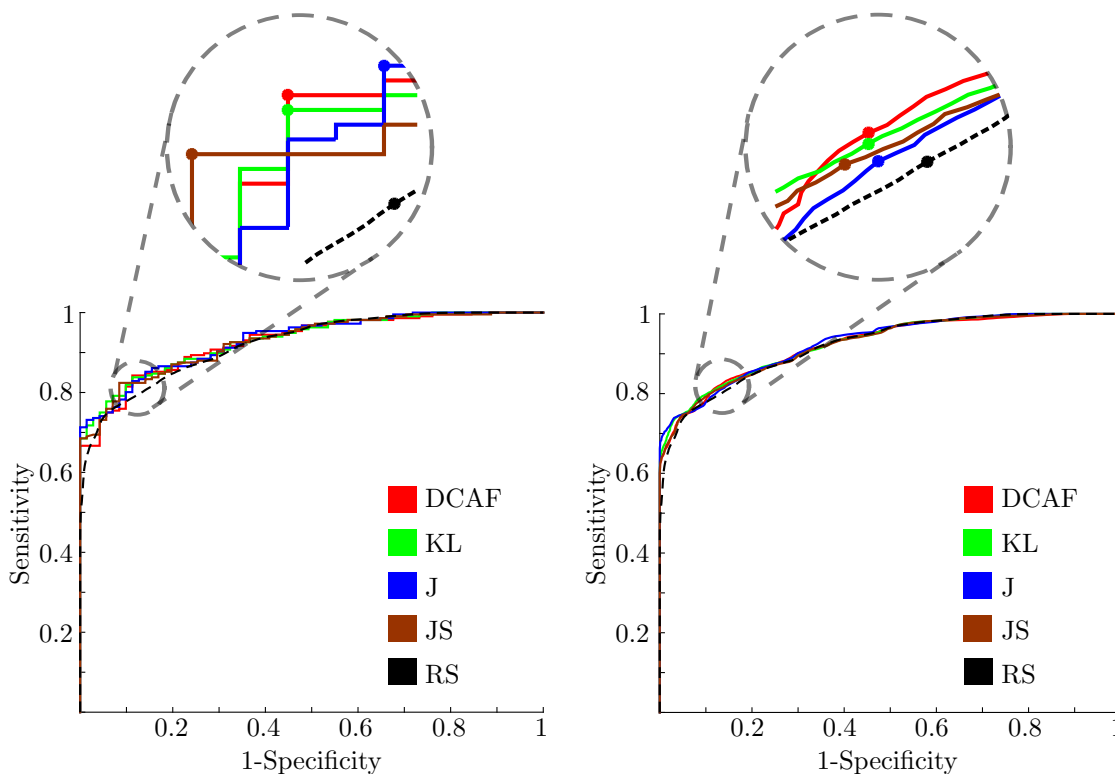


Figure 12: ROC curves corresponding to the performance measures described in Tables 5 and 6.

584 8 Conclusions

585 Sparse representations of signals constitute a powerful technique which yields high accuracy rates in
 586 the detection of apnea-hypopnea events. In this work the difference of conditional activation frequency
 587 (DCAF) measure was successfully used for accurately pointing out discriminative atoms in a full dic-
 588 tionary. Additionally, we compared the performance of the DCAF with four widely used discrepancy
 589 measures. It was found that the DCAF and three other discrepancy measures (KL, J y JS divergences)
 590 outperform the random selection of atoms, unlike F score. Additionally, DCAF is cheaper to compute.
 591 Discriminative sub-dictionaries were successfully constructed by taking the best ranked atoms of full dic-
 592 tionaries according to their discriminative power. Results show that sparse representations of signals in
 593 terms of discriminative sub-dictionaries result in better performances than the ones obtained in terms of
 594 full dictionaries in the detection of apnea-hypopnea events by using only pulse oximetry signals. In this
 595 context, it was found that more sparse solutions almost always yielded in better performances. Addi-
 596 tionally, it was observed that larger dictionary over-completeness worsens the performance of the system.

597 Future research lines include more analysis of the DCAF measure, the study of its properties and an
598 extension of such a measure to multi-class problems, among others.

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