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Security-Constrained Unit Commitment Problem including thermal and pumped storage units: An MILP formulation by the application of linear approximations techniques

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ABSTRACT

This paper presents a new approach for solving Security Constrained Unit Commitment (SCUC) problems based on the application of several linear approximation techniques. Model considers thermal units, transmission constraints, and Pumped Storage Units (PSUs), which are important in power systems during peak and off-peak demand periods. Particularly, the proposed MILP model takes into account the hydraulic heads of PSUs for generating and pumping modes through linear methods of operation point selection. Therefore, a more realistic model is presented in comparison with other models available in the literature. To verify the effectiveness of the proposed approach, a modified IEEE 31-bus power system with two PSUs is tested. Our results indicate that the implementation of PSUs can achieve production cost savings compared with power systems in which thermal generation supplies the entire electricity demand. For improving the power system operations and maintenance, different indicators obtained from results are reported in graphs. This information is useful to identify the most critical parts of the systems and make recommendations for corrective or future actions.

1. Introduction

The electricity production optimization has become a crucial task in the power systems operations. According to Ref. [1], the global demand for electricity has been doubled in the last forty years. During the early decades of this period all attention of power production methods was focused on how to get energy, as economically as possible. The full effects of these methods were not taken into account and there was a clear predominance of fossil fuel sources, nearly 90% of the energy resources. Starting in the mid-80s, a change in the energy production paradigm was evidenced due to fluctuations in crude oil prices and investigations on the effects of fossil fuel pollution. This situation continued until the present; there are currently numerous organizations which strictly monitor the effects of pollution, particularly CO_2 emissions. Nowadays, fossil fuels remain to be the predominant energy source, with over 86%.

Fossil fuels as an energy source presents two disadvantages: the pollution and the eventual depletion of fossil fuels. There are two possible solutions for improving this situation: increasing the participation of renewable sources of energy and enhancing the efficiency of the electric power production.

The problem that determines the most effective combination of a number of generating units in order to meet the forecast demand at minimum cost, is known as Unit Commitment Problem (UC) [2]. These problems have been researched for a long time: from the work of Ref. [3], which the author determines the feasibility of applying Dynamic Programming five decades ago, to new investigations as Ref. [4], where models have been enhanced to account for large scale power systems with 118 buses, 54 thermal units, and 186 lines.

In Ref. [5], the problem than includes hydraulic generation is called as Hydraulic Unit Commitment Problem (HUC), and mixed integer nonlinear algorithms are developed in this work to account power systems with an important participation of these units.

However, if a solution of an UC problem is applied to a real system with transmission lines, inconveniences can appear such as overloads of lines or even unfeasibility of the proposed solution. In literature, there are UC-HUC problems that are solved without considering network constraints. If these constraints are consid-

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Nomenclature

Indexes

i,x,c,t,l	Index for thermal unit, PSU, load, time period, line
bu, bu_i, bu_o	Index for bus, input bus, output bus
xg,n	Index for generation segment, hydraulic head range

Constants

a_i, $b_{i,}c_{i,}$	Coefficients of the fuel cost function of unit *i*
I,X,C,BU,T,XG	Total number of thermal units, PSUs, loads, buses, periods, segment of generation
$p_{i,t}^{UP}/e_{x,t}^{UP}$	Upper limit of power output of thermal unit/PSU [MW]
$dkt_{c,t}/R_{c,t}$	Active load/spinning reserve [MW]
x_l	Line reactance [p.u.]
$\bar{H}_x$	Maximum water level of reservoir [m]
$Hg_{xg,n,x}/Pg_{xg,n,x}/Qg_{xg,n,x}$	Head/power output/discharge of point (xg,n) for PSU *x* in generating mode [m]/[MW]/[m^3/s]
$Pp_{n,x}/Qp_{n,x}$	Consumption/water flow pumped for head n [MW]/[m^3/s]
$\overline{v^{up}}/\underline{v^{up}}/\overline{v_x^{lo}}/\underline{v_x^{lo}}$	Max/min. limit for upper/lower reservoir volume [m^3]
$\mu_{in}^{up}/\mu_{out}^{up}/\mu_{in}^{lo}/\mu_{out}^{lo}$	River water inflow/outflow for upper/lower reservoir [m^3/s]
$c\_slop^{up}/c\_slop_x^{lo}$	Slope of the curve of upper/lower reservoir volume
$c\_ord^{up}/c\_ord_x^{lo}$	Position coefficient for upper/lower reservoir

Variables

$u_{i,t}$	Binary variable 1 if a thermal unit *i* is on, 0 if it is off
$p_{i,t}/e_{x,t}$	Active power for unit *i/x* at time *t* [MW]
$cu_{i,t}/cd_{i,t}$	Startup/shutdown cost of unit i in time *t* [$]
$\theta_{bu_i-bu_o}$	Difference of voltage angle between buses $bu_i - bu_o$ [rad]
$pg_{x,t}/pp_{x,t}$	Power output/consumption, unit *x* at time *t* [MW]
$h_{x,t}$	Hydraulic head for unit *x* at time *t* [m]
$qg_{x,t}/qp_{x,t}$	Water flow discharged/pumped [m^3/s]
$wg_{xg,n,x,t}$	Weight of operational point of PSU in generating mode
$wp_{x,n,t}$	Weight of operational point of PSU in pumping mode
$v_t^{up}/v_{x,t}^{lo}$	Upper/lower reservoir volume at time *t* [Mm^3]
$z_{x,t}^{gen}/z_{x,t}^{pump}$	Binary variable for generating/pumping mode
$\alpha_t^{gen}/\alpha_t^{pump}$	Binary variable for exclusivity of mode
$wl_t^{up}/wl_{x,t}^{lo}$	Water elevation of the upper/lower reservoir [m]

ered, the problem is called Security Constrained Unit Commitment (SCUC). There are two main ways for modeling transmission constraints in the literature: the AC and DC power flow models. A complete description of the AC model is presented in Ref. [6], this model is known by the high level of precision, but the disadvantage is the elevated demand of computational requirement due to nonlinearities and the amount of variables. The DC model is a linearization of the AC model applying assumptions and the most competitive advantage is the substantial computational saving. A detailed description is in Ref. [7].

Within the hydraulic generation field, the Pumped Storage Units (PSUs) are gaining importance at global scale. They propose a number of challenges which are correctly labeled by Ref. [8]. PSUs store energy in the form of water, reducing the production costs. When the energy price is low, usually at nighttime or during the off-peak periods, PSUs pump water from a lower reservoir to an upper one. Then, during peak periods, the water pumped is converted into electrical power as in a conventional hydropower plant. A complete review of PSUs global capacities, technological developments, and recommendations, is presented in Ref. [9].

PSUs differ from the traditional hydraulic power plants since they consume power during pumping cycles, with an efficiency range of 75–85%. The implementation of PSUs has technical and economic benefits because they take advantage of fluctuations in the electricity market prices, and they can provide power grid ancillary services. Older PSUs have single speed and modern PSUs have adjustable speed. Some of the benefits of the adjustable speed technology are reductions on the unit startups and shutdowns during the pumping mode, decreases of disturbances on the power grid, and enhancing of efficiency of power systems.

Mathematical programming is a tool that helps to solve problems. The objective is to choose the best option from a group of available alternatives. In recent times, some linear approximations have been developed in the literature to represent nonlinear problems. In Ref. [10], authors present computational advances that have been contributing to the efficient solution of mixed-integer linear programming (MILP) problems, presenting advantages such as reductions in computational times, global optimality, and the flexibility to add constraints. Linear approximations of UC problems are developed for power systems with thermal units in Ref. [11], but models do not include hydraulic generation. For modeling conventional hydraulic plants, a complete model is presented in Ref. [12], however, PSUs are not included. Linear models for PSUs are presented in Ref. [13], without transmission constraints. This approach takes into account hydraulic heads in generating mode. The hydraulic head is a measure of the elevation and the water pressure at a point in a reservoir that represents the total energy of the water, and its influence that must also be noted. Regard to SCUC problems, transmission constraints are modeled with a linear approximation based on the DC model in Ref. [14], but in this approach, the model is applied only to thermal generation.

In literature, there are approaches that apply different methods to addressing SCUC problems considering some of the previously mentioned items in an integrated way. In Ref. [15], authors propose a strategy based on the stochastic price-based Unit Commitment (PBUC), but unfortunately, head effects are not considered. Responding to these problems, this paper presents a new MILP formulation in order to solve the SCUC problem with realistic linear models, including power generation of base thermal units, cycling units, gas turbine units, and PSUs. This proposal considers characteristics as real generation–discharge curves, hydraulic head effects of PSUs for both operating modes, and a detailed description of reservoirs behaviors. Transmission constraints included are modeled using the DC power flow model to guarantee the feasibility of solutions.

This work aims to find the best solution using an optimization model and taking into account all constraints. For these reasons, the proposed model also differs from others techniques which present iterative, simulation or penalization methods. To illustrate the effectiveness of the new model, a modified IEEE 31-bus with 2 PSUs system is tested. The model has been programmed in GAMS, using CLPEX and Gurobi. Results obtained from the test cases are presented graphically with aims to improve their interpretation. Graphs illustrate the performance of lines, generators, reservoirs, and hydraulic heads. This constitutes a helpful tool in the decision-making processes because the proposed approach allows identifying critical elements to emphasize the maintenance and operations of electric-hydraulic systems.

This paper is organized as follows. Section 2 describes the mathematical model for hydrothermal generation, PSUs, and transmission constraints. Approximations for nonlinearities are presented in Section 3. Section 4 reports numerical testing results and comparisons. Finally, some relevant conclusions are drawn in Section 5.

2. SCUC model, hydrothermal generation and PSUs

The aim of the SCUC is to meet the energy demand, minimizing the production cost. The electric power system modeled considers *I* thermal units, *X* PSUs, *BU* buses, *L* transmission lines, and *C* loads. The objective function presented in Eq. (1) represents the operation cost, including power output costs, shutdown costs and startup costs of all thermal units during the programming horizon *T* (all symbols are defined in nomenclature). The objective function originally presents a nonlinear term.

$$\min C = \sum_{i=1}^{I}\sum_{t=1}^{T}\left[\left(a_i u_{i,t} + b_i p_{i,t} + c_i {p_{i,t}}^2\right) + cu_{i,t} + cd_{i,t}\right] \tag{1}$$

Load system balance is presented in Eq. (2) as follows:

$$\sum_{c=1}^{C} dkt_{c,t} \le \sum_{i=1}^{I} p_{i,t} + \sum_{x=1}^{X} e_{x,t}, \quad t = 1, \ldots, T \tag{2}$$

Spinning reserve is the online power generation capacity available but unloaded that can respond quickly to compensate outages from others units, and is modeled in Eq. (3) as follows:

$$\sum_{c=1}^{C} dkt_{c,t} + \sum_{c=1}^{C} R_{c,t} \le \sum_{i=1}^{I} p_i^{UP} u_{i,t} + \sum_{x=1}^{X} e_{x,t}^{UP} z_{x,t}^{gen}, \quad t = 1, \ldots, T \tag{3}$$

2.1. Thermal generation

Thermal unit constraints which are implemented in this work are presented in Ref. [16], where authors present constraints regarding limit power output, initial status, minimum up/down times, ramp rates, hot/cold startup costs, and shutdown costs. In Ref. [16], a lower amount of variables is needed to solve UC problems, compared with other methods.

2.2. Hydraulic system constraints

PSU power $e_{x,t}$ at time *t* is defined in Eq. (4). It has positive values if the PSU is producing power, negative values when the PSU is pumping water, and zero if the PSU is offline.

$$e_{x,t} = pg_{x,t} - pp_{x,t}, \quad x = 1, \ldots\ldots X, t = 1, \ldots, T \tag{4}$$

Complete equations for the power output and power consumption of each PSU is defined in Ref. [17], which depend on the water flow discharge/pumped by the PSU and the hydraulic head. These equations are nonlinear and will be linearized in the next section.

Constraint (5) shows the exclusivity of different PSU modes (generating, pumping, or offline) and avoids overlaps between modes of PSUs at the same time period.

$$z_{x,t}^{gen} + z_{x,t}^{pump} \le 1, \quad x = 1, \ldots\ldots X, t = 1, \ldots, T \tag{5}$$

Binary variable α_t^{gen} is implemented for ensuring that if a PSU is in pumping mode, the rest of PSUs must not be in generating mode. This implies that if $z_{x,t}^{pump} = 1$, thus all $z_{x,t}^{gen} = 0$. Otherwise, if $z_{x,t}^{pump} = 0$, then the rest of $z_{x,t}^{gen} = 0$ or 1. These constraints are modeled by Eqs. (6) and (7) as follows:

$$\sum_{x=1}^{X} z_{x,t}^{gen} \le X\alpha_t^{gen}, \quad t = 1, \ldots, T \tag{6}$$

$$z_{x,t}^{pump} \le X\left(1 - \alpha_t^{gen}\right), \quad t =, \ldots, T \tag{7}$$

Similar reasoning is expressed in Eqs. (8) and (9). Variable α_t^{pump} is set up to ensure that if a PSU is in generating mode, the rest of PSUs cannot be in pumping mode.

$$\sum_{x=1}^{X} z_{x,t}^{pump} \le X\alpha_t^{pump}, \quad t = 1, \ldots, T \tag{8}$$

$$z_{x,t}^{gen} \le X\left(1 - \alpha_t^{pump}\right) \quad t = 1, \ldots, T \tag{9}$$

Eqs. (10) and (11) calculate the water level in upper and lower reservoirs, taking into account their volumes. Constants *c_slop* and *c_ord* are obtained from the geometrical data of each reservoir.

$$wl_t^{up} = c\_slop^{up} v_t^{up} + c\_ord^{up}, \quad t = 1, \ldots, T \tag{10}$$

$$wl_{x,t}^{lo} = c\_slop_x^{lo} v_{x,t}^{lo} + c\_ord_x^{lo}, \quad x = 1, \ldots, X, t = 1, \ldots, T \tag{11}$$

Eq. (12) defines the hydraulic head for each PSU as the difference between reservoir water levels.

$$h_{x,t} = wl_t^{up} - wl_{x,t}^{lo}, \quad x = 1, \ldots, X; t = 1, \ldots, T \tag{12}$$

Maximum and minimum limits of reservoir volumes are determined by Eqs. (13) and (14).

$$\overline{v^{up}} \le v_t^{up} \le \underline{v^{up}}, \quad t = 1, \ldots, T \tag{13}$$

$$\overline{v_x^{lo}} \le v_{x,t}^{lo} \le \underline{v_x^{lo}}, \quad x = 1, \ldots, X, t = 1, \ldots, T \tag{14}$$

Eqs. (15) and (16) calculate upper and lower reservoir volumes. These values are affected by the water flows which are pumped or discharged by PSUs, in addition to the water flows from the rivers. The water inflows and outflows are assumed as constant values for the purposes of this approach.

$$v_t^{up} = v_{t-1}^{up} + \mu_{in}^{up} + \mu_{out}^{up} - \sum_{x=1}^{X} qg_{x,t} + \sum_{x=1}^{X} qp_{x,t}, \quad \mathrm{x = 1, \ldots, X, t = 1, \ldots, T} \tag{15}$$

$$v_{x,t}^{lo} = v_{x,t-1}^{lo} + \mu_{x,in}^{lo} + \mu_{x,out}^{lo} + qg_{x,t} - qp_{x,t}, \quad \mathrm{x = 1, \ldots, X, t = 1, \ldots, T} \tag{16}$$

Cyclic conditions are imposed to the upper reservoir in Eq. (17).

$$v_{t=T}^{up} \ge v_{t=1}^{up}, \quad t = 1, \ldots, T \tag{17}$$

2.3. Transmission constraints with AC power flow model

This model presents four variables which are associated to each bus (active power, reactive power, bus voltage, and bus voltage angle). Ref. [18] details their advantages of precision. The complete formulation of the AC power flow model is presented in Ref. [6].

3. The MILP model for solving the SCUC problem

This section presents linearization techniques which are applied to nonlinear equations of the SCUC problem including PSUs.

3.1. Linear approximation of the objective function

An accurate piecewise linear approximation of the fuel cost function is presented in Ref. [19], but only thermal generation is included. The solutions obtained with MILP models are very close to the nonlinear optimal solutions as detailed in Ref. [20].

3.2. Linear approximations for PSUs

Accurate linear approximations of PSU generation are difficult due to the nonlinearity and non-concavity of the hydraulic unit performance curve [21]. Initially, the effects of head variation were ignored in literature in order to avoid nonlinearities. However, this can be an inconvenience and may conduct to inaccuracies. To guarantee against the above problems, our model applies the triangular tessellation in a two-dimensional space method. This method considers nine operation points and three curves which represent three hydraulic head ranges H_1, H_2, and H_3, respectively. Ref. [22] suggests minimum and maximum output limits to ensure that the PSU operates in the safety zone, avoiding problems as cavitation, mechanical vibrations, and low efficiency.

Then, the three curves on the safety zone are approximated by three segments in two dimensions that constitute nine data points. These points are denoted in the x–y axis and the space is divided into eight triangles. By adopting this method, our model only requires three binary variables to represent a point within a triangle by the weighting sum of the three corner points.

Eq. (18) determines the weighting sum of the nine points. The weighting sum is equal to 1 when a PSU is operating in generating mode.

$$\sum_{xg=1}^{XG}\sum_{n=1}^{N} wg_{xg,n,x,t} = z^{gen}_{x,t},$$
$$x = 1,\ldots,X,\ t = 1,\ldots,T \tag{18}$$

The hydraulic head for each PSU is determined by Eqs. (19) and (20). If a PSU is in generating mode, the head value is equal to the sum of the weighted head of each data point. The maximum water level of reservoir is included in Eq. (19) to deactivate this constraint when the PSU is not operating in generating mode.

$$h_{x,t} \le \sum_{xg=1}^{XG}\sum_{n=1}^{N} wg_{xg,n,x,t}\mathrm{Hg}_{xg,n,x} + \bar{H}_x(1 - z^{gen}_{x,t}),$$
$$\mathrm{x = 1,\ldots,X,\ t = 1,\ldots,T} \tag{19}$$

$$h_{x,t} \ge \sum_{xg=1}^{XG}\sum_{n=1}^{N} wg_{xg,n,x,t}Hg_{xg,n,x},$$
$$\mathrm{x = 1,\ldots,X,\ t = 1,\ldots,T} \tag{20}$$

The PSU power output at time t is defined by Eq. (21).

$$pg_{x,t} = \sum_{xg=1}^{XG}\sum_{n=1}^{N} wg_{xg,n,x,t}Pg_{xg,n,x},$$
$$x = 1,\ldots,X,\ t = 1,\ldots,T \tag{21}$$

The water discharge for the PSU x at the time t is denoted in Eq. (22).

$$qg_{x,t} = \sum_{xg=1}^{XG}\sum_{n=1}^{N} wg_{xg,n,x,t}Qg_{xg,n,x},$$
$$\mathrm{x = 1,\ldots,X,\ t = 1,\ldots,T} \tag{22}$$

In pumping mode, N hydraulic head ranges are also considered. A three point method is applied in the following equations for linearizing the consumption–pumping relationship. In Eq. (23), the weighted sum indicates if the PSU is in pumping mode.

$$\sum_{n=1}^{N} wp_{x,n,t} = z^{pump}_{x,t}, \quad x = 1,\ldots,X,\ t = 1,\ldots,T \tag{23}$$

The power consumed and the pumped water flow by PSUs at time t are defined in Eqs. (24) and (25), respectively.

$$pp_{x,t} = \sum_{n=1}^{N} wp_{x,n,t}Pp_{n,x}, \quad x = 1,\ldots,X,\ t = 1,\ldots,T \tag{24}$$

$$qp_{x,t} = \sum_{n=1}^{N} wp_{x,n,t}Qp_{n,x}, \quad x = 1,\ldots,X,\ t = 1,\ldots,T \tag{25}$$

3.3. Transmission constraints with the DC power flow model

Often, precision requirements are less important than the speed requirements, especially in real power systems. Due to this reason, the DC power flow model is developed by linearizing the AC model based on three assumptions, as is detailed in Ref. [7]: (1) transmission losses are not included, because the line resistances are considered as negligible compared with the line reactances. (2) The voltage difference between buses is minimal. For this reason, all bus voltages are assumed as equal to 1 [p.u.]. (3) The voltage angle differences between neighboring buses are minimal.

Considering that line l connects buses bu_i and bu_o, active power balance is formulated with the DC model by Eq. (26).

$$\sum_{bu=1}^{BU}\sum_{i=1}^{I} p_{i,bu,t} + \sum_{bu=1}^{BU}\sum_{x=1}^{X} e_{x,bu,t} + \sum_{bu_i=1}^{BU}\sum_{bu_o=1}^{BU}\left(\frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_l}\right)$$
$$-\sum_{bu_o=1}^{BU}\sum_{bu_i=1}^{BU}\left(\frac{\theta_{bu_i,t} - \theta_{bu_o,t}}{x_l}\right) = \sum_{bu=1}^{BU}\sum_{c=1}^{C} dkt_{c,bu,t},\ t = 1,\ldots,T \tag{26}$$

4. Numerical tests

A modified IEEE 31-bus with two PSUs is presented to illustrate the effectiveness of the MILP model. The one-line diagram is presented in Fig. 1. The power system is composed of sixteen thermal units, forty three transmission lines, eleven power loads, and two PSUs. Models are programmed in GAMS using the linear solvers CPLEX and Gurobi on a computer with an Intel i5 750 (2.67 GHz) processor and 3 GB of RAM. The relative gap is set to zero for all test cases. The time horizon is one day divided by 24 h and the spinning reserve adopted is 10% of the total load demand.

The information related to the whole power system can be found in Ref. [23]. Only the modified generation data for thermal units

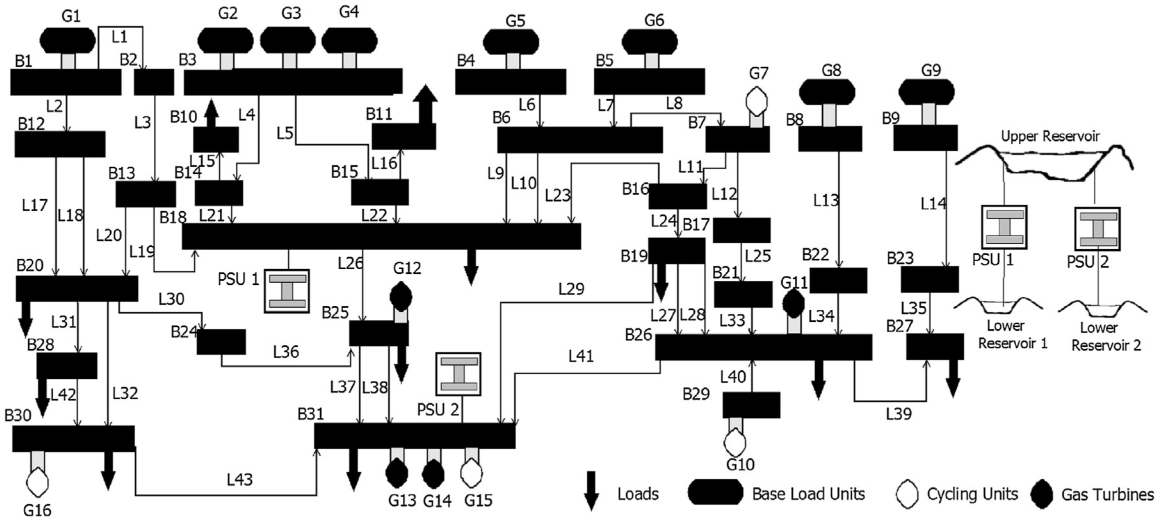

Fig. 1. IEEE 31-bus power system one-line diagram and two PSUs.

Table 1
Modified thermal generation data for the IEEE 31-bus system.

Unit	p_i^{UP} [MW]	a_i [$/h]	b_i [$/MWh]	c_i [$/MW^{2}h]	Unit	p_i^{UP} [MW]	a_i [$/h]	b_i [$/MWh]	c_i [$/MW^{2}h]
1	675	62	2.92	0.000512	9	312	65	2.92	0.000523
2	810	64	2.55	0.0005241	10	202	295	188.11	0.001955
3	810	65	2.55	0.0005311	11	607	57	406.71	0.063123
4	810	78	2.55	0.000534	12	1125	126	334.01	0.059512
5	937	80	2.02	0.000411	13	1406	147	292.32	0.052511
6	937	65	2.02	0.000421	14	353	110	455.11	0.066534
7	1080	192	136.08	0.001523	15	270	214	219.32	0.001812
8	337	94	2.92	0.000512	16	750	220	143.15	0.001712

is presented in Table 1. Two peaks of load demand are distinguished: (1) between hours 9 and 13, and (2) between hours 17 and 20. This paper implements power generation data and water discharge linked to PSUs along with reservoir information as proposed in Ref. [24]. The hydraulic system is composed of two PSUs, with a common upper reservoir and two lower reservoirs. Regularly, upper reservoir is operated with a volume greater than 80% of the maximum capacity to keep energy storage in the form of water. The variation of water elevation for upper reservoir is lower than 10%. Capacity of upper reservoir is 150 [Mm^3], and capacities of two lower reservoirs are 8 and 12 [Mm^3], respectively. The pumping efficiency rates of each PSU are 82.2%, 83.2%, and 83.3%, respectively, corresponding to each hydraulic head range.

Three test cases are tested: (*i*) 31-bus power system—thermal generation. (*ii*) 31-bus power system—hydrothermal generation (without pumping mode). (*iii*) 31-bus power system—hydrothermal generation (with pumping mode). Table 2 presents a comparison between three test cases, showing the main results.

For (*i*), variable $e_{x,t}$ is set to zero and a total amount of 129,613.07 [MW] is produced by thermal units to meet the load demand. Base units 1-6, 8, and 9 produce 83% of the total power output. This high value is due to the lowest cost for each MW produced by these units and their high startup costs. Thus, the obtained solution implies that units 1-6, 8, and 9 are online 24 h with power outputs close to the maximum capacity. Then, particular emphasis must be placed on the maintenance. If any of these units is damaged, the forecast demand will be satisfied by other of the sixteen thermal units that have higher production costs. Thus, the total cost will increase. On the other hand, the combination of units 11, 12 and 14 produces less than 1% of the total power output. According to the aforementioned characteristics, this set of units is committed to meet peaks of demand. Even unit 12 is offline along the programming horizon.

For (*ii*), capacity of pumped storage is disabled, thus the binary variable $z_{x,t}^{pump}$ is set to zero. There are three lapses of the programming horizon in which the two PSUs produce power: hour 1, hours 8–21, and hours 23–24. Hours 8–21 match with the peak load period. This means that PSUs help to relieve the power production of base units during these hours, in order to decrease the total cost. In this case, hydropower generation represents 4.6% of the total power output through the time horizon.

In (*iii*), the pumping mode is enabled. The sixteen thermal units produce 96% of the total power generation. In this case, the power consumption by PSUs that operate in pumping mode is added to the forecast demand. Regarding hydraulic power generation, two lapses are distinguished: the first hour, and hours between 8 and 24. As in (*ii*), peak loads are seen start for hour 8. Hydro generation have an increase of 25.1% in comparison with case (*ii*). This increase is given by PSUs, which are operating in pumping mode during off-peak periods at hours 2–6 and consume 2359 [MW] to pump water. With aims of illustrating case (*iii*) Figs. 2–4 are presented.

Fig. 2 presents the power generation for thermal units 1–16 and the two PSUs along the programming horizon. In the figure, each line of the graph represents the power dispatched form each unit to meet the load demand.

Fig. 3 illustrates power flows transmitted by each transmission line at hour 12 including the maximum capacities. Hour 12 is chosen to be analyzed because the highest value of hourly load demand is produced during this hour. Three lines are occupied at full capacity: lines 24, 27, and 28. In this regard, it is important for the system operator to emphasize the maintenance of these lines. An outage in one of them could lead several complications as overloads in other lines, increases of operational costs, or even a lack of opportunity

Table 2
Comparison between three test cases. 31-bus system.

Case	(*i*) (thermal)	(*ii*) (not pumping)	(*iii*) (pumping)
Operating cost (% cost saving)	$4,150,036 (0%)	$3,194,982 (23%)	$3,074,826 (26%)
Hydro generation	0 [MW]	6358 [MW]	7954 [MW]
CPU time for CPLEX/Gurobi	40.4/30.6 [s]	90.9/123.0 [s]	116.9/125.1 [s]
Total variables (binary variables)	3697 (384)	5425 (840)	5473 (888)

Table 3
Comparison between three methods for solving case (*iii*).

Model	MILP (a)	MINLP (b)	MILP (c)
Solver	CPLEX/Gurobi	SBB	CPLEX/Gurobi
Operating cost	$3,074,826	–	$3,021,157
CPU time [s]	116.9/125.1	3600 (limit)	179.4/156.4
Total variables (binary variables)	5473 (888)	9822 (571)	7548 (790)

Table 4
Comparison of unit utilization and commitment factors for three test cases.

U	(*i*)		(*ii*)		(*iii*)		U	(*i*)		(*ii*)		(*iii*)		U	(*i*)		(*ii*)		(*iii*)	
	UF	CF	UF	CF	UF	CF		UF	CF	UF	CF	UF	CF		UF	CF	UF	CF	UF	CF
1	1	0.8	1	0.8	1	0.8	7	0.5	0.3	0.5	0.3	0.5	0.2	13	0.5	0.1	0.3	0	0.5	0.1
2	1	0.8	1	0.8	1	0.8	8	1	1	1	1	1	1	14	0.1	0	0.1	0	0	0
3	1	0.8	1	0.8	1	0.8	9	1	0.8	1	0.8	1	0.8	15	0.3	0.3	0.2	0.1	0.3	0.1
4	1	0.8	1	0.8	1	0.8	10	0.4	0.3	0.4	0.3	0.4	0.3	16	0.5	0.5	0.5	0.4	0.5	0.3
5	1	0.8	1	0.8	1	0.8	11	0.2	0	0.3	0	0.3	0	P1	–	–	0.4	0.3	0.7	0.6
6	1	0.8	1	0.8	1	0.8	12	0	0	0	0	0	0	P2	–	–	0.7	0.6	0.6	0.6

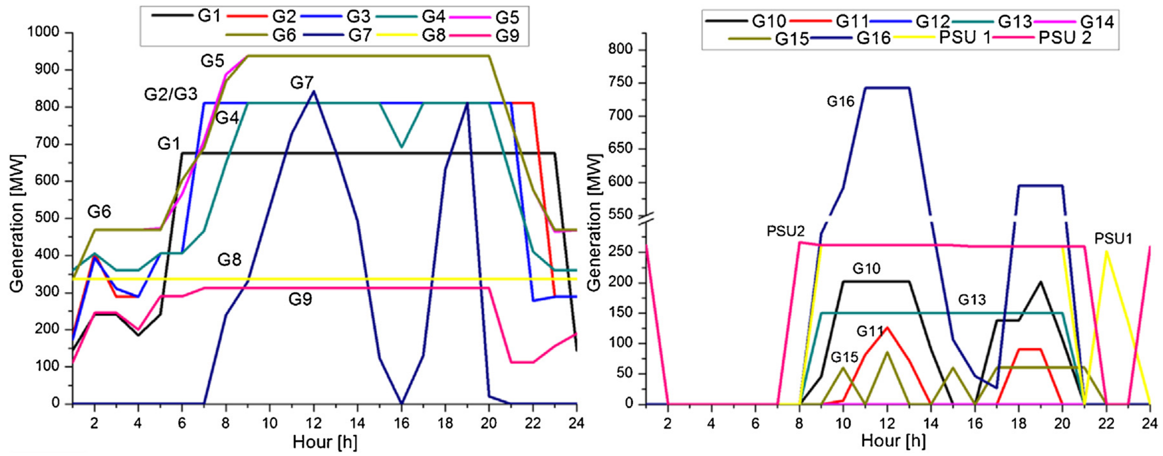

Fig. 2. Generation of 16 thermal units and the two PSUs. Case (*iii*).

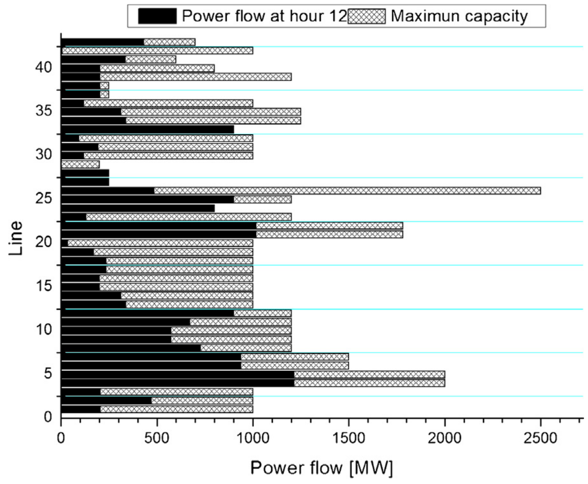

Fig. 3. Occupation of 43 transmission lines at hour 12. Case (*iii*).

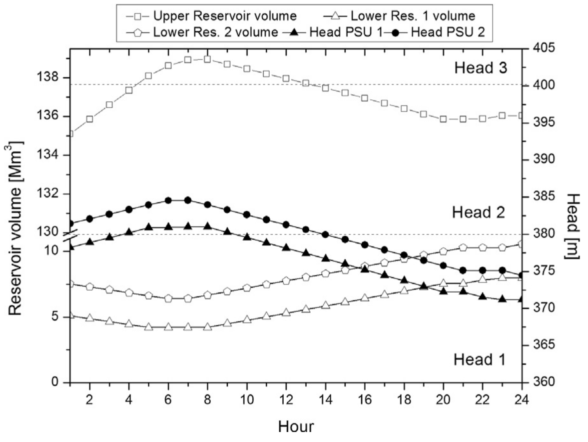

Fig. 4. Reservoir volumes and hydraulic heads. Case (*iii*).

to transmit power flows to load centers because buses 16 and 19 are connected only by line 24 (see Fig. 1). The line utilization factor is the amount of power flow through the line divided by the maximum line capacity. There are fourteen lines with utilization factor percentages between 81% and 50%: lines 4–8, 11–12, 21–22, 25, 37–38, 41, and 43. These lines are not at the limit of their capacity, but the percentages are high and, if there is an increase in the hourly demand, lines can take critical values of utilization factor. The rest of lines have percentages lower than 50% and they have enough idle capacity to support increases in power flows without technical difficulties. They also serve to alleviate the transmission of power flows in lines closer to the maximum capacity.

In connection with the hydraulic system, results about reservoir volumes and the PSU heads are presented in Fig. 4. It can be observed that upper reservoir volume is inversely proportional to the sum of volumes from the two lower reservoirs. The two PSUs pump 2.027 [Mm^3] of water to upper reservoir between hours 2–6. During the rest of the programming horizon, at least one PSU is producing power and the total amount of water discharged by the two PSUs is 8.464 [Mm^3]. Water flows of rivers also have an impact on reservoir volumes. Upper reservoir receives 7.2 [Mm^3] of water and spills 0.072 [Mm^3], while the total values of water inflow and outflow for the two lower reservoirs along the programming horizon are 0.168 and 0.12 [Mm^3], respectively. The figure also illustrates the hydraulic heads for both PSUs. PSU 1 is in the head range 2 (380–390 [m]) between hours 4–9 and PSU 2 between hours 1–13. The rest of the programming horizon, both PSUs are in the head range 1 (380 [m] or less). Due to the fact that PSU 2 remains in the range 2 for seven hours more than PSU 1, the total power generation of PSU 2 is 11.6% higher than PSU 1 along the hours in which both PSUs are operating in generating mode (hours 1, 9–13, 15, 17–20, 24).

Table 3 presents a comparison between three models for (*iii*): (a) the MILP model presented in this paper in Section 3, (b) a mixed integer nonlinear model (MINLP, implementing PSUs power output and consumption equations defined in Ref. [17], and the AC power flow model for transmission constraints given in Ref. [6]), and (c) other linear model based on Ref. [13] (adding transmission constraints with the DC model, and equations for hydrothermal generation). Results indicate that solver SBB was unable to find a feasible solution for the nonlinear model (b) with a time limit of 3600 s. This is due to the hard computational requirement to solve nonlinear equations of AC model and the nonlinear equations for the generation–discharge curve of PSUs.

Compared to linear model (c), the model presented in this paper (a) presents two important differences. First, reductions in the CPU times due to the implementation of the triangular tessellation in a two-dimensional space for PSUs in generating mode. This approximation technique presents several advantages which were previously announced in order to linearization. The second difference is that model (a) considers the head effects for generating and pumping modes of PSUs, while model (c) only considered it in generating mode. As a result, there are differences in the values of total operating costs when PSUs operate in pumping mode along the programming horizon. This implies that the proposed model is more appropriate to be applied in real hydrothermal power systems.

Table 4 presents comparisons of two factors relating to unit operation for three test cases. The unit utilization factor (UF) is the sum of power produced by each unit along the programming horizon, divided by $24p_i^{UP}$. The unit commitment factor (CF) is the sum of hours during which a unit is online divided by 24 h. Comparing (*i*) with (*iii*), and (*ii*) with (*iii*), it can be seen that UF and CF factors for thermal units decrease in (*iii*). It means that PSUs alleviate thermal generation. Instead, UF and CF factors for PSUs are higher in (*iii*) because hydro generation is higher due to the pumping mode.

5. Conclusion

This paper presents an MILP formulation for the SCUC problem with hydrothermal generation and pumped storage capacity. These problems are hard to solve due to the nonlinearities, particularly curves of generation–discharge for hydraulic units and transmission constraints for the AC power flow model. Accurate solutions can be obtained by applying linearizations and auxiliary variables. In consequence, the novel approach presented in this paper applies linear approximation techniques in order to obtain accurate MILP models of electric-hydraulic systems. Effects of hydraulic heads in both operating modes are taken into account, giving a more realistic model in comparison with other approaches in the literature. Results of test cases indicate that the utilization of PSUs achieves cost savings of up to 26%, compared with cases without pumped storage. Besides, solutions are obtained with convenient CPU times of 91 s.

The interpretation of results constitutes a helpful tool for the maintenance and investment operations. Presented tables and figures allow to identify critical elements, improving the performance of power-hydraulic systems, classifying lines into three groups considering the utilization factor, and explaining the hydraulic heads effects.

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