Speed of light demonstration using Doppler beat

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Abstract.

From an apparatus previously designed for measuring the Doppler shift using a rotating mirror, an improved, versatile version was developed for speed-of-light demonstrations in a classroom or a teaching laboratory. By adding a second detector and adequate beam-splitter and mirrors three different configurations are easily assembled. One configuration is used for time-of-flight measurements between a near and a far detector, allowing to measure the speed of light provided that the path length between detectors is known. Another variation is the interferometric method obtained by superposing the far and near signals, in such a way that a minimum of the combined signal is obtained when the time delay makes the signals arrive out of phase by π radians. Finally, the standard Doppler configuration allows the measurement of the frequency beat as a function of the rotation frequency. The main advantages of the apparatus are: a) the experimental setup is simple and completely accessible to undergraduate students, b) the light is visible, students can see the rays, which, with the use of appropriate screens, can be blocked at any point along their paths, c) the experiment can take place entirely within the teaching laboratory or demonstration room (using the interferometric method, the shortest distance to the far mirror was as small as 0.5 m), d) different configurations can be built, including some economical setups within the budget of teaching laboratories.

1. Introduction

There are many methods that are valuable for teaching light propagation. For example, using present day technology, it is possible to reproduce old methods such as Fizeau [1] or Foucault [2] schemes.

Among the methods for demonstrating the speed of light, the time-of-flight technique (TOF) is one of the most common in undergraduate courses. The measurement is direct, as students can measure the time delay of light propagating through a given distance. Further, they can see how the delay changes when the distance between detectors is modified.

There are mainly two types of TOF schemes: those involving short light pulses and those that measure the phase shift (or delay) of a modulated light. The idea of producing a short duration pulse (or a train of pulses) is more attractive because results are simpler to interpret: the separation of the pulses (as seen in an oscilloscope screen) is proportional to the travel time. By knowing the distance between detectors, the value of the speed of light is readily calculated. Light pulses have been generated by means of rotating mirror [3], Pockels-cell modulator [4], acousto-optic modulator [5], or modulated lasers [6, 7, 8].

As a drawback, in order to reach a good resolution, the pulse duration has to be as short as possible, in which case a large bandwidth detector is needed. Bandwidth is the more expensive feature of an oscilloscope. Further, in case the pulse is deformed, students have difficulties in associating a given characteristics in the received pulse, with the corresponding feature in the emitted pulse.

On the other hand, when using a modulated beam at a constant frequency, it is necessary to have a stabilized modulation. Phase shift measurements include amplitude modulated light beams produced by means of Kerr cell [9], modulated LEDs [10, 11], laser pointers [12, 13], or modulated laser [14, 15]. Recently, Pegna [16] has published an apparatus that modulates a collimated red laser at a frequency of 433.92 MHz making possible a speed of light demonstration within a sub-meter measuring base.

In order to reach a reasonable accuracy, many data points per cycle, and as many cycles as possible should be registered. It is thus recommended that a deep memory oscilloscope be used. Since a deep memory oscilloscope is cheaper than a large bandwidth one, the modulation technique, whether measuring the phase or the time difference, should be preferred.

In a previous paper[17] we described the measurement of the Doppler beat produced by a rotating mirror. Due to the fact that the measured beat frequency ranged from 10 to 80 MHz we suggested the use of the apparatus for speed of light demonstrations. By adding a second detector and adequate beam-splitter and mirrors, the modified apparatus can be used for time-of-flight speed of light demonstrations. The two different configurations described above are easily set up. It is possible to emulate the techniques described in Refs. [3, 11], that is, an improved version of the rotating mirror technique and a variation of the interferometer of Ref. [9, 10, 11].

As described in Ref. [17], a Doppler beat is obtained by a rotating mirror, in which one part of a beam is reflected from the advancing side and the other part is reflected from the receding part of the rotating mirror. The Doppler beat frequency produced in the recombined beam, f_b, is given by

$$f_b = \frac{4\pi h F}{c} f_0 \tag{1}$$

where F is the frequency of the rotating mirror, h the beam separation, f_0 the frequency of the light source, and c the speed of light. The above expression is valid to first order in

hF/c. Therefore, the linearity of the beat frequency with the laser frequency is warranty in this kind of experiments where h is in the cm range and F in the 100 Hz range.

Note that for a 80 MHz beat frequency its wavelength is 3.75 m, therefore it is possible to measure different phase shifts ranging from zero to one or more wavelengths within the classroom or laboratory. By improving the motor we were able to reach 200 MHz of beat frequency, thus having a wavelength around 1 meter, which means that measurements can be performed in the range of few meters.

The Doppler beat method exhibits a modulation efficiency of more than 80% which is independent of the frequency, in sharp contrast with other external modulation method. Also, the Doppler method is free from the problems inherent to direct methods as, for example, frequency response, relaxation oscillations, or chirp. As a drawback, large rotating mirrors at high speeds may exhibit problems with air resistance, vibrations, and acoustic noise.

Another advantage of the Doppler beat is that the beat frequency is proportional to the light frequency, while the frequency produced by other modulated methods bears no relation to it. In the present configuration, different sources may be used to investigate the dependence, if any, with the wavelength of the light source.

2. Apparatus

As above mentioned, the beat frequency is a function of the rotation frequency of the mirror. Therefore, in order to increase the beat frequency, we have replaced the old brushed DC motor by a 30,000 rpm outrunner brushless motor (used for model airplanes), which was located at the bottom of the mirror support. Although there are many commercial electronic controllers, we preferred to built a controller with the capability of vary the rotation frequency from 10 Hz to 600 Hz at 1 Hz step. For a beam separation of 1.5 cm and green light, the beat frequency ranges from 3.5 to 200 MHz in steps of $\approx$ 350 kHz. The controller is just an aid to the students for setting up the rotation speed, the actual value is measured by an auxiliary photodetector.

In Fig. 1 the mirror mount, that works up to 100 Hz, is shown at rest (left) and operating at 50 Hz (center), where the reflected laser beams are also visible. For higher speeds up to 600 Hz, a one-piece handcrafted mirror was used (right). It was constructed from a 20 mm side cube of cobalt-iron alloy, of which only one face was polished.

The beat signals are almost free of mechanical noise, because, after recombination, any perturbation in the reflection of one beam is almost cancelled out by the perturbation in the reflection of the other beam.[17] This is one of the strongest point in favour of this rotating mirror method relative to other modulating methods. The other point is the variable modulation frequency in a relatively ample range.

In Fig. 2 the apparatus for Doppler beat measurements is shown. The recombined light that reflects off the rotating mirror is directed, by means of a mirror (M), to a photodetector (PD1). An auxiliary photodiode is used to measure the frequency of the rotating mirror by capturing a reflection from it. Depending on the particular scope of

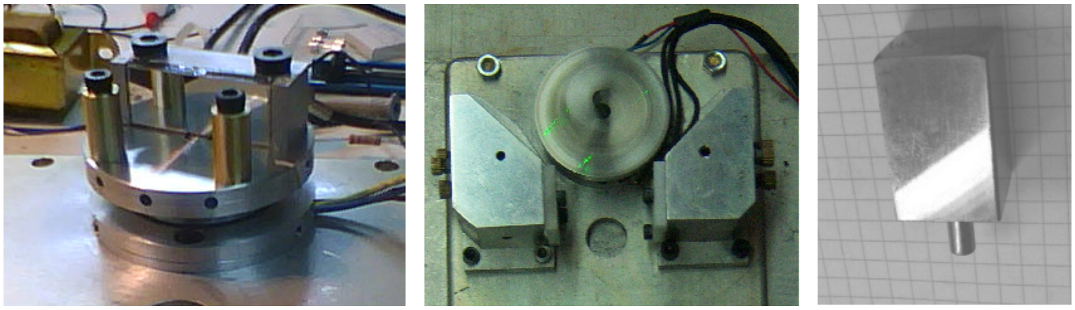


Figure 1. Mount and driver of the rotating mirror (left). The mirror working at 50 Hz (center) in between the optical positioners (the green laser beams are also visible). One-piece handcrafted mirror designed to work up to 600 Hz (right).

the measurement, we have used either a photomultiplier or a fast photodiode. Also, the He-Ne laser can be replaced by a diode laser in all the experiments here described.

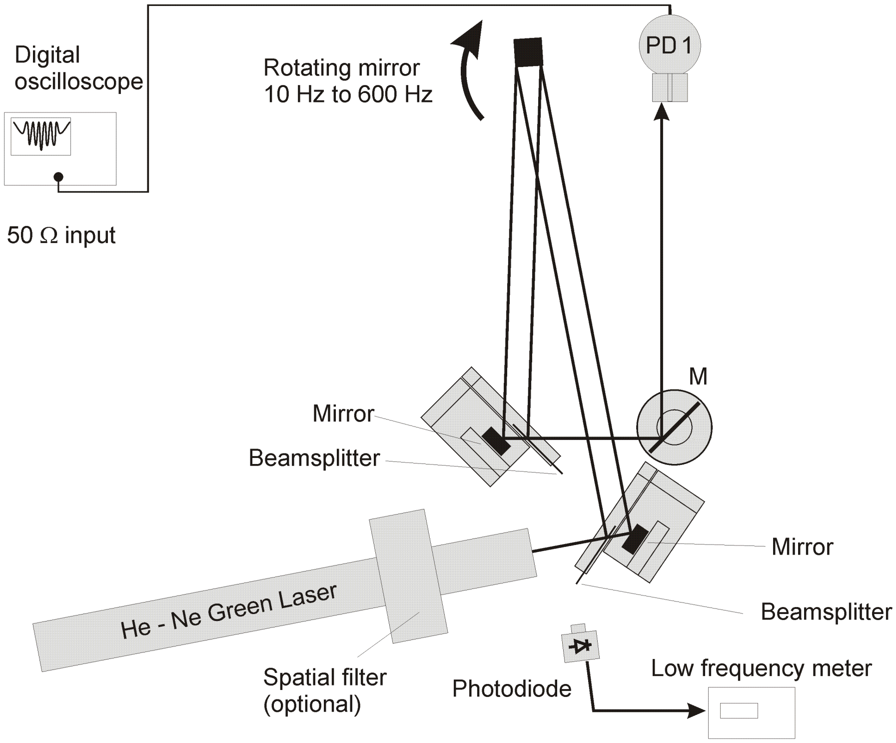


Figure 2. Simple experimental arrangement of the rotating mirror scheme (not to scale). M is a mirror that directs the recombined light to the photodetector, PD1. At the oscilloscope the beat pattern of the two beams that reflects off the mirror is recorded. Arrowheads on light paths indicate direction of observed light. Arrows by rotating mirror represent rotation.

By wearing adequate protective equipment, the beam separation, h, can directly

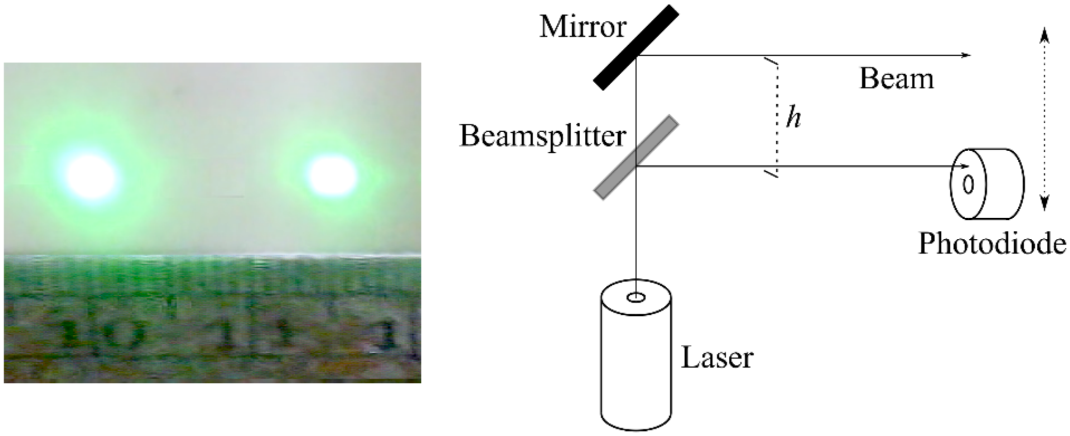


Figure 3. Picture of the two beams showing the beam separation, h (left). An accurate measurement of h is performed by using a micropositioner for displacing a photodiode in a direction perpendicular to the beam (right).

be measured using a standard rule, see Fig. 3 (left). A more accurate measurement can be achieved by placing a photo-detector on a micropositioner which is displaced on a direction perpendicular to the beam, Fig. 3 (right). The intensity as a function of the micropositioner position is perfectly fitted by a Gaussian, Fig. 4. Defining h as the distance between the peaks of the two Gaussian the separation is obtained with an error below 0.5%. An alternate method is by means of a CCD camera, measuring the distance between spots in a similar way to the one described by Voros and Weihs.[2]

Students can record the beat frequency as a function of the rotation frequency, as in the example given in Fig. 5. The comparison of the measured values with the theoretical values, given by (1), is excellent. It is also worth mentioning, the fact that the beat signals are almost free of noise[17] produces a low statistical error. The overall error is below 1%.

3. TOF Measurements

From the setup of Fig. 2, by replacing the mirror (M) by a beamsplitter (BS1), and adding a couple of mirrors (M1 and M2, also a cube corner reflector may be used to this purpose) at a given distance, b, it is possible to have two beams, both modulated by Doppler beat, but with different delay, as shown in Fig. 6. Using a second photodetector (PD2) for collecting the retarded beam, the time difference between the signals of both photodetectors allows for the speed of light determination.

Although both detectors are physically located close to each other, the "far" photodetector, PD2, is the one that collects the retarded signal reflected off the mirrors M1 and M2, whereas the "near" signal is detected by PD1. Signal on PD2 is delayed, relative to PD1, by a time $t = 2b/c$ that can be directly measured on the oscilloscope.

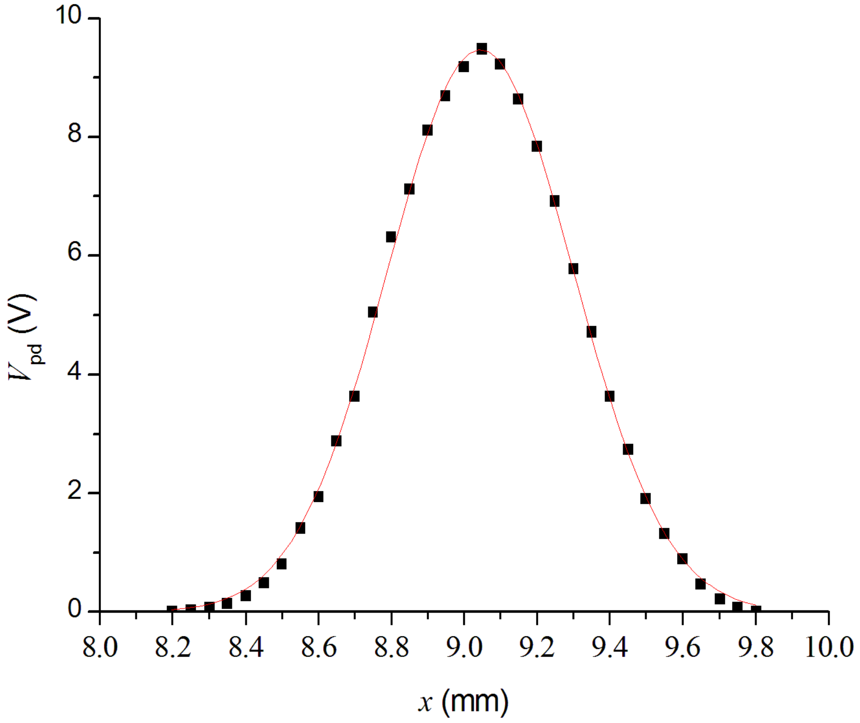


Figure 4. Voltage (black squares) recorded on a photodiode mounted on a micropositioner, when moved across a beam. Also the Gaussian fit is shown (red full line).

The distance b can be varied.

Notice that the present TOF measurement loosely resembles the technique described in Ref. [3] that uses a rotating mirror. A main pulse is produced by the sweep of the rotating beam over the photodetector slit (see Fig. 2, Ref. [3]). The difference in the present approach is that, inside the main sweeping pulse, there is a Doppler beat frequency modulation, thus making a more precise determination of the delay between the far and near signals. In Fig. 7 sample beats produced by a rotation frequency of ≈ 36.4 Hz are shown. Note that at this rotation frequency, the sweeping pulse is of few μs width, thus, the duty cycle is very low ($< 10^{-4}$). Anyway, this parameter is not important to the present demonstrations where the oscilloscope is used in repetitive mode.

The Doppler beat helps in making simpler the measurement. In the previously quoted rotating mirror technique [3] great care was necessary in aligning that part of the sweep falling on the far detector that corresponds to the position of the beam being reflected by the beam-splitter into the near detector. In our apparatus neither the alignment nor the width of the sweep pulse (i.e., the far and near signals may have different width) are critical when using coherent light. Doppler beat is produced in the

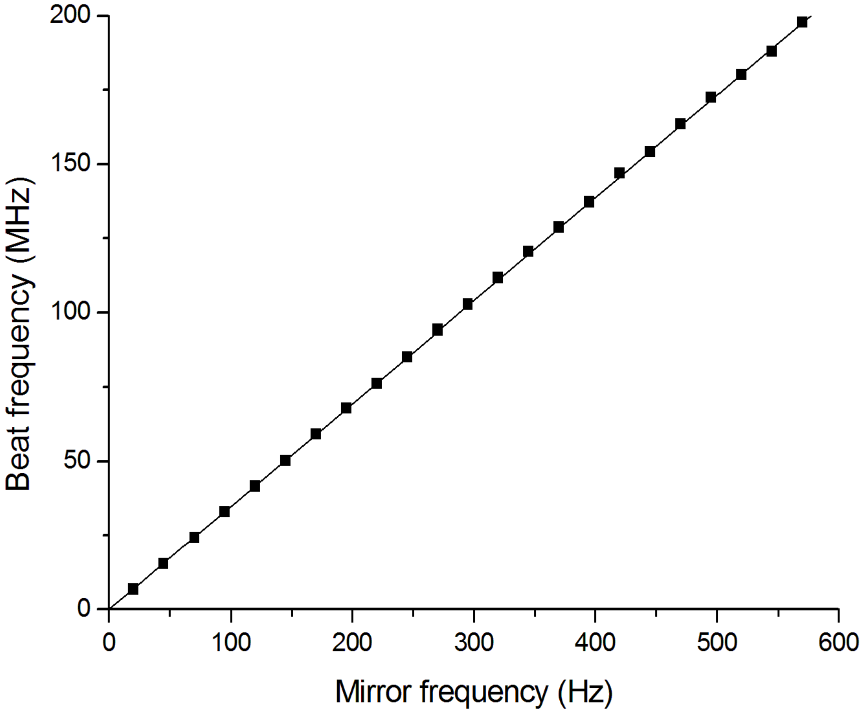


Figure 5. Sample data of the beat frequency (solid squares) as a function of the rotation frequency. Also the theoretical values given by (1) are plotted (solid line). Errors bars are within the squares.

whole beam, therefore a perfect alignment between the sweeping pulses of the far and the near detector is not required. The time delay information is contained in the phase difference between Doppler beats, rather than in the sweeping pulse delay.

Alignment is conducted with the rotating mirror at rest. Optical components and photodetectors are displaced until a maximum intensity is measured. Then, rotating the mirror by hand (equivalent to ≈ 0.1 Hz rotation speed) minor adjustments are performed to photdetectors, mirrors and beamspltters until the Doppler beats are clearly visible in the signals. Third-year students were able to align and run the experiment after some guidance and practice. All the results presented here were obtained by students in different semesters using different configurations.

With the TOF configuration we were able to produce good results working at a rotation frequency from 15 to 570 Hz, which represent beat frequencies from 5 to 200 MHz. The far mirrors were usually located at a distance between 1.5 and 8 meter from the apparatus, thus the separation of the near and far path difference was varied from 3 to 16 meter.

Students can use the oscilloscope cursors to manually measure the time delay between the near and the far signals. An alternate way is by measuring the relative

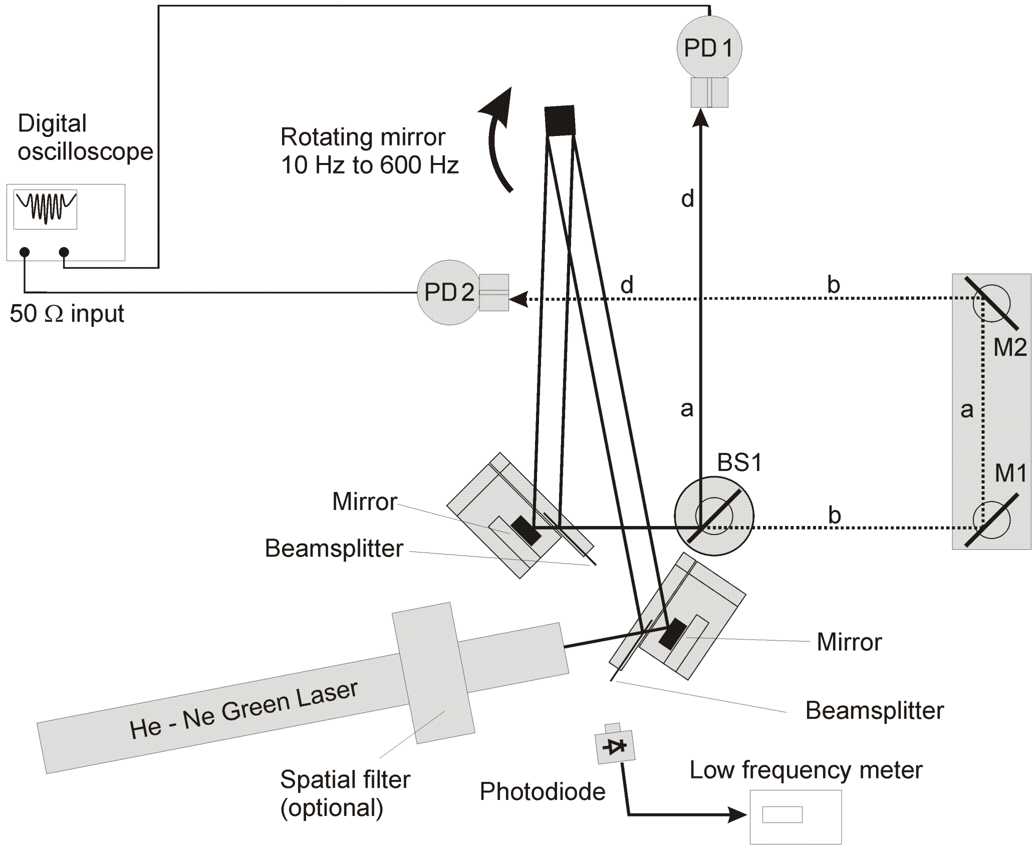
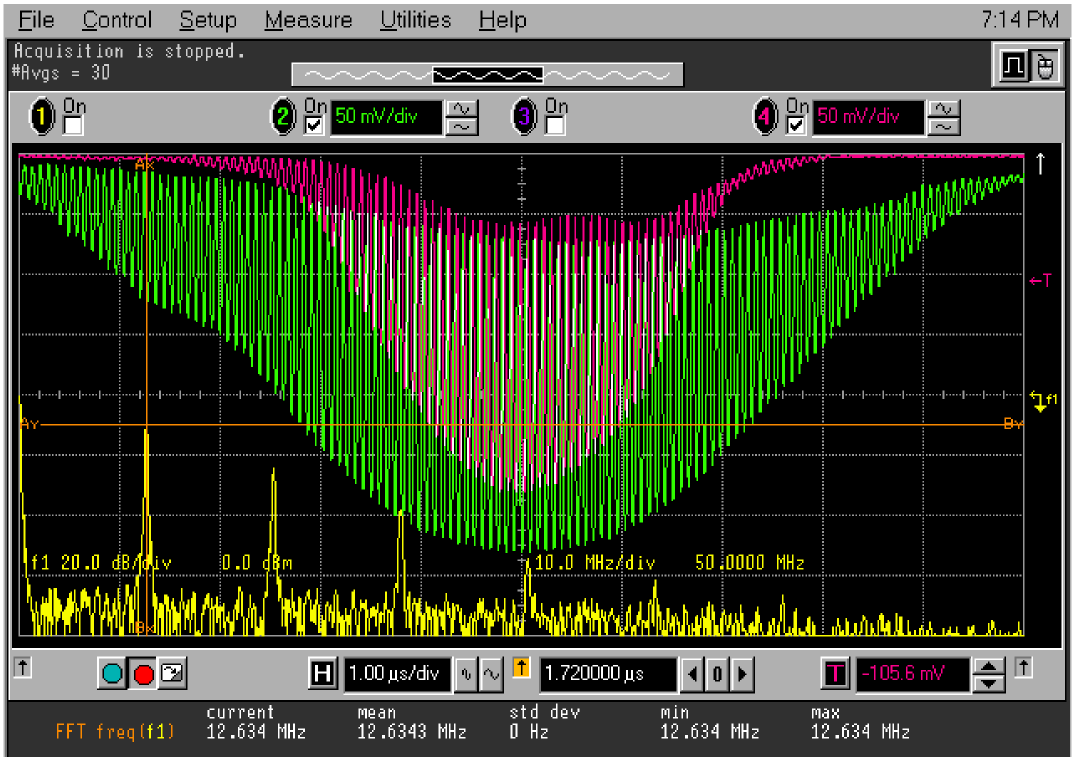


Figure 6. Experimental arrangement for TOF measurements (not to scale). Beamsplitter BS1 divides the beam into the "near" signal, detected by photodetector PD1, and the "far" signal that arrives to photodetector PD2 after reflecting off the mirrors M1 and M2, located at a distance b from the beamsplitter. Notice that by symmetry of the configuration, the path difference between beams is $2b$. Usually b is much greater than either distance a or d. Arrowheads on light paths indicate direction of observed light. Arrows by rotating mirror represent rotation.

phase between the two waveforms (for example, using the Lissajous method, or the product method). Or by a cross correlation technique, if a more precise determination of the time difference is seek. Once the delay was measured, and knowing the path difference between detectors, the speed of light was obtained.

The measurements proceeded as follows. For each position of the far mirrors, the speed of the rotating mirror was varied from 15 to 570 Hz. At approximately 10 Hz interval both photodetector signals (the near and far signals) were saved to the computer together with the photodiode measurement of the mirror frequency. Using a Hewlett Packard Infinium digitizer, 500 MHz, 1GSa/s, also the Fast Fourier Transform (FFT) of each signal was recorded. In order to analyse the data, we developed a simple computer program that automatically evaluates the delay between the signals and gives the speed of light.

In Fig. 8 we show the delay between the far and the near signal at a fixed distance



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Acquisition is stopped.
#Avgs = 30
```

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               current       mean          std dev     min          max
FFT freq(f1)   12.634 MHz    12.6343 MHz   0 Hz        12.634 MHz   12.634 MHz
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Figure 7. Sample screen capture of the oscilloscope traces. The Doppler beat (≈ 12.6 MHz) is clearly seen inside both sweeping pulses: the near detector (upper trace, in red), and the far detector (middle trace, in green). Also the FFT corresponding to the near signal is plotted (lower trace, in yellow).

$b = 4.645$ m ($L = 9.29$ m) for different beat frequencies. As it can be seen, the time delay, ≈ 31 ns, is fixed and equals the time of flight of the path difference between the near and far detector. Since the time delay is fixed, then, the phase shift depends on the beat frequency, ranging from $\pi/2$ to 2π in the example of Fig. 8.

A typical set of data measured by third-year students gave the following results: first run, over 84 different frequencies, separation $b = (4.645 \pm 0.005)$ m, mean speed $c = (3.002 \pm 0.014) \times 10^8$ m/s; second run, on 73 different frequencies, separation $b = (7.985 \pm 0.005)$ m, mean speed $c = (2.989 \pm 0.007) \times 10^8$ m/s . A group of students worked on more that 10 different distances, the shortest distance they used with this scheme was $b = (1.465 \pm 0.005)$ m. The overall mean speed obtained was $c = (2.999 \pm 0.008) \times 10^8$ m/s that agrees with the expected value of $c = 2.9971 \times 10^8$ m/s in air.

A low measurement error is obtained if a sufficient number of cycles are present in the signals. When the delay was below 10 ns, the accuracy of the method dropped since the jitter of the transit time inside the photodetectors was of the order of 0.2 to 0.3 ns. In order to get an error below 1% using the present TOF configuration and detectors, a separation of at least 3 meter is required (ie, a path difference of 6 m).

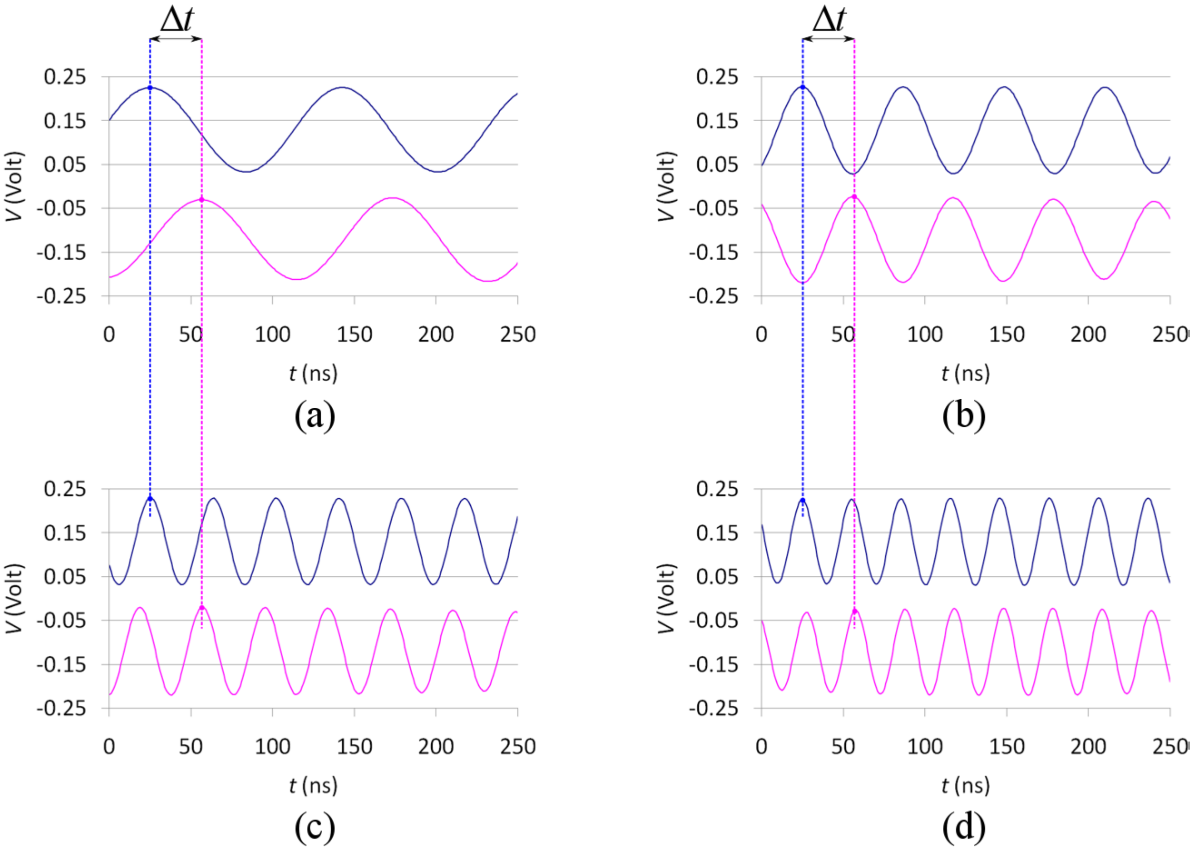

Figure 8. Far (lower magenta full line) and near (upper blue full line) signals for a distance $b = 4.645$ m (that corresponds to a separation $L = 9.290$ m), and beat frequency (a) 8.431 MHz, (b) 16.204 MHz, (c) 26.588 MHz, and (d) 33.081 MHz. From the signals, it is apparent that the time delay, Δt, does not depend on the beat frequency.

4. Interference measurements

In order to avoid the error due to the difference of the transit time between the two photodetectors, a configuration using only one detector has been used. This is equivalent to an interferometer having two arms of different length similar to that of Fig. 1 of Ref. [11].

The interference configuration is achieved by adding a second beamsplitter, BS2, to the previous configuration, leading to the setup shown in Fig. 9. In this configuration both photodetectors receive the light intensity of the interference of the near and the far beams, allowing to exploit the concept of interferometry. A minimum signal is obtained when the time delay makes the signals arrive out of phase by π radians. Actually, only one detector is needed. Having two detectors is useful because the beams have different intensities, and we can chose in which one the interference minimum is deeper.

Locating the far mirrors at a fixed distance the idea is to find the frequency that achieves the destructive interference of the AC component of the two beating signals.

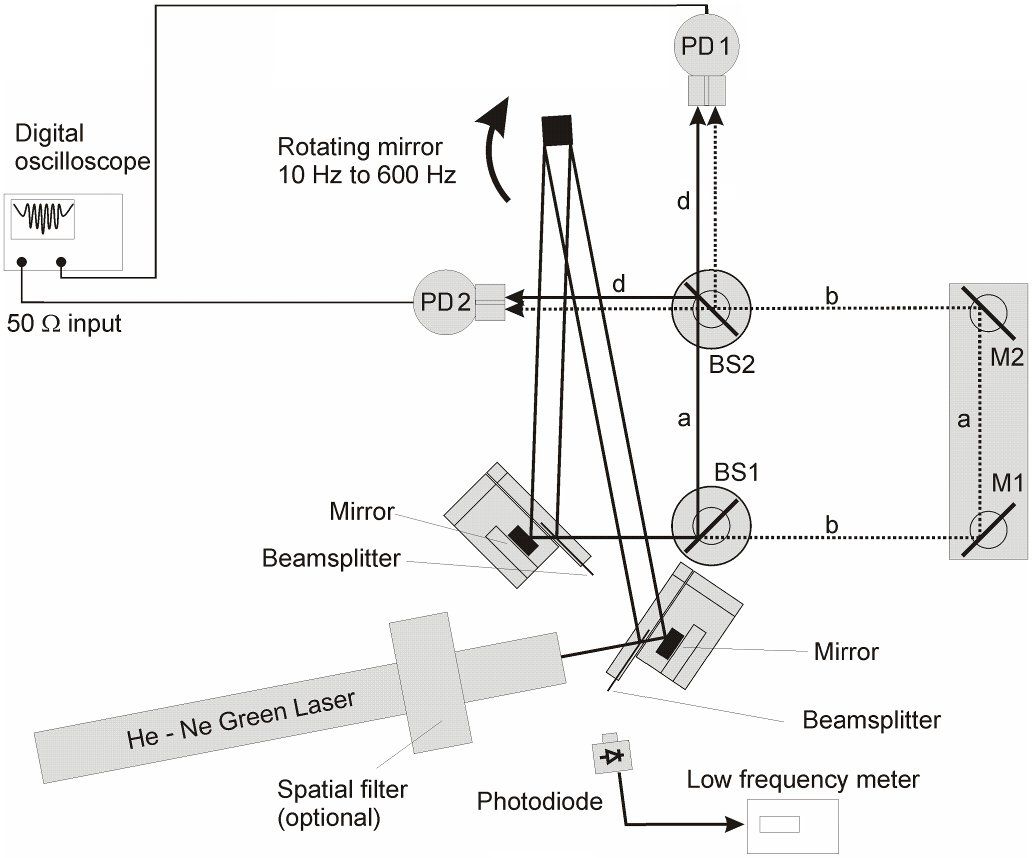

Figure 9. Experimental arrangement for interference (not to scale). A second beamsplitter, BS2, is conveniently located in the path of the far and the near beams in the configuration of Fig. 6. Both photodetectors should record a similar signal as they both receive the interference between the two Doppler beats: the near signal and the far signal. If they arrive in phase, a maximum intensity is recorded. The extinction of the signal is produced when they are in counter phase. Beamsplitters were selected in order to have a similar intensity in both beams. Arrowheads on light paths indicate direction of propagation. Arrows by rotating mirror represent rotation.

Since we were limited to 200 MHz beat frequency (that corresponds to the maximum speed of the motor, $\approx$ 600 Hz), at the shortest used distance ($b = 0.495$ m) we were able to see just one destructive interference corresponding to half wavelength. On the other hand, at the longest used distance ($b = 4.645$ m) up to 6 destructive interference frequencies were found.

An example of the intensity of the superimposed signals as a function of the beat frequency is shown in Fig. 10. In this example, 420 data points were collected between 20 and 30 MHz beat frequency. This has been accomplished by measuring several times at the same nominal rotation frequency. As the frequency approaches the first interferometric minimum, the visibility goes to zero. In this situation, half wavelength occupies the distance $2b$. From the best fit of the photodetector intensity as a function of the rotation frequency, we get the minimum frequency $f_b = (25.64 \pm 0.10)$ MHz.

Since the distance of the far mirrors is $b = (2.930 \pm 0.005)$ m, then, the wavelength is $\lambda = 11.72$ m, and the speed of light in air is $c = \lambda f_b = (3.005 \pm 0.016) \times 10^8$ m/s.

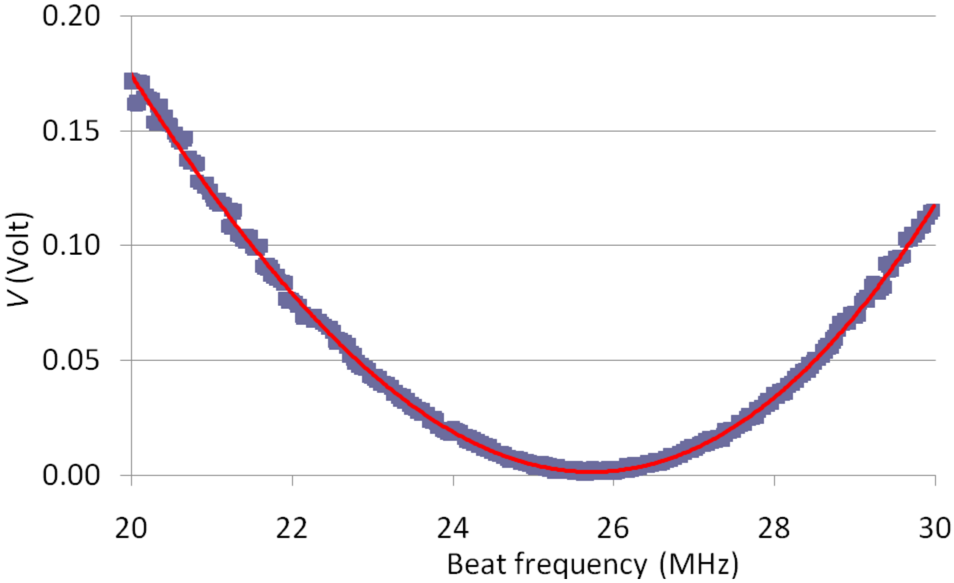

Figure 10. Visibility curve as a function of the Doppler beat frequency at a fixed distance of the far mirrors $b = 2.930$ m (the blue squares are the measured values, while the red full line is the best fit). The minimum visibility correspond to a beat frequency $F = 25.64$ MHz, thus the speed of light is $c = (3.005 \pm 0.016) \times 10^8$ m/s .

Using the shortest distance, $b = 0.495 \pm 0.005$ m, the destructive frequency was $f_0 = (151.2 \pm 1.0)$ MHz resulting in a speed of light of $c = (2.994 \pm 0.032) \times 10^8$ m/s. Notice that, in this case, the far mirrors were located only half meter apart from the apparatus, producing a measurement with 1% error. In other words, the interference method produces better results at the shortest distances.

5. Discussion

Using the beat between two beams with a Doppler beat produced by a rotating mirror, whose beat frequency can easily be varied in an ample range, it is possible to have a wavelength that fits within a classroom or laboratory, thus the delay or destructive interference can be easily seen. Different configurations may be developed based on this idea. Working up to 60 MHz of beat frequency (that corresponds to a 5 m wavelength) together with a diode laser, a low budget variation of the experiments can be developed. More expensive configurations, as the one described here which allows a variation of the modulation frequency from 3.5 to 200 MHz in steps of 350 kHz, give a wider modulation range than most common amplitude modulation apparatuses.

Actually, three different configurations are easily assembled from the described Doppler apparatus, namely: Doppler configuration, TOF configuration and interference configuration. The present layouts were designed to be operated by third-year students, with a background on elementary lab courses such as Mechanics, Acoustics, Optics, Electricity and Magnetism.

Doppler configuration is similar to the original one,[17] that can be reproduced in the present apparatus by removing the second beam-splitter BS2 and the far mirrors, M1 and M2 (see Fig. 9). The Doppler beat is detected on the near photodetector, as it has been done in the previous work.[17]

TOF measurements use a beamsplitter and two far mirrors. A qualitative demonstration can be performed by varying the rotating frequency of the mirror while keeping the far mirrors at a fixed position, and observing that the delay between the far and near photodetector signals remains fixed. Alternatively, the delay can be varied by modifying the distance between the far and near detectors. A quantitative measurement is obtained by directly measuring the delay between the far and the near signals in the oscilloscope signals.

Using the interference configuration, we were able to determine the speed of light in a distance just below 0.5 m, with less than 1% error. Even a shorter distance is possible, for example, increasing the beat frequency by improving the motor in order to drive larger mirrors (which will allow a larger beam separation), and/or by using an adequate optical path with multiple reflection in the rotatory mirror. For example, with $h = 4$ cm and 4 reflections in a rotating mirror at 600 Hz, the beat frequency will be ≈ 2 GHz. This means that it should be possible to measure the speed of light in air, with a reasonable error, in a distance as short as several cm. Of course, this kind of apparatus will be more expensive and will require more expensive detectors and digitizer.

Although the method seems to be very simple, because there is no need of extremely careful alignment, we found two main problems. The first one, common to any TOF technique, is the delay in cables and detectors. In our case the offset difference was 0.86 ns due to a slightly different cable length of each channel. Of course this delay can be measured by swapping the far and near detectors, and obtaining a mean value over a large sample.

The second problem, that was more difficult to deal with, was related to the jitter of the photodetector. We observed that the transit time in the detector depends on the light intensity. Therefore, in order to avoid spurious results a similar intensity should arrive to both photodetectors. This is not simple to implement and probably it is the most difficult part of the method. Of course the extra, spurious delay was irrelevant in the results at larger distance, but becomes important at shorter distance when the TOF delay is small, like in the $b = 1.465$ m case where the delay is below 10 ns.

In conclusion, the apparatus described here is a good alternative to other time of flight measurements because it fulfils most of the requirements for this kind of experiments, namely:

a) The experimental setup is simple and completely accessible to undergraduate students.

b) Both, modulated frequency and path difference can be varied in an ample range of values at small steps.

c) The light is visible, students can see the rays, which, with the use of appropriate screens, can be blocked at any point along their paths. In particular, doing so close to the rotating mirror, allows turning on-off the beat phenomenon.

d) The experiment can take place entirely within the teaching laboratory or demonstration room. To reach a 6 m separation, a mirror could be placed 3 m away from the apparatus.

e) Inexpensive configurations can be built which requires no special equipment beyond what is normally available in physics department laboratories.

Acknowledgements

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