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A multiperiod model for production planning and design in a multiproduct batch environment

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ABSTRACT

A general multiperiod model to optimize simultaneously production planning and design decisions applied to multiproduct batch plants is proposed. This model includes deterministic seasonal variations of costs, prices, demands and supplies. The overall problem is formulated as a mixed-integer linear programming model by applying appropriate linearizations of non-linear terms. The performance criterion is to maximize the net present value of the profit, which comprises sales, investment, inventories, waste disposal and resources costs, and a penalty term accounting for late deliveries. A noteworthy feature of this approach is the selection of unit dimensions from the available discrete sizes, following the usual procurement policy in this area. The model simultaneously calculates the plant structure (parallel units in every stage, and allocation of intermediate storage tanks), and unit sizes, as well as the production planning decisions in each period (stocks of both product and raw materials, production plans, policies of sales and procurement, etc.).

1. Introduction

Nowadays, one of the most important challenges faced by business is the adjustment of the firm resources in order to satisfy market requirements subjected to fluctuations over time, mainly costs, prices, existences, demands, etc. In many industries, products have distinctive demand patterns that vary due to market or seasonal changes coupled with raw material supplies that also undergo changes. Because of these variations over time, there has been an increased interest in the development of multiperiod optimization models in recent years.

Flexible production is receiving increased attention in the chemical processing industry. This flexible production leads to faster responses to the market fluctuations and is most commonly achieved in batch plants. In this work, efforts are focused on multiproduct batch production environment, where several different products are produced sharing the same equipment operating in the same sequence.

A batch process refers to a general non-continuous process that consists of multiple stages employing a combination of identical parallel batch units. In a multiproduct batch plant each product is produced at a time. Batch units are characterized by a processing time and no simultaneous feed and removal is performed. Also, intermediate storage tanks may be available between successive stages of operation in order to decouple the production process. Fig. 1 shows a plant configuration of this type of industry.

Most of the previous approaches in batch plants used to pose models that worked with a single long time period and constant conditions without considering variations due to seasonal or market fluctuations. Also, these previous efforts

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Notation

Subscripts

c	Raw material
i	Product
j	Batch stage
m	Number of parallel units at batch stages
s	Discrete sizes for batch stages
t	Time period
v	Discrete sizes for storage tanks

Superscripts

d	Downstream
L	Lower bound
p	Subprocess
u	Upstream
U	Upper bound

Parameters

co_{it}	Operating cost coefficient of product i in period t.
cp_{it}	Cost coefficient for late delivery of product i in period t.
CT	Number of common ingredients for producing each final product i.
DE_{it}	Demand of product i at period t.
F_{cit}	Parameter that accounts conversion of raw material c to produce i at period t.
g_j	Number of discrete sizes available for storage tanks at stage j.
H	Time horizon.
H_t	Net available production time for all products at period t.
k_j	Number of discrete sizes available for batch units at stage j.
np_{it}	Price of product i at period t.
S_{ijt}	Size factor of product i in stage j for each period t.
SI_j	Set of available discrete sizes for the storage tanks allocated after stage j.
ST_{ijt}	Size factor for product i for an intermediate storage tank in the location j.
SV_j	Set of available discrete sizes for the batch units at stage j.
pt_{ijt}	Processing time of product i in batch stage j in period t.
wp_{it}	Waste disposal cost coefficient per product i.
wr_{ct}	Waste disposal cost coefficient per raw material c.
α_j	Cost coefficient for a batch unit in stage j.
β_j	Cost exponent for a batch unit at stage j.
ε_{ct}	Inventory cost coefficient for raw material c in period t.
κ_{ct}	Price for the raw material c in period t.
v_{js}	Standard volume of size s for batch unit at stage j.
π_j	Cost coefficient for an intermediate storage tank allocated in position j.
σ_{it}	Inventory cost coefficient for product i in period t.
τ_j	Cost exponent for an intermediate storage tank allocated in position j.
vt_{jv}	Standard volume of size v for storage tank allocated in position j.
ζ_c	Time periods during which raw materials have to be used.
χ_i	Time periods during which final products have to be used.
Λ_{ij}	Parameter that represents the maximum difference of number of batches.

Binary variables

d_j	It is 1 if the tank is allocated in position j.
st_{jv}	It is 1 if storage at position j has size v.
y_{jm}	It is 1 if batch stage j has m units of the same size in parallel.
z_{js}	It is 1 if equipment at batch stage j has size s.

Integer variables

M_j	Number of parallel units operating out of phase at batch stage j.

Continuous variables

B_{ijt}	Batch size of product i in stage j in period t.
C_{ct}	Amount of raw material c purchased in period t.
e_{ijst}	Continuous variable that represents the product of the variables $q_{it}\ z_{js}$.
f_{ijvt}	Continuous variable that represents the product of the variables $q_{it}\ st_{jv}$.
IM_{ct}	Inventory of raw material c at the end of a period t.
IP_{it}	Inventory of final product i at the end of a period t.
n_{ijt}	Number of batches of product i in stage j at period t.
PW_{it}	Product i wasted at period t due to the limited product lifetime.
q_{it}	Amount of product i to be produced in period t.
QS_{it}	Amount of product i sold at the end of period t.
r_{jms}	Continuous variable that represents the product of the binary variables $z_{js}y_{jm}$.
RM_{ct}	Raw material c used for production in period t.
RM_{cit}	Raw material c consumed in period t to manufacture product i.
RW_{ct}	Raw material c wasted at period t due to the limited product lifetime.
T_{it}	Total time for producing product i in period t.
TL_{it}^{p}	Limiting cycle time of product i for a subprocess p in period t.
V_j	Size of a batch unit at stage j.
VT_j	Size of the intermediate storage tank allocated in position j.
w_{ijmt}	Continuous variable that represents the product of the variables $n_{ijt}y_{jm}$.
ϑ_{it}	Amount of late delivery for product i in period t.

usually decouple the design and planning problems and solve only one problem making several assumptions over the other. In the design problem, the production requirement of each product and the total production time available are specified. A procedure is generated in order to determine the plant configuration and equipment sizes to minimize the capital cost. Different formulations have been developed and solved through different methodologies [1–5]. Moreover, several approaches with varying degrees of detail have been introduced in the past years to solve the planning problem [6–8].

Unlike previously cited works, a smaller number of articles have posed models for multiperiod scenarios. In general, these works follow the same trend only focusing on one problem at a time. Multiproduct batch facilities in a multiperiod scenario have been studied by Birewar and Grossmann [9] that presented a multiperiod linear programming model for production planning of batch plants that considers benefits and product inventory cost, but design decisions are not included in that approach. Voudouris and Grossmann [10] developed a cyclic MILP problem where synthesis, sizing and scheduling issues were integrated, including intermediate storage sizing and allocation. Van den Heever and Grossmann [11] considered a multiperiod nonlinear optimization model posed through general disjunctive programming for the design, and capacity expansion of general chemical process systems. They proposed two algorithms for the solution of the model in order to reduce the solution times of MINLP problems.

Taking into account modeling and resolution difficulties in previous works in process industry, problems are generally decomposed into simpler steps: design, operation, planning, scheduling, etc. These problems, however, are related and they should be solved together, at least some of them. The trade-offs among them depend on several elements: time horizon, product lifetime, characteristics of facilities, supply policies, etc. Most of these works used to pose models working with only one time period with constant conditions. These alternatives can be improved if the problem elements can vary over time in a multiperiod context.

Two main contributions have been addressed in this work. On one hand, concurrent design and production planning decisions for multiproduct batch plants have been simultaneously posed, so as to assess the trade-offs between them. On the other hand, the multiperiod effect has been explicitly taken into account. In this way, the changes caused by market and seasonal fluctuations in decision variables in every period are considered, using deterministic values proposed by the decision maker.

In contrast to previous works, the optimal design is determined by considering the units available in discrete sizes which correspond to the real procurement of equipment. In order to get an MILP model, a linearization method is applied over bilinear terms. In this way, the original nonlinear and non-convex model is transformed to obtain a linear formulation that can be solved to global optimality with reasonable computational effort. In short, this general model handles deterministic seasonal variations of product demands and prices, the raw material and investment costs, takes into account discrete sizes of batch units and storage tanks, and considers inventories of both final products and raw materials. This is a valuable MILP model since it corresponds to a more realistic case. Decision makers can simultaneously assess different elements from the strategic and tactic points of view of the operations management. The implementation of the proposed model is demonstrated through its application in several examples.

The paper is structured as follows. The main characteristics of the problem are discussed in Section 2. Section 3 presents a description of the proposed mathematical formulation. Illustrative examples are included in Section 4 and their results are discussed. Finally, some concluding comments are presented in Section 5.

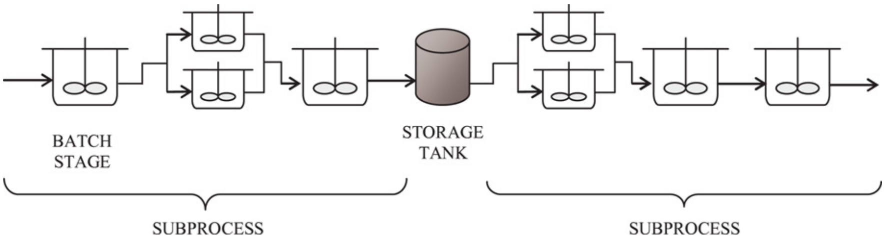

Fig. 1. Flowsheet of the batch plant.

2. Overall description

This approach poses a model for the simultaneous production planning and design of noncontinuous multiproduct plants on a multiperiod scenario. The plant includes $j = 1, 2, \dots, J$ batch processing stages to produce $i = 1, 2, \dots, I$ products over $t = 1, 2, \dots, T$ time periods. In every period t, the production of product i in batch stage j requires a given processing time pt_{ijt}, and a material balance factor S_{ijt}, called the Size Factor, which specifies the volume (or mass) of material which must be processed at stage j to produce a unit volume (or mass) of final product i. Both processing times and size factors are known values that depend on the production recipe. Time horizon H is discretized into a number of time periods T with a specified length H_t not necessarily equal.

In each period t, the limiting cycle time of product i, TL_{it}, is the largest processing time in the plant for product i, (i.e. $TL_{it} = \max_j\{pt_{ijt}\}$) and corresponds to the time between two consecutive batches. Thus, the production rate for each product is determined by the largest processing time at stage j and the remaining stages that must operate at this rate has idle times. These idle times can be reduced in either of the following two ways. In the first, a duplication of units in parallel at the batch stage j with the longest processing time is adopted. If M_j units operate out-of-phase, then the processing time is divided by the number of units at that stage. Another way to reduce idle times consists of the introduction of intermediate storage tanks. Storage tanks do not act as long term storage, so the Finite Intermediate Storage (FIS) policy is adopted.

The plant operates in single product campaign (SPC) mode in every time period. With the single product campaign approach, all batches of a product are successively processed without overlapping with other products. When storage tanks are not allocated, batches are transferred from stage to stage without delay. This is called Zero Wait transfer policy (ZW) in the scheduling context.

This model considers design decisions, involving the selection of sizes for batch units V_j and storage tanks VT_j and the number of units to be duplicated in batch stages. Following the usual procurement policy, the batch unit size of stage j, V_j, is restricted to take values from the set $SV_j = \left\{v_{j1}, v_{j2}, \dots, v_{jk_j}\right\}$, where k_j is the given number of available discrete sizes from the commercial point of view for that stage. In the same way, variable VT_j is restricted to take values from the set $SI_j = \left\{vt_{j1}, vt_{j2}, \dots, vt_{jg_j}\right\}$, where g_j is the given number of discrete sizes available for the storage tank in that position. The available values in SV_j and SI_j are proposed by the designer.

On the other hand, production planning decisions allow specifying at each period t and for each product i, the amount to be produced q_{it}, the number of batches n_{ijt} at stage j, and the total time T_{it} to produce product i. Furthermore, at the end of every period t, the inventory levels of both final products i, IP_{it}, and raw materials c, IM_{ct}, are obtained. Total sales of product i, QS_{it}, the amount of purchased raw material c, C_{ct}, and the amount of raw material c to be used for the production, RM_{ct}, in each period t are included in this formulation.

The performance criterion of this model is to maximize the net present value of the profit along the global time horizon, taking into account incomes from product sales, expenditures from raw material purchases and inventory, sales penalty costs and investment costs. If time periods are equal, waste disposal costs are also added to the objective function.

3. Mathematical formulation

This section presents an MILP formulation that simultaneously solves design and planning problems in a multiperiod context.

3.1. Batch equipment

At batch stage j, the available unit size V_j must be sufficient to process a batch B_{ijt} of product i over all time periods t, as follows:

$$V_j \geq S_{ijt}\, B_{ijt} \quad \forall i, j, t \tag{1}$$

where S_{ijt} is the size factor corresponding to product i at stage j which can vary in each period t due seasonal effects (e.g. variable solute concentrations).

Let q_{it} be the amount of product i produced at time period t and n_{ijt} the number of batches of product i to be processed in batch stage j over time period t. Then,

$$q_{it} = B_{ijt}\, n_{ijt} \quad \forall i, j, t. \tag{2}$$

By combining Eqs. (1) and (2) the following constraints are obtained:

$$n_{ijt} \geq \frac{S_{ijt}\, q_{it}}{V_j} \quad \forall i, j, t. \tag{3}$$

In this work, unit sizes V_j for batch stages are considered available in discrete sizes υ_{js}, with $s = 1, 2, \ldots, k_j$, which corresponds to the usual commercial procurement of equipments. To rigorously tackle this situation, a binary variable z_{js} is introduced whose value is one if equipment at batch stage j has size s; otherwise zero. Using these discrete variables the following expressions hold:

$$\frac{1}{V_j} = \sum_s \frac{z_{js}}{\upsilon_{js}} \quad \forall j \tag{4}$$

$$\sum_s z_{js} = 1 \quad \forall j. \tag{5}$$

By substituting Eq. (4) into (3), new constraints can be formulated that restrict the unit sizes to discrete values. This operation generates bilinear products $q_{it}\, z_{js}$, with continuous and discrete variables, which can be eliminated by introducing the nonnegative continuous variable e_{ijst} that represents those bilinear products in order to reformulate the constraints as linear ones [12]:

$$n_{ijt} \geq \sum_s \left(\frac{S_{ijt}}{\upsilon_{js}}\right) e_{ijst} \quad \forall i, j, t \tag{6}$$

$$e_{ijst} \leq q^U_{it}\, z_{js} \quad \forall i, j, s, t \tag{7}$$

$$q_{it} = \sum_s e_{ijst} \quad \forall i, j, t \tag{8}$$

where q^U_{it} represents the upper bound for q_{it}.

3.2. Intermediate storage

When an intermediate tank is allocated, the original process is decoupled into two subprocesses, upstream and downstream of the tank, each one presenting its own batch size and limiting cycle time for each product. For this reason, a multiproduct plant can be viewed as a series of subprocesses p which are separated by intermediate storage tanks, each one with the same batch size and limiting cycle time for every product i. In this way, there are $J - 1$ possible positions for a storage tank, where the j-th location is between batch stages j and $j + 1$. Productivity of both batch subprocesses must be the same to avoid material accumulation in the storage tank.

The storage sizing constraint proposed by [1] implies that storage vessel size, VT_j, has to be twice as large as the largest of the up and downstream batch sizes. As no a priori tank allocation is given, binary variables d_j are used to select their allocation, with value one if the tank is allocated, or zero otherwise.

$$VT_j \geq 2\, ST_{ijt}\, B_{ijt}\, d_j \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{9}$$

$$VT_j \geq 2\, ST_{ijt}\, B_{i,j+1,t}\, d_j \quad \forall i, j = 1, 2, \ldots, J-1, t. \tag{10}$$

The parameter ST_{ijt} refers to the size factor for storage tank and its value depends on the production recipe. Thus, previous equations specify the capacity of the tank (if any).

Without any intermediate storage allocation, the number of batches is equal for consecutive stages. If a storage tank is located between stages j and $j + 1$, the number of batches of the two stages is allowed to be different. The difference is not allowed to be larger than a given value $\Lambda_{i,j}$ dependent on the problem. This can be expressed mathematically as:

$$n_{ijt} - \Lambda_{i,j}\, d_j \leq n_{i,j+1,t} \leq n_{ijt} + \Lambda_{i,j}\, d_j \quad \forall i, j = 1, 2, \ldots, J-1. \tag{11}$$

If no storage is allocated in position j (i.e. $d_j = 0$) there is no difference between consecutive numbers of batches. If a storage tank is allocated (i.e. $d_j = 1$) the difference should not be more or less than $\Lambda_{i,j}$, and then Eqs. (9) and (10) must be fulfilled.

By introducing Eq. (2), the above Eqs. (9) and (10) can be expressed as

$$n_{ijt} \geq 2\left(\frac{ST_{ijt}\, q_{it}}{VT_j}\right) d_j \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{12}$$

$$n_{i,j+1,t} \geq 2\left(\frac{ST_{ijt}\, q_{it}}{VT_j}\right) d_j \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{13}$$

$$n_{i,j+1,t} \geq n_{ijt} - \Lambda_{i,j}\, d_j \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{14}$$

$$n_{i,j+1,t} \leq n_{ijt} + \Lambda_{i,j}\, d_j \quad \forall i, j = 1, 2, \ldots, J-1, t. \tag{15}$$

Here, Eq. (11) has been split into two expressions.

The size of the storage tank, VT_j, is also considered to be available in standard sizes. For this reason the binary variable st_{jv} is one if storage at position j has size v, zero otherwise. Thus, variable VT_j is restricted to take values from the set $SI_j = \{vt_{j1}, vt_{j2}, \ldots, vt_{jg_j}\}$, where vt_{jv} represents discrete size v for storage tank at position j. It is assumed that $v = 1$ represents size 0, or, in other words, that no storage vessel is used. Thus, the binary variable d_j is replaced by the variable st_{jv}.

The following constraints are posed to define the storage tank size:

$$\sum_v st_{jv} = 1 \quad \forall j = 1, 2, \ldots, J-1 \tag{16}$$

$$\frac{1}{VT_j} = \sum_{v \neq 1} \frac{st_{jv}}{vt_{jv}} \quad \forall j = 1, 2, \ldots, J-1. \tag{17}$$

After Eq. (17) is introduced into Eqs. (12) and (13), there arise cross products $q_{it}\, st_{jv}$ which are then replaced by continuous nonnegative variables f_{ijvt}, that represents those products, and the following linear constraints are obtained:

$$n_{ijt} \geq 2 \sum_{v \neq 1}\left(\frac{ST_{ijt}}{vt_{jv}}\right) f_{ijvt} \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{18}$$

$$n_{i,j+1,t} \geq 2 \sum_{v \neq 1}\left(\frac{ST_{ijt}}{vt_{jv}}\right) f_{ijvt} \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{19}$$

$$f_{ijvt} \leq q_{it}^U\, st_{jv} \quad \forall i, v, j = 1 \ldots J-1, t \tag{20}$$

$$q_{it} = \sum_v f_{ijvt} \quad \forall i, j = 1 \ldots J-1, t. \tag{21}$$

By using the binary variables st_{jv} replacing variables d_j, the constraints (14) and (15) can be written as:

$$n_{i,j+1,t} \geq n_{ijt} - \Lambda_{i,j}\, \left(1 - st_{j1}\right) \quad \forall i, j = 1, 2, \ldots, J-1, t \tag{22}$$

$$n_{i,j+1,t} \leq n_{ijt} + \Lambda_{i,j}\, \left(1 - st_{j1}\right) \quad \forall i, j = 1, 2, \ldots, J-1, t. \tag{23}$$

When binary variable st_{j1} is 1 this indicates that a storage tank with size 0 is selected, i.e. no tank is allocated after stage j, so the number of batches between stages j and $j+1$ has to be equal. Whereas, if binary variable st_{j1} is 0 a storage tank with a size different of zero is allocated between stages j and $j+1$, then the difference on number of batches cannot be larger that the parameter $\Lambda_{i,j}$.

3.3. Time constraints

A batch is processed to completion in a unit on a batch stage j during a processing time pt_{ijt} and transferred to the next stage. The shortest possible time between two batches of a product i leaving a stage j is called the limiting cycle time TL_{it}. As was already mentioned, the allocation of intermediate storage tanks allows the limiting cycle times of the subprocesses, TL_{it}^p, can be chosen independently of each other. Moreover, if M_j identical parallel units operating out of phase are added in stage j so as to reduce the limiting cycle time, the following expression holds:

$$TL_{it}^p \geq \frac{pt_{ijt}}{M_j} \quad \forall i, p, j \in p, t. \tag{24}$$

In this way, every subprocess can have different number of batches and cycle times. Tank allocation would increase the cost of the plant, but due to the decoupling of subprocesses, subprocesses with smaller TL_{it}^p can operate with smaller equipment sizes.

In order to avoid material accumulation, the production time assigned to product i at each period t, T_{it}, has to be equal to the production time that is assigned to this product i in every subprocess p. Then, the following expression holds:

$$T_{it} = n_{ijt}\, TL_{it}^p \quad \forall i, p, j \in p, t. \tag{25}$$

Production rates (ratio of amount produced and total time to produce product i) of all subprocesses in each time period must be the same to avoid material accumulation in the storage tank. Taking into account that q_{it} and T_{it} are equal for all the subprocesses, the production rate upstream and downstream of the storage tank must be the same.

Also, the total production time for all products cannot exceed the total available time horizon for period t.

$$\sum_i T_{it} \leq H_t \quad \forall t. \tag{26}$$

For the continuous variables M_j to take integer values, binary variables y_{jm} are used. Their value is 1 if batch stage j has m identical units in parallel; otherwise zero. So the following constraints are posed:

$$\frac{1}{M_j} = \sum_m \frac{y_{jm}}{m} \quad \forall j \tag{27}$$

$$\sum_m y_{jm} = 1 \quad \forall j. \tag{28}$$

By multiplying Eq. (24) by n_{ijt}, and taking into account Eqs. (25) and (27) these constraints can be expressed as

$$T_{it} \geq \sum_m \frac{pt_{ijt}\, n_{ijt}}{m} y_{jm} \quad \forall i, j, t. \tag{29}$$

The nonlinearities in constraint (29) are eliminated by introducing new variables w_{ijmt} to represent the cross product $n_{ijt}y_{jm}$. These equations can now be written as

$$T_{it} \geq \sum_m \left(\frac{pt_{ijt}}{m}\right) w_{ijmt} \quad \forall i, j, t \tag{30}$$

$$w_{ijmt} \leq n^U_{ijt} y_{jm} \quad \forall i, j, m, t \tag{31}$$

$$n_{ijt} = \sum_m w_{ijmt} \quad \forall i, j, t \tag{32}$$

where n^U_{ijt} represent upper bounds for the variables n_{ijt}.

3.4. Planning and inventory constraints

The following constraints manage inventories and force total production to meet product demands, over all the time periods t. This model assumes that the process handles $c = 1, 2, \ldots, CT$ common ingredients for producing each final product i.

The stock of product i at the end of period t, IP_{it}, is equal to the amount in storage at end of the previous period, $IP_{i,t-1}$, plus the production during this period q_{it}, less the amount sold QS_{it} and less the waste due to expired product shelf life, PW_{it}:

$$IP_{it} = IP_{i,t-1} + q_{it} - QS_{it} - PW_{it} \quad \forall i, t \tag{33}$$

where the amount sold QS_{it} is bounded by maximum demands DE^U_{it}.

Eq. (34) poses the inventory of raw material c at the end of a time period t, IM_{ct}, that is equal to the stock in the previous period, $IM_{c,t-1}$, plus the purchases during period t, C_{ct}, less the consumption for production, RM_{ct}, and less the waste due to limited product lifetime, RW_{ct}.

$$IM_{ct} = IM_{c,t-1} + C_{ct} - RM_{ct} - RW_{ct} \quad \forall c, t. \tag{34}$$

When the problem takes into account time periods of equal length, lifetime considerations of both raw materials and product can be added into the formulation [13]. Let ζ_c and χ_i be the given time periods for raw material c and product i, respectively, during which they have to be used. Thus, to guarantee that the stock of both raw material and product in each period cannot be used after the next ζ_c or χ_i time periods respectively, the following constraints are imposed:

$$IP_{it} \leq \sum_{\Omega=t+1}^{t+\chi_i} QS_{i\Omega} \quad \forall i, t \tag{35}$$

$$IM_{ct} \leq \sum_{\Omega=t+1}^{t+\zeta_c} RM_{c\Omega} \quad \forall c, t. \tag{36}$$

Eq. (35) ensures the lifetime of product i by enforcing that it is sold in less than χ_i time periods from when it is stored while Eq. (36) ensures that raw material c is processed in less than ζ_c time periods. It should be mentioned that the above constraints make sense only when time periods are equal in length, as well as the corresponding term in the objective function and the last terms in Eqs. (33) and (34).

Furthermore, stocks of both final product i and raw material c stored during period t cannot exceed the respective maximum available storage capacities, IP_{it}^{U} and IM_{ct}^{U}:

$$0 \le IP_{it} \le IP_{it}^{U} \quad \forall i, t \tag{37}$$

$$0 \le IM_{ct} \le IM_{ct}^{U} \quad \forall c, t. \tag{38}$$

The initial inventories of both raw material and product IM_{c0}, IP_{i0}, at the beginning of the time horizon are assumed to be given. The use of IM_{c0} and IP_{i0} have a strong impact when this model is only used for production planning without considering design, for example in an existing plant.

The amount consumed of raw material c in period t to manufacture product i, RM_{cit}, is obtained from a mass balance:

$$RM_{cit} = F_{cit}\, q_{it} \quad \forall c, i, t \tag{39}$$

where F_{cit} is a given parameter that accounts for the process conversion of raw material c to produce product i during period t. This parameter may suffer variations in every period t because of changes in composition of raw materials.

The total consumption of raw material c for production in period t, RM_{ct} is obtained from the following expression:

$$RM_{ct} = \sum_{i} RM_{cit} \quad \forall c, t. \tag{40}$$

If a given batch of product i late meeting a minimum product demand DE_{it}^{L}, then a late delivery ϑ_{it} take place in that period [13]. Late deliveries are undesirable; therefore they can be quantified by an appropriate penalty cost term which is minimized in the objective function. This term takes into account expenditures due to delay in satisfying the agreed demand.

$$\vartheta_{it} \ge \vartheta_{i,t-1} + DE_{it}^{L} - QS_{it} \quad \forall i, t. \tag{41}$$

3.5. Objective function

The objective function (42) of the problem is the maximization of the benefit of the net present value of the project taking into account the difference between total sales and costs. Total costs include purchases of raw materials, investments, inventories, operation, late delivery penalties and waste disposal costs.

$$\begin{aligned}\psi = {} & \sum_{t}\sum_{i} np_{it}\, QS_{it} - \sum_{t}\sum_{c} \kappa_{ct}\, C_{ct} - \sum_{j} M_j\, \alpha_j\, V_j^{\beta_j} - \sum_{j}^{J-1} \pi_j\, VT_j^{\tau_j} \\ & - \sum_{t}\left[\sum_{c} \varepsilon_{ct}\left(\frac{IM_{c,t-1}+IM_{ct}}{2}\right) H_t + \sum_{i} \sigma_{it}\left(\frac{IP_{i,t-1}+IP_{it}}{2}\right) H_t\right] \\ & - \sum_{t}\sum_{i}\left(co_{it}\, q_{it} + cp_{it}\, \vartheta_{it} + wp_{it}\, PW_{it}\right) - \sum_{t}\sum_{c} wr_{ct}\, RW_{ct}.\end{aligned} \tag{42}$$

Final products i are sold at price np_{it} and raw materials c are acquired at price κ_{ct} in every period t. Investment costs correspond to batch units and storage tanks and are calculated using a power law expression on the capacity [1]. Parameters α_j, and π_j are cost factors and β_j, τ_j are cost exponents for batch and stage tanks respectively. Inventory costs are assessed using an expression used by [9] and include both raw material and final product inventories with its corresponding cost coefficients ε_{ct} and σ_{it}. Operating costs include energy consumption in the process (steam, electricity, etc.) which are proportional to the production through cost coefficients co_{it}. Late delivery penalties are included with a cost coefficient cp_{it}. Finally, if time periods are equal, waste disposal costs wp_{it} per product i and wr_{ct} per raw material c are also considered.

The previous expression considers continuous sizes for units in the investment costs. In order to restrict it to the available discrete sizes, earlier defined binary variables are included in these terms. Thus, considering the equipment costs ψ_{EQ} separately,

$$\psi_{EQ} = \sum_{j} M_j\, \alpha_j\, V_j^{\beta_j} + \sum_{j}^{J-1} \pi_j\, VT_j^{\tau_j}\, st_{jv} \tag{43}$$

and replacing with the appropriate discrete variables:

$$\psi_{EQ} = \sum_{j}\sum_{m}\sum_{s} m\, \alpha_j\, v_{js}^{\beta_j}\, y_{jm}\, z_{js} + \sum_{j}^{J-1}\sum_{v\ne 1} \pi_j\, vt_{jv}^{\tau_j}\, st_{jv}. \tag{44}$$

Applying suitable transformations, the following expression is obtained:

$$\psi_{EQ} = \sum_{j}\sum_{m}\sum_{s} cb_{jms}\, r_{jms} + \sum_{j}^{J-1}\sum_{v\ne 1} ct_{jv}\, st_{jv}. \tag{45}$$

Table 1
Data for example 1.

	Batch stage Size factors, S_{ijt} (L/kg)						Processing time, pt_{ijt} (h)						Storage Size factor, ST_{ijt} (L/kg)				
i	1	2	3	4	5	6	1	2	3	4	5	6	St_1	St_2	St_3	St_4	St_5
P1	5.0	2.6	1.6	3.6	2.2	2.9	9.3	5.4	4.2	2.0	1.5	1.3	2.0	2.5	1.2	2.4	3.0
P2	4.7	2.3	1.6	2.7	1.2	2.5	8.5	5.8	4.1	2.5	1.4	1.5	2.5	2.0	1.2	2.5	1.2
P3	4.2	3.6	2.4	4.5	1.6	2.1	9.7	5.5	4.3	2.1	1.2	1.3	2.0	3.3	1.4	2.2	1.6

The terms $cb_{jms} = m\,\alpha_j\, v_{js}^{\beta_j}$ represent the cost of standard batch vessels and $ct_{jv} = \pi_j\, vt_{jv}^{\tau_j}$ the cost of standard storage vessels. New variables r_{jms} are introduced to eliminate the product of binary variables $z_{js}y_{jm}$ through the constraints:

$$r_{jms} \geq z_{js} + y_{jm} - 1 \quad \forall j, m, s. \tag{46}$$

Variable r_{jms} is equal to one when both variables y_{jm} and z_{js} take the value one. These new variables can be settled as continuous if the following bounds are added:

$$0 \leq r_{jms} \leq 1. \tag{47}$$

3.6. Formulation summary

In conclusion, the entire MILP formulation described in this paper is the maximization of the objective function (42) using Eq. (45) as the term of investment costs, subject to constraints (5)–(8), (16), (18)–(23), (26), (28), (30)–(41), (46) and (47) and the necessary bounds.

An important feature of this model is that the discrete variables only depend on plant design and are independent of time periods, which allows handling large problems with less computational effort.

4. Examples

The presented model can be applied in very different cases and contexts. Depending on product lifetime, characteristics of the production facilities, management objectives, etc., different elements of the model can be emphasized, not taken into account or even rejected. For example, in a very long term problem, with one-year periods, inventory policies are less significant. On the other hand, in a short term model, with one-week periods, equipment investment is likely to be less valuable for inclusion. Nevertheless, the cases with a close integration between design and planning are here emphasized. Then, several factors must be analyzed when the model is formulated. The significance of the elements contemplated in the model and the trade-offs among them are strongly related with the specific modeled scenario. Several model assumptions are affected, for example by the number of time periods and their length.

In this section, an example is proposed to illustrate the key features of the approach described in the previous sections for multiproduct batch plant design and operation planning in a multiperiod scenario. Different approaches can be posed with this model depending on the manager requirements and objectives. Here, using the same data, two problems are presented in order to show the versatility and usefulness of this formulation. Also, the computational performance is assessed.

4.1. Example 1

A multiproduct batch plant is considered that involves the production of three products P1–P3 using two raw materials C1 and C2 processed in six stages over a planning horizon of 2 years. Due to the seasonal provision of raw materials, a discretization interval of 3 months is used for the multiperiod MILP model, resulting in 8 time periods ($H_t = 1500$ h). Size factors and processing times for every product are given in Table 1. As was already mentioned, the units in each stage can be duplicated, thus, batch stages may consist of up to two parallel items. Available discrete sizes to perform every stage involved in the plant are shown in Table 2. Data for raw materials as well as parameters F_{cit}, which are assumed to be equal for all time periods, are shown in Table 3. Bounds on demands, prices of products, and costs of each ingredient in every period are presented in Table 4.

The inventory cost coefficient for all final products is \$0.4/(ton h) and the product lifetime in number of periods is 4. Cost coefficients for late delivery it is assumed as a 50% of product prices.

The problem involves 2088 continuous and 67 binary variables in 3136 constraints. Results and optimal sizes are summarized in Table 5. Fig. 2 shows the plant structure where batch stage 1 has two parallel units which operate out of phase and one in the others stages. In this way, the limiting cycle time determined by stage 1 is reduced. Finally, a storage tank is allocated between batch stages 3 and 4 which decouples the plant operation, allowing a reduction in the size units of the equipments that belong to the downstream subprocess. Thus the capital cost is also decreased. Table 6 shows the optimal production q_{it} for each product during the time horizon. A detailed analysis of the economic results is summarized in Table 7. The solution was obtained using GAMS/CPLEX 9.0 in a CPU time of 29.20 s with a 0% margin of optimality in a Pentium(R) IV CPU (3.00 GHz).

Table 2
Available standard sizes.

Option	Batch stages Discrete volumes, v_{js} (L)						Storage tanks Discrete sizes, vt_{jv} (L)	
	1	2	3	4	5	6		
1	2000	1500	1000	1000	500	500		0
2	2500	2000	1250	1500	750	750		1500
3	3000	2500	1500	2000	1000	1000		2000
4	3500	3000	2000	2500	1250	1250		2500
5	4000	3500	2500	3000	1500	1500		3000
	Cost coefficient			α_j	1250		π_j	950
	Cost exponent			β_j	0.6		τ_j	0.6

Table 3
Parameters, initial inventories and cost of raw materials.

	Parameter F_{ci}			Initial inventory (kg)	Storage cost ($/(ton h))	Lifetime (time periods)
c	P1	P2	P3	IM_{c0}	ε_c	ζ_c
C1	0.5	1.0	0.7	20 000	0.05	3
C2	1.5	1.2	1.0	40 000	0.05	3

Table 4
Costs, prices and demand bounds for example 1.

	Costs of raw materials, ($/kg)		Prices of products, ($/kg)			Bounds on demands, ($\times 10^3$ kg)		
	κ_{ct}		np_{it}			$DE_{it}^L - DE_{it}^U$		
t	C1	C2	P1	P2	P3	P1	P2	P3
1	1.0	0.5	2.05	2.60	2.00	25–50	22.5–45	20–40
2	1.5	0.8	2.25	2.60	2.20	26.5–53	24–48	21–42
3	1.5	0.5	2.25	2.40	2.20	27.5–55	25.5–51	22.5–45
4	1.0	0.8	2.05	2.40	2.00	28.5–57	26.5–53	24–48
5	1.0	0.5	2.05	2.60	2.00	31.5–63	28.5–57	26–52
6	1.5	0.8	2.25	2.60	2.20	34–68	29.5–59	26.5–53
7	1.5	0.5	2.25	2.40	2.20	36–72	31–62	27.5–55
8	1.0	0.8	2.05	2.40	2.00	36–72	31–62	27.5–55

Table 5
Results of example 1.

	Stage					
	1	2	3	4	5	6
V_j	3000	2000	1250	1000	500	750
VT_j			1500			
M_j	2	1	1	1	1	1

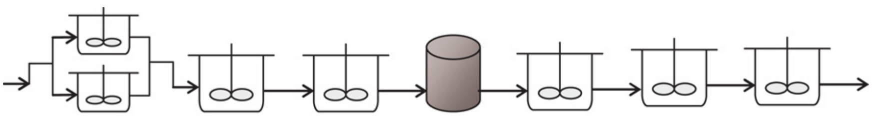

Fig. 2. Optimal flowsheet of the plant.

Table 6
Example 1—Optimal productions of each product in every period.

i	Period								Total (kg)
	1	2	3	4	5	6	7	8	
A	50 000	53 000	98 608	13 391	63 000	68 000	72 000	58 909	476 908
B	67 629	25 370	51 000	53 000	82 396	33 603	31 000	44 640	388 638
C	40 000	73 814	13 185	84 056	15 943	54 560	53 439	39 111	374 108

Assessing the computational performance, a critical point in multiperiod models is the number of time periods. Table 8 compares number of variables, number of constraints and CPU time for the solution of this problem considering different

Table 7
Example 1—Economic evaluation results.

Description	Optimal value
Sales incomes	2789 021.86
Supply costs	1626 893.70
Investment cost for batch units	711 922.07
Investment cost for storage tanks	76 450.15
Raw material inventory costs	102 498.80
Product inventory costs	24 159.99
Operating costs	123 966.02
Waste disposal costs	0.00
Late delivery penalties	0.00
Total: Profit ($)	123 131.12

Table 8
Example 1—Comparison for different time periods considered.

Number of periods	Binary variables	Continuous variables	Constraints	CPU time (s)
4	67	1108	1680	3.750
8	67	2088	3136	29.203
12	67	3068	4592	95.296
16	67	4048	6048	136.828

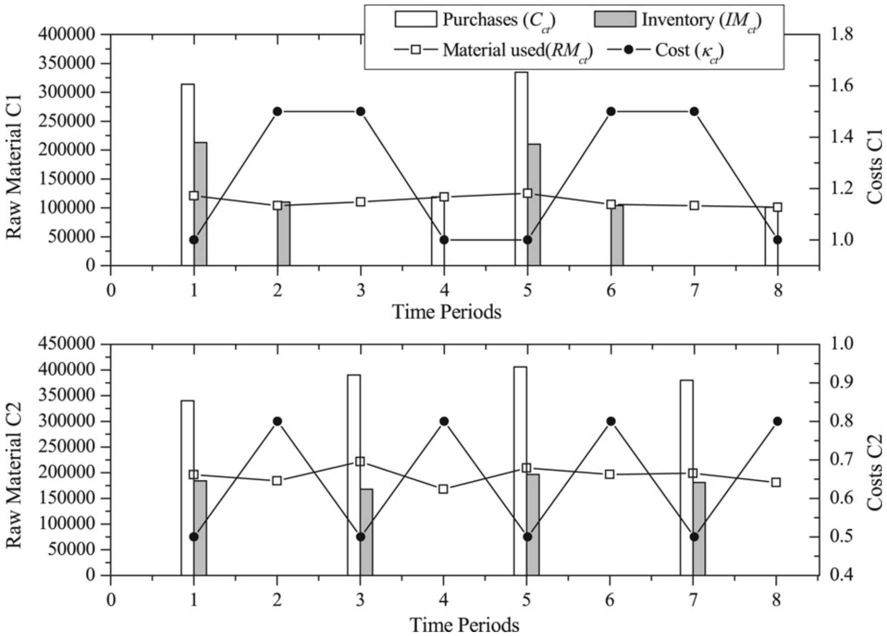

Fig. 3. Profiles for raw materials.

number of periods. It can be noted that the number of binary variables remain the same in all cases since they do not depend on time periods, and the problems can be solved on a PC in a few seconds.

Finally, the detailed optimal production plan is shown through Figs. 3–6. Namely, they show the results for products and raw materials in which production, inventory, purchases and sales profiles are disaggregated by periods at the optimal solution.

Fig. 3 is divided in two diagrams corresponding to raw materials C1 and C2. The first diagram shows that purchases of C1 are made only on periods 1, 4, 5 and 8 where costs have the lowest value. Also, the extra material purchased in periods 1 and 5 is kept as inventory for production in the subsequent periods. Note that material purchased in periods 4 and 8 is totally consumed in those time periods. In the second diagram, costs of raw material C2 have a cyclical variation that results in purchases being made only in periods where the cost is lower and the extra material is kept as inventory to be used in production in the following period.

Fig. 4 shows that product P1 is produced in all time periods; but in time period 3, the amount in excess is stored in order to satisfy maximum demand in the next time period where production reaches its lowest value. Also, maximum demands are satisfied in all periods except in the last one.

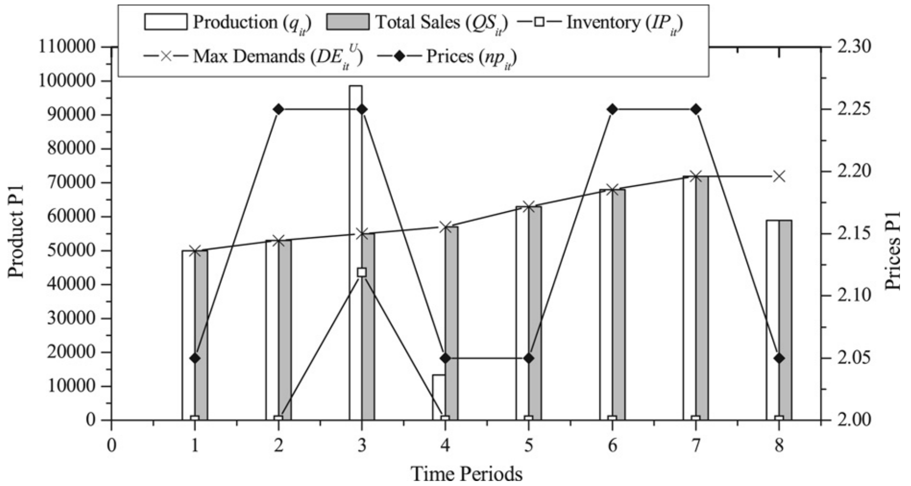

Fig. 4. Profile for product P1.

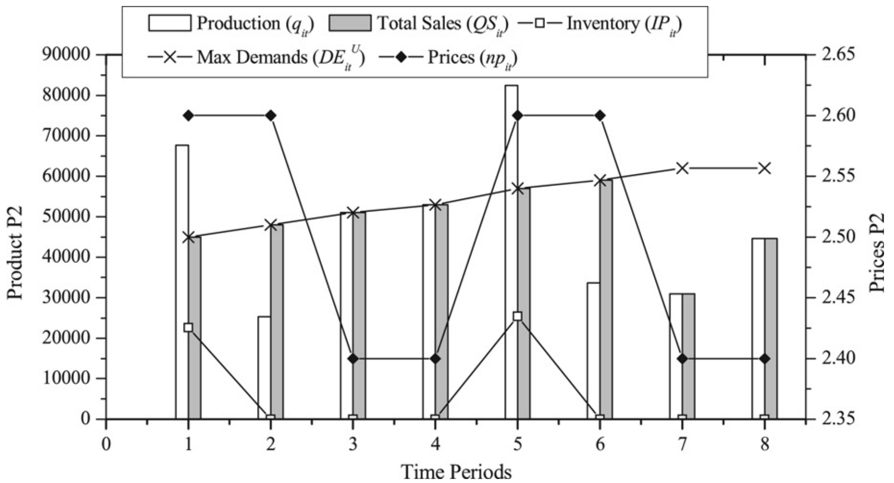

Fig. 5. Profile for product P2.

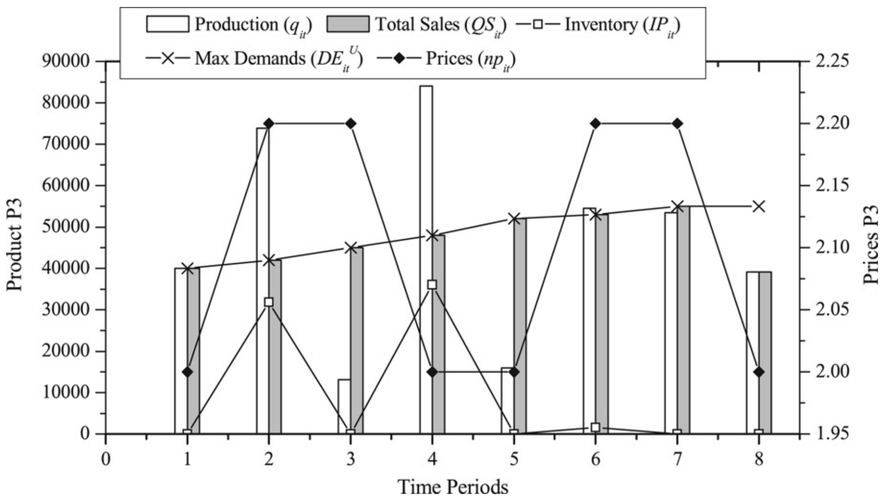

Fig. 6. Profile for product P3.

Table 9
Costs, prices and demand bounds for example 2.

	Costs of raw materials, ($/kg)		Prices of products, ($/kg)			Bounds on demands, ($\times 10^3$ kg)		
	κ_{ct}		np_{it}			$DE_{it}^L - DE_{it}^U$		
t	C1	C2	P1	P2	P3	P1	P2	P3
1	1.0	0.5	2.05	2.60	2.00	8–16	8.43–16.8	7.5–15
2	1.5	0.8	2.25	2.60	2.20	8.55–17.1	9–18	7.87–15.7
3	1.5	0.5	2.25	2.40	2.20	8.92–17.8	9.56–19.1	8.43–16.8
4	1.0	0.8	2.05	2.40	2.00	9.25–18.5	9.93–19.8	8.5–17
5	1.0	0.5	2.05	2.60	2.00	9.52–19	9.75–19.5	8.75–17.5
6	1.5	0.8	2.25	2.60	2.20	10–20	10–20	8.9–17.8
7	1.5	0.5	2.25	2.40	2.20	10.5–21	10–20	8–16
8	1.0	0.8	2.05	2.40	2.00	11–22	10–20	8.75–17.5
9	1.0	0.5	2.05	2.60	2.00	11.7–23.5	10.5–21	8.75–17.5
10	1.5	0.8	2.05	2.60	2.00	12.2–24.5	10.8–21.7	9.62–19.2
11	1.5	0.5	2.25	2.60	2.20	12.5–25	11.2–22.5	10–20
12	1.0	0.8	2.25	2.60	2.20	13.5–27	11.6–23.2	10.5–21

Table 10
Example 2–Optimal production planning.

	P1 ($\times 10^3$ kg)			P2 ($\times 10^3$ kg)			P3 ($\times 10^3$ kg)			C1 ($\times 10^3$ kg)		C2 ($\times 10^3$ kg)	
t	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	C_{ct}	IM_{ct}	C_{ct}	IM_{ct}
1	16.00	16.00	0.0	21.29	16.87	4.42	15.00	15.00	0.0	94.89	75.09	87.34	62.78
2	17.10	17.10	0.0	13.58	18.00	0.0	20.84	17.75	5.09	0.0	38.37	0.0	0.0
3	22.00	17.85	4.15	19.12	19.12	0.0	11.78	16.87	0.0	0.0	0.0	130.7	63.02
4	14.34	18.50	0.0	19.87	19.87	0.0	17.65	17.00	0.65	39.41	0.0	0.0	0.0
5	19.05	19.05	0.0	25.35	19.50	5.86	8.82	9.47	0.0	114.7	73.66	132.6	64.84
6	20.00	20.00	0.0	14.14	20.00	0.0	17.87	17.87	0.0	0.0	37.00	0.0	0.0
7	21.00	21.00	0.0	15.30	15.30	0.0	16.00	16.00	0.0	0.0	0.0	133.8	68.02
8	22.00	22.00	0.0	20.00	20.00	0.0	11.02	11.02	0.0	38.71	0.0	0.0	0.0
9	17.22	17.22	0.0	27.22	21.00	6.22	8.75	8.75	0.0	121.3	79.35	128.8	61.60
10	12.25	12.25	0.0	19.66	21.75	4.14	19.62	9.62	10.0	0.0	39.83	0.0	0.0
11	29.90	25.00	4.90	24.87	22.50	6.51	0.0	10.00	0.0	0.0	0.0	141.7	67.03
12	22.09	27.00	0.0	16.74	23.25	0.0	13.81	13.81	0.0	37.45	0.0	0.0	0.0

Fig. 5 illustrates product P2 profile, where clearly can be seen that production in time periods 1 and 5 are higher than demands because the prices of both raw materials are lower. The extra amount is held as inventory to meet maximum demands in the following intervals. Maximum demands are not satisfied only in the last two periods where minimum demands are covered because of the lower price.

Results for product P3 are shown in Fig. 6. The excess of production made in periods 2, 4 and 6 is carried forward as inventory for satisfying demands in subsequent periods. Only in the last period, do sales of product P3 not satisfy maximum demands.

It should be noted that late deliveries and wastage of raw material or final product are not fined in any of the time periods in this case.

Using the results of example 1, the managers can assess different alternatives taking into account demand forecasts, products, market structure, time horizon, prices, etc. working with the same model, that are not contemplated in this article. In the first version, these alternatives must be evaluated through deterministic data. Future versions should include probability distribution functions to assess fluctuations of the problem parameters.

4.2. Example 2

In order to demonstrate the versatility of this approach, only the case of production planning is considered. It is assumed that plant structure is given by the optimal solution obtained for Example 1, and the other problem elements can vary. In the example 2, the same products as in the previous example are processed over a planning horizon of 1 year divided into 12 equal time periods ($H_t = 500$ h) allowing managers to get a detailed plan of production for each month.

Data related to demand patterns, raw materials costs and product prices are given in Table 9. The data for parameters F_{cit}, processing times, size factors, and cost coefficients are the same in example 1.

The model involves 3068 continuous variables and 4592 constraints. It is worth mentioning that the problem was solved in CPU time of only 0.12 s since binary variables are not involved, so a linear problem is formulated.

An optimal objective function value of $537306.30 was obtained. The detailed production planning decisions, i.e. purchases, production, sales, and inventories for each product and raw materials to be made in every period, are summarized in Table 10. The economic results of the optimal solution for this problem are summarized in Table 11.

Table 11
Example 2—Economic evaluation results.

Description	Optimal value
Sales incomes	1445 442.64
Supply costs	824 137.77
Raw material inventory costs	18 266.02
Product inventory costs	2 597.82
Operating costs	63 134.73
Waste disposal costs	0.00
Late delivery penalties	0.00
Total: Profit ($)	537 306.30

From Table 10, it can be seen that raw materials C1 and C2 are purchased during periods where their costs are the lowest ones. Also, the extra amount acquired in those periods is held as inventory for the production of final products in the following periods. All products are produced in all time periods except product P3 in the eleventh period, where the minimum demand is satisfied with the amount in inventory from the previous period.

5. Conclusions

A general model has been presented in this paper to simultaneously address the problems of multiproduct batch plant production planning and design over a multiperiod scenario. The original non-linear formulation has been transformed so as to obtain a mixed integer linear programming formulation which can be solved to global optimality.

Several features can be stressed. The model considers design and production planning decisions at the same time. Previous efforts used to solve these problems separately which hinders the interactions between both types of decisions. Also, the model considers a multiperiod scenario. Then, seasonal and market variations can be taken into account.

From the design point of view, the model considers several interesting elements: different configuration options are included (duplication of batch units, intermediate storage tanks allocation) and a real procurement policy is adopted, with units available in discrete units. From the production planning point of view, all the usual decisions are contemplated: inventories, sales, purchases, etc. In this first approach, deterministic fluctuations have been considered.

This is an interesting formulation that allows managers to have a feedback about the impact of his or her decisions, considering interactions between design, commercial, production, sales and inventory policies simultaneously.

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