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Multiperiod optimization for the design and planning of multiproduct batch plants

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Abstract

This paper presents a general multiperiod optimization model, which simultaneously solves the design and planning decisions in multiproduct batch plants. Therefore, the trade-offs between both problems are taken into account as well as variations due to seasonal effects, demand patterns, etc.

From the design point of view, the model is formulated considering batch and semicontinuous units, the allocation of intermediate storage, and structural decisions. Following the usual procurement policy, equipment is provided using discrete sizes. From the planning point of view, the formulation takes into account both products and raw materials inventories, product demands and raw materials supplies that vary seasonally in a multiperiod approach.

The objective is the maximization of an economic function, which considers incomes, and both investment and operation costs. A plant that produces five oleoresins in seven stages is used to illustrate this approach.

Keywords: Multiperiod optimization; Multiproduct batch plants; Batch process design

1. Introduction

In the last years, the growing importance of batch processing has been widely known. The rapid development of new products, the suitability of these processes to produce complex and high value products in low volume and its flexibility have encouraged a rapid development in this area.

Multiproduct batch plants are characterized by the production of multiple products with similar recipes. In the design problem, the production requirement of each product and the total production time available are specified. A procedure is generated in order to determine the plant configuration and the equipment sizes to minimize the capital cost. Several works have considered this problem assuming several conditions: given recipes, generally single product campaigns, only one time period, etc. Using these assumptions, several models were developed and solved through different methodologies: mathematical programming (Montagna, Vecchietti, Iribarren, Pinto, & Asenjo, 2000; Ravemark & Rippin, 1998), heuristics methods (Dietz, Azzaro-Pantel, Pibouleau, & Domenech, 2006; Huang & Wang, 2002; Jayaraman, Kulkarni, Karale, & Shelokar, 2000; Patel, Mah, & Karimi, 1991; Wang, Quan, & Xu, 1996) or simulation (Maruejouls, Azzaro-Pantel, Schirlin, Pibouleau, & Domenech, 2002; Petrides, Koulouris, & Lagonikos, 2002). The development of this area was called "filling in the hole" by Rippin (1993), from works that achieved small successive improvements.

Several works have recognized the trade-offs between design decisions and synthesis, operation and scheduling problems, and have developed more complex models to assess them. For example, Birewar and Grossmann (1989, 1990) have considered the relationship between design, synthesis and scheduling problems. Voudouris and Grossmann (1993) have addressed a model to determine the optimal configuration and cyclic operation of multiproduct batch plants. Xia and Macchietto (1997) have solved the design and synthesis of batch plants through a stochastic method. Iribarren et al. (2004) have posed a model using disjunctive programming to solve the synthesis and design for the production of multiple recombinant proteins.

Most of the works formulate the models considering a time horizon composed by only one period. This approach hinders to consider processes that operate under variations in the model

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Nomenclature

a_{ku} binary variable that denotes if equipment at semicontinuous stage k has size u
B_{ijt} batch size of product i in stage j in period t
C_{it} amount of raw material for producing i purchased in period t
d_j binary variable that indicates if the tank is allocated in position j
D_{ikt} duty factor of product i in stage k in period t
DE_{it} demand for product i in period t
E_i extent of extraction for product i
f_{ijvt} continuous variable that represents the product of the variables $q_{it}st_{jv}$
fx product concentration in the solid feed P
fy product concentration in the extraction solvent L
F_{it} parameter that accounts for the process conversion of i in period t
G_k number of semicontinuous units in parallel in phase in stage k
h_{ikugt} continuous variable that represents the product of the variables $q_{it}a_{ku}x_{kg}$
H time horizon
H_t net available production time for all products in period t
IM_{it} inventory of raw material i at the end of a period t
IP_{it} inventory of final product i at the end of a period t
L extraction solvent
m_j number of discrete sizes available for stage k
M_j number of batch units in parallel out of phase in stage j
n_{ijt} number of batches of product i in stage j in period t
n_j number of discrete sizes available for stage j
np_{it} price of product i in period t
p_j number of discrete sizes available for storage tanks
P solid feed
$\mathrm{PE}_{i,t-1}$ factor that accounts the loss of raw materials
PR_{it} productivity of i in time period t
q_{it} amount of product i to be produced in period t
QS_{it} amount of product i sold at the end of period t
r_{jms} continuous variable that represents the product of the binary variables $z_{js}y_{jm}$
R_k size of semicontinuous item k
RM_{it} raw material inventory for product i in period t
$stjv$ binary variable that denotes if storage at position j has size v
S_{ijt} size factor of product i in stage j for each period t
ST_{ijt} size factor for product i for an intermediate storage tank in the location j
t_{ijt} processing time of product i in batch stage j in period t
T_{it} total time for producing product i in period t
TL_{it}^{p} limiting cycle time of product i for a subprocess p in period t
T_{ijt} time required to process a batch of product i in stage j in period t
V_j size of a batch unit at stage j
VT_j size of the intermediate storage tank allocated in position j
w_{itjk} continuous variable that represents the product of the variables $n_{ijt}y_{jk}$
x_{kg} binary variable that denotes if semicontinuous stage k has g units of the same size in parallel
y_{jm} binary variable that denotes if batch stage j has m units of the same size in parallel
z_{js} binary variable that denotes if equipment at batch stage j has size s

Greek letters

α_j cost coefficient for a batch unit in stage j
β_j cost exponent for a batch unit at stage j
γ_k cost coefficient for a semicontinuous unit in stage k
δ_k cost exponent for the semicontinuous unit in stage k
ε_i inventory cost coefficient for raw material i
η_i extraction factor for product i
θ_{ikt} processing time of product i for semicontinuous unit k in period t
κ_{it} price for the raw material of product i in period t
λ_{ijvt} continuous variable that represents the product of the variables $n_{ijt}st_{jv}$
μ_{ibjmt} continuous variable that represents the product of the variables $\xi_{ibt}y_{jm}$
ν_{js} standard volume of size s for batch unit at stage j
ξ_{ibt} variable that represents the product of $\phi_{ibt}n_{ijt}$
π_j cost coefficient for an intermediate storage tank allocated in position j
ρ_{kgu} continuous variable that represents the product of the binary variables $a_{ku}x_{kg}$
σ_i inventory cost coefficient for product i
τ_j cost exponent for an intermediate storage tank allocated in position j
υ_{jv} standard volume of size v for storage tank allocated in position j
ϕ_{ibt} operating time of a semicontinuous subtrain b for product i in period t
Φ Maximum ratio allowed between consecutive batch sizes
ω_{ku} Standard size u for semicontinuous unit at stage k

Subscripts

b semicontinuous subtrain
g number of parallel units at semicontinuous stages
i product
j batch stage
k semicontinuous stage

m	number of parallel units at batch stages
s	discrete sizes for batch stages
t	time period
u	discrete sizes for semicontinuous stages
v	discrete sizes for storage tanks

Superscripts

d	downstream
f	final
i	initial
L	lower bound
n	number of stages in the extraction
p	subprocess
u	upstream
U	upper bound

parameters along the time horizon. Costs, raw materials and demands typically vary from period to period due to market or seasonal reasons. Some authors have posed solutions considering the uncertainty in the model parameters (Petkov & Maranas, 1998; Tarifa & Chiotti, 1995). For example, Ierapetritou and Pistikopoulos (1996) proposed a formulation using stochastic programming for simultaneously solving the design and operation of batch plants with uncertainty in process parameters and product demands. Another approach to consider variability is the development of multiperiod optimization models that involve designing plants that operate under variations in the model parameters along all the periods in time horizon. Costs and demands typically vary from period to period. Varvarezos, Grossmann, and Biegler (1992) resorted to decomposition methods to solve multiperiod MINLP. Van den Heever and Grossmann (1999) considered a general disjunctive multiperiod nonlinear optimization model for the design, operation planning and capacity expansion of general chemical process systems.

Taking into account these previous efforts, a new formulation is considered for the design of multiproduct batch plants. The proposed model considers simultaneously design and planning decisions on a multiperiod context. In this way, variations of prices, costs and product demands due to seasonal reasons are considered. Also, the multiperiod approach presented with different period lengths overcomes the strong limitation posed by previous models that considered single product campaigns. Here, a long time horizon can be decomposed in shorter periods where the single product campaigns assumption is reasonable. From the design point of view, all the units usually considered in batch process are involved: storage tanks, batch and semicontinuous units.

Also, the proposed model considers discrete sizes for the units. This is a usual procurement policy in this industry. Besides, it is an assumption used by other authors (Dietz, Azzaro-Pantel, Pibouleau, & Domenech, 2005; Ierapetritou & Pistikopoulos, 1996; Sparrow, Forder, & Rippin, 1975; Tan & Mah, 1998; Voudouris & Grossmann, 1992) that, from the mathematical point of view, allows a mixed integer linear program (MILP) formulation. Therefore, a realistic design case is posed, which can be solved to global optimality with reasonable computational effort.

The paper is organized as follows. First, the description of the problem is presented. Then, a multiperiod model is formulated which incorporates all the elements of the design and operation planning problems. Finally, the solution of the MILP is illustrated with a specific example of a multiperiod batch plant that produces Oleoresins.

2. Problem description

The multiperiod design and operation planning problem for a multiproduct noncontinuous plant can be stated as follows. It is assumed that a total of I products is to be produced in a multiproduct batch plant which consists of J batch processing stages, K semicontinuous stages that form L semicontinuous subtrains. Intermediate storage tanks can be allocated between batch stages. The products are manufactured over a time horizon H, which is divided in T time periods of specified length H_t not necessarily equal.

The basic data for units are the size/duty factors for each batch/semicontinuos stage for each product i in every period t, S_{ijt}/D_{ikt}; the processing times for each product i at a batch stage j, t_{ijt}. For each product i, the lower and upper bounds on its demands in every period t, $DE_{it}^{\mathrm{L}}/DE_{it}^{\mathrm{U}}$, are known.

The objective function ψ to be maximized is the net present value of the profit along the global time horizon, taking into account incomes from product sales, expenditures from raw materials purchases, inventory and investments costs.

Following are the key assumptions, which are usual for this kind of problem:

1. The size/duty factors are constant for each product.
2. When multiple parallel units are considered at a stage, they have the same size.
3. The plant operates in single product campaign (SPC) mode in each time period.
4. Batch units in parallel operate out of phase, while semicontinuous units operate in phase.
5. When storage tanks are not allocated, ZW (zero wait) policy is employed.
6. If intermediate storage tanks are employed in the process, their role is only to decouple the operations of the stages upstream from those downstream to the tank; they do not act as a long-term storage, so FIS (finite intermediate storage) policy is adopted.

Three levels of decisions are considered in the solution of this problem:

1. Structural decisions.
2. Design decisions.
3. Production planning decisions.

The first two decisions are independent from the time periods, and involve investment costs, while the last one is valid for every time period. The first level decides the number of parallel units, which operates out of phase at batch stage j and the number of parallel units that operate in phase at the semicontinuous stage k. The allocation of intermediate storage tank is made at this level too. The allocation and sizing of intermediate storage has been included in the model to obtain a more efficient plant design. For this reason, a multiproduct plant can be viewed as a series of subprocesses, which are separated by intermediate storage. Design decisions involve the selection of equipment sizes for both batch and semicontinuous units, and intermediate storage tanks among available discrete sizes.

Finally, the last level determines at each period t and for each product i the amount to be produced, the number of batches at stage j, and the total time to produce product i. Furthermore, inventory considerations are taken into account in the plant operation because of the possible seasonal variations of raw materials availability and product demands. They may be maintained in stock until they are consumed. Then, for each product and raw material at the end of every period t, inventories levels are obtained. Moreover, total sales, amount of raw material purchased, and raw material to be used for the production of product i in each period t are determined with this formulation. In this model two cases are considered, i.e., when the elaboration of product i depends on only one raw material, and the general case, when production requires several raw materials.

3. Model formulation

The problem involves optimizing the process over all time periods $t=1, \ldots, T$ and has the object of maximizing the profit subject to the constraints described below.

3.1. Batch equipment

The general batch process literature (Biegler, Grossmann, & Westerberg, 1997) describes the batch unit size of stage j, V_j, through a sizing equation that is applied for each product i as follows:

$$V_j \geq S_{ij} B_i, \quad \forall i, j \tag{1}$$

where B_i is the batch size for product i, e.g. kg of product exiting from the last stage; and S_{ij} is the size factor of product i for stage j, i.e., the size needed at stage j to produce 1 kg of final product i.

The allocation of a storage tank decouples the process into two subprocesses: one upstream from the tank, and the other downstream. This in turn allows batch sizes on either side of the tank to be chosen independently. Therefore, the previously unique B_i is changed to batch sizes B_{ij} defined for product i in stage j. Appropriate constraints adjust the batch sizes among different units. Considering the multiperiod character, the size of a unit in a batch stage j is given by:

$$V_j \geq S_{ijt} B_{ijt}, \quad \forall i, j, t \tag{2}$$

where B_{ijt} and S_{ijt} denote the batch size and size factor, respectively of product i in stage j for each period t.

The amount of product i produced in time period t, q_{it} depends on the number of batches n_{ijt}, as defined by:

$$q_{it} = B_{ijt} n_{ijt}, \quad \forall i, j, t \tag{3}$$

By combining Eqs. (2) and (3) the following constraints are obtained:

$$n_{ijt} \geq \frac{S_{ijt} q_{it}}{V_j}, \quad \forall i, j, t \tag{4}$$

As already mentioned, variable V_j is restricted to take values from the set $\mathrm{SV}_j = \{\nu_{j_1}, \nu_{j_2}, \ldots, \nu_{jn_j}\}$, where ν_{js} represents the discrete size s for batch equipment j and n_j is the given number of discrete sizes available for stage j. To rigorously tackle this situation for batch stages, the following binary variable is introduced:

$$z_{js} = \begin{cases} 1 & \text{if equipment at batch stage } j \text{ has size } s \\ 0 & \text{otherwise} \end{cases}$$

Using the above definition, $1/V_j$ can be expressed in terms of the defined discrete variables as follows:

$$\frac{1}{V_j} = \sum_s \frac{z_{js}}{\nu_{js}}, \quad \forall j \tag{5}$$

$$\sum_s z_{js} = 1, \quad \forall j \tag{6}$$

In this way, by substituting Eq. (5) into Eq. (4), new constraints can be formulated that restrict the volumes to discrete sizes

$$n_{ijt} \geq \sum_s \left(\frac{S_{ijt} q_{it}}{\nu_{js}} \right) z_{js}, \quad \forall i, j, t \tag{7}$$

Constraint (7) is nonlinear because of the bilinear terms $q_{it} z_{js}$. In order to eliminate these bilinearities, a new nonnegative continuous variable e_{ijst} is defined to represent this cross-product (Ierapetritou & Pistikopoulos, 1996; Voudouris & Grossmann, 1992). Then the following linear constraints are obtained:

$$n_{ijt} \geq \sum_s \left(\frac{S_{ijt}}{\nu_{js}} \right) e_{ijst}, \quad \forall i, j, t \tag{8}$$

$$e_{ijst} \leq q_{it}^{\mathrm{U}} z_{js}, \quad \forall i, j, s, t \tag{9}$$

$$q_{it} = \sum_s e_{ijst}, \quad \forall i, j, t \tag{10}$$

where q_{it}^{U} represents the upper bound for q_{it}.

3.2. Semicontinuous equipment

Semicontinuous units operate continuously with periodic startups and shutdowns. The processing time of product i for semicontinuous unit k in period t, θ_{ikt}, considering that G_k parallel units in phase are available, can be calculated by:

$$\theta_{ikt} = \frac{D_{ikt} B_{ijt}}{G_k R_k}, \quad \forall i, k, t \tag{11}$$

where j is the batch stage adjoining unit k, D_{ikt} the duty factor of product i in period t, i.e. the size needed in stage k to obtain 1 kg of final product i, and R_k is the size of the semicontinuous item k, usually a processing rate. This size depends on the type of unit, for example, in pumps it is the power, in heat exchangers is the exchange area, etc.

A series of semicontinuous equipment with no batch unit or intermediate storage among them is a semicontinuous subtrain. All the units belonging to subtrain b must operate for the same length of time to avoid accumulation of material. In this way, the operating time of a semicontinuous subtrain is the maximum operating time of all the semicontinuous units that belong to that subtrain. Thus, the operating time of a semicontinuous subtrain b for product i in period t, ϕ_{ibt}, is given by:

$$\phi_{ibt} = \max_{k \in b}(\theta_{ikt}), \quad \forall i, b, t \tag{12}$$

By substituting Eq. (3) into Eq. (11), the above relation can be expressed as

$$\phi_{ibt} \geq \frac{D_{ikt} q_{it}}{G_k R_k n_{ijt}}, \quad \forall i, b, k \in b, t \tag{13}$$

By setting the operating time of a semicontinuous subtrain to be equal to the longest operating time of all the stages in the subtrain, the requirement that all the stages operate for the same length of time can be satisfied. The rates of the remaining semicontinuous stages in the semicontinuous subtrain can always be reduced during the operation of the plant by increasing their operating time.

In a similar way, the size of the semicontinuous unit R_k is restricted to take values from the set $\mathrm{SR}_k = \{\omega_{j1}, \omega_{j2}, \ldots, \omega_{km_k}\}$, where ω_{ku} denote discrete size u for semicontinuous unit k and m_k is the number of discrete sizes available for stage k. Thus, the following binary variable is defined:

$$a_{ku} = \begin{cases} 1 & \text{if equipment at semicontinuous stage } k \text{ has size } u \\ 0 & \text{otherwise} \end{cases}$$

Again, the inverse of the semicontinuous sizes can be expressed in terms of discrete variables as

$$\frac{1}{R_k} = \sum_u \frac{a_{ku}}{\omega_{ku}}, \quad \forall k \tag{14}$$

$$\sum_u a_{ku} = 1, \quad \forall k \tag{15}$$

As suggested by Kocis and Grossmann (1988), parallel units can be treated by introducing the discrete variable:

$$x_{kg} = \begin{cases} 1 & \text{if semicontinuous stage } k \text{ has } g \text{ units} \\ & \quad \text{of the same volume in parallel} \\ 0 & \text{otherwise} \end{cases}$$

These constraints are used to represent an integer number for G_k where the available and feasible options g are considered for each stage:

$$\frac{1}{G_k} = \sum_g \frac{x_{kg}}{g}, \quad \forall k \tag{16}$$

$$\sum_g x_{kg} = 1, \quad \forall k \tag{17}$$

Then, introducing (14) and (16) into Eq. (13), the following expression is obtained:

$$\phi_{ibt} n_{ijt} \geq \sum_g \sum_u \left(\frac{D_{ikt} q_{it}}{g \omega_{ku}} \right) a_{ku} x_{kg}, \quad \forall i, b, k \in b, t \tag{18}$$

In order to reformulate the constraints for the operating time of semicontinuous subtrains as linear ones, first a new variable has to be introduced in the formulation

$$\xi_{ibt} = \phi_{ibt} n_{ijt}, \quad \forall i, b, t \tag{19}$$

where n_{ijt} corresponds to the stage j which is immediately adjoining the subprocess b.

Constraint (18) can now be written as

$$\xi_{ibt} \geq \sum_g \sum_u \left(\frac{D_{ikt} q_{it}}{g \omega_{ku}} \right) a_{ku} x_{kg}, \quad \forall i, b, k \in b, t \tag{20}$$

The cross-product $q_{it} a_{ku} x_{kg}$ can be eliminated by introducing the following variable

$$h_{ikugt} = \begin{cases} q_{it} & \text{if } a_{ku} \text{ and } x_{kg} \text{ are } 1 \\ 0 & \text{otherwise} \end{cases}$$

with which constraint (20) can be replaced by the linear expression:

$$\xi_{ibt} \geq \sum_g \sum_u \left(\frac{D_{ikt}}{g \omega_{ku}} \right) h_{ikugt}, \quad \forall i, b, k \in b, t \tag{21}$$

To define the variables h_{ikugt}, the following equivalence constraints are introduced (Voudouris & Grossmann, 1993):

$$\sum_g h_{ikugt} \leq q_{it}^{\mathrm{U}} a_{ku}, \quad \forall i, k, u, t \tag{22}$$

$$\sum_u h_{ikugt} \leq q_{it}^{\mathrm{U}} x_{kg}, \quad \forall i, k, g, t \tag{23}$$

$$q_{it} = \sum_g \sum_u h_{ikugt}, \quad \forall i, k, t \tag{24}$$

3.3. Time constraints

A batch unit is periodically operated with steps of filling, processing, discharging, and possibly waiting. The time required to process a batch of product i in stage j in each period t, T_{ijt}, is the sum of the processing time of product i in batch stage j, t_{ijt}, and both the filling time $\phi_{ib_j^u t}$, and the emptying time $\phi_{ib_j^d t}$, corresponding to the upstream b_j^u and downstream b_j^d semicontinuous subtrains, respectively, if they are included in the plant. Considering the existence of M_j parallel units operating out of phase in a batch stage j and, upstream and downstream semicontinuous subtrains, T_{ijt} is given by:

$$T_{ijt} = \frac{\phi_{ib_j^u t} + t_{ijt} + \phi_{ib_j^d t}}{M_j}, \quad \forall i, j, t \tag{25}$$

The following discrete variables are introduced in order to treat parallel units in at batch stages

$$y_{jm} = \begin{cases} 1 & \text{if batch stage } j \text{ has } m \text{ units of the same volume in parallel} \\ 0 & \text{otherwise} \end{cases}$$

To represent an integer number for M_j the following constraints are posed where the available and feasible options m are considered for each stage:

$$M_j = \sum_m \frac{y_{jm}}{m}, \quad \forall j \tag{26}$$

$$\sum_m y_{jm} = 1, \quad \forall j \tag{27}$$

The limiting cycle time for product i in a subprocess p in a period t, TL_{it}^p represents the time between two successive batches of product i in the subprocess p. It can be calculated as the maximum of all stage cycle times in that subprocess, thus:

$$\mathrm{TL}_{it}^p = \max_{j \in p, b \in p} (T_{ijt}, \phi_{ibt}), \quad \forall i, p, t \tag{28}$$

that can be expressed as:

$$\mathrm{TL}_{it}^p \geq \frac{\phi_{ib_j^u t} + t_{ijt} + \phi_{ib_j^d t}}{M_j}, \quad \forall i, p, j \in p, t \tag{29}$$

$$\mathrm{TL}_{it}^p \geq \phi_{ibt}, \quad \forall i, p, b \in p, t \tag{30}$$

As has been mentioned, the allocation of storage tanks divides the process into two subprocesses. In this way, every subprocess can have different number of batches and limiting cycle times. Tank allocation would increase the cost of the plant, but due to the decoupling of subprocesses, subprocesses with smaller TL_{it}^p can operate with smaller equipment sizes.

As Modi and Karimi (1989) noted, the subprocesses are coupled through the requirement that the production time allocated to product i be the same for all the subprocesses. Therefore, the total times for producing i, T_{it}, or alternatively, production rates PR_{it}, in each time period t must be equal in all the subprocesses in order to avoid accumulation of material. The total time for producing product i in time period t is defined as:

$$T_{it} = n_{ijt}\mathrm{TL}_{it}^p, \quad \forall i, p, j \in p, t \tag{31}$$

The productivity of i in time period t is

$$\mathrm{PR}_{it} = \frac{B_{ijt}}{\mathrm{TL}_{it}^p}, \quad \forall i, p, j \in p, t \tag{32}$$

In expressions (31) and (32), the number of batches n_{ijt} and the batch size B_{ijt} must be the same in all the units, which belong to each subprocess.

By introducing Eq. (3) the above equations can be expressed as

$$\mathrm{PR}_{it} = \frac{q_{it}}{n_{ijt}TL_{it}^p} = \frac{q_{it}}{T_{it}}, \quad \forall i, t \tag{33}$$

The fact that q_{it} and T_{it} are equal for all the subprocesses ensure that the production rate upstream and downstream of the storage tank be the same.

By multiplying Eqs. (29) and (30) by n_{ijt}, and taking into account Eqs. (26), (31) and (19), these constraints can be expressed as

$$T_{it} \geq \sum_m \left(\frac{\xi_{ib_j^u t} + t_{ijt}n_{ijt} + \xi_{ib_j^d t}}{m} \right) y_{jm}, \quad \forall i, j, t \tag{34}$$

$$T_{it} \geq \xi_{ibt}, \quad \forall i, b, t \tag{35}$$

The nonlinearities in constraint (34) are eliminated by introducing new variables w_{itjk}, μ_{ibjmt} to represent the cross-product $n_{ijt}y_{jk}$ and $\xi_{ibt}y_{jm}$, respectively. This equation can now be written as

$$T_{it} \geq \sum_m \left(\frac{\mu_{ib_j^u jmt}}{m} \right) + \sum_m \left(\frac{t_{ijt}}{m} \right) w_{ijmt} + \sum_m \left(\frac{\mu_{ib_j^d jmt}}{m} \right), \quad \forall i, j, t \tag{36}$$

$$w_{ijmt} \leq n_{ijt}^{\mathrm{U}} y_{jm}, \quad \forall i, j, m, t \tag{37}$$

$$n_{ijt} = \sum_m w_{ijmt}, \quad \forall i, j, t \tag{38}$$

$$\mu_{ibjmt} \leq \xi_{ibt}^{\mathrm{U}} y_{jm}, \quad \forall i, b, j, m, t \tag{39}$$

$$\xi_{ibt} = \sum_m \mu_{ibjmt}, \quad \forall i, b, j, t \tag{40}$$

where n_{ijt}^{U} and ξ_{ibt}^{U} represent upper bounds for the variables n_{ijt} and ξ_{ibt}, respectively. The upper bound for ξ_{ibt} can be obtained analytically from the constraint (20).

Considering the case of SPC-ZW policy in the period t, the production time horizon constraint can be expressed as:

$$\sum_i T_{it} \leq H_t, \quad \forall t \tag{41}$$

The length of each time period H_t can vary and it is possible to aggregate many periods or divide them as necessary. For this reason the constraint of SPC is not too restrictive from a practical point of view. With this approach it is possible to get more flexible production programs and a more realistic formulation for the design problem.

3.4. Intermediate storage

The allocation of an intermediate storage tank between two batch stages causes the process to be divided into two subprocesses upstream and downstream of the tank. Therefore, for J batch stages there exist, at most $J-1$ possible positions for storage tanks to be allocated between two consecutive batch stages.

The capacity constraints for the storage tanks are simplified mass balances around the storage vessels. Different expressions have been proposed. Modi and Karimi (1989) presented a relatively tight upper bound for the size of storage vessels given by the following constraint:

$$\mathrm{VT}_j \geq \mathrm{ST}_{ij}(B_{ij} + B_{i,j+1}), \quad \forall i, j = 1, \ldots, J-1 \tag{42}$$

where VT_j is the size of the storage tank allocated after batch stage j, and ST_{ij} is the size factor corresponding to the intermediate storage tank.

Another expression to obtain an upper bound for the storage vessels, but not as tight as the previous expression, is given by the following constraints (Ravemark, 1995; Voudouris & Grossmann, 1993) applied to every time period t:

$$\mathrm{VT}_j \geq 2\mathrm{ST}_{ijt} B_{ijt} d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{43}$$

$$\mathrm{VT}_j \geq 2\mathrm{ST}_{ijt} B_{i,j+1,t} d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{44}$$

Here, as no a priori tank allocation is given, binary variables d_j are used to select them. The value of variable d_j is 1 if a tank is placed in position j, or zero otherwise.

These last Eqs. (43) and (44) are preferred. Even though they are less tight than (42), simpler transformations exist for these expressions.

If the storage tank does not exist between two consecutive stages, then their batch sizes are constrained to be equal. This effect is imposed by the following constraints (Ravemark, 1995):

$$1 + \left(\frac{1}{\Phi} - 1\right) d_j \leq \frac{B_{ijt}}{B_{i,j+1,t}} \leq 1 + (\Phi - 1) d_j, \quad \forall i, j = 1, \ldots, J-1 \tag{45}$$

where Φ is a constant value corresponding to the maximum ratio allowed between consecutive batch sizes. If no storage is available (i.e. $d_j = 0$) the ratio of consecutive batch sizes is one. If a storage tank is allocated (i.e. $d_j = 1$) the ratio can vary between $1/\Phi$ and Φ.

By introducing Eq. (3), Eqs. (43)–(45) can be expressed as:

$$n_{ijt} \geq 2\left(\frac{\mathrm{ST}_{ijt} q_{it}}{\mathrm{VT}_j}\right) d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{46}$$

$$n_{i,j+1,t} \geq 2\left(\frac{\mathrm{ST}_{ijt} q_{it}}{\mathrm{VT}_j}\right) d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{47}$$

$$n_{i,j+1,t} \leq n_{ijt} + (\Phi - 1) n_{ijt} d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{48}$$

$$n_{i,j+1,t} \geq n_{ijt} + \left(\frac{1}{\Phi} - 1\right) n_{ijt} d_j, \quad \forall i, j = 1, \ldots, J-1, t \tag{49}$$

where Eq. (45) has been split into two expressions.

As has been previously mentioned, variable VT_j is restricted to take values from the set $\mathrm{ST}_j = \{\upsilon_{j1}, \upsilon_{j2}, \ldots, \upsilon_{jp_j}\}$, where $\upsilon_{j\nu}$ represents discrete size ν for storage tank at position j and p_j is the given number of discrete sizes available for storage tanks. Then, the following binary variables are introduced:

$$\mathrm{st}_{j\nu} = \begin{cases} 1 & \text{if storage at position } j \text{ has size } \nu \\ 0 & \text{otherwise} \end{cases}$$

The following constraint holds,

$$\sum_{\nu} \mathrm{st}_{j\nu} = 1, \quad \forall j = 1, \ldots, J-1 \tag{50}$$

In this model it has been assumed that in all the positions j the alternative $\nu = 1$ represents size 0, or in other words that no storage tank is allocated. Right side of Eqs. (46) and (47) involves the inverse of the tank sizes. So, the following expression is posed:

$$\frac{1}{\mathrm{VT}_j} = \sum_{\nu \neq 1} \frac{\mathrm{st}_{j\nu}}{\upsilon_{j\nu}}, \quad \forall j = 1, \ldots, J-1 \tag{51}$$

where $\nu \neq 1$ stands for the nonzero volume sizes. The number of batches n_{ijt} and $n_{i,j+1,t}$ are given by Eqs. (46) and (47). Taking into consideration Eq. (51), the following storage capacity constraints are obtained:

$$n_{ijt} \geq 2\sum_{\nu \neq 1}\left(\frac{\mathrm{ST}_{ijt} q_{it}}{\upsilon_{j\nu}}\right) \mathrm{st}_{j\nu}, \quad \forall i, j = 1, \ldots, J-1, t \tag{52}$$

$$n_{i,j+1,t} \geq 2\sum_{\nu \neq 1}\left(\frac{\mathrm{ST}_{ijt} q_{it}}{\upsilon_{j\nu}}\right) \mathrm{st}_{j\nu}, \quad \forall i, j = 1, \ldots, J-1, t \tag{53}$$

By introducing the new continuous nonnegative variable $f_{ij\nu t}$ to represent the cross-product $q_{it}\mathrm{st}_{j\nu}$, Eqs. (52) and (53) are then replaced by the following linear constraints:

$$n_{ijt} \geq 2\sum_{\nu \neq 1}\left(\frac{\mathrm{ST}_{ijt}}{\upsilon_{j\nu}}\right) f_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{54}$$

$$n_{i,j+1,t} \geq 2\sum_{\nu \neq 1}\left(\frac{\mathrm{ST}_{ijt}}{\upsilon_{j\nu}}\right) f_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{55}$$

$$f_{ij\nu t} \leq q_{it}^{\mathrm{U}} \mathrm{st}_{j\nu}, \quad \forall i, \nu, j = 1, \ldots, J-1, t \tag{56}$$

$$q_{it} = \sum_{\nu} f_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{57}$$

By using the binary variable $\mathrm{st}_{j\nu}$ in place of the variable d_j the constraints (48) and (49) can be written as

$$n_{i,j+1,t} \geq n_{ijt} + \left(\frac{1}{\Phi} - 1\right)\sum_{\nu \neq 1} \mathrm{st}_{j\nu} n_{ijt}, \quad \forall i, j = 1, \ldots, J-1, t \tag{58}$$

$$n_{i,j+1,t} \leq n_{ijt} + (\Phi - 1)\sum_{\nu \neq 1} \mathrm{st}_{j\nu} n_{ijt}, \quad \forall i, j = 1, \ldots, J-1, t \tag{59}$$

The nonlinearity in these equations is eliminated with the definition of a new variable $\lambda_{ij\nu t}$. In this way the following linear constraints are obtained:

$$n_{ij+1t} \geq n_{ijt} + \left(\frac{1}{\Phi} - 1\right)\sum_{\nu \neq 1}\lambda_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{60}$$

$$n_{ij+1t} \leq n_{ijt} + (\Phi - 1)\sum_{\nu \neq 1}\lambda_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{61}$$

$$\lambda_{ij\nu t} \leq n_{ijt}^{\mathrm{U}}\mathrm{st}_{j\nu}, \quad \forall i, \nu, j = 1, \ldots, J-1, t \tag{62}$$

$$n_{ijt} = \sum_{\nu}\lambda_{ij\nu t}, \quad \forall i, j = 1, \ldots, J-1, t \tag{63}$$

Taking into account the structure of the plant, its batch and semicontinuous stages, different constraints must be considered depending on the position for the allocation of a storage tank. For example, if there are semicontinuous subtrains and the storage tanks are allocated immediately after a batch stage, before its downstream semicontinuous subtrain, Eq. (34), and then (36) should be modified. In this case, it is necessary to formulate the following constraints to replace Eq. (34):

$$T_{it} \geq \sum_{m}\left(\frac{\xi_{ib_j^u t} + t_{ijt}n_{ijt}}{m}\right)y_{jm}, \quad \forall i, j, t \tag{64}$$

$$T_{it} \geq \sum_{m}\left(\frac{\xi_{ib_j^u t} + t_{ijt}n_{ijt} + \xi_{ib_j^d t}}{m}\right)y_{jm} - \mathrm{BM}(1 - \mathrm{st}_{j1}),$$
$$\forall i, j = 1, \ldots, .J-1, t \tag{65}$$

Constraint (65) is of the Big-M type, where BM is a large constant that has an appropriate value that will ensure that if the storage tank does not exist, then Eq. (34) is considered. Formulations considering other structural alternatives are also feasible.

3.5. Planning constraints

Two cases are presented taking into account the relation between products and raw materials.

Case a: In this case, it is assumed that the elaboration of each product requires a unique raw material that it is not shared by other products. This assumption is valid for the solved example, an oleoresins plant.

IP_{it}, the inventory of final product i at the end of a period t depends on the inventory that is left from the previous interval, $\mathrm{IP}_{i,t-1}$, the quantity produced q_{it}, and the total sales, QS_{it}:

$$\mathrm{IP}_{it} = \mathrm{IP}_{i,t-1} + q_{it} - \mathrm{QS}_{it}, \quad \forall i, t \tag{66}$$

Sales depend on the lower and upper bounds for products demands, $DE_{it}^{\mathrm{L}}/DE_{it}^{\mathrm{U}}$. Then:

$$DE_{it}^{\mathrm{L}} \leq \mathrm{QS}_{it} \leq DE_{it}^{\mathrm{U}}, \quad \forall i, t \tag{67}$$

IP_{it} has an upper bound, which corresponds to the maximum inventory capacity for each product in the plant.

$$0 \leq \mathrm{IP}_{it} \leq \mathrm{IP}_{it}^{\mathrm{U}}, \quad \forall i, t \tag{68}$$

In the same way, the inventory of raw material IM_{it} at the end of a time period t, depends on the inventory that is left from the previous interval, $\mathrm{IM}_{i,t-1}$, the quantity purchased C_{it}, and the amount used in the production process, RM_{it}:

$$\mathrm{IM}_{it} = \mathrm{PE}_{i,t-1}\mathrm{IM}_{i,t-1} + C_{it} - \mathrm{RM}_{it}, \quad \forall i, t \tag{69}$$

The parameter $\mathrm{PE}_{i,t-1}$ is a factor smaller than or equal to 1 that takes into account the loss of raw materials between two consecutive periods, to consider the degradation of natural raw materials.

IM_{it} has an upper bound, which is the maximum inventory capacity for each raw material in the plant.

$$0 \leq \mathrm{IM}_{it} \leq \mathrm{IM}_{it}^{\mathrm{U}}, \quad \forall i, t \tag{70}$$

The initial amount of raw material in the inventory IM_{i0} for each product at the beginning of the time horizon is assumed to be given. Idem for the initial product inventory, IP_{i0}. The use of IM_{i0} and IP_{i0} have a strong impact when this model is only used for operation planning without considering design, for example in an existing plant.

The raw material necessary for the production of the product i is obtained from a mass balance:

$$\mathrm{RM}_{it} = F_{it}q_{it}, \quad \forall i, t \tag{71}$$

where F_{it} is a parameter that accounts for the process conversion, e.g. ratio of solvent to solids, time of contact, etc.

Case b: The approach is generalized for cases which involve several raw materials for producing each product, as occurs in several industries. Then, the process handles $c = 1, \ldots, CT$ ingredients to manufacture the products. Let F_{cit} be a parameter that accounts for the process conversion of raw material c to make product i during period t. RM_{cit}, the amount consumed of raw material c in period t to elaborate product i, is obtained from a mass balance. Then,

$$\mathrm{RM}_{cit} = F_{cit}q_{it}, \quad \forall c, i, t \tag{72}$$

The total consumption of raw material c for production in period t, RM_{ct} is obtained from

$$\mathrm{RM}_{ct} = \sum_{i}\mathrm{RM}_{cit}, \quad \forall c, t \tag{73}$$

In this case, Eqs. (69) and (70) must be rewritten considering each ingredient c in period t, i.e. C_{ct}, and IM_{ct}.

3.6. Objective function

The objective function to be optimized is,

$$\psi = \sum_{t}\sum_{i}np_{it}\mathrm{QS}_{it} - \sum_{t}\sum_{i}\kappa_{it}C_{it} - \sum_{j}M_j\alpha_j V_j^{\beta_j}$$
$$- \sum_{k}G_k\gamma_k \leq R_k^{\delta_k} - \sum_{j}^{J-1}\pi_j\mathrm{VT}_j^{\tau_j}$$

$$-\sum_t\sum_i \varepsilon_i \left(\frac{\mathrm{IM}_{i,t-1}+\mathrm{IM}_{it}}{2}\right) H_t$$
$$-\sum_t\sum_i \sigma_i \left(\frac{\mathrm{IP}_{i,t-1}+\mathrm{IP}_{it}}{2}\right) H_t \quad (74)$$

The first term corresponds to the incomes due to sales, where np_{it} is the price of product i in period t. The second term is the cost of raw materials with κ_{it} the price for the ingredient of product i in period t, which takes into account market fluctuations for harvest, transportation, cooling facilities, etc. Here the case (a) posed in the Planning Constraints has been considered. Similar expression can be posed for case (b). The third, fourth and fifth terms are investment cost corresponding to batch units, semicontinuous units, and storage tanks in the plant, where α_j, β_j, γ_k, δ_k, π_j and τ_j are appropriate cost coefficients that depend on the type of equipment being considered (Ravemark & Rippin, 1998). As proposed by Birewar and Grossmann (1990) the inventory can be expressed as an average at each period. So, the last two terms correspond to both raw materials and final products inventory costs, where ε_i and σ_i are inventory cost coefficients for raw materials and products, respectively.

Let ψ_{EQ} be the term of capital costs for all equipments in function (74).

$$\psi_{\mathrm{EQ}} = \sum_j M_j \alpha_j V_j^{\beta_j} + \sum_k G_k \gamma_k R_k^{\delta_k} + \sum_j^{J-1} \pi_j \mathrm{VT}_j^{\tau_j} \mathrm{st}_{j\nu} \quad (75)$$

In a general approach, it is calculated through a power law expression where the sizes V_j, R_k and VT_j are considered as continuous values. However, in this work, units are selected from a set of available discrete sizes. Thus, maintaining the law expression and considering the available discrete sizes, the term of capital cost is posed as follows:

$$\psi_{\mathrm{EQ}} = \sum_j\sum_m\sum_s m\alpha_j \nu_{js}^{\beta_j} y_{jm} z_{js} + \sum_k\sum_g\sum_u g\gamma_k \omega_{ku}^{\delta_k} x_{kg} a_{ku}$$
$$+ \sum_j^{J-1}\sum_{\upsilon\neq 1} \pi_j \upsilon_{j\nu}^{\tau_j} \mathrm{st}_{j\nu} \quad (76)$$

to obtain finally:

$$\psi_{\mathrm{EQ}} = \sum_j\sum_m\sum_s cb_{jms} r_{jms} + \sum_k\sum_g\sum_u cs_{kgu} \rho_{kgu}$$
$$+ \sum_j^{J-1}\sum_{\upsilon\neq 1} ct_{j\nu} \mathrm{st}_{j\nu} \quad (77)$$

where the terms $cb_{jms} = m \cdot \alpha_j \cdot \nu_{js}^{\beta_j}$ represent the cost of standard batch vessels, $cs_{kgu} = g \cdot \gamma_k \cdot \omega_{ku}^{\delta_k}$ the cost of standard semicontinuous sizes and $ct_{j\nu} = \pi \cdot \upsilon_{j\nu}^{\tau_j}$ the cost of standard storage vessels. New variables r_{jms}, ρ_{kgu} are introduced to eliminate the product of binary variables $z_{js}y_{jm}$, $a_{ku}x_{kg}$, respectively, through the constraints:

$$r_{jms} \geq z_{js} + y_{jm} - 1, \quad \forall j, m, s \quad (78)$$

$$\rho_{kgu} \geq x_{kg} + a_{ku} - 1, \quad \forall k, g, u \quad (79)$$

These new variables can be settled as continuous if the following bounds are added:

$$0 \leq r_{jms} \leq 1 \quad (80)$$

$$0 \leq \rho_{kgu} \leq 1 \quad (81)$$

The final model is a MILP which consists of maximizing the objective function represented by Eq. (73) using Eq. (76) as the term of capital cost and subject to constraints (6), (8)–(10), (15), (17), (21)–(24), (27), (35)–(41), (50), (54)–(57), (60)–(63), (66)–(71), (78)–(81) and the necessary bounds for the case (a) of planning constraints. A similar model should be posed for the case (b).

An important feature of the model is that the discrete variables only depend on plant design and are independent of time periods, which allows handling large problems with less computational effort.

4. An illustrative example: computational results

In order to illustrate the multiperiod MILP model, a multiproduct batch plant is considered that manufactures five oleoresins ($I = 5$), sweet bay (A), oregano (B), pepper (C), rosemary (D), and thyme (E) oleoresins. All the products are obtained via the following processing stages: (1) grinding, where a size reduction is realized; (2) extraction in a four stages countercurrent arrangement which produces the dissolution of active principles into an organic solvent; (3) expression, where hydraulic pressing is used for the recovery of liquid extract; (4) evaporation, solvent separation from fluid end products; (5) thickening, solvent separation from semisolid end products; (6) blending, task in which the extract is mixed with diluents, solubilizing agents, and/or essential oils to strength the aroma and (7) canning, packing of end products (Fig. 1). Stages 1, 4, 5 and 7 are semicontinuous stages and they can be duplicated up to three items. Stages 2, 3 and 6 are batch stages, each of them may consist of up to two parallel units. Storage tanks can be allocated after batch stages 2 (position 1) and 3 (position 2). This model corresponds to the case (a) previously posed in the Planning Constraints Section.

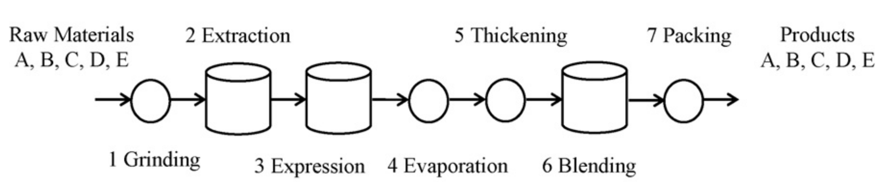

Fig. 1. Flowsheet of the multiproduct batch plant for the production of oleoresins.

Table 1
Data for the example

i	Size factors, S_{ijt} (L/kg)			Processing time, t_{ijt} (h)			Duty factor, D_{ikt}				Storage size factor, ST_{ijt} (L/kg)	
	2	3	6	2	3	6	1	4	5	7	Position 1	Position 2
A	20	15	1.5	1.5	1	0.5	0.3	0.045	0	0.023	25	20
B	80	55	1.5	1.5	1	0.5	1.2	0.18	0	0.094	90	60
C	20	15	1.5	2.5	2	2	0.3	0.045	0.110	0.023	25	18
D	40	25	1.5	1.5	1	1	0.6	0.090	0.225	0.047	50	25
E	30	20	1.5	1.5	1	1	0.43	0.065	0.160	0.034	35	30
	$\beta_j=0.6$	$\beta_j=0.6$	$\beta_j=0.6$				$\gamma_k=0.22$	$\gamma_k=0.40$	$\gamma_k=0.62$	$\gamma_k=0.4$	$\tau_j=0.5$	$\tau_j=0.5$
	$\alpha_j=592$	$\alpha_j=582$	$\alpha_j=457$				$\delta_k=370$	$\delta_k=250$	$\delta_k=210$	$\delta_k=250$	$\pi_j=450$	$\pi_j=450$

Table 2
Prices and demand bounds for the example

t	Costs of raw materials, κ_{it} (\$/kg)					Prices of products, np_{it} (\$/kg)					Bounds on demands, $DE_{it}^{\mathrm{L}} - DE_{it}^{\mathrm{U}}$ ($\times 10^2$ kg)				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
1	2.2	0.5	1.2	0.6	0.7	55	45	40	42	48	3.5–20	4.5–25	2.5–30	3.5–25	3.0–25
2	2.2	0.5	1.2	0.6	1.6	55	48	40	42	48	2.0–20	4.0–24	3.0–25	3.5–25	3.5–24
3	2.2	0.5	1.2	0.6	1.6	55	48	40	42	48	3.0–24	4.0–20	2.0–30	3.5–25	3.0–24
4	2.2	1.5	1.2	0.6	1.6	55	48	40	42	48	3.5–20	4.0–20	4.0–30	3.5–25	2.5–30
5	1.5	1.5	2.5	1.8	1.6	52	48	44	45	52	2.5–15	3.5–20	3.0–24	3.5–24	2.0–35
6	1.5	1.5	2.5	1.8	1.6	52	48	44	45	52	3.0–15	4.0–24	4.0–25	3.0–24	2.0–35
7	1.5	1.5	2.5	1.8	1.6	52	48	44	45	52	4.0–20	4.0–24	2.0–24	3.0–24	2.5–30
8	1.5	1.5	2.5	1.8	1.6	52	48	44	45	52	4.0–20	3.5–24	2.0–25	4.0–24	2.0–35
9	2.2	1.5	2.5	0.6	1.6	55	48	44	42	48	3.5–20	3.5–24	3.5–20	4.0–25	2.0–24
10	2.2	1.5	2.5	0.6	1.6	55	48	44	42	48	3.0–15	4.0–30	2.0–24	4.0–25	2.0–25
11	2.2	1.5	1.2	0.6	0.7	55	45	40	42	48	2.5–15	4.0–25	2.5–30	4.0–25	4.5–20
12	2.2	1.5	1.2	0.6	0.7	55	45	40	42	48	3.0–15	3.5–25	4.0–30	4.0–25	3.0–24

Table 3
Standard sizes available for each stage

	Discrete volumes, ν_{js}			Discrete sizes, ω_{ku}				Discrete sizes, υ_{jv}
	2	3	6	1	4	5	7	Storage
1	500	500	50	5	0.7	0.7	2.5	0
2	1000	750	100	10	1	1	5	1000
3	1500	1000	150	15	1.5	1.5	10	2000
4	2000	1200	200	20	2	2	15	4000
5	2500	1500	250	25	2.5	2.5	20	5000
6	3000	2000	500	30	3	3	30	

In order to obtain the parameter F_{it}, Eq. (71), the next equations are used:

$$fx_{it}^{n+1}[1 + E_i(1 - \eta_i)] = fx_{it}^{n}(1 + E_i - \eta_i) + \eta_i fx_{it}^{1}, \quad \forall i, t$$

$$F_{it} = \frac{1}{fx_{it}^{n+1} - fx_{it}^{1}}, \quad \forall i, t$$

where E_i is the extraction factor, η_i the extent of the extraction and fx_{it} is the product concentration in the vegetable solid feed. The index n is the number of each stage for the n staged countercurrent extraction (see Appendix A).

Tables 1–4 contain the data for this example. The duty factors in Table 1 are in kW/(kg h) for stages 1 and 7, in m^2/(kg h) for stages 4 and 5. The final product inventory cost coefficient is \$1.5/(t h) and the raw material inventory cost is \$1/(t h) for all products. The parameter $\mathrm{PE}_{i,t}$ is taken equal to 1 for this example.

The developed MILP model has been solved on a Pentium(R) 4 CPU 3.00 GHz with the GAMS package, using the CPLEX solver, with the data shown in Tables 1–4. A time horizon of 6000 h has been considered, that has been divided in 12 equal periods, as shown in Table 2. The example results for the

Table 4
Data for the example

	Extraction parameters			Initial inventory
	fx^{n+1}	E	η	IM_0
A	0.1	1	0.85	2000
B	0.025	1.2	0.99	2000
C	0.1	0.9	0.90	2000
D	0.05	1.4	0.95	2000
E	0.07	1	0.75	2000

Table 5
Results for example considering 12 time periods

t	A ($\times 10^2$ kg)				B ($\times 10^2$ kg)				C ($\times 10^2$ kg)				D ($\times 10^2$ kg)				E ($\times 10^2$ kg)			
	q_{it}	QS_{it}	IP_{it}	IM_{iz}	q_{it}	QS_{it}	IP_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	IM_{it}
1	20.0	20.0	0.0	0.0	70.3	25.0	45.3	0.0	30.0	30.0	0.0	0.0	25.0	25.0	0.0	0.0	82.8	25.0	57.8	1670
2	20.0	20.0	0.0	0.0	64.3	24.0	85.6	0.0	41.6	25.0	16.6	0.0	25.0	25.0	0.0	0.0	82.8	24.0	116.5	0.0
3	24.0	24.0	0.0	0.0	53.2	20.0	118.8	0.0	79.3	30.0	65.8	0.0	72.0	25.0	47.0	0.0	0.0	24.0	92.5	0.0
4	20.0	20.0	0.0	0.0	0.0	20.0	98.8	0.0	79.3	30.0	115.1	371.9	74.0	25.0	96.0	0.0	0.0	30.0	62.5	0.0
5	15.0	15.0	0.0	0.0	0.0	20.0	78.8	0.0	26.9	24.0	118.0	0.0	0.0	24.0	72.0	0.0	0.0	35.0	27.5	0.0
6	15.0	15.0	0.0	0.0	0.0	24.0	54.8	0.0	0.0	25.0	93.0	0.0	0.0	24.0	48.0	0.0	7.5	35.0	0.0	0.0
7	50.2	20.0	30.2	0.0	0.0	24.0	30.8	0.0	0.0	24.0	69.0	0.0	0.0	24.0	24.0	0.0	30.0	30.0	0.0	0.0
8	54.7	20.0	65.0	0.0	0.0	15.8	15.0	0.0	0.0	25.0	44.0	0.0	0.0	24.0	0.0	0.0	35.0	35.0	0.0	0.0
9	0	20.0	45.0	0.0	0.0	3.5	11.5	0.0	0.0	20.0	24.0	0.0	25.0	25.0	0.0	0.0	24.0	24.0	0.0	0.0
10	0	15.0	30.0	0.0	0.0	4.0	7.5	0.0	0.0	24.0	0.0	0.0	25.0	25.0	0.0	0.0	25.0	25.0	0.0	0.0
11	0	15.0	15.0	0.0	0.0	4.0	3.5	0.0	30.0	30.0	0.0	0.0	25.0	25.0	0.0	0.0	20.0	20.0	0.0	0.0
12	0	15.0	0	0.0	0.0	3.5	0.0	0.0	30.0	30.0	0.0	0.0	25.0	25.0	0.0	0.0	24.0	24.0	0.0	0.0

Table 6
Optimal solution problem with intermediate storage tank

	Stage						
	1	2	3	4	5	6	7
V_j		2500 L	2000 L			150 L	
R_k	25 HP			3 m^2	3 m^2		30 HP
VT_j			5000 L				
M_j		2	1			2	
G_k	3			2	3		1

products are summarized in Table 5. Table 6 shows the optimal sizes and number of parallel units obtained. Also, Fig. 2 illustrates the optimal plant structure of the problem. For this case, two parallel units have been selected for the extractor and the mixer and a tank is allocated between expression and blending batch stages. The selection of parallel units allows the reduction of the idle time for the stages. For stages of grinding and thickening, three parallel units have been selected and two units for the evaporation. These semicontinuous units operate in phase and are indicated by overlapped units in Fig. 2. The provision of intermediate storage immediately after expression stage decouples the plant operation which allows a size reduction, and thus in the capital cost, of the equipments that belong to the downstream subprocess.

The optimal solution for both products B and C are illustrated in Figs. 3 and 4. They show inventory, production, sales and purchase profiles at the optimal solution.

Product B is the less convenient to produce because it is in a very small concentration in its raw material (see Table 4) and its reduced profit. The first diagram of Fig. 3 shows that raw material for B is purchased during the initial three periods where prices of raw materials are lower. The second diagram shows that the production of product B is made only during the first three periods, because the costs are lower manly due to the lower raw material price. Most of the amount produced in these periods is held in inventory for satisfying maximum demands in the seven subsequent intervals and minimum demands in the last four periods.

On the other hand, in the optimal solution, the production of C is larger (see Table 5), since it is one of the most profitable products. The first diagram of Fig. 4 shows that the purchase profile of C reaches the maximum value in the fourth period because of the lower price of raw materials. When the price suddenly rises the purchases are first stopped, but later restarted when raw materials prices fall down in the last periods. The extra amount of raw material purchased in fourth period is maintained as inventory. The second diagram shows that the production of C occurs in almost all of the time periods. Furthermore, product C builds up inventory during the first five periods because demands are lower than the production capacity of these periods, and most of this amount is consumed to satisfy the demands in the following periods when the production has stopped. Production is restarted in the last two periods to satisfy the maximum demands.

In order to assess the computational performance several examples were also solved with different number of periods.

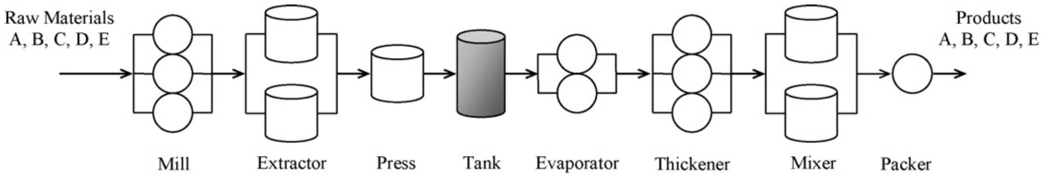

Fig. 2. Optimal configuration of the plant showing the units in parallel and intermediate storage tanks.

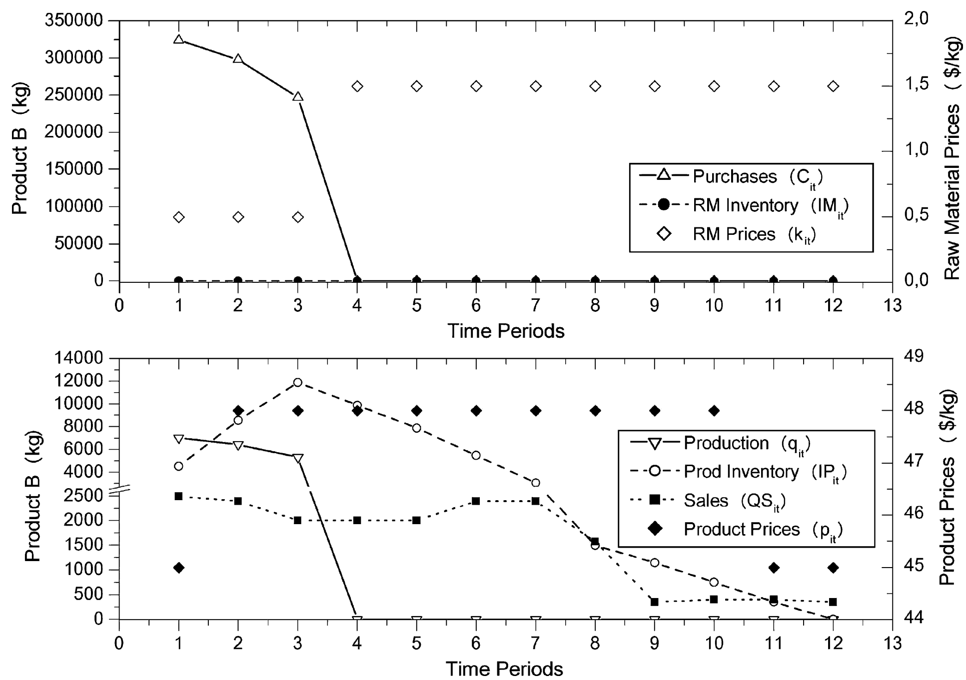

Fig. 3. Results for product B.

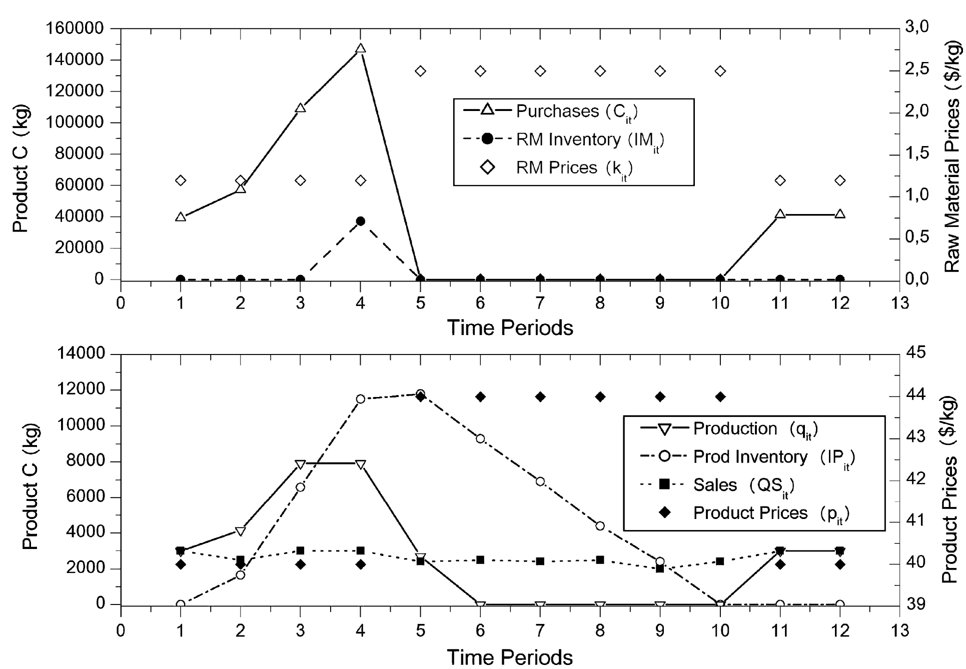

Fig. 4. Results for product C.

Table 7
Problem sizes for different time periods considered

Number of periods	Objective $	Discrete variables	Continuous variables	Constraints	CPU time (s)	Relative gap (%)
3	3486336.44	70	2234	2102	4.203	0.0
5	3386141.62	70	3604	3420	11.187	0.0
10	3265426.40	70	7029	6715	82.390	0.0
12	3270299.45	70	8399	8033	118.406	0.0
15	3447807.24	70	10454	10010	458.906	0.0
20	3367474.89	70	13879	13305	408.500	0.0
24	3626317.35	70	16619	15941	547.687	0.0

The sizes of the different problems considered with respect to the number of both discrete and continuous variables, the constraints and the CPU time as well as the number of time periods are shown in Table 7. It can be noted that, although the number of continuous variables increases with the number of time periods considered, for a given number of available discrete sizes for the units, the number of discrete variables are the same for all the problems. The computational results show a significant increase when a greater number of periods are considered. However, reasonable computation times have been obtained.

In multiperiod problems where the binary variables increase with each additional time period, Van den Heever and Grossmann (1999) showed that the MILP solution time increases nearly exponentially. Otherwise, the formulation here presented is not as time consuming as their formulations because only the number of continuous variables increases with the time periods since there are not variations in the plant configuration during the time horizon.

In order to assess the effect of the multiperiod scenario, one example was solved considering a given demand over one period. Then the obtained optimal plant was used for planning plant operation over several periods. Assuming a multiperiod formulation with five time periods which maximum demands are summarized in Table 8, the corresponding total profit was \$1741459.91. The optimal plant structure is $(V_2, V_3, V_6) = (1000, 750, 100)$ with two units in the first stage and one unit in the others for batch stages, and $(R_1, R_4, R_5, R_7) = (30, 2, 3, 30)$ for semicontinuous stages, with 1, 2, 3 and 1 units operating in parallel, respectively. A storage tank after second batch stage $(\mathrm{VT}_2) = (2000)$, is allocated. Considering only one period, this problem is solved for the following cases: (i) the first time period, where demands are the lowest ones, and (ii) the fifth time period with the highest demands. The solution of the problem considering case (i) results in the selection of one unit for all batch stages with sizes $(V_2, V_3, V_6) = (500, 500, 50)$ and $(R_1, R_4, R_5, R_7) = (30, 1.5, 3, 2.5)$ for semicontinuous sizes, with 3, 1, 1 and 1 units operating in parallel respectively; and also a tank is allocated after second batch stage with size $(\mathrm{VT}_2) = (1000)$. On the other hand, in case (ii) the optimal plant corresponds to the selection of equipment sizes $(V_2, V_3, V_6) = (1500, 1000, 100)$ for batch stages with one unit in each, and $(R_1, R_4, R_5, R_7) = (25, 2, 3, 5)$ for semicontinuous stages with 2, 2, 3 and 1 units in parallel. No intermediate storage tank is selected here.

Table 8
Example with five time periods

t	Maximum demands DE_{it}^{U}				
	A	B	C	D	E
1	2000	2000	2000	2000	1000
2	3000	3000	3000	2000	2000
3	4000	3000	5000	1000	3000
4	3000	2000	3000	3000	4000
5	4000	4000	5000	3000	5000

Table 9
Different number of discrete units

Number of discrete units	Continuous variables	Discrete variables	CPU time (s)
3	4300	45	13.078
6	7029	70	82.390
10	11139	108	405.375

If the problem with five time periods is restricted to take the values of the units obtained in the solution of case (i), it has a total benefit of \$1257581.96 corresponding to a 28% reduction, whereas in case (ii) the profit was \$1692670.513, which correspond to a reduction of 2.8%. In the first case, differences are due to reduced production levels taking into account smaller units because of the design considered the lowest demands. In the last case, in spite of using equipment with the appropriate sizes to satisfy the highest demands, a better planning is obtained using the proposed methodology. Note that these lower values of total profits demonstrate the superiority of the proposed multiperiod approach, which takes into account fluctuations over demands in every period.

The model performance is also affected by the number of available discrete sizes for each stage. Considering the example where the horizon time was divided into 10 time periods, Table 9 was performed by changing the number of available discrete sizes for the units in every stage. It shows the number of both continuous and discrete variables and CPU times obtained.

Table 10
Computational times

BM	Number of periods		
	5	10	12
T^{U}	11.187	82.390	118.406
$2T^{\mathrm{U}}$	15.531	88.531	105.750
$5T^{\mathrm{U}}$	18.593	91.468	124.984
$10T^{\mathrm{U}}$	15.796	105.546	162.140

As Gupta and Karimi (2003) noted, the solution times depend heavily on the value of parameters BM used in constrains as Eq. (65). Thus, even though BM can be any large positive value it has to be selected judiciously. In this work, several values of BM were tried to evaluate its effect on CPU times. In general, the smallest CPU time corresponds to the value of BM adopted as the maximum of the time available for each period (see Table 10).

5. Conclusions

A multiperiod model for optimizing the simultaneous design and operation planning of a multiproduct batch plant has been developed. This model explicitly accounts for the effect of seasonal or market variations of products demands and raw materials availability. Both raw materials and products inventory costs are readily accounted for. From the structure of the plant point of view, the presented model is general, involving batch, semicontinuous and storage tanks. The usual options to increase the efficiency of the batch plant design, such as unit duplication, are addressed.

Multiperiod MILP formulation involves discrete decisions for structure selection and continuous decisions for operation at each period at the plant. Furthermore, the model shows the interaction between design decisions and commercial, production, sales and inventory policies simultaneously. In general, previous models only considered one period with fixed amounts to be produced in the time horizon.

Results were obtained for a plant that produces five oleoresins in seven processing stages. Solutions over an increasing number of time periods were also provided and analyzed to assess the model performance. Through this example, the effect of multiperiod context was evaluated in order to justify this approach. Very different and poor solutions were obtained if only one period is considered. On the other hand, planning decisions severely affect the final design. So the simultaneous assessment of design and planning decisions in a multiperiod context is a useful approach.

Appendix A. Batch extraction

We want to predict final concentrations from the concentrations in the initial feed. To do so, we need

$$m_\mathrm{e} = \frac{fy^\infty}{fx^\infty} = \frac{n_L^\infty P}{L n_P^\infty} \quad \text{the equilibrium constant} \tag{A.1}$$

$$E = \frac{n_L^\infty}{n_P^\infty} = m_\mathrm{e}\left(\frac{L}{P}\right) \quad \text{the extraction factor} \tag{A.2}$$

$$n_L^i + n_P^i = n_L^f + n_P^f$$

mass balance on the solute at finite time of contact (A.3)

$$n_L^i + n_P^i = n_L^\infty + n_P^\infty$$

mass balance on the solute at infinite time of contact (A.4)

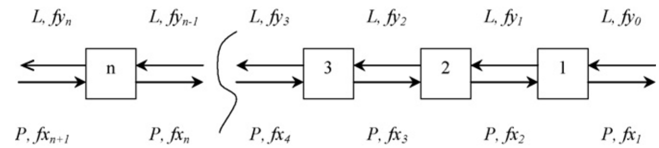

Fig. A.1. A staged countercurrent extraction.

$$\eta = \frac{n_L^f - n_L^i}{n_L^\infty - n_L^i} = \frac{n_P^i - n_P^f}{n_P^i - n_P^\infty} \quad \text{the extent of extraction} \tag{A.5}$$

where fy^∞ and fx^∞ are the product concentrations in the extraction solvent L and the solid feed P, respectively at infinite time (equilibrium). By the symbols n_P and n_L we describe the amounts (kg) of solute in P and L, and we call n_P^∞ and n_L^∞ to the amounts at the equilibrium.

The model is based in a staged countercurrent extraction as shown in Fig. A.1.

We can combine Eqs. (A.5) and (A.3), through the final amount of solute in the solvent after finite time, and introducing the extraction factor from Eq. (A.2), we get

$$n_P^i - n_P^f + n_L^i = E\eta n_P^\infty + n_L^i(1-\eta) \tag{A.6}$$

Let us obtain the product $\eta \cdot n_P{}^\infty$ from Eq. (A.5), which leads, after algebraic manipulation to:

$$n_P^i[1+E(1-\eta)] = n_P^f(1+E) - \eta n_L^i \tag{A.7}$$

For the n stage, the result is

$$n_P^{n+1}[1+E(1-\eta)] = n_P^n(1+E-\eta) + \eta n_P^1 - \eta n_L^0 \tag{A.8}$$

$$fx^{n+1}[1+E(1-\eta)] = fx^n(1+E-\eta) + \eta fx^1 - \eta\frac{E}{m_\mathrm{e}} fy^0 \tag{A.9}$$

This is the desired result. It relates the feed concentration fx^{n+1} to the exhausted effluent concentration fx^1. In general, the solvent that enters to the first stage does not contain solute, so $fy^0 = 0$ and:

$$fx^{n+1}[1+E(1-\eta)] = fx^n(1+E-\eta) + \eta fx^1 \tag{A.10}$$

The total mass balance in Fig. A.1, is

$$Lfy^n + Pfx^1 = Pfx^{n+1} + Lfy^0 \tag{A.11}$$

$$P = \frac{Lfy^n}{fx^{n+1} - fx^1} = \frac{B_\mathrm{out}}{fx^{n+1} - fx^1} \tag{A.12}$$

To get the total raw material, we have to multiply the above equation by the number of batches

$$\mathrm{RM} = \frac{1}{fx^{n+1} - fx^1} q \tag{A.13}$$

Applying the above equation to several products and representing the ratio with the variable F_i it can be expressed as

$$\mathrm{RM}_i = F_i q_i \tag{A.14}$$

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