New Alternatives in the Design and Planning of Multiproduct Batch Plants in a Multiperiod Scenario

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New alternatives for the simultaneous design and planning of multiproduct batch plants in a multiperiod scenario are presented in this article. This formulation allows the flexible configuration of the plant in every time period for each product considering the assignment of parallel units of different sizes operating either in-phase or out-of-phase. Capacity expansion during the time horizon is also allowed in order to satisfy new variable requirements. For each batch stage, following the usual procurement policy, units are selected from a set of standard and discrete sizes that are available to perform each operation. The model is formulated through a mixed-integer linear programming (MILP) formulation that maximizes the net present value of profit. From the planning point of view, product sales, raw materials purchases, inventories, waste disposal, and late deliveries are taken into account. Thus, this model simultaneously solves both design and production planning for given forecasts of product demands and pricing in each time period.

1. Introduction

Continuous growth in complexity, competitiveness, and uncertainty of the market of high-added-value chemicals and food products with a short life cycle have renewed the interest in batch operations and the development of optimization models. The main advantage of batch plants in this context is their inherent flexibility to use the various available resources for manufacturing relatively small amounts of several different products within the same facilities. Since the demands for such products typically vary from period to period because of market or seasonal changes, in the last few years an increased amount of research effort has been made to develop multiperiod optimization models.

This is an area with many works that reflect the intense rate of research production. Then, it is very difficult to summarize the most outstanding articles. However, most published studies deal with restricted formulations of multiperiod problems. For example, some authors have only considered either the planning or the design problem in their formulations. Birewar and Grossmann[1] proposed a nonlinear programming (NLP) model for simultaneous planning and scheduling in multiproduct batch plants that considers benefits and product inventory costs. Sahinidis and Grossmann[2] proposed a mixed-integer linear programming (MILP) formulation for selecting capacity expansion policies for both continuous and batch operation modes. Norton and Grossmann[3] described a simplified, high-level, multiperiod MILP planning investment model that maximizes the net present value of network's operations and expansion decisions with dedicated and flexible plants. Iyer and Grossmann[4] reported multiperiod operation planning for utility systems. Ryu[5] highlighted the different time scales between demands and capacity expansion in multiperiod planning considering capacity augmentation.

Some studies consider design and planning problems simultaneously. Varvarezos et al.[6] proposed a decomposition method for solving MINLP multiperiod design problems. Voudouris and Grossmann[7] integrated synthesis, design, operation planning, and scheduling issues in an MILP model considering repeated cycles in the design horizon, which avoids the time period dependence of the problem. Several articles studied the optimal design of multiproduct plants under uncertainty in the product demands assuming probability distributions.[8,9] For utility systems, Iyer and Grossmann[10] presented a model for synthesis and multiperiod operational planning. An extension of this work was proposed by Oliveira Francisco and Matos[11] to include global emissions of atmospheric pollutant issues. Van den Heever and Grossmann[12] treated the design of multiproduct plants by means of a disjunctive multiperiod nonlinear optimization model that simultaneously incorporates design and operation as well as expansion planning.

Different tools and approaches have been used to pose and solve these problems. Most of them have preferred a deterministic approach, but several works also pose models where data, for example demands, are uncertain.[13−16] Different methods have been used to formulate the corresponding models. Most of the works pose a mathematical model, generally MILP[17] or MINLP[18] formulations. However, there exist works that resort to simulation[19] or heuristic[20] approaches.

A multiperiod MILP model that simultaneously optimizes design and production planning decisions applied to multiproduct batch plants was presented in a previous study.[21] These authors developed an optimization model that maximizes the net profit of the plant accounting for parameters variation due to seasonal or market fluctuations.

In a multiproduct batch plant, one product is manufactured at a time, in a sequence of one or more processing steps. Each step is carried out in a single equipment unit or in several parallel units. Processing of other products is carried out using the same equipment in successive campaigns. The unit with the minimum capacity limits the batch size, while the limiting cycle time is fixed by the stage with the longest processing time.

In order to reduce the investment cost of multiproduct plants, several unit arrangements are available; the usual one is a parallel unit working either in- or out-of-phase.[22,23] If parallel units operate out-of-phase, this reduces the cycle time of the stage, i.e., the time elapsed between successive batches leaving

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the stage. This also decreases the idle time of the other stages when the duplicated stage represents the bottleneck for the production train, thus reducing the size of these stages. If parallel units operating in-phase are adopted, they operate simultaneously as if they were the same unit. The batch fed to the stage is split among the available units, while the batches leaving these units are merged after their processing. This arrangement is particularly useful when the batch size exceeds the upper-bound capacity of the equipment.

In this paper, a new mixed-integer linear programming (MILP) model is addressed, which can simultaneously handle design and planning decisions in a multiperiod approach. This article expands the previous study by Moreno et al.[21] in order to present a more flexible formulation. From the design point of view, the capacity expansion is allowed in the new model, and then new units can be added in different time periods, accounting for the tradeoff between the scale-economy savings of large initial capacities and the cost of installing the capacity before it is required. Unlike previous approaches, the units operating in parallel at each stage can have different sizes. Also, this model takes into account flexible plant configurations, where available units at every stage can be arranged in different structures for each product in every time period. Thus, performance can be improved since units can be configurated in different ways for every product, increasing either its production rate (units working out-of-phase) or its capacity (units working in-phase). Finally, discrete sizes of equipment items are assumed and a set of sizes is available for each unit operation. This assumption is justified, since most of the available equipment in fine chemistry has standard sizes. Thus, by making choices among available discrete sizes, the model determines the optimal design that corresponds to the real procurement of equipments.

From the planning point of view, variations in prices, costs, product demands, and raw material supplies due to market and seasonal fluctuations are included. Moreover, this approach considers different period lengths and inventories of both final products and raw materials.

In addition, this approach can be also applied to the retrofit problem when redesigning the facilities is necessary to expand the existing capacity in order to accommodate the increased demand or to manufacture new products.

The remainder of this paper is organized as follows. First, the main characteristics of this problem are presented in detail in Section 2. Then, Section 3 describes the multiperiod formulation including all the elements of the design and planning problems. The versatility of the proposed approach is demonstrated via its application to representative examples in Section 4. Finally, some remarkable conclusions are presented in Section 5.

2. Problem Definition

The problem addressed in this paper can be stated as follows. In a multiperiod scenario, a multiproduct batch plant processes $i = 1, 2, ..., I$ products. Every product follows the same production sequence throughout the $j = 1, 2, ..., J$ batch processing stages of the plant. Each stage j may consist of one or more units $k = 1, 2, ..., K_j$, where K_j is the maximum number of units that can be added at stage j. These duplicated units can have different sizes, operating either in-phase to increase capacity or out-of-phase to decrease the cycle time.

Since this is a multiperiod problem, the time horizon H is discretized into $t = 1, 2, ..., T$ specified time periods H_t, not necessarily of the same length. Production product i at stage j in every period t requires a given processing time t_{ijt} and a material balance factor S_{ijt} called the size factor, which specifies the volume required at stage j to produce a unit mass of final product i. For each product i, lower and upper bounds on its demands in every period t, DE_{it}^{L}/DE_{it}^{U}, are known. Costs and availability of raw materials vary from period to period and are also assumed to be known.

Regarding design decisions, this model involves the selection of the size for each batch unit k at stage j, V_{jk}, which are restricted to take values from a set $SV_j = \{\upsilon_{j1}, \upsilon_{j2}, ..., \upsilon_{jn_j}\}$ of available discrete sizes, where n_j is the number of available sizes at stage j.

Yoo et al.[24] developed a generic retrofit model that presented a generalized superstructure with the interesting concept of group. Group is defined as a set of units, which are operated in-phase, but those in different groups are operated out-of-phase. The division of units of each stage into groups differs from product to product; i.e., for each product i, units can be grouped in different ways. New and old units can be used in-phase and out-of-phase, forming groups. Lee et al.[25] developed a model for the capacity-expansion problem of multisite batch plants, where this concept was used again. A generalized superstructure was generated to assign units to groups in each considered plant. Montagna[26] extended the model proposed by Yoo et al.[24] through the allocation of intermediate storage tanks. Then, a more realistic formulation is obtained, although with an increased level of resolution complexity.

Following the concept of group introduced in the above studies, in this article the configuration of groups using the available batch units must be determined at each stage j for every product i in each time period t.

One key feature of this approach is that units can be added in different time periods t, for example, in order to satisfy the projected expansion of the plant or to fulfill higher demands through the time horizon.

It will be assumed that the plant operates in single-product campaign (SPC) mode in every period. Also, batches are transferred from stage to stage without delay, i.e., a zero-wait (ZW) transfer policy is used. Taking into account the adopted multiperiod approach and depending on the periods length, this assumption does not constitute such a serious constraint as in previous formulations.

In the plant design, the model incorporates the batch units V_{jk}, selecting among available discrete sizes υ_{js}. At each time period t, the model determines the number of groups G_{ijt} at stage j and which of the existing units in that period are assigned to each one for every product i.

In this model, two scenarios are considered. In the first one, the elaboration of product i depends on a unique raw material that is identified with the same subscript i of the product. In the second general scenario, production of product i requires a set of raw materials CT.

On the other hand, production planning decisions allow determining at each period t and for each product i the amount of product to be produced q_{it}, the number of batches n_{it}, and the total time T_{it} required to produce product i. Moreover, at the end of every period t, the levels of both final product IP_{it} and raw material inventories IM_{it} are obtained. Also, the total sales QS_{it}, the amount of purchased raw material C_{it}, and the amount of raw material to be used for the production RM_{it} of product i in each period t are determined with this formulation.

Then the model simultaneously considers the design and the production planning of the plant. The performance criterion is to maximize the net present value along the global time horizon, taking into account incomes from product sales, expenditures

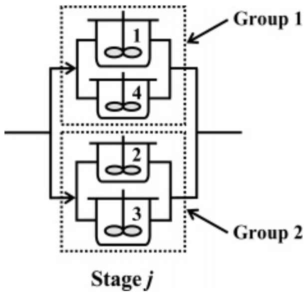

Figure 1. Groups at stage j.

from raw materials purchases, inventories, penalties, and investment costs. If time periods are equal, waste disposal costs are also added to the objective function.

3. Problem Formulation

This section describes the basic constraints and major characteristics of the mathematical formulation.

3.1. Unit and Group Assignment Constraints. Several variables are introduced to determine the plant structure. Since the units can be added at any time period, a binary variable w_{jkt} is used. The value of this variable is 1 if unit k is included in the plant structure at stage j in period t; otherwise, the value is zero. Each unit k at stage j can be added only in one period:

$$\sum_{t}^{T} w_{jkt} \leq 1 \qquad \forall\, j, k \tag{1}$$

The units are included in a sequential manner in order to avoid alternative optimal solutions with the same value for the objective function:

$$\sum_{\tau=1}^{t} w_{jk\tau} \geq \sum_{\tau=1}^{t} w_{j,k+1,\tau} \qquad \forall\, j, k = 1, 2, ..., K_j - 1, t \tag{2}$$

These constraints ensure that a unit is incorporated only if all the previous ones are also included.

The concept of group introduced by Yoo et al.[24] and extended by Montagna[26] is used to handle the simultaneous consideration of parallel batch units working in- and out-of-phase. In order to illustrate the unit arrangements to conform groups, Figure 1 shows an example of four units ($K_j = 4$) at stage j. These units must not be necessarily identical. In this way, up to four groups of one unit each could exist at stage j. Units can be arranged in different ways to determine groups. Figure 1 is one option where the units have been arranged into two groups. Both groups 1 and 2 operate out-of-phase. Units closed by the dotted line form a group, e.g., units 1 and 4 form group 1 and operate in-phase.

The variables related to both existence of groups and units allocations to groups are defined in the same way as in previous articles, adapted to the multiperiod approach adopted in this study.

Since the units can be grouped in different ways at each stage depending on the product and the time period, the binary variable y_{ijkgt} is introduced. This variable is equal to 1 if unit k of stage j is assigned to group g for product i in period t; otherwise; the variable is equal to zero. Each unit k at stage j can be assigned at most to one group g for product i in period t:

$$\sum_{g=1}^{G_j^{\mathrm{T}}} y_{ijkgt} \leq 1 \qquad \forall\, i, j, k, t \tag{3}$$

Parameter G_j^{T} is the maximum number of groups allowed at stage j. A pseudobinary variable y_{ijgt} is also introduced to indicate whether group g exists or not at stage j in period t for product i. Group g is generated at stage j in period t only if at least one unit k is assigned to it in that period:

$$y_{ijgt} \leq \sum_{k=1}^{K_j} y_{ijkgt} \qquad \forall\, i, j, g, t \tag{4}$$

If unit k is assigned to group g at stage j in period t, the group must exist:

$$y_{ijkgt} \leq y_{ijgt} \qquad \forall\, i, j, k, g, t \tag{5}$$

By introducing constraint 6 into the formulation, the continuous variable y_{ijgt} behaves like a binary variable since it is bounded by binary variables through eqs 4 and 5.

$$y_{ijgt} \leq 1 \qquad \forall\, i, j, g, t \tag{6}$$

If unit k is assigned to group g at stage j for product i in period t, the unit must exist in that period:

$$y_{ijkgt} \leq \sum_{\tau=1}^{t} w_{jk\tau} \qquad \forall\, i, j, k, g, t \tag{7}$$

If unit k exists at stage j in period t, then it must be included in a group g. This is written as

$$\sum_{\tau=1}^{t} w_{jk\tau} = \sum_{g=1}^{G_j^{\mathrm{T}}} y_{ijkgt} \qquad \forall\, i, j, k, t \tag{8}$$

In order to avoid redundant assignment of units to groups, which results in the same value for the objective function, the following constraints are added:[24]

$$\sum_{k=1}^{K_j} 2^{K_j-k} y_{ijkgt} \geq \sum_{k=1}^{K_j} 2^{K_j-k} y_{ijk,g+1,t} \qquad \forall\, i, j, g = 1, 2, ..., G_j^{\mathrm{T}} - 1, t \tag{9}$$

This constraint orders the different groups through a weight 2^{K_j-k} assigned to each unit k. The order of the group is obtained by adding the weights of all units in the group.

3.2. Design Constraints. As mentioned in the problem definition, the unit sizes V_{jk} are available in discrete sizes υ_{js}, which correspond to the real commercial procurement of equipment. To rigorously tackle this situation, the binary variable z_{jks} is introduced, whose value is 1 if unit k at stage j has size s; otherwise, it is zero. The variable V_{jk} is restricted to take values from the set $\mathrm{SV}_j = \{\upsilon_{j1}, \upsilon_{j2}, ..., \upsilon_{jn_j}\}$, where n_j is the number of discrete sizes available for each stage. Using the previous definition, V_{jk} can be expressed in terms of discrete variables as

$$V_{jk} = \sum_{s}^{n_j} \upsilon_{js} z_{jks} \qquad \forall\, j, k \tag{10}$$

If unit k at stage j is added in some period t, it must take a size s for the volume from the available sizes at that stage:

$$\sum_{s}^{n_j} z_{jks} = \sum_{t=1}^{T} w_{jkt} \qquad \forall\, j, k \tag{11}$$

Only one of the available sizes at stage j must be selected if unit k at stage j exists:

$$\sum_{s}^{n_j} z_{jks} \leq 1 \qquad \forall\, j, k \tag{12}$$

The amount of product i produced in time period t, q_{it}, depends on the number of batches n_{it} and the batch size B_{it} of final product i processed in that period as follows:

$$q_{it} = B_{it}\, n_{it} \qquad \forall\, i, t \tag{13}$$

The sizing equation described in general literature that relates the unit size with the batch size of a product i at each stage j and regarding the multiperiod approach is

$$V_j \geq S_{ijt}\, B_{it} \qquad \forall\, i, j, t \tag{14}$$

where S_{ijt} is the size factor at stage j for product i, which can vary in each period t taking into account seasonal effects. This is the minimum capacity required at this stage for producing one unit mass of product i.

By combining eq 13 and eq 14, the constraints take the following form:

$$q_{it} \leq V_j \frac{n_{it}}{S_{ijt}} \qquad \forall\, i, j, t \tag{15}$$

The aforementioned constraints must be modified to consider not only the volume of each unit k at every stage j, V_{jk}, but also the volume of the units in a group, that is to say, units operating in-phase. The volume of a group is equal to the total volume of all units that are operated in-phase. Thus, the unit sizes included in group g at stage j must be added. Then, the allocation of units to groups must be taken into account: volumes V_{jk} must be related to the binary variable y_{ijkgt}, as shown by the following expression:

$$q_{it} \leq \sum_{k=1}^{K_j} (V_{jk}\, y_{ijkgt}) \frac{n_{it}}{S_{ijt}} + BM_{ijt}\,(1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{16}$$

Equation 16 is a Big-M constraint that guarantees that batches can be processed in the batch stage j if group g exists for product i in time period t; otherwise, the constraint is redundant because of the large value of BM_{ijt}. The value of BM_{ijt} can be calculated by

$$\mathrm{BM}_{ijt} = K_j \max_s(v_{js}) \max_t\, (n_{it}^{\mathrm{U}}/S_{ijt}) \qquad \forall\, i, j, t \tag{17}$$

By substituting eq 10 into eq 16, new constraints can be formulated that restrict the volumes to discrete sizes:

$$q_{it} \leq \sum_{k=1}^{K_j} \sum_{s}^{n_j} \left(\frac{v_{js}}{S_{ijt}} z_{jks}\, y_{ijkgt}\, n_{it} \right) + \mathrm{BM}_{ijt}\,(1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{18}$$

Constraint 18 is nonlinear because of the product between binary and continuous variables. In order to reformulate these constraints as linear ones, the cross-product $z_{jks} y_{ijkgt} n_{it}$ can be eliminated by introducing the continuous variable h_{ijkgst} that is equal to n_{it} if z_{jks} and y_{ijkgt} are 1; otherwise, the variable is equal to zero. The following constraints must be posed,

$$q_{it} \leq \sum_{k=1}^{K_j} \sum_{s}^{n_j} \left(\frac{v_{js}}{S_{ijt}} \right) h_{ijkgst} + \mathrm{BM}_{ijt}\,(1 - y_{ijgt}) \qquad \forall\, i, j, g, t \tag{19}$$

$$\sum_{s}^{n_j} h_{ijkgst} \leq n_{it}^{\mathrm{U}}\, y_{ijkgt} \qquad \forall\, i, j, k, g, t \tag{20}$$

$$h_{ijkgst} \leq n_{it}^{\mathrm{U}}\, z_{jks} \qquad \forall\, i, j, k, g, s, t \tag{21}$$

$$\sum_{s}^{n_j} h_{ijkgst} \leq n_{it} + \mathrm{BM2}_{it}\,(1 - y_{ijkgt}) \qquad \forall\, i, j, k, g, t \tag{22}$$

$$\sum_{s}^{n_j} h_{ijkgst} \geq n_{it} - \mathrm{BM2}_{it}\,(1 - y_{ijkgt}) \qquad \forall\, i, j, k, g, t \tag{23}$$

where n_{it}^{U} is the upper bound for n_{it} and the value of $\mathrm{BM2}_{it}$ is the upper bound n_{it}^{U}.

3.3. Timing Constraints. The time t_{ij} during which a batch of a product i is processed in a unit at stage j and then transferred to the next one is defined as the processing time of product i at stage j. The maximum time between two successive batches of product i in the process determines the limiting cycle time of product i, TL_i. Considering the multiperiod approach of this formulation, the limiting cycle time of product i in period t is given by

$$\mathrm{TL}_{it} \geq t_{ijt} \qquad \forall\, i, j, t \tag{24}$$

As pointed out in the Introduction, the addition of units operating out-of-phase at time-limiting stages reduces idle times and increases the units utilization. In this way, if stage j has groups of parallel units, TL_{it} can be calculated by the division between processing time t_{ijt} and the number of out-of-phase groups for product i in every period t:

$$\mathrm{TL}_{it} \geq \frac{t_{ijt}}{\sum_{g=1}^{G_j^{\mathrm{T}}} y_{ijgt}} \qquad \forall\, i, j, t \tag{25}$$

The total time for producing product i in time period t is defined as

$$T_{it} = \mathrm{TL}_{it}\, n_{it} \qquad \forall\, i, t \tag{26}$$

By multiplying eq 25 by the number of batches n_{it}, the expression takes this form:

$$T_{it} \geq \frac{t_{ijt}\, n_{it}}{\sum_{g=1}^{G_j^{\mathrm{T}}} \mathrm{y}_{ijgt}} \qquad \forall\, i, j, t \tag{27}$$

Equation 27, however, is nonlinear. In order to obtain a linear expression, the number of groups at each stage j in every period t can be expressed by the following constraint,

$$\sum_{g=1}^{G_j^{\mathrm{T}}} y_{ijgt} = \sum_{g=1}^{G_j^{\mathrm{T}}} g\, u_{ijgt} \qquad \forall\, i, j, t \tag{28}$$

$$\sum_{g=1}^{G_j^T} u_{ijgt} = 1 \qquad \forall\ i, j, t \qquad (29)$$

where the variable binary u_{ijgt} is 1 if there are g groups operating out-of-phase in time period t for product i at stage j. By substituting y_{ijgt} for u_{ijgt} in eq 27, the following expression can be posed as follows:

$$T_{it} \geq \sum_{g=1}^{G_j^T} \left(\frac{t_{ijt}\, n_{it}}{g}\right) u_{ijgt} \qquad \forall\ i, j, t \qquad (30)$$

This constraint is also nonlinear. To eliminate bilinear terms $n_{it}u_{ijgt}$, a new non-negative continuous variable e_{ijgt} is defined to represent this cross-product.[8,27] Then, the following linear constraints are obtained:

$$T_{it} \geq \sum_{g=1}^{G_j^T} \left(\frac{t_{ijt}}{g}\right) e_{ijgt} \qquad \forall\ i, j, t \qquad (31)$$

$$e_{ijgt} \leq n_{it}^{U}\, u_{ijgt} \qquad \forall\ i, j, g, t \qquad (32)$$

$$\sum_{g=1}^{G_j^T} e_{ijgt} = n_{it} \qquad \forall\ i, j, t \qquad (33)$$

By considering the case of SPC−ZW policy in period t, the total time required to produce all batches scheduled within a period cannot exceed the length of period H_t:

$$\sum_{i=1}^{I} n_{it}\, \mathrm{TL}_{it} \leq H_t \qquad \forall\ t \qquad (34)$$

By taking into account eq 26, the following expression is obtained:

$$\sum_{i=1}^{I} T_{it} \leq H_t \qquad \forall\ t \qquad (35)$$

The length of each time period H_t is adopted by the designer. It is possible to aggregate many periods or divide them as necessary depending on the specific scenarios to be assessed. For this reason, the constraint of SPC is not too restrictive from a practical point of view. With this approach, it is possible to obtain more flexible production programs and a more realistic formulation for the design problem.

3.4. Planning and Inventory Constraints. The following planning constraints manage raw materials and products inventories and force total production to meet product demands, over all time periods t. As was previously mentioned, this model assumes two scenarios.

3.4.1. Scenario 1: One Raw Material. In this case, each product requires only one ingredient that is processed to obtain the final product, and it is not shared by other products. Here, the unique ingredient being used is identified with subscript i of the product. This case is applied when only one raw material is purified, isolated, or extracted to obtain the final product. A representative example is a vegetable-extraction process.

In constraint 36, the amount of final product i stored at the end of period t, IP_{it}, depends on the stock at the previous time period, $\mathrm{IP}_{i,t-1}$; the net amount produced during this period, q_{it}; the amount sold, QS_{it}; and the amount wasted due to the expired product shelf life, PW_{it}, as follows:

$$\mathrm{IP}_{it} = \mathrm{IP}_{i,t-1} + q_{it} - \mathrm{QS}_{it} - \mathrm{PW}_{it} \qquad \forall\ i, t \qquad (36)$$

The stock of raw material i at the end of a time period t, IM_{it}, depends on the amount stored in the previous period, $\mathrm{IM}_{i,t-1}$; the purchases during period t, C_{it}; the consumption for production, RM_{it}; and the wastes due to the limited product lifetime, RW_{it}:

$$\mathrm{IM}_{it} = \mathrm{IM}_{i,t-1} + C_{it} - \mathrm{RM}_{it} - \mathrm{RW}_{it} \qquad \forall\ i, t \qquad (37)$$

Furthermore, stocks of both raw material and final product stored during period t cannot exceed the respective maximum available storage capacities, IP_{it}^{U} and IM_{it}^{U}:

$$0 \leq \mathrm{IP}_{it} \leq \mathrm{IP}_{it}^{U} \qquad \forall\ i, t \qquad (38)$$

$$0 \leq \mathrm{IM}_{it} \leq \mathrm{IM}_{it}^{U} \qquad \forall\ i, t \qquad (39)$$

At the beginning of the time horizon, the initial inventories of both raw material and product, IM_{i0} and IP_{i0}, are assumed to be given. The uses of IM_{i0} and IP_{i0} have a strong impact when this model is only used for production planning without considering design, for example, in an existing plant.

The raw material necessary for the production of product i is obtained from a mass balance,

$$\mathrm{RM}_{it} = F_{it}\, q_{it} \qquad \forall\ i, t \qquad (40)$$

where F_{it} is a parameter that accounts for the process conversion and may suffer variations in every period t, for example, because of changes in composition of raw materials.

3.4.2. Scenario 2: Two or More Raw Materials. In a more general case, the previous approach can be easily extended for plants that involve several raw materials for producing each product. In this case, the process handles c = 1, 2, ..., CT common ingredients to manufacture the products. Let F_{cit} be a parameter that accounts for the process conversion of raw material c to produce product i during period t. The amount of raw material c consumed in period t to elaborate product i, RM_{cit}, is obtained from a mass balance. Then,

$$\mathrm{RM}_{cit} = F_{cit}\, q_{it} \qquad \forall\ c, i, t \qquad (41)$$

The total consumption of raw material c for production in period t, RM_{ct}, is obtained from the following expression:

$$\mathrm{RM}_{ct} = \sum_{i=1}^{I} \mathrm{RM}_{cit} \qquad \forall\ c, t \qquad (42)$$

Then, eqs 37, 39, and 44 must be rewritten using new variables that consider each ingredient c in every period, i.e., C_{ct}, IM_{ct}, RW_{ct}, and RM_{ct}.

3.4.3. Lifetime Considerations. When the problem takes into account time periods of equal length, lifetime considerations of both raw materials and products can be added into the formulation.[31] Let ζ_i and χ_i be the time periods during which they have to be used. Thus, to guarantee that the stock of both raw materials and products in each period cannot be used after the next ζ_i or χ_i time periods, respectively, the following constraints are imposed:

$$\mathrm{IP}_{it} \leq \sum_{\tau=t+1}^{t+\chi_i} \mathrm{QS}_{i\tau} \qquad \forall\ i, t \qquad (43)$$

$$\mathrm{IM}_{it} \le \sum_{\tau=t+1}^{t+\zeta_i} \mathrm{RM}_{i\tau} \qquad \forall\ i, t \tag{44}$$

Equation 43 ensures the lifetime of product i by enforcing that it is sold in less than χ_i time periods from its storage, while eq 44 ensures that raw material i is processed in less than ζ_i time periods. It should be mentioned that the above constraints have sense only when the time periods are equal in length, as well as the last term in the objective function and the last terms in eqs 36 and 37.

3.4.4. Penalty Constraint. By using appropriate penalty constraints, failures to fulfill commitments can be quantified. If a given batch of product i meets a minimum product demand $\mathrm{DE}_{it}^{\mathrm{L}}$ with delay, then a late delivery ϑ_{it} takes place in that period.[31,32] Late deliveries are undesirable; therefore, they can be quantified through the variables ϑ_{it} by an appropriate penalty function that is minimized in the objective function.

$$\vartheta_{it} \ge \vartheta_{i,t-1} + \mathrm{DE}_{it}^{\mathrm{L}} - \mathrm{QS}_{it} \qquad \forall\ i, t \tag{45}$$

3.5. Objective Function. The objective function for this model, ψ, is the maximization of the net present value of the benefit over the horizon time. It takes into account the value of the products, the cost of the purchased raw materials, the inventory cost for final products and raw materials, and the investment costs for the units. Furthermore, operation, waste disposal, and late delivery costs are included in the objective. Each of these terms are considered separately below for scenario 1, i.e., only one ingredient for producing each product.

3.5.1. Value of Products and Cost of Raw Materials.

$$\psi_{\mathrm{P}} = \sum_{t}^{T} \sum_{i}^{I} \mathrm{np}_{it}\, \mathrm{QS}_{it} \tag{46}$$

$$\psi_{\mathrm{RM}} = \sum_{t}^{T} \sum_{i}^{I} \kappa_{it}\, C_{it} \tag{47}$$

The summations in the above expressions are taken over all products and periods considered. The first equation corresponds to the incomes for saleable products, where np_{it} is the price of final product i in period t. The following expression represents the cost of purchases of raw materials where κ_{it} is the unit price of the ingredient used to manufacture product i in every period t.

3.5.2. Inventory Costs. To determine the cost of holding inventory, the variation of each material kept in storage during the time horizon must be considered. Birewar and Grossmann[1] proposed an average in period t, which is used here. Thus, the inventory costs for raw materials and final products, respectively, can be expressed as

$$\psi_{\mathrm{I}} = \sum_{t}^{T} \sum_{i}^{I} \left[\epsilon_{it} \left(\frac{\mathrm{IM}_{i,t-1} + \mathrm{IM}_{it}}{2} \right) H_t + \sigma_{it} \left(\frac{\mathrm{IP}_{i,t-1} + \mathrm{IP}_{it}}{2} \right) H_t \right] \tag{48}$$

where ϵ_{it} and σ_{it} are inventory cost coefficients for every product and raw material in each period, respectively.

3.5.3. Operation, Waste Disposal, and Late Delivery. As has been already mentioned, considerations of costs due to waste disposal, late delivery, and operation are included in the objective function. Mathematically, these terms take the following form

$$\psi_{\mathrm{W}} = \sum_{t}^{T} \sum_{i}^{I} (\mathrm{wp}_{it}\, \mathrm{PW}_{it} + \mathrm{wr}_{it}\, \mathrm{RW}_{it}) \tag{49}$$

$$\psi_{\mathrm{D}} = \sum_{t}^{T} \sum_{i}^{I} \mathrm{cp}_{it}\, \vartheta_{it} \tag{50}$$

$$\psi_{\mathrm{O}} = \sum_{t}^{T} \sum_{i}^{I} \mathrm{co}_{it}\, q_{it} \tag{51}$$

By taking into account that wp_{it} and wr_{it} are the waste disposal coefficients costs per product and per raw material, respectively, the first equation corresponds to the total waste disposal costs. The following expression represents the costs incurred due to late deliveries through the cost coefficient cp_{it}. The final term corresponds to the cost of operation where co_{it} is the cost coefficient for each product in every period.

3.5.4. Investment Costs. The investment cost of the batch units is obtained using a power law expression on the capacity where α_{jt} and β_j are specific cost coefficients for each stage j in every period t.

$$\psi_{\mathrm{EQ}} = \sum_{t}^{T} \sum_{j}^{J} \sum_{k}^{K_j} (\gamma_{jkt} + \alpha_{jt}\, V_{jk}^{\ \beta_j})\, w_{jkt} \tag{52}$$

Coefficients α_{jt} take into account the allocation periods. γ_{jkt} corresponds to the fixed cost associated with each unit k at stage j added in period t.

As can be seen, the above function involves nonlinear terms. Replacing the unit sizes V_{jk} with the appropriate discrete sizes using constraint 13, the following expression can be obtained:

$$\psi_{\mathrm{EQ}} = \sum_{t}^{T} \sum_{j}^{J} \sum_{k}^{K_j} \sum_{s}^{n_j} (\gamma_{jkt} + \alpha_{jt}\, \upsilon_{js}^{\ \beta_j})\, z_{jks}\, w_{jkt} \tag{53}$$

The nonlinear terms involved in the above expression can be substituted by equivalent linear ones after some straightforward transformations. New continuous variables r_{jkst} are introduced to eliminate the product of binary variables $z_{jks}w_{jkt}$ through the constraints

$$r_{jkst} \ge z_{jks} + w_{jkt} - 1 \qquad \forall\ j, k, s, t \tag{54}$$

Note that variable r_{jkst} is equal to 1 when both variables z_{jks} and w_{jkt} are equal to 1. The following bounds are added to force the variables r_{jkst} to take these values:

$$0 \le r_{jkst} \le 1 \tag{55}$$

By using variables r_{jkst} and terms $c_{jkst} = \gamma_{jkt} + \alpha_{jt}\, \upsilon_{js}^{\beta_j}$, which represent the costs of standard batch vessels, eq 53 can be replaced by the following linear expression:

$$\psi_{\mathrm{EQ}} = \sum_{t}^{T} \sum_{j}^{J} \sum_{k}^{K_j} \sum_{s}^{n_j} c_{jkst}\, r_{jkst} \tag{56}$$

Thus, by considering the above expressions, the complete function objective is outlined below:

$$\max \psi = \psi_{\mathrm{P}} - \psi_{\mathrm{RM}} - \psi_{\mathrm{EQ}} - \psi_{\mathrm{I}} - \psi_{\mathrm{W}} - \psi_{\mathrm{D}} - \psi_{\mathrm{O}} \tag{57}$$

All the parameters in the above equations are based on given present values. Both income and outcome terms of the sum-

Table 1. Process Data of Example 1

	size factors S_{ijt} (L/kg)				processing time t_{ijt} (h)				conversion factor	initial inventory
i	1	2	3	4	1	2	3	4	F_{it}	IM_0
A	20	15	12	1.5	3.8	1.4	2.3	0.5	13.382	2500
B	20	15	12	1.5	3.2	2	2.5	2	13.811	2500
C	40	25	24	1.5	2.9	1	2.2	1	22.409	2500

Table 2. Available Standard Sizes of Example 1

	batch stages discrete volumes, ν_{js} (L)			
option	1	2	3	4
1	50	200	200	25
2	100	400	400	50
3	250	600	600	100
4	500	800	1000	250
5	1000	1000	1500	500
α_j	350	548	430	350
fixed installation cost		γ_j	5,050	
cost exponent		β_j	0.6	

mation that defines the objective function are discounted at the specified interest rate.

3.6. Summary of Formulation. The final formulation for the multiperiod model of a multiproduct batch plant presented in this paper involves the maximization of the objective function represented by eq 57 and subject to the constraints in eqs 1−9, 11, 12, 19−23, 28, 29, 31−33, 35−40, 43−45, 54, and 55, plus the bounds constraints that may apply. Bilinear terms have been eliminated through an efficient method in order to generate a MILP model that can be solved to global optimality.

4. Examples

In this section, two examples that illustrate the use of the proposed model will be discussed. The first case is an oleoresins plant, where only one main raw material is used for the production of each final product. A second example is considered where production of each product depends on two raw materials. Besides the usual design problem, two retrofit cases involving an existing multiproduct batch plant will be considered in order to demonstrate the potential of the proposed approach. Finally, the advantages of the multiperiod formulation over a single-period one considering simultaneously design and planning decisions are demonstrated.

4.1. Example 1. This case involves the design and planning of a batch plant producing three oleoresins, specifically, sweet bay (A), pepper (B), and rosemary (C) oleoresins. Each product recipe requires the following stages: (1) extraction in a four-stage countercurrent arrangement, (2) expression, (3) evaporation, and (4) blending. All of these stages can be duplicated up to two either identical or not identical units; therefore, the maximum number of groups that can exist at a stage is also two. A global horizon time of 3 years has been considered, which is divided into six equal time periods of 6 months each (3000 h).

In order to obtain the parameter F_{it} necessary for eq 40, the mass balances for batch extraction have to be made.[21] The data for parameter F_{it}, processing times, and size factors are given in Table 1. For simplicity purposes, these values for all time periods are assumed to be equal. Table 2 shows the available discrete sizes for each stage and cost coefficients associated. Coefficients α_{jt} are calculated by using the values of α_j (that correspond to one period) and taking into account the periods involved. Fixed costs γ_{jkt} are considered identical in all periods, for all units and stages. Prices of raw materials and final

Table 3. Prices and Demand Bounds of Example 1

	costs of raw materials κ_{it} ($/kg)			products prices np_{it} ($/kg)			maximum demands DE^U_{it} ($\times 10^2$ kg)		
t	A	B	C	A	B	C	A	B	C
1	1.2	1.2	1.4	20	38	35	20	30	25
2	1.2	1.2	1.4	20	38	36	25	35	30
3	1.3	1.5	1.4	20	42	37	45	50	40
4	1.5	1.5	1.5	25	42	38	50	50	45
5	1.5	1.8	1.5	25	45	39	55	60	50
6	1.6	1.8	1.6	25	45	40	60	60	60

products, and maximum bounds on demand forecasts over these periods, are given in Table 3. Minimum product demands in each period are assumed as 50% of maximum product demands. The discount rate is 10% annually. Products and raw materials lifetimes in time periods are 3 and 2, respectively. The inventory coefficient costs per ton of both final products and raw materials are $1.5/(ton h) and $0.2/(ton h), respectively.

The mathematical formulation involves 2687 variables, 516 of which are binary variables; also, it involves 4439 linear constraints. The problem was solved with the modeling system GAMS via CPLEX 9.0 solver with a 0% margin of optimality. The optimal solution was obtained after a CPU time of 143.0 s on a Pentium (R) IV processor (3.00 GHz).

An optimal objective function value of $1,533,821.3 was obtained. The produced final amounts produced and sales, besides inventories of both raw materials and products for each product in every period, are summarized in Table 4. Table 5 shows the equipment sizes at each stage. Between brackets, the period is indicated when the unit is allocated.

The following conclusions can be obtained from Table 4. No inventory of final product A exists because it is produced in all time periods and the amount produced in each period meets the maximum demands. On the other hand, for product B in time periods 2 and 4, an extra amount is produced that is kept as inventory to satisfy maximum demands in the subsequent periods where production is smaller. Production of product C in time periods 1 and 5 is higher than maximum demands. This excess of production is stored to meet demands in the following intervals. Moreover, no raw material inventory exists for all products.

Figure 2 corresponds to the optimal structure of the plant in each period for every product. In this figure, units closed by a dotted line are included in the same group.

As can be seen, only one unit is added at all stages in time period 1. In period 3, a second unit is incorporated at stage 1, and those with unit 1 are operated out-of-phase, conforming two groups for all products in order to reduce their processing times (see Table 1). In period 4, a new unit is added at stage 3. For both products B and C, the units at stages 1 and 3 are divided into two groups, i.e., they are operated out-of-phase and the same unit structure is maintained until period 6. On the other hand, for product A in period 4, units at stage 1 are operated out-of-phase, while at stage 3, the units are grouped, i.e., they are operated in-phase. In the last two time periods, duplicated units conform two groups working out-of-phase.

4.2. Example 2. A batch plant manufactures products A, B, and C through six different stages using two raw materials R1 and R2. A planning horizon of 2 years with four 6-month periods (3000 h) is considered. It is assumed that a maximum of two groups may exist at each stage, and thus, up to two units can be added.

The process data for the design problem are given in Table 6. Available discrete sizes to perform every stage involved in the plant are shown in Table 7. Data related to maximum

Table 4. Optimal Plan for Example 1

	A (× 10^2kg)					B (× 10^2kg)					C (× 10^2kg)				
t	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}	q_{it}	QS_{it}	IP_{it}	C_{it}	IM_{it}
1	20.0	20.0	0.0	242.6	0.0	30.0	30.0	0.0	389.3	0.0	32.55	25.0	7.55	704.2	0.0
2	25.0	25.0	0.0	334.5	0.0	46.8	35.0	11.8	646.3	0.0	22.45	30.0	0.0	503.2	0.0
3	45.0	45.0	0.0	602.2	0.0	38.2	50.0	0.0	527.5	0.0	40.0	40.0	0.0	896.3	0.0
4	50.0	50.0	0.0	669.1	0.0	63.75	50.0	13.75	880.4	0.0	45.0	45.0	0.0	1008	0.0
5	55.0	55.0	0.0	736.0	0.0	46.25	60.0	0.0	638.7	0.0	61.38	50.0	11.38	1375	0.0
6	60.0	60.0	0.0	802.9	0.0	60.0	60.0	0.0	828.6	0.0	48.62	60.0	0.0	1089	0.0

Table 5. Optimal Unit Sizes of Example 1

	stages (L)			
unit	1	2	3	4
k_1	250 (t_1)	200 (t_1)	200 (t_1)	25 (t_1)
k_2	250 (t_3)		200 (t_4)	

Table 6. Process Data of Example 2

	size factors S_{ijt} (L/kg)						processing time t_{ijt} (h)					
i	1	2	3	4	5	6	1	2	3	4	5	6
A	5.0	2.6	1.6	3.6	2.2	2.9	9.3	5.4	4.2	2.0	1.5	1.3
B	4.7	2.3	1.6	2.7	1.2	2.5	8.5	5.8	4.1	2.5	1.4	1.5
C	4.2	3.6	2.4	4.5	1.6	2.1	9.7	5.5	4.3	2.1	1.2	1.3

demand patterns, raw material costs, and final product sales prices for all products are given in Table 8. Minimum product demands in each period are assumed as 50% of maximum product demands. Parameters F_{cit} and data of raw materials are given in Table 9.

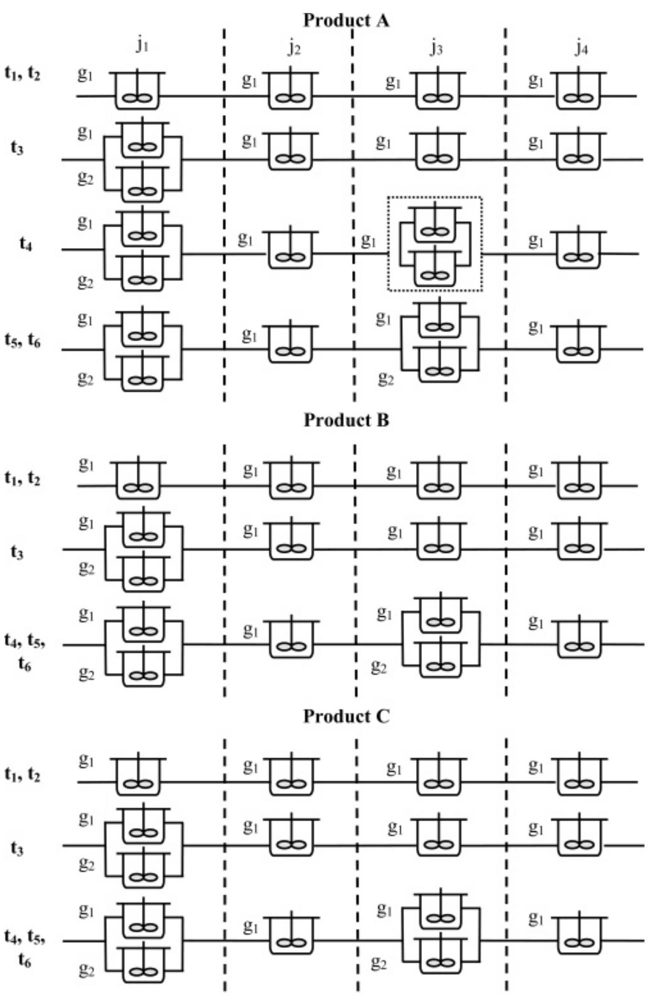

Figure 2. Optimal structure of the plant for example 1.

Table 7. Available Standard Sizes of Example 2

	batch stages discrete volumes, ν_{js} (L)					
option	1	2	3	4	5	6
1	1500	500	400	700	500	500
2	2000	750	700	1000	750	750
3	2500	1000	1250	1500	1000	1000
4	3000	1500	1500	2500	1250	1250
5	3500	2000	2000	3000	1500	1500
α_j	135	148	140	150	150	145
fixed installation cost			γ_j	2,500		
cost exponent			β_j	0.6		

Table 8. Costs, Prices, and Demand Bounds of Example 2

	costs of raw materials (\$/kg) κ_{it}		products prices (\$/kg) np_{it}			bounds on demands DE^U_{it} (× 10^3 kg)		
t	R1	R2	A	B	C	A	B	C
1	1.0	0.5	2.20	2.80	2.10	50.0	45.0	40.0
2	1.5	0.8	2.25	2.70	2.30	55.0	51.0	45.0
3	1.6	0.6	2.20	2.80	2.10	63.0	53.0	52.0
4	1.1	0.9	2.25	2.70	2.30	72.0	59.0	55.0

Table 9. Conversion Factors, Initial Inventories, and Costs of Raw Materials of Example 2

	conversion factor F_{ci}			initial inventory (kg)	storage cost (\$/(ton h))	lifetime (time periods)
	A	B	C	IM_0	ϵ_c	ζ_i
R1	0.5	1.0	0.7	20 000	0.05	2
R2	1.5	1.2	1.0	40 000	0.05	2

Table 10. Economic Evaluation Results of Example 2

description	optimal value
sales incomes	1,317,286.9
raw material costs	777,309.4
investment cost for batch units	256,517.7
raw material inventory costs	98,435.7
product inventory costs	2,805.4
operating costs	55,120.2
waste disposal costs	0.0
late delivery penalties	0.0
total	127,098.4

Table 11. Optimal Unit Sizes of Example 2

	stages (L)					
unit	1	2	3	4	5	6
k_1	1500 (t_1)	750 (t_1)	700 (t_1)	1000 (t_1)	750 (t_1)	1000 (t_1)
k_2	1500 (t_1)	750 (t_3)				

The inventory cost coefficient for all final products is 0.4\$/(ton h), and the product lifetime is 4 periods. Cost coefficients for late delivery are assumed as 50% of product prices. An annual discount rate of 10% is employed here.

The above example was modeled using GAMS modeling system coupled with CPLEX 9.0 for the MILP optimization. A 0% margin of optimality was used during the branch-and-bound solution procedure.

Table 12. Optimal Plant Configuration of Example 2 for Each Period and Product

	time periods											
	t_1			t_2			t_3			t_4		
stage	A	B	C	A	B	C	A	B	C	A	B	C
1	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$
2	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$	$(k_1)-(k_2)$
3	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)
4	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)
5	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)
6	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)	(k_1)

The resulting mathematical model, which comprises 4403 equations, 534 binary variables, and 2091 continuous variables, was solved in a CPU time of 215.89 s. The optimal solution has a value of \$127,098.4. The detailed analysis of the economic results for this case is shown in Table 10, and the optimal unit assignment is summarized in Table 11, including the period when the unit is allocated.

Table 12 shows the different unit configurations for every product in each time period. In this table, the units between brackets are included in the same group, i.e., they are units in parallel operating in-phase. In the first time period, there is one unit in all stages except at stage 1, where a second unit is added. Also, a second unit at stage 2 is aggregated in the third time period. At these stages, the units form two groups operating out-of-phase to decrease the limiting cycle time for all products.

Finally, the optimal flows of all products and raw materials are illustrated in Figures 3−7. Figures 3 and 4 show the prices and quantities used, purchased, and held as inventory for both raw materials R1 and R2 through every period. Both raw materials are purchased in periods where costs are the lowest ones. For raw material R1, the extra material purchased in period 1 is kept as inventory in periods 1 and 2 for production in following periods.

As shown in Figure 5, almost all maximum demands of product A are satisfied mainly with the production made in every time period except for the last one. In time period 3, a small extra amount is produced and is held as inventory that is used in the following period. For product B, Figure 6 shows that production in period 1 is higher than maximum demand for that period because of the lowest values of raw materials. Thus, the amount in excess is stored in inventory to satisfy maximum demand in the second period. Figure 7 shows that production of C occurs in all periods. Only in time period 2, the production is slightly lower than the corresponding maximum demand for that interval.

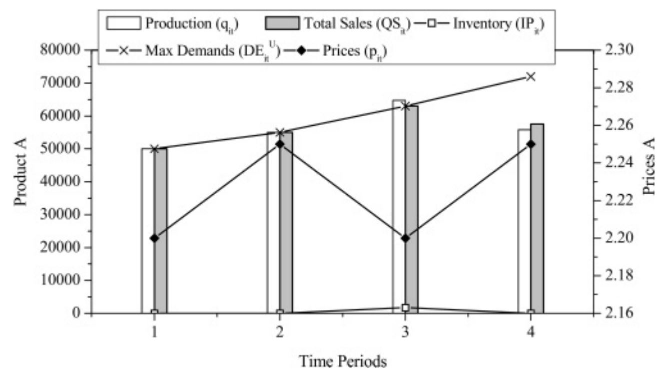

Figure 5. Profile for product A.

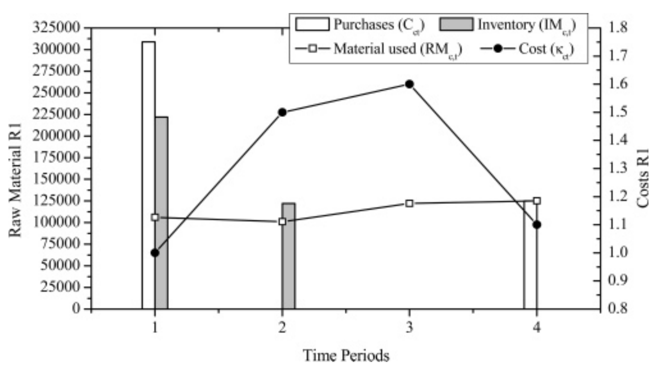

Figure 3. Profiles for raw material R1.

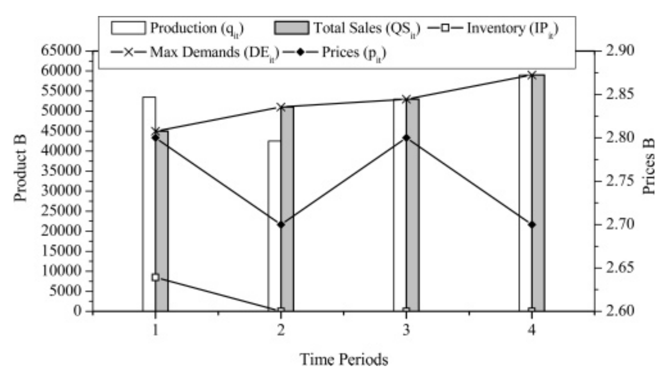

Figure 6. Profile for product B.

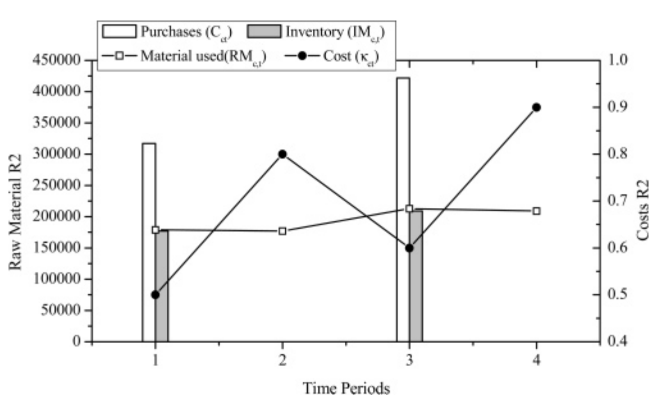

Figure 4. Profiles for raw material R2.

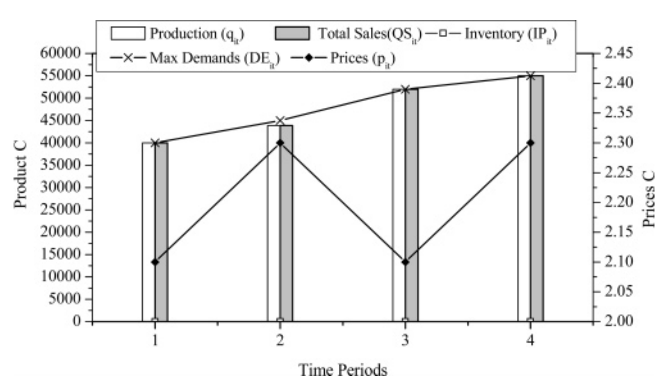

Figure 7. Profile for product C.

Table 13. Costs, Prices, and Demand Bounds for the Retrofit Problem

	costs of raw materials ($/kg) κ_{it}		prices of products ($/kg) np_{it}			bounds on demands DE_{it}^{U} (× 10^3 kg)		
t	R1	R2	A	B	C	A	B	C
1	1.6	0.6	2.25	2.90	2.20	75.0	65.0	60.0
2	1.1	0.9	2.30	2.80	2.40	78.5	67.0	62.5
3	1.7	0.7	2.25	2.90	2.20	90.0	80.0	75.0
4	1.2	1.0	2.30	2.80	2.40	92.0	83.0	80.0

Table 14. Economic Evaluation Results for Case i of the Retrofit Problem

description	optimal value
sales incomes	1,929,385.7
raw material costs	1,308,314.3
investment cost for batch units	51,207.9
raw material inventory costs	99,743.8
product inventory costs	510.3
operating costs	77,901.0
waste disposal costs	0.0
late delivery penalties	0.0
total	391,708.2

4.2.1. Retrofit Problems. The posed model is also valid for a retrofit approach.[24,26,28−30] In this problem, starting from an existing plant, a new structure has to be determined because of modifications in the original condition. In this example, two cases are considered: (i) new demand patterns for all products and (ii) the manufacture of a new product. Only added units are considered in the objective function. Unlike previous cited works in retrofit, this formulation also includes planning decisions (inventories, raw material purchases, etc.)

4.2.1.1. Case i. Using the above example, new production targets and selling prices have to be considered for subsequent periods. A retrofit problem is formulated to fulfill them. New units can be added and the net benefit is maximized by taking into account design, operation, and planning decisions.

Assuming an existing plant like the optimal solution obtained in Table 11, the following 2 years are assumed as a global horizon that is divided into four equal time periods. Size factors, processing times, and available discrete sizes are the same as those in the previous problem (Tables 6 and 9). New data on maximum demands, raw materials, and product prices are given in Table 13.

The solution of this problem involving 4117 continuous and 1058 binary variables in 8525 constraints results in the addition of a new unit at stages 1 and 3 in the second period with a profit of $391,708.2. The optimal sizes of these units are 1500 L at stage 1 and 750 L at stage 3. The solution was obtained in a CPU time of 280.51 s.

The economic results of the optimal solution for this problem are summarized in Table 14. The detailed production planning decisions obtained for the solution are shown in Table 15. As can be seen, the extra amounts of both raw materials purchased in time periods with the lowest costs are maintained as inventory. All products are produced in all time periods in order to satisfy specified maximum demands for most periods, except for product C in the first time period.

As is shown in Table 16, different arrangements of the units are proposed for each product. In this table, units between brackets form a group. In the first period, the structure of the plant does not change and duplicated units at stages 1 and 2 operate out-of-phase to reduce the limiting cycle time for all products. As mentioned above, the optimal structure is obtained by adding in the second period one unit at both stages 1 and 3. For product B, two groups exists at stage 1 where units 1 and 2 are grouped, while for both products A and C, three groups with one unit each are generated that operate out-of-phase. At stage 3, parallel units operate out-of-phase for products A and C, while for product B, they conform a group, i.e., they are operated in-phase. For all the subsequent time periods, at both stages 2 and 3 there are two groups with one unit each and the three units at stage 1 conform three groups, to reduce the limiting cycle time for all products.

4.2.1.2. Case ii. An existing plant corresponding to the optimal solution of example 2 is assumed that currently manufactures the before-mentioned products A, B, and C. In this case, a new product D is introduced to be produced in this plant in a horizon time of 2 years divided into four time periods. Maximum demands for products A, B, and C in all periods are equal to the amounts corresponding to those in the last time period in example 2 (Table 8), while for product D, they show a growing trend along the intervals, i.e., 42 000, 47 000, 55 000, and 60 000 kg, respectively. Costs of raw materials and prices of final products A, B, and C are the same as those in case i (see Table 13). Data for new product D are given in Table 17, and the selling price of product D is $2.6 in all time periods.

The optimal plant requires the allocation of a new unit of 1500 L at stage 1 and a unit of 750 L at stage 3 in the second time period, with an expected profit of $390,861.2. Table 18 shows the optimal production planning decisions, and Table 19 summarizes the economics results for this case. The detailed conformation of units for the new product D is shown in Figure 8. The other products present the same structure as that in Table 17 except for product B in time period 2, for which all parallel units at stages 1 and 3 operate out-of-phase.

As shown in Figure 8, duplicated units in stages 1 and 2 in the first period conform one group, i.e., they are operated in-phase to produce a larger amount of D. In period 2, a third unit is incorporated at stage 1 that, with units 1 and 2, conforms a group. At stage 3, the new added unit operates out-of-phase with unit 1 conforming two groups. For the last two periods, all parallel units in stages 1, 2, and 3 are divided into groups of one unit each, which operate out-of-phase to reduce the limiting cycle time.

4.2.2. Comparing Approaches. The advantages of a multiperiod formulation versus a single-period approach are illustrated here as well as the simultaneous consideration of design and planning decisions through two examples, the differences between the presented formulation and previous approaches.

First, a single-period problem (1) consisting of the product demands of the first time period of example 2 (the lowest demands) is solved. Then, a second problem (2) with similar characteristics is formulated, where the demands coincide with the highest values of the last period. For both problems, demands

Table 15. Optimal Production Planning for Case i of the Retrofit Problem

	A (× 10^3 kg)			B (× 10^3 kg)			C (× 10^3 kg)			R1 (× 10^3 kg)		R2 (× 10^3 kg)	
t	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	C_{ct}	IM_{ct}	C_{ct}	IM_{ct}
1	75.00	75.00	0.0	65.00	65.00	0.0	37.75	37.75	0.0	108.9	0.0	448.1	259.9
2	78.00	78.00	0.0	67.00	67.00	0.0	62.50	62.50	0.0	328.2	178.5	0.0	0.0
3	92.02	90.00	2.02	80.00	80.00	0.0	75.00	75.00	0.0	0.0	0.0	623.6	314.6
4	89.98	92.00	0.0	83.00	83.00	0.0	80.00	80.00	0.0	183.9	0.0	0.0	0.0

Table 16. Optimal Unit Conformation for the Case (i) of the Retrofit Problem

	time periods					
	t_1			t_2		
stage	A	B	C	A	B	C
1	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)−(*k*3)	(*k*1,*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)
2	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)
3	(*k*1)	(*k*1)	(*k*1)	(*k*1)−(*k*2)	(*k*1, *k*2)	(*k*1)−(*k*2)
4	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)
5	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)
6	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)

	time periods					
	t_3			t_4		
stage	A	B	C	A	B	C
1	(*k*1)−(*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)	(*k*1)−(*k*2)−(*k*3)
2	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)
3	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)	(*k*1)−(*k*2)
4	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)
5	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)
6	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)	(*k*1)

Table 17. Size Factors and Processing Times for Product D

	stages					
product D	1	2	3	4	5	6
S_{ijt} (L/kg)	5.5	4.2	1.5	2.9	1.4	2.3
t_{ijt} (h)	5.5	5.2	3.9	2.7	1.5	1.9

have to be satisfied as usual in design problems. The resulting optimal plant structures for these problems are summarized in Table 20. In this way, both solutions show the results with a traditional approach in the limiting periods.

Taking into account the results of Table 20, both plant structures are adopted in order to satisfy the demands posed in the problem with four periods. First, the problems are solved without considering planning decisions, for example, inventories of raw materials and products are not allowed. The optimal solutions are \$39,953.3 and \$17,768.4 (Table 21). These solutions are significantly lower than the optimal profit of \$127,098.4 previously obtained.

Second, the original data of example 2 are used by taking into account the plant structures of Table 20, but now planning alternatives are considered. The optimal economical assessment of both solutions is presented in Table 21. For problem 1, the optimal solution is \$117,706.2, which is lower than the best profit of \$127,098.4 attained in the original example 2. This reduction is due to the fact that a small plant has been obtained and it cannot satisfy the growing demands of the following periods. In this case, the plant has been designed without forecasts of next periods, and only the data of period 1 has been taken into account. Comparing Tables 10 and 21, the incomes for sales have been decreased. In the case of problem 2, the

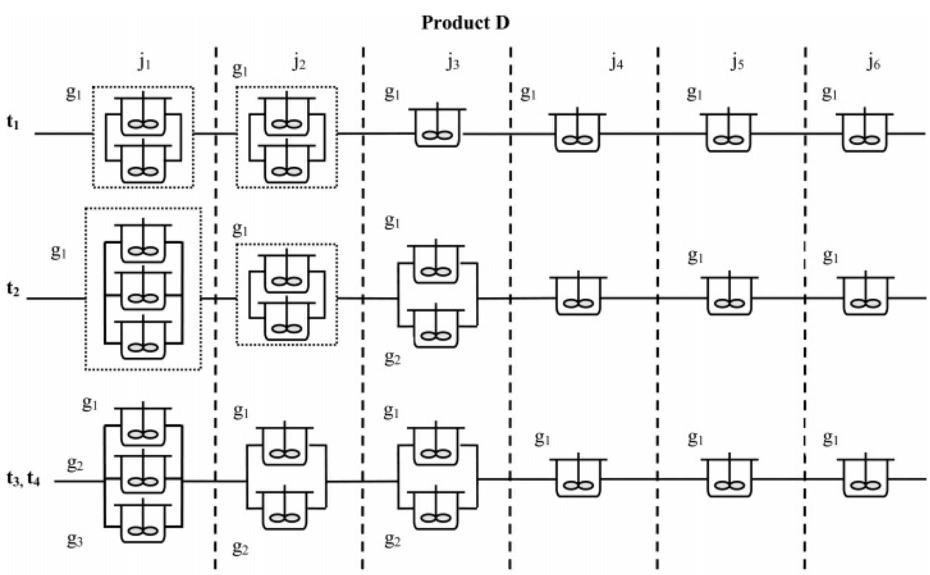

Figure 8. Optimal structure of units for product D in every period.

Table 18. Optimal Production Planning for Case ii of the Retrofit Problem

	A (× 10³ kg)			B (× 10³ kg)			C (× 10³ kg)			D (× 10³ kg)		
t	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}	q_{it}	QS_{it}	IP_{it}
1	72.00	7200	0.0	59.00	59.00	0.0	28.95	28.95	0.0	21.00	21.00	0.0
2	72.00	72.00	0.0	59.00	59.00	0.0	55.00	55.00	0.0	48.65	47.00	1.64
3	72.00	72.00	0.0	59.00	59.00	0.0	55.00	55.00	0.0	53.35	55.00	0.0
4	63.00	63.00	0.0	59.00	59.00	0.0	55.00	55.00	0.0	60.00	60.00	0.0

Table 19. Economic Evaluation Results for Case ii of the Retrofit Problem

description	optimal value
sales incomes	1,968,376.1
raw material costs	1,350,412.8
investment cost for batch units	51,207.9
raw material inventory costs	96,766.6
product inventory costs	436.3
operating costs	78,691.3
waste disposal costs	0.0
late delivery penalties	0.0
total	390,861.2

Table 20. Optimal Sizes for Both Problems 1 and 2

		stage (L)					
problem	unit	1	2	3	4	5	6
1	k_1	1500	750	700	1000	750	750
	k_2	1500					
2	k_1	2000	1000	700	1500	1000	1250
	k_2	2000					

optimal profit, $110,075.3, is also lower than the optimal solution in Table 10. If results of Table 21 are analyzed, greater incomes are achieved. However, the investment costs for batch units are also greater than those in previous solutions. Taking into account a larger raw materials supply to satisfy higher demands, a poorer result is obtained in the last case. These results are reasonable when taking into account that the plant has been designed to fulfill larger demands. However, the greater investment costs, without considering the demand evolution, result in a lower profit.

In this way, this example emphasizes that the simultaneous assessment of design and planning decisions over a multiperiod scenario yields better solutions. Planning decisions allow for taking advantage of several problem elements (inventories, prices, etc.) to effectively use the units. The multiperiod approach takes into account fluctuations in the problem conditions. Previous solutions could be easily nonsense when the problem conditions change and these modifications have not been addressed.

Note that above both solutions are lower than the best profit of $127,098.4 obtained in the original example 2. Table 21 present the optimal economic evaluation of both problems 1 and 2 corresponding to the multiperiod approach.

In particular, the worst performance corresponds to problem 2. Although it was solved for the highest demands of time period 4 and income for sales are similar to that in Table 16, it required a bigger plant than the original example 2. The small plant for problem 1 satisfies the requirements of the new multiperiod scenario, but the sales incomes have been severely reduced because upper bounds on product demands are not met in several time periods. In this way, this example shows the effectiveness of simultaneous production planning and design decision considerations over a multiperiod scenario.

5. Conclusions

The context where a multiproduct batch plant operates is dynamic. Several market and seasonal fluctuations affect the optimal structure adopted during the plant lifetime. Then, the units included in a plant and their arrangement cannot be maintained constant during the time horizon. Also, there exist critical trade-offs between design and production decisions in a stable scenario that are strengthened in the multiperiod case. However, most of the articles in this area assume constant conditions during the time horizon and prioritize decomposition approaches solving separated problems.

In this context, a formulation that simultaneously takes into account all these elements is quite useful to assess the trade-offs among them. In detail, the planning model considers variations in prices, product demands, costs, and raw materials availability due to seasonal or market fluctuations. A realistic design case has been posed that considers available units in discrete sizes. With this model, a flexible design of the plant is obtained with the incorporation of units in every period, which can have different sizes. Through the use of groups, the units at every stage can be operated either in-phase or out-of phase, allowing each product to present different operation configurations in each period. In this first version, fluctuations in the elements of the problem during the time periods have been taken into account through deterministic values. Future works will introduce uncertainty in this formulation.

The overall problem was formulated as an MILP model that can be solved to global optimality. Thus, this MILP model clearly integrates design, planning, and commercial decisions optimizing the structure of the plant besides purchases, production, sales, and inventory policies for each product in every period.

The model has been assessed through two examples. However, by taking into account the great number of variables involved in this formulation and the strong trade-offs among them, very different applications can be distinguished. Further on the standard problem solved in this article, several scenarios can be posed. For example, the available units in a given plant can be configurated and the production can be planned in a problem focused on planning. Also, a retrofit approach can be solved where new units are incorporated if they are required from an original plant. All these scenarios are affected by several factors: the number and the length of the periods, the dimension of fluctuations, product lifecycles, etc. Then, the use of this model should also be adjusted and studied in depth, taking into account specific contexts.

Table 21. Economic Evaluation Results for Both Problems 1 and 2

	optimal values			
description	problem 1 without planning	problem 2 without planning	problem 1 considering planning	problem 2 considering planning
sales incomes	1,164,704.1	1,346,440.9	1,176,963.7	1,346,440.9
raw material costs	840,325.1	992,222.3	687,991.6	796,364.5
investment cost for batch units	235,962.6	280,082.2	235,962.6	280,082.2
raw material inventory costs	0.0	0.0	86,156.0	96,322.9
product inventory costs	0.0	0.0	432.8	7,112.2
operating costs	48,465.0	56,368.0	48,714.4	56,483.7
waste disposal costs	0.0	0.0	0.0	0.0
late delivery penalties	0.0	0.0	0.0	0.0
total	39,951.3	17,768.4	117,706.2	110,075.3

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Nomenclature

Subscripts

g = group
i = product
j = batch stage
k = equipment
s = discrete sizes for the units
t = time period
τ = time period

Superscripts

T = total
L = lower bound
U = upper bound

Parameters

co_{it} = operating cost coefficient of product i at period t
DE_{it} = demand for product i in period t
F_{ci} = conversion of raw material c to produce i at period t
G_j^T = total number of groups at stage j
H = time horizon
H_t = net available production time for all products in period t
K_j = maximum number of units that can be added at stage j
n_j = number of discrete sizes available for stage j
np_{it} = price of product i in period t
S_{ijt} = size factor of product i at stage j for each period t
t_{ijt} = processing time of product i in batch stage j in period t
wp_i = waste disposal cost coefficient per product i
wr_i = waste disposal cost coefficient per raw material i
α_j = cost coefficient for a batch unit in stage j
β_j = cost exponent for a batch unit at stage j
ϵ_i = inventory cost coefficient for raw material i
κ_{it} = price for the raw material of product i in period t
υ_{js} = standard volume of size s for batch unit at stage j
σ_i = inventory cost coefficient for product i
ζ_i = time periods during which raw materials have to be used
χ_i = time periods during which products have to be used

Binary Variables

u_{ijgt} = it is 1 if at stage j for product i there are g groups in period t
w_{jkt} = it is 1 if unit k at stage j is added in period t
y_{ijgt} = it is 1 if group g at stage j exists in period t for product i
y_{ijkgt} = it is 1 if unit k at batch stage j is assigned to group g for product i in period t
z_{jks} = it is 1 if equipment k at batch stage j has size s

Integer Variables

G_{ijt} = number of groups at stage j for every product i in a period t

Continuous Variables

B_{it} = batch size of product i in period t
C_{it} = amount of raw material for producing i purchased in period t
e_{ijgt} = continuous variable that represents the product of the variables $n_{it}u_{ijgt}$
h_{ijkgst} = continuous variable that represents the product of the variables $z_{jks}y_{ijkgt}n_{it}$
IM_{it} = inventory of raw material i at the end of a period t
IP_{it} = inventory of final product i at the end of a period t
n_{it} = number of batches of product i in period t
PW_{it} = product i wasted at period t due to the limited product lifetime
q_{it} = amount of product i to be produced in period t
QS_{it} = amount of product i sold at the end of period t
RM_{it} = raw material inventory for product i in period t
RW_{it} = raw material i wasted at period t due to the limited product lifetime
r_{jkst} = continuous variable that represents the product of the binary variables $z_{js}w_{jkt}$
T_{it} = total time for producing product i in period t
TL_{it} = limiting cycle time of product i in period t
V_{jk} = size of a batch unit k at stage j
ϑ_{it} = late delivery for product i in period t

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