PROCESS DESIGN AND CONTROL

Heuristic Method for the Optimal Synthesis and Design of Batch Plants Considering Mixed Product Campaigns

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In this paper, a heuristic method is presented for the simultaneous solution of the synthesis and design problems of batch plants. A detailed nonlinear program (NLP) model is developed that considers a superstructure to represent all the configuration options for the plants. Usually, similar works in this area assume as a hard constraint the use of single-product campaigns. In this work, mixed campaigns are introduced to pose problems where this is a significant condition. Specific scheduling constraints are formulated, and a resolution strategy is presented to solve the problem. This formulation is valid for multiproduct batch plants and a special type of multipurpose plants where products follow different production paths sharing some but not all the stages. The approach is implemented for a Torula yeast, brandy, and bakery yeast production plant. To assess the method, different mixed campaigns are modeled. Economical and synthesis, design, and operational results are also reported.

1. Introduction

In a multiproduct/multipurpose batch plant, several products are manufactured following the same or different production sequences, sharing the equipment, raw materials and other production resources. The inherent operational flexibility of multiproduct/multipurpose plants gives rise to considerable complexity in the design and synthesis of such plants. In many cases, scheduling strategies are not incorporated or well integrated. Usually the simplest scheduling sequence, a single-product campaign, is considered, which may lead to overdesign or underdesign. To ensure that any resource incorporated in the design can be used as efficiently as possible, detailed considerations of plant scheduling must be taken into account at the design stage. Therefore, it is important to consider design, synthesis, and scheduling simultaneously.

In recent years, several authors have incorporated scheduling constraints to the synthesis problem of multiproduct and multipurpose batch plants. Birewar and Grossmann[1,2] have addressed the problem of simultaneous sizing and scheduling of a multiproduct batch plant that accounts for the unlimited intermediate storage (UIS) and zero wait (ZW) policies with mixed product campaigns. In that work, they developed an NLP model for fixed time and size factors. Their later work incorporates the structural decision of parallel units for some stages resulting in a mixed integer nonlinear program (MINLP) model.

Zhang and Sargent[3,4] developed a general scheduling formulation based on a variable event time representation. This continuous time formulation for scheduling can be easily extended to the design and synthesis of batch plants. However, when nonlinear task models (processing time, utility usage, and unit availability) and nonlinear capital cost functions are considered, a nonconvex MINLP problem will arise. Even for the locally linearized models, a large number of auxiliary variables and constraints for the linearization of bilinear terms of integer and continuous variables must be typically introduced to reduce the MINLP into an MILP, which makes the problem very large in scale and difficult to solve.

Xia and Macchietto[5] presented a formulation based on the variable event time scheduling model of Zhang and Sargent.[3,4] A stochastic method is used to solve the resulting nonconvex MINLP problem directly, instead of the introduction of a large number of auxiliary variables and constraints to reduce the MINLP into an MILP. Lin and Floudas[6] extended the continuous-time scheduling formulation proposed by Ierapetritou and Floudas[7] and Ierapetritou et al.[8] to address the problem of integrated design, synthesis, and scheduling of multipurpose batch plants. They studied both linear and nonlinear cases, which resulted in MILP and MINLP problems, respectively.

In this paper, the synthesis, design, and operational issues for a sequential multipurpose batch plant are considered simultaneously in an NLP model. In a sequential multipurpose plant, it is possible to recognize a specific direction in the plant floor that is followed by the production paths of all the products.[9] However some processing units are used only by some products. Obviously, the model presented is also valid for the multiproduct batch plant where all the products use all the stages. In addition, alternatives for the number of units in series are introduced. The configuration options are explicitly considered in terms of a superstructure.[10]

The consideration of simultaneous optimization is not an usual approach in the literature. In general, these problems are treated in separate form: first the plant configuration problem, then the sizing problem, and last the campaign determination problem. This leads to suboptimal solutions.

The proposed methodology solves in the first place a relaxed model where scheduling constraints for mixed campaigns are not considered with the purpose of obtaining the ratios among the number of batches of the different products considered. With

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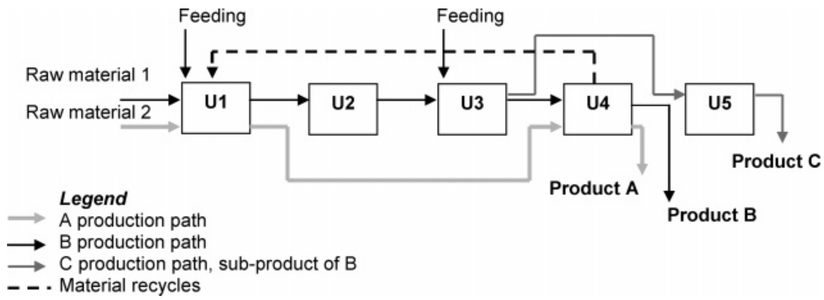

Figure 1. Flowsheet of a sequential multipurpose batch plant.

these ratios, it is possible to envisage different campaign configurations. Then, it is possible to propose different structures for the campaign. Taking into account that the approach is applied to problems with moderate number of products and stages where the model detail is emphasized, the number of campaigns to plan is manageable.

Once the campaign is defined, the appropriate sequence constraints are added to the relaxed model, and thus the design and operation integrated problem of a sequential multipurpose plant is solved. When alternative campaigns are designed, changeover times between different products can be considered. The plant configuration obtained in the relaxed model solution is adopted for the mixed product campaign model; therefore the alternative configurations remaining are deactivated in this later model.

The incorporation of an extra plant is considered in this work to provide the material and power streams that the multipurpose plant requires, so that these resources are bounded.

The objective function employed in this formulation is the maximization of the net annual profit as given by the earnings of selling products and savings from unused resources (those produced and available in the mother plant and not used in the multipurpose plant) minus the annualized investment and operating costs.

Several examples of different mixed campaigns are stated for a Torula yeast, brandy, and bakery yeast production plant to assess the proposed approach.

2. Model Assumptions

The problem considered in this paper has the following characteristics: (i) The plant has batch and semicontinuous units. (ii) N_p products are processed in the plant. (iii) Not all the products follow the same production path, it is a sequential multipurpose batch plant[9] (see Figure 1). (iv) The production path for each product is known. (v) The product demands are upper and lower bounded. (vi) The processing times are continuous variables, and the time horizon is given. (vii) The mixing, splitting, and recycle of batches are allowed. (viii) The production of subproducts is also considered. (ix) The material and energy resources are bounded. (x) The unit sizes are continuous variables.

The objective is to determine the optimal plant design and operation to meet a specified economic criterion.

Figure 1 shows a sequential multipurpose plant where two products, A and B, and a subproduct of B, product C, are produced. Product A follows the production path U1 → U4, and U1 receives an extra feeding (blend of batches) and a recycled batch from U4. Product B follows the path U1 → U2 → U3 → U4. At U3, the batches are split to produce product C through U5. U3 also has an extra feeding.

3. Solution Procedure

The simultaneous optimization of the configuration, design, operation, and scheduling of a sequential multipurpose batch plant results in a very large-scale problem and is difficult to solve as it was stated in the introduction section. So, a heuristic procedure is proposed to solve this simultaneous optimization. The main idea lies in solving first a model without mixed product campaign constraints and then, according to the optimal number of batches of each product obtained in the first model solution, determining the possible campaign configurations. For each campaign configuration proposed, an NLP model is formulated for the optimal plant configuration obtained in the first model solution. The heuristic approach is resumed in the following steps: (i) First, a model whose constraints consider the design and operation of a multipurpose plant without considering the tasks scheduling constraints is solved. This model is a relaxation of the mixed campaigns problem and is solved as an NLP problem. The model has an embedded superstructure that considers different configuration options for the plant synthesis.[10] The solution of the relaxed model provides the estimated number of batches of each product and the plant configuration. (ii) Relationships between the number of batches of each product, which are obtained from the relaxed model, are established, and the possible sequences of the multiproduct campaigns are selected for the plant synthesis obtained in the relaxed model solution. (iii) For each proposed campaign configuration, an NLP problem is modeled and solved. In this model, a novel set of tasks scheduling constraints are added to the relaxed model to ensure that the production processes of two different products do not overlap in the same unit. In this way, a model for each mixed product campaign is formulated and solved, for the optimal plant configuration obtained from the relaxed model solution, as an NLP problem. At this step, the plant configuration is fixed, and the sizing problem is solved. (iv) The campaign with the best objective function value is chosen as the optimal solution.

The first model represents a relaxation for the second one. Therefore, the objective function value of the relaxed model solution represents an upper bound for the objective value of the mixed product campaign model. In the studied cases presented below, the gap between these values is very tight, which ensures that the solution obtained for the mixed product campaign is optimal.

4. Mathematical Modeling

A plant with N_j batch units and N_k semicontinuous units is considered. N_p products are manufactured in the plant not necessarily following the same production path.

Both problems (the relaxed one and the problem with scheduling constraints) include a detailed modeling for all the products and the batch and semicontinuous units.

4.1. Relaxed Model. The components and total mass balances at each stage, the connection constraints between stages, and the design equations for each stage for each product are considered as a detailed model. If there are recycles or interconnections between the production processes, as really happens in the study cases, also the balances that correspond to these connections are considered. Mass balances for some units are given by differential equations such as

$$\frac{dC_{xij}}{dt} = h(t,x) \quad \forall x \tag{1}$$

where C_{xij} is the concentration of component x (biomass, substrate, product, etc.) at stage j of production process i. These dynamic equations are discretized and included in the overall model. Note that the discretized equations involve the processing time of the batch item and the time integration step, all of which are considered variables. The number of grid points is problem data, but since the final processing time is variable, the discretization step length is also an optimization variable determined according to the final time for each unit. For these models, the trapezoidal method was adopted.[11] For example, if the biomass balance is

$$\frac{dX_{ij}}{dt} = (\mu_{ij} - \upsilon_{ij})X_{ij} \tag{2}$$

where X represents the biomass concentration, μ is the specific growth rate of biomass, and υ represents the biomass death rate. The corresponding set of algebraic equations is

$$X_{ij}^{(p+1)} = X_{ij}^{(p)} + \frac{l_{ij}}{2}((\mu_{ij}^{(p+1)} - \upsilon_{ij})X_{ij}^{(p+1)} + (\mu_{ij}^{(p)} - \upsilon_{ij})X_{ij}^{(p)}) \tag{3}$$

where l is the step length and $p = 1, ..., P$ are the grid points.

In addition, for the stages that are shared by several products, the following constraints are considered.

For batch item j and product i

$$V_j \geq V_{ij} \quad \forall i = 1,..., N_p, \forall j \in \mathrm{EB}_i \tag{4}$$

For semicontinuous item k and product i

$$V_k \geq V_{ik} \quad \forall i = 1,..., N_p, \forall k \in \mathrm{ES}_i \tag{5}$$

where V are the batch and semicontinuous sizes and EB_i and ES_i represent the set of batch and semicontinuous units in the production path of product i.

Let t_{ij} be the processing time for product i at stage j, θ_{ik} the processing time for product i at semicontinuous stage k, CT_i the cycle time for the production of product i, and Nb_i the number of batches of product i over the horizon time HT, then

$$T_{ij} = \theta_{ik'} + t_{ij} + \theta_{ik''} \quad \forall i = 1,..., N_p, \forall j \in \mathrm{EB}_i \tag{6}$$

$$\mathrm{CT}_i \geq T_{ij} \quad \forall i = 1,..., N_p, \forall j \in \mathrm{EB}_i \tag{7}$$

Note that eq 6 defines the time that the batch unit j will be occupied with product i, which contemplates the material loading ($\theta_{ik'}$) and unloading ($\theta_{ik''}$) time if this unit is located between semicontinuous units. It is worth noting that in this approach it is assumed that variables t_{ij} and $\theta_{ik'}$ are involved in detailed submodels, some of them written as differential equations and included in the actual model as was presented in eqs 1−3.

Several consecutive semicontinuous units give rise to a semicontinuous subtrain. In this paper, only perfectly synchronized semicontinuous subtrains are considered, then

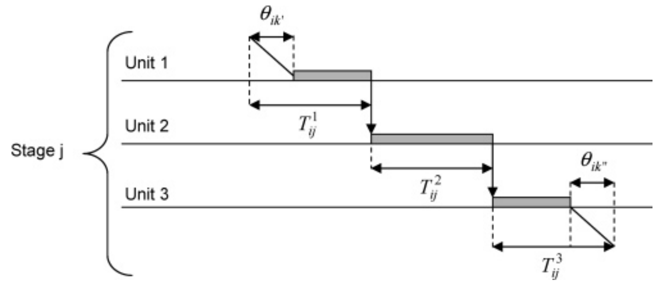

Figure 2. In series unit configuration for a batch stage.

$$\theta_{ik} = \theta_{i,k+1} \quad \forall i = 1,..., N_p \tag{8}$$

where k and $k+1$ belong to the same subtrain.

For products i that share the unit j ($i \in I_j$), the following constraints are considered

$$\sum_{i \in I_j} Nb_i T_{ij} \leq HT \quad \forall j = 1,..., N_j \tag{9}$$

In the same way, for all the products i that share the unit k ($k \in I_k$)

$$\sum_{i \in I_k} Nb_i \theta_{ik} \leq HT \quad \forall k = 1,..., N_k \tag{10}$$

If all the products follow the same production path, then eq 10 becomes redundant because the batch processing time considers the semicontinuous processing times upstream and downstream of the batch unit.

A characteristic of this model is that, for certain batch stages, the number of units in series is a priori unknown. For these stages, a superstructure that contemplates all the possible configurations, or those chosen by the designer as feasible, is modeled and embedded in the global model. Then, if the stage is preceded by a semicontinuous unit, the first unit in the series has to consider the filling time in its operating time or the emptying time if this stage has a downstream semicontinuous unit (see Figure 2).

In this case, the cycle time of stage j is given by

$$T_{\mathrm{ij}} = \max_u \{T_{ij}^u\} \text{ for each } u \in Nu_j \tag{11}$$

or in a continuous formulation

$$T_{\mathrm{ij}} \geq T_{ij}^u \text{ for each } u \in Nu_j \tag{12}$$

where Nu_j is the set of units in series at stage j.

To simplify the result analysis, in this work only units in series are considered as possible configurations. The incorporation of units in parallel on the superstructure model can also be done as proposed in Corsano et al.,[10] and it does not represent a model limitation.

The material and energy resources s required for each production process can be obtained from another plant that belongs to the same industrial complex called the "mother plant" or can be imported from another plant. The unused amount of resource s, that is, the amount of s that is not consumed by the multipurpose plant, can be sold to other complexes. If F_s^{prod}, F_s^{imp}, and F_s^{ex} are the amount per hour of produced, imported, or exported resource s, respectively, and if f_{sij} and f_{sik} are the amount of s consumed for producing product i at the stage j or k respectively, then

$$F_s + F_s^{\text{imp}} = \sum_{i=1}^{N_p}\left(\sum_{j=1}^{N_j}\frac{f_{sij}}{CT_i} + \sum_{k=1}^{N_k}\frac{f_{sik}}{CT_i}\right) + F_s^{\text{ex}} \qquad (13)$$

The production rate constraints for each product are

$$\frac{Nb_iB_i}{HT} = Q_i \quad \forall i = 1,...,N_p \qquad (14)$$

$$Q_i^{\min} \le Q_i \le Q_i^{\max} \quad \forall i = 1,..,N_p \qquad (15)$$

where Q_i is the production rate of product i which is bounded by $Q_i^{\min}$ and $Q_i^{\max}$, and B_i is the batch size of product i.

The selected objective function is the maximization of the net annual profit (NAP), given by the sum of the earnings from products sales and the exported resources (Sl) minus the total annualized cost (TAC) given by investment (CInv) and operating (CO) costs. The considered operating costs are the raw material, power resources, and disposal costs

$$\text{NAP} = Sl - \text{TAC} \qquad (16)$$

$$\text{Sl} = \sum_{i=1}^{N_p} G_iB_iNb_i + \sum_{s} G_sF_s^{\text{exp}}\,HT \qquad (17)$$

Where G_i represents the product i sale price (\$ ton^{-1}), B_i is the produced batch of product i (ton), G_s represents the resource price of s (\$ ton^{-1}), and U_s is the amount of unused resource s (ton)

$$\text{TAC} = \text{CInv} + \text{CO} \qquad (18)$$

$$\text{CInv} = C_{ann}\left(\sum_j \alpha_jV_j^{\beta_j} + \sum_k \alpha_kV_k^{\beta_k}\right) \qquad (19)$$

$$\text{CO} = \sum_{i=1}^{N_p} Nb_i\text{Res}_i + HT\left(\sum_s C_s^{\text{prod}}F_s^{\text{prod}} + \sum_s C_s^{\text{imp}}F_s^{\text{imp}}\right) \qquad (20)$$

where C_s is the cost of resource s, and Res_i is the disposal cost of product process i that varies according to the effluent.

The relaxed model solution provides the optimal plant configuration and design and the number of batches of each product.

It is worth noting that the relaxed model resembles a single-product campaign one, in which all the batches of a product are processed without overlapping with others products. The difference is that in the relaxed model, the scheduling constraint of a single-campaign model

$$\sum_{i=1}^{N_p} \text{CT}_iNb_i \le HT$$

is not necessarily satisfied because of eq 9.

Figure 3 schows the relaxed model solution of a plant that processes two products with different production path. The product A follows the path stage 1 → stage 2 → stage 4, while the product B goes through stage 1 → stage 3 → stage 4.

4.2. Multiproduct Campaign Model. In many cases, a stream in the production of a product can be recycled to some previous stage in the process of the same or another product. In such a case, the single-product campaign is impracticle because the material to be recycled should be stored. In addition single-product campaigns require high inventory values, furthermore many products cannot be stored because they are degraded in short time. From the point of view of the model design they raise greater challenges than the formulations for single campaigns. The first decision to be considered is how the multiproduct campaign will be configured. In this work, this decision is imposed by the designer from the estimation of the ratio of number of batches of the different products elaborated in the plant.

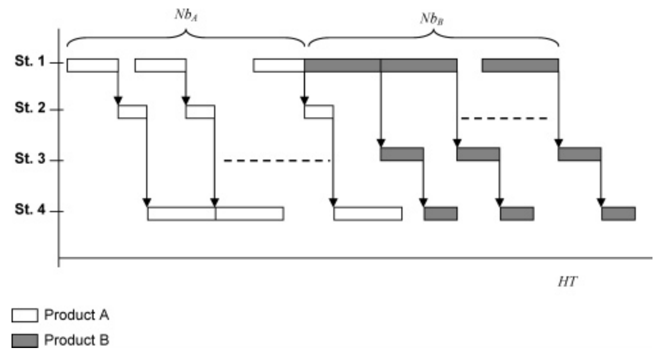

Figure 3. Gantt chart for multipurpose plant relaxed model.

Let i' be the product with the small number of batches in the relaxed model solution. Define r_i = round($\text{Nb}_i/\text{Nb}_{i'}$), as the rounding of the relation between the number of batches of each product i and product i', obtained in the relaxed model solution, so that r_i is a model parameter.

Let Nb be the number of times that the mixed campaign is repeated. For the mixed campaign model, the following constraints are imposed

$$\text{Nb} = \text{Nb}_{i'} \qquad (21)$$

$$\text{Nb} = \text{Nb}_ir_i^{-1} \quad \forall i = 1,..., N_p \qquad (22)$$

Nb is an optimization variable, since Nb_i is an optimization variable for each product i.

For example, if three products, A, B, and C, are processed in the sequential multipurpose plant with $N_A = 100$, $N_B = 120$, and $N_C = 310$ in the relaxed model solution, then $\text{Nb} = N_A$, $r_B = 1$, and $r_C = 3$, that is, the campaign A−B−C−C−C or some of its permutations is established.

Taking into account that this strategy is applied to detailed models with a reduced number of products, this is an affordable method. Therefore, this procedure estimates the proportion among the number of batches of all products. Different campaigns can be proposed by the designer.

Now, new constraints have to be developed to formulate the different conditions that arise from that type of campaigns. All the following constraints are posed for a determined mixed campaign.

Let SL_{ij} be the idle time at unit j after processing a batch of product i and before processing the next batch and CT_j be the cycle time for unit j defined by

$$\text{CT}_j = \sum_{i \in I_j} (T_{ij} + \text{SL}_{ij}) \quad \forall j = 1,..., N_j \qquad (23a)$$

Analogously

$$\text{CT}_k = \sum_{i \in I_k} (\theta_{ik} + \text{SL}_{ik}) \quad \forall k = 1,..., N_k \qquad (24)$$

If there is more than one batch of some product in the campaign, the processing time and idle time for that product must be added as many times as repetitions occur.

If a stage j has more than one unit in series as a result of the superstructure optimization model performed in the relaxed

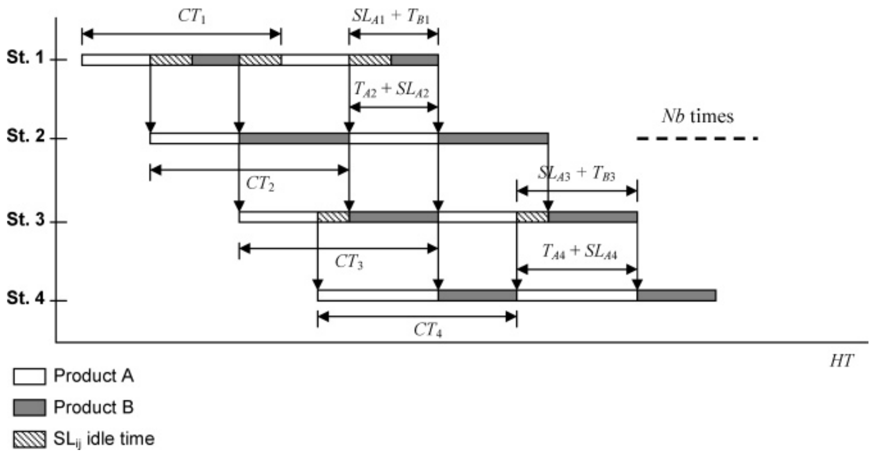

Figure 4. Gantt chart for product productions that follow the same path: case *i*.

model, the following constraints have to be added

$$CT_j^u = \sum_{i \in I_j} (T_{ij}^u + SL_{ij}^u) \quad \forall u \in Nu_j \tag{23b}$$

The modeler must establish for each unit the order in which products will be processed, using the relationship between the number of batches previously determined. Next the constraints that must be implemented according to the production path that each product follows are settled. These constraints are established for two consecutive products to ensure that the production process of two different products does not overlap in the same unit.

Because of the ZW transfer policy adopted

$$CT_j = CT_{j+1} \quad \forall j = 1, ..., N_j \tag{25}$$

$$CT_k = CT_{k+1} \quad \forall k = 1, ..., N_k \tag{26}$$

$$CT_j = CT_k \text{ for some } j = 1, ..., N_j \text{ and } k = 1, ..., N_k \tag{27}$$

Suppose that in the production path, the product $i + 1$ is processed in unit j immediately after product i and both follow the path $j \rightarrow j + 1$. Three cases are presented.

Case *i*. If all the products follow the same production path, the following constraints are added to the model with the objective of avoiding task superposition at the same processing unit

$$T_{i+1,j} + SL_{ij} = T_{i,j+1} + SL_{i,j+1} \quad \forall i = 1, ..., N_p - 1, \forall j = 1, ..., N_j - 1 \tag{28}$$

$$CT_j Nb \leq HT \text{ for some } j \tag{29}$$

The constraints (eq 28) were used by Birewar and Grossmann[2] but with a different definition of the idle time at stage *j*. In their work, they defined SL_{ijk} as the idle time between the batches of products *i* and *k* in processing unit *j*. In that model, the processing times and size factors are fixed; the campaign configuration is obtained as a result of the model solution, and the model is solved as an MINLP problem.

Since eqs 25−27 establish that the unit cycle times are equal, the constraint (eq 29) written for some *j* means that it will hold for every unit *j*.

This first case is shown in Figure 4.

It is worth noting that if stage *j* has more than one unit in series (as a result of the superstructure model optimization), for each of these units, the constraints (eq 28) must be satisfied, and CT_j is defined by eq 23b.

Case *ii*. If a production path is different, the following constraints are imposed for two consecutive products with different production paths. Let A and B be two consecutive products in the production campaign such that A follows the path $l \rightarrow s$ and B the path $m \rightarrow s$ and let J_A and J_B be the set of processing units that are utilized in the process production of A and B respectively; then (a) if the processing order is conserved at stage *s*, the following constraint is added to avoid task superposition at stage *s*

$$T_{Ap} + SL_{Ap} + \sum_{\substack{j \in J_B \\ p \leq j \leq m}} T_{Bj} - \sum_{\substack{j \in J_A \\ p \leq j \leq s}} T_{Aj} \geq 0 \tag{30}$$

where *p* represents the first stage shared by both products. The two first terms indicate that the B process production at stage *p* begins after the time $T_{Ap} + SL_{Ap}$.

Figure 5 shows this situation for the processing of two products, A and B, where the production path of A is $J_A = \{1, 2, 4\}$ and that of B is $J_B = \{1, 2, 3, 4\}$, that is, $l = 2$, $s = 4$, $m = 3$, and $p = 1$, so the equation that must be added is

$$T_{A1} + SL_{A1} + \sum_{\substack{j \in J_B \\ j \leq 3}} T_{Bj} - \sum_{\substack{j \in J_A \\ j \leq 4}} T_{Aj} \geq 0$$

Figure 6 shows another case where the sequence order is conserved, but the first processed product has more stages than the second one and $p \neq 1$. The production path for A is $J_A = \{1, 2, 3, 4\}$ and that for B is $J_B = \{2, 4\}$, that is, $l = 3$, $s = 4$, $m = 2$, and $p = 2$, and the constraint added is

$$T_{A2} + SL_{A2} + \sum_{\substack{j \in J_B \\ 2 \leq j \leq 2}} T_{Bj} - \sum_{\substack{j \in J_A \\ 2 \leq j \leq 4}} T_{Aj} \geq 0.$$

(b) If the designer chooses to change the production order at stage *s*, the constraint that must be added instead of eq 30 is

$$\sum_{\substack{j \in J_A \\ p \leq j \leq l}} T_{Aj} - (T_{Ap} + SL_{Ap} + \sum_{\substack{j \in J_B \\ j \leq s}} T_{Bj}) \geq 0 \tag{31}$$

In this way, because the processing times are variables, the distribution will be different. The designer can evaluate both solutions and choose the best economical solution as the optimal solution.

Figure 7 shows the scheduling production for two products that change the production order. A follows the production path $J_A = \{1, 2, 3, 4\}$, while for B, $J_B = \{1, 2, 4\}$, tht is, $l = 3$, *s*

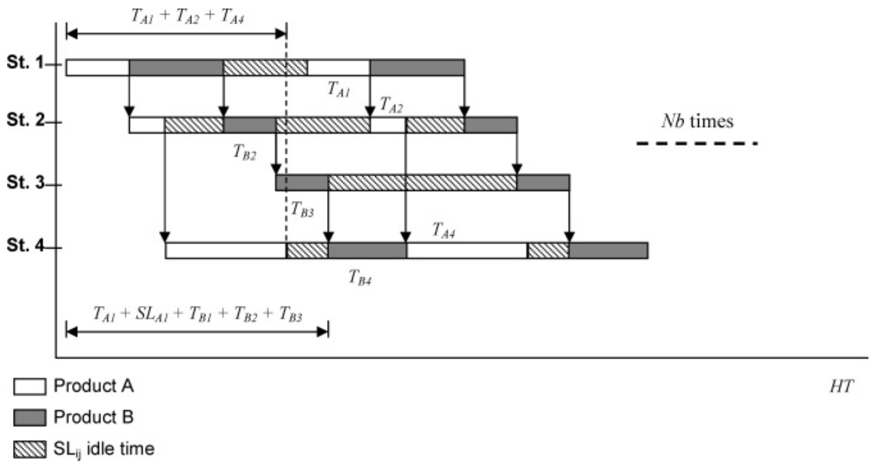

Figure 5. Gantt chart for case *ii*-a without changing the production sequence.

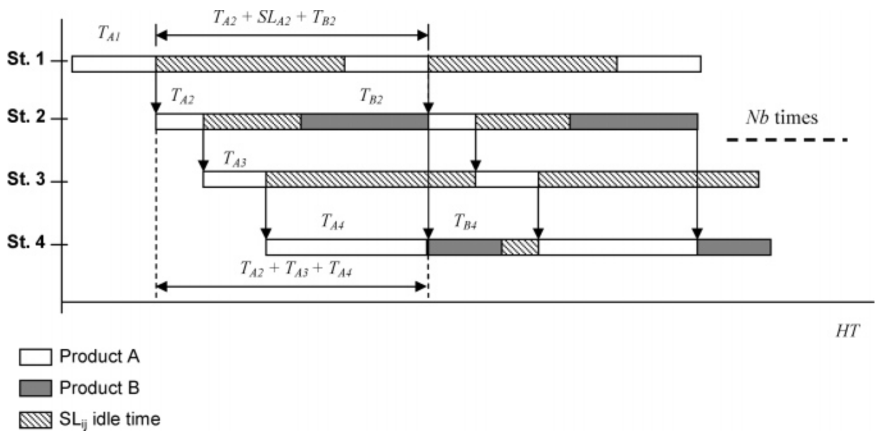

Figure 6. Gantt chart for case *ii*-a with $p \neq 1$.

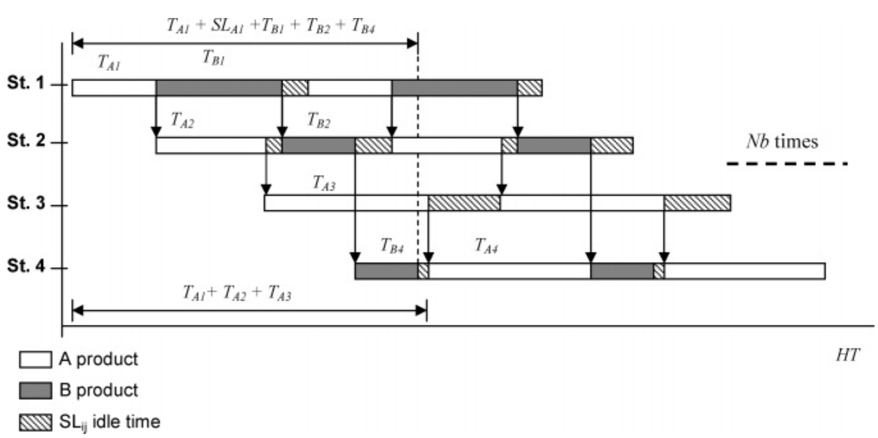

Figure 7. Gantt chart for case *ii-b* changing the production sequence.

$= 4$, $m = 2$, and $p = 1$. In stage 1 and 2, product A is processed before product B, while in stage 4, this order is changed.

If stage s is a semicontinuous unit, eqs 30 and 31 are valid in each case, with the reservation that T_{ij} represents the batch processing time with the loading and unloading times, and therefore, the operating times of the semicontinuous units must not be added because they are contemplated on T_{ij}.

For all the situations described for case *ii*, constraints 28 and 29 must be added for those products that follow the same path in two consecutive units. For example, for Figure 5 the following constraint must be added

$$\mathrm{SL}_{\mathrm{A1}} + T_{\mathrm{B1}} = T_{\mathrm{A2}} + \mathrm{SL}_{\mathrm{A2}}$$

Case *iii*. If two consecutive products A and B in the production campaign are such that A follows the path $s \rightarrow l$ and B the path $s \rightarrow m$, no constraints are added because they follow independent paths.

All the constraints considered in the different cases are added to the previous relaxed model where only the optimal plant configuration is active, that is, the plant structure is fixed.

5. Study Case, Sequential Multipurpose Plant: Torula Yeast, Brandy, and Bakery Yeast Production Integrated to a Sugar Plant

The integration of several processes into a sugar complex is considered. The sugar plant produces sugar and bagasse for sale,

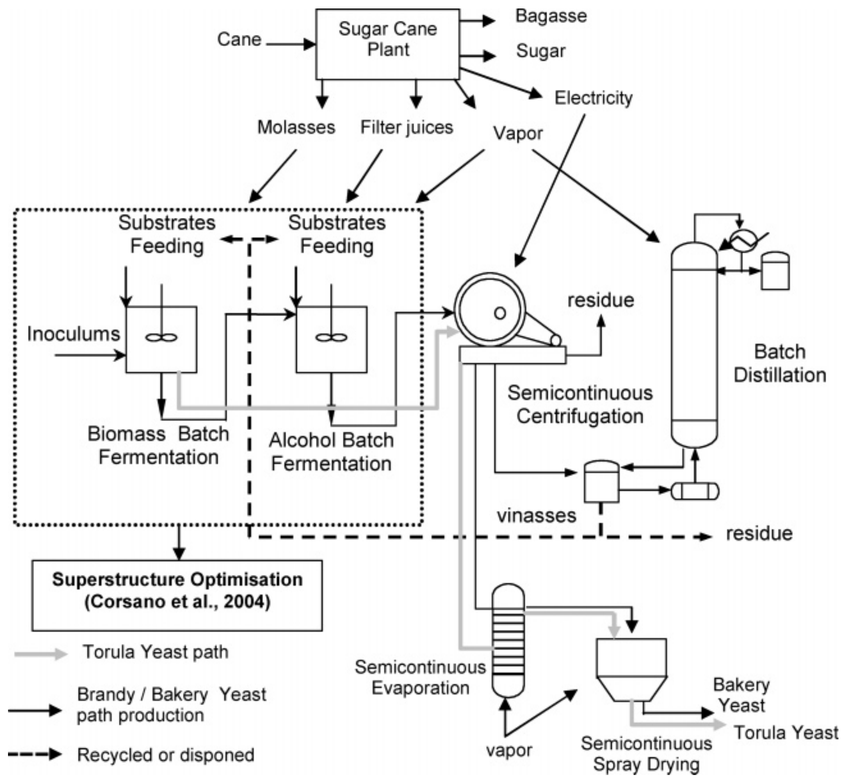

Figure 8. Flowsheet for sugar cane complex integration.

Table 1. Decision Variables Description

synthesis	design	operation	scheduling
plant configuration	unit sizes	batch blending, batch splitting, and batch recycling flow rates within the same production process	cycle time of each production process
number of units in series	heating and cooling areas	flow rate recycles from one process to another	units cycle time
blend and recycle allocation	power consumption (vapor and electricity)	material and energy resources allocation from mother plant to the different production processes	units idle time
	stage number of distillation column	production rates	number of batches
		component concentrations	mixed campaign configuration
		unit processing times for each product	

and molasses, filter juices, vapor, and electricity that are used in the derivatives plant. The derivatives plant is a sequential multipurpose plant with batch and semicontinuous units to produce Torula yeast, brandy, and bakery yeast. The bakery yeast is a subproduct of brandy production that is obtained by the evaporation and drying of the centrifugation residue of this process. Figure 8 shows the integration scheme.

The molasses and filter juices produced in the sugar plant serve as sugaring substrates for the biomass and alcohol fermentations. In addition, water and vinasses are added to the fermentation feed. The vinasses are a non-distilled waste of brandy production. The electricity generated in the sugar plant is used in the centrifuge of the derivatives plant, whereas the fermentors, the evaporator, the spray dryer, and the distillation column consume the steam. In addition, if it is necessary, steam can be imported from other power stations with operative cost imputed on the total annual cost. The vapor and the electricity that are not consumed by the derivatives plant can be sold. For the fermentation stages, the superstructure optimization model proposed by Corsano et al.[10] integrated to the overall model is adopted. For a detailed model of the brandy process model, see Corsano et al.[12] Therefore, a synthesis, design, operation, and scheduling problem is solved for the sequential multipurpose plant integrated to the sugar plant as an NLP model.

For the sugar plant, the model optimizes the amount of extracted filter juices. The sugar plant is considered as an existing mother plant, and the amount of extracted filter juices is a process variable. The production of sugar, molasses, vapor, and electricity depends on the amount of filter juice extraction. If more filter juice is extracted, the molasses and sugar productions are diminished. The consumption of vapor and electricity in the sugar production process is also decreased, and therefore, the amount of electricity and vapor available for derivatives and the bagasse for sale are increased.

Table 1 lists the most important synthesis, design, operating, and scheduling decisions considered in this study case to have a brief view of the model complexity, but in the results, only

Table 2. Optimal Variables for Sequential Multipurpose Plant Relaxed Model

variable	description	optimal value
Q_T	Torula production rate (ton h^{-1})	1.87
Q_{BY}	bakery yeast production rate (ton h^{-1})	1.36
Q_B	brandy production rate (ton h^{-1})	5.00
Nb_T	Torula batches	281
Nb_B	brandy and bakery yeast batches	305
CT_T	cycle time for Torula production (h)	16.0
CT_B	cycle time for brandy production (h)	24.5
CT_{BY}	cycle time for bakery yeast production (h)	11.9
NAP	net annual profit (\$ h^{-1})	6903.

some of these variables are reported since the objective of this paper is to focus the heuristic approach.

The models were implemented and solved in GAMS[13] in a Pentium IV, 1.60 GHz. The code CONOPT2 was employed for solving the NLP problems.

First the relaxed model is solved. Table 2 shows the description and optimal values for some optimization variables. Minimum and maximum production rates were fixed for each product ($Q_i^{min} = 1$ ton h^{-1} and $Q_i^{max} = 5$ t h^{-1}). Figure 9 shows the optimal plant synthesis. The fermentation stage configuration consists of one biomass fermentor and two alcohol fermentors in series. The fermentors size is upper bounded by 750 m^3.

As can be observed in Table 2, the number of Torula batches is 281, while for brandy and bakery Yeast, the number is is 305. So, for the mixed campaign model, the campaign Brandy−Torula (B−T) is proposed, that means, one batch of each product. It is a reasonable campaign in the sense that the vinasses produced upon brandy production would be used in the Torula fermentation. The vinasses cannot be stored for long periods of time because of degradation and inventory considerations, so campaign B−T seems a good option.

In addition to this campaign, other alternatives can be assessed via the addition of the corresponding constraints to the relaxed problem, as shown below.

According to the proposed methodology, the mixed product campaign model adopts the plant configuration obtained in the relaxed model optimal solution.

Figure 10 shows the Gantt chart for the relaxed model solution.

5.1. B−T Sequence Campaign for Fermentation Stage and T−B for Semicontinuous Stages (B−T/T−B). The mixed campaign model consists of the relaxed model plus the corresponding constraints to the B−T campaign for the plant synthesis obtained in the relaxed model solution. The sizing problem is solved in this stage. Because brandy production uses alcohol fermentors that Torula production does not employ, the sequence campaign at the semicontinuous subtrain (centrifuge, evaporator, and dryer) is changed. This campaign is denoted by B−T/T−B. Then, the following constraints are added to the relaxed model with the synthesis options fixed. Campaign definition says that the number of batches of B and T must be the same

$$Nb = Nb_B = Nb_T$$

To avoid task overlapping at the semicontinuous train

$$(T_{B,ferbio} + T_{B,fer\_al1} + T_{B,fer\_al2} - T_{B,cen}) - (T_{B,ferbio} + SL_{B,ferbio} + T_{T,ferbio}) \geq 0$$

The subscripts B and T refer to the brandy and Torula production respectively, while ferbio, fer_al1 and fer_al2 represent the biomass fermentor and alcohol fermentors 1 and 2, respectively. The first term in parentheses represents the time needed to process the brandy batch up to the alcohol fermentation second unit, while the second parenthesis represents the time when the Torula batch finished processing on the centrifuge. T_{B,fer_al2} and $T_{T,ferbio}$ consider the centrifuge loading time; so in the first case, $T_{B,cen}$ must be subtracted. The units that share both productions are the biomass fermentor and the semicontinuous subtrain; therefore, no more constraints are added.

The results for some optimal variables are presented in Table 3, and the production schedule is displayed in a Gantt chart in Figure 11. The optimal design variables are displayed in Table 7.

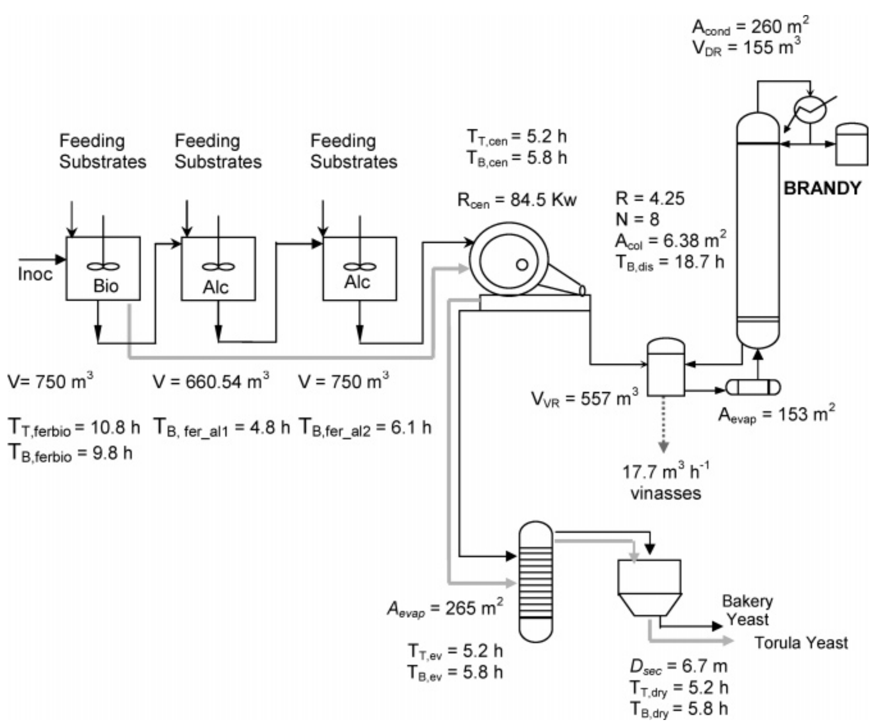

Figure 9. Optimal flowsheet for relaxed model plant design.

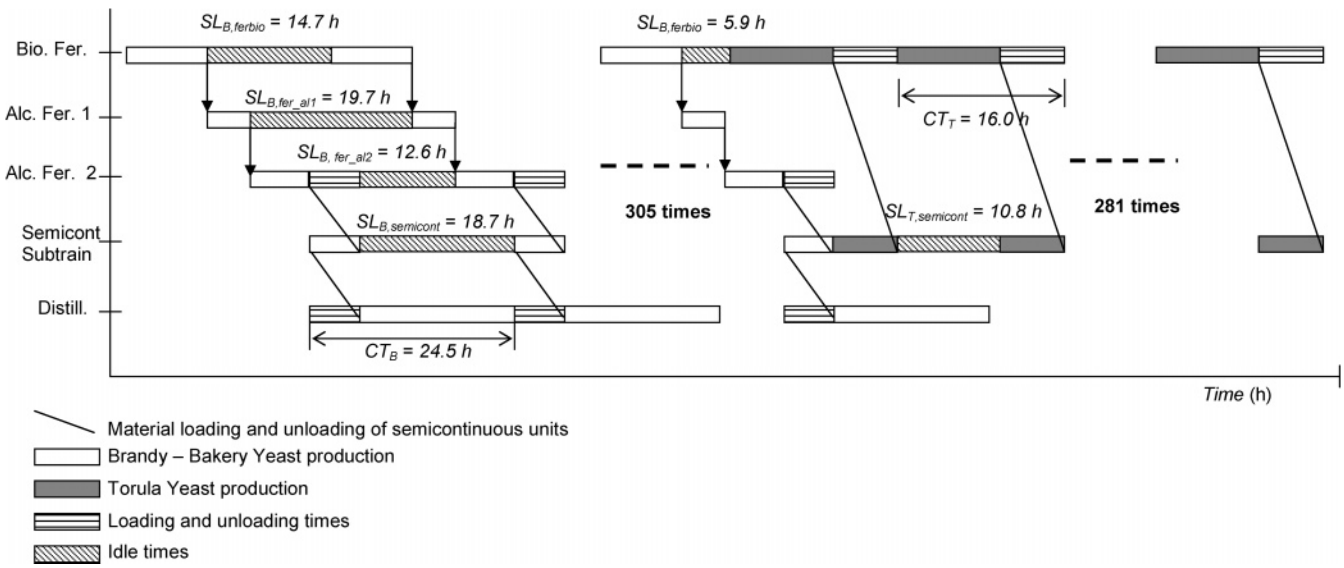

Figure 10. Gantt chart for Torula yeast, bakery yeast, and brandy production relaxed model.

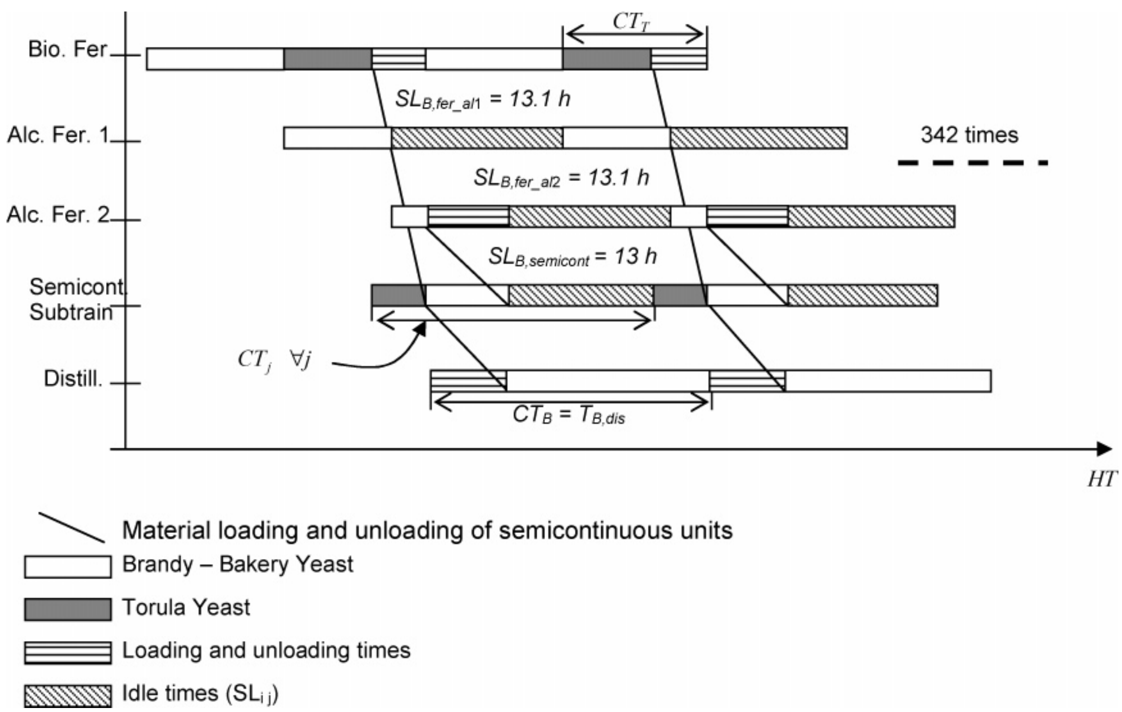

Figure 11. Gantt chart for B−T/T−B mixed product campaign.

Table 3. Optimal Variables for the B−T/T−B Model

variable	description	optimal value
Q_T	Torula production rate (ton h^{-1})	2.2
Q_{BY}	bakery yeast production rate (ton h^{--1})	1.38
Q_B	brandy production (ton h^{-1})	5.00
Nb	no. of times that the mixed campaign is repeated	342
CT_T	cycle time for Torula production (h)	12.4
CT_B	cycle time for brandy production (h)	22.0
CT_{BY}	cycle time for bakery yeast production (h)	9.6
NAP	net annual profit ($ h^{-1})	6890.

A novel result of this sequential multipurpose plant model with the mixed-product campaign is the fact that because some units are not used by all products, the operating times of such stages are larger, and therefore, the operating and investment costs of these stages are reduced. This means that a better use of equipment is achieved, as occurs at the distillation stage. This would not happen if the process adopts a single-product campaign, which is usually the case.

The objective functions of relaxed and mixed-product campaign models are not comparable because of the fulfillment of constraints 9 and 10 in the relaxed model do not imply that the total production of Torula yeast, brandy, and bakery yeast hold in the horizon time. In this case, for the relaxed model solution

$$CT_B\,Nb_B + CT_T\,Nb_T = 24.5 \times 305 + 16 \times 281 = 11968.5 \geq 7500 = HT$$

However, the total production of Torula yeast, brandy, and bakery yeast is completed in the mixed product campaign model in 7500 h (HT) because, for all the stages, the cycle time is 22 h and the campaign is repeated 342 times.

Table 4 shows the processing and idle times.

5.2. B−T Sequence Campaign for All the Stages. In this example, a change is introduced with respect to the previous one. In the semicontinuous subtrain the same sequence as in the other stages is employed.

Table 4. Processing and Idle Times for the B−T/T−B Campaign

	brandy		Torula	
	processing time (h)	idle time (h)	processing time (h)	idle time (h)
biomass fermentor	9.6	0	8.7[a]	0
alcohol fermentor 1	8.9	13.1		
alcohol fermentor 2	3.6[a]	13.1		
semicontinuous subtrain	5.3	13.0	3.7	0
distillation	16.7[a]	0		

[a] Without consideration of the loading and unloading times

Table 5. Processing and Idle Times for the B−T Campaign

	brandy		Torula	
	processing time (h)	idle time (h)	processing time (h)	idle time (h)
biomass fermentor	8.9	1.1	10.8[a]	0
alcohol fermentor 1	5.9	17.3		
alcohol fermentor 2	3.6[a]	17.2		
semicontinuous subtrain	2.4	0	2.4	18.4
distillation	20.8[a]	0		

[a] Without consideration of the loading and unloading times.

For this campaign the following constraints are added to the relaxed model: The campaign definition is

$$\mathrm{Nb} = \mathrm{Nb}_{\mathrm{B}} = \mathrm{Nb}_{\mathrm{T}}$$

where Nb_{B} and Nb_{T} are variables.

To avoid task overlapping at the semicontinuous train

$$(T_{\mathrm{B,ferbio}} + \mathrm{SL}_{\mathrm{B,ferbio}} + T_{\mathrm{T,ferbio}} - T_{\mathrm{T,cen}}) - (T_{\mathrm{B,ferbio}} + T_{\mathrm{B,fer\_al1}} + T_{\mathrm{B,fer\_al2}}) \geq 0$$

The first term in parentheses represents the time when the brandy batch arrives to the centrifuge, while the second one is the time when the Torula batch leaves the centrifuge.

The model is solved for the same production rates as the B−T/T−B campaign to establish a comparison. The optimal solution for this sequence campaign increases the investment cost by 7% and the idle times by 30% for the same production rate. Table 5 shows the processing and idle times, and Figure 12 shows the Gantt chart for the B−T campaign. The optimal design variables values are displayed in Table 7 and the economical results in Table 8.

As can be seen from comparing Tables 4 and 5, the semicontinuous subtrain processing time is shorter than that in the previous case, so the equipment sizes are increased and the investment cost is increased too. The negative terms in Table 8 mean that there is unused electricity that is sold to increase the profit.

5.3. B−B−T Sequence Campaign for All the Stages. This mixed product campaign does not adjust to the ratio between the number of batches of Torula and brandy products previously determined. However, it is solved to assess the effectiveness of the proposed strategy.

For this campaign, two consecutive batches of brandy are processed, and one Torula batch is processed. Therefore, superposition of tasks must be avoided in the alcohol fermentors and distillation stage between the two brandy batches and in the semicontinuous subtrain between the brandy and Torula batches. The following constraints are added to the relaxed model with the plant configuration fixed to the optimal configuration obtained in the relaxed model solution:

To define the campaign

$$\mathrm{Nb} = \mathrm{Nb}_{\mathrm{T}}$$

$$\mathrm{Nb} = {}^{1}/_{2}\mathrm{Nb}_{\mathrm{B}}$$

where Nb_{B} and Nb_{T} are variables. To avoid the task overlapping at the first alcohol fermentation stage of the brandy

$$T_{\mathrm{B,ferbio}} + \mathrm{SL}_{\mathrm{B1,ferbio}} = T_{\mathrm{B,fer\_al1}} + \mathrm{SL}_{\mathrm{B1,fer\_al1}}$$

To avoid the task overlapping at the second alcohol fermentation stage of the brandy

$$T_{\mathrm{B,fer\_al1}} + \mathrm{SL}_{\mathrm{B1,fer\_al1}} = T_{\mathrm{B,fer\_al2}} + \mathrm{SL}_{\mathrm{B1,fer\_al2}} \qquad (32)$$

To avoid the task overlapping at the distillation stage of brandy

$$T_{\mathrm{B,fer\_al2}} + \mathrm{SL}_{\mathrm{B1,fer\_al2}} = T_{\mathrm{B,dis}} + \mathrm{SL}_{\mathrm{B,dis}}$$

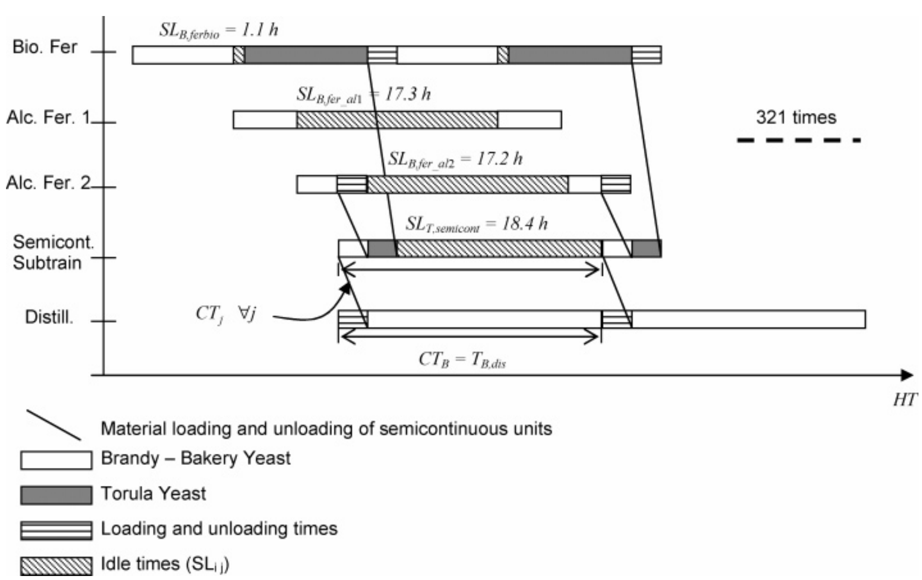

Figure 12. Gantt chart for B−T mixed product campaign.

Table 6. Processing and Idle Times for the B−B−T Campaign

	Torula		brandy		
	processing time (h)	idle time (h)	processing time (h)	idle time 1 (h)[a]	idle time 2 (h)[b]
biomass fermentor	10.2	0	5.6	0	0
alcohol fermentor 1			5.1	0.5	13.3
alcohol fermentor 2			3.7	0.5	13.3
semicontinuous subtrain	2.6	13.8	1.4	4.2	0.6
distillation			4.2	0	12.8

[a] Idle time after first batch. [b] Idle time after second batch.

Table 7. Optimal Design Variables for Sequential Multipurpose Plant Models

unit	B−T/T−B campaign	B−T campaign	B−B−T campaign
biomass fermentor (m^3)	750	750	750
alcohol fermentor 1 (m^3)	631.8	655.7	344.4
alcohol fermentor 2 (m^3)	724.8	750	391.5
centrifuge (Kwh)	118.3	184.5	172
evaporator (m^2)	389	571.4	539.61
dryer (m of diameter)	7.2	13.	11.3
distillation			
condenser area (m^2)	305.6	272.3	715.7
evaporator area (m^2)	180.8	161.1	423.4
stages number	9	10	9
reflux ratio	5.18	5.5	5.66
transversal column area (m^2)	7.53	6.71	17.64

To avoid the task overlapping between the second brandy batch and the Torula batch in the semicontinuous train

$$(2T_{B,ferbio} + SL_{B1,ferbio} + SL_{B2,ferbio} + T_{T,ferbio} - T_{T,cen}) - (2T_{B,ferbio} + SL_{B1,ferbio} + T_{B,fer\_al1} + T_{B,fer\_al2}) \geq 0$$

$SL_{B1,unit}$ and $SL_{B2,unit}$ represent the idle time after the first and second brandy batches, respectively, in the campaign at that unit.

It is not necessary to add constraints to avoid task overlapping between the two consecutive Brandy batches at the semicontinuous subtrain because the second alcohol fermentor considers the centrifugation time that represents the fermentor unloading time. Therefore, eq 32 ensures that there will not be task overlapping in the semicontinuous subtrain.

Table 8. Economical Comparison between Different Campaigns

	B−T/T−B campaign	B−T campaign	B−B−T campaign
investment costs ($ h^{-1})			
biomass fermentors	68.27	68.27	68.27
alcohol fermentors	29.22	29.69	22.19
centrifuge	20.62	27.94	26.61
evaporator	22.94	28.13	27.29
dryer	58.26	76.9	71.98
distillation	117.11	117.56	147.35
operating costs ($ h^{-1})			
inoculums	78.11	65.04	150
water (fresh and cooling)	31.33	33.23	33.56
vinasses disposal	16.65	17.34	25.45
profit for sale of unused electricity ($ h^{-1})	−10.70	− 4.4	− 5.74
total ($ h^{-1})	431.81	459.71	566.97

For the B−B−T campaign, the optimal solution is worse by about a 31% than the objective function obtained in section 5.1 for the same production rate. Table 6 shows the processing and idle times for this campaign. As can be observed, the distillation processing time is reduced more than three times, so the investment cost is also increased (Table 8). The unit sizes for this campaign can be observed in Table 7. The total idle time is increased by 26%. Figure 13 shows the Gantt chart for this example.

Because the earnings for sales are the same in all cases since the production rates are fixed to the optimal value obtained in example 5.1, Table 8 compares the investment and operation costs of the three campaigns. In all cases, there is unused electricity that is considered to be a benefit.

Another disadvantage of this campaign is that the biomass fermentor is suboccupied in brandy production (the brandy batch size in biomass fermentor is 308.64 m^3, while the biomass fermentor size is 750 m^3 because it reaches this value for Torula production). This occurs because the same production rate that first was reached in a batch now is reached in two batches.

Only the alcohol fermentation stage is cheaper in this case because the unit sizes are smaller. This occurs because each brandy batch size is smaller since two batches are processed.

Table 7 shows the unit sizes for the different studied examples.

Table 9 presents the solution times and the number of variables and constraints of each studied case. The reduction of the variables and constraint number in mixed-product

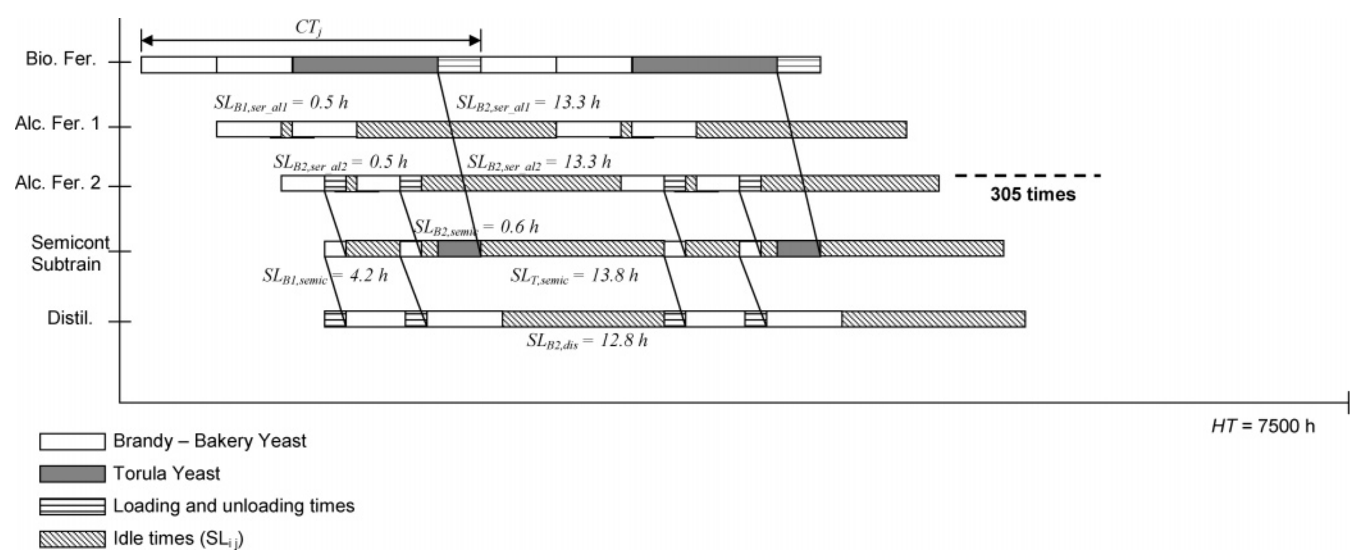

Figure 13. Gantt chart for the B−B−T mixed product campaign.

Table 9. Studied Cases: Computational Results

	relaxed model	B−T/T−B campaign	B−T campaign	B−B−T campaign
no. of variables	2398	761	761	772
no. of constraints	2244	704	705	729
CPU (s)	161.4	42.9	60.6	69.9

campaign models is because in those models the plant configuration is fixed, and therefore the superstructure model is not considered.

6. Conclusions

In this paper, a detailed model for the optimal synthesis, design, operation, and scheduling of a sequential multipurpose noncontinuous plant was developed. The resolution strategy considers two steps. First, a relaxed model is solved to obtain the number of batches of each product and the plant synthesis. Then the designer chooses the mixed campaign on the basis of the ratio between the number of batches. A novel set of task-scheduling constraints is proposed to avoid task superposition at the processing units.

The problem was formulated as an NLP model, and a superstructure model was embedded to solve the synthesis problem. There are not previous published works dealing with the synthesis, design, and scheduling problem simultaneously solved as an NLP model.

Another characteristic of these models is the high level of detail reached in the processing unit description, some of them by means of ordinary differential equations. Batch blending, batch splitting, and recycles are allowed as novel components for the multipurpose plant model, decisions taken in this work as optimization variables.

The model was implemented for a Torula yeast, brandy, and bakery yeast production plant. The model was formulated and solved according to the proposed strategy. The optimal solution was compared with different campaign sequences. In all cases, the investment and operative costs and the idle times were increased. Economical and synthesis, design, and operational results are also reported. The gap between the objective function value of the relaxed model solution and the objective value of the mixed-product campaign model is very tight, which ensures that the solution obtained for the mixed-product campaign is optimal, and it serves to verify the proposed heuristic methodology.

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