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Robust MPC suitable for closed-loop re-identification, based on probabilistic invariant sets

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ABSTRACT

This work extends a recent set-based Model Predictive Control (MPC) scheme for closed loop re-identification that solves the potential conflict between the simultaneous persistent excitation of the system and the stabilization of the closed-loop system.

Based on the original scheme proposed in González et al. (2014), this manuscript extends those results by taking into account model uncertainties and by exploiting the knowledge of the probability distribution of the excitation signal used to identify the plant.

The robust extension solves the main drawback of the previous work, which was limited to a nominal analysis while the need of re-identification assumes the presence of model uncertainties. In addition, the probabilistic analysis allows the use of smaller target sets computed as *Probabilistic Invariant Sets* (PIS), improving the system performance during the identification procedure.

Simulation results show the practical benefits of the novel robust strategy.

1. Introduction

Model predictive control (MPC) is a popular control technique that mainly bases its performance on a simplified model of the plant. In most applications, this model requires a periodic update (re-identification) that must be performed in a closed-loop fashion in order to not to stop the process under control. The main problem of the closed-loop model update is that the control objectives are opposite to those of exciting the system for identification. From the controller point of view, the persistent excitation required by any identification procedure is a disturbance that should be rejected. On the other hand, from the identification point of view, the control actions produce undesirable correlations between the signals used to obtain the model parameters. Several strategies were developed to perform closed-loop re-identification under MPC controllers. The interested reader may refer to [1,2], where the main problem of a closed-loop identification is explained and studied, i.e. the dynamic control objectives are incompatible with the identification objectives, or [3,4] in which a method is proposed to handle these opposite objectives, and how to perform the re-identification in a closed-loop system [5–7].

The main theoretical drawback of all these schemes is the lack of formal feasibility and attractivity/stability analysis. In [8], a MPC scheme suitable for re-identification is proposed, which overcomes this drawback and ensures recursive feasibility and stability, performing at the same time a safe closed-loop re-identification. The basic idea was to design a MPC that steers the system states to the interior of a (target) invariant set while they are outside that set, and once the states reach the set, to persistently excite the system. The MPC problem formulation uses the concept of generalized distance from a point (the state and input trajectory) to a set (target invariant set and input excitation set). So, the two tasks of convergence and excitation can be spatially separated in the state space. The method was also tested in a polymerization reactor in [9], where a proper form to compute invariant sets for uncertain systems is presented.

The method proposed in [8], however, is only developed for the nominal case (although the terminal set can be robustly computed), which is a strong drawback given that re-identification scenarios are precisely given when the prediction model is no longer valid. To this aim, and as a first contribution of this work, we propose a robust formulation of the controller presented in [8], which is based on the well-known tube-based MPC formulation [10]. We select this approach since [10] shows that this form of robust MPC

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has the same order of online complexity as the one shown by conventional MPC for deterministic linear systems, quadratic cost and additive uncertainties, among other advantages. To do that, the parametric-uncertainty of the model is properly converted into additive uncertainties. Although we use the tube-based MPC formulation to treat the uncertainties of the system, there are several ways to extend the nominal MPC of the previous work to the robust case, we can refer for example the following significant works [11–15], of robust MPCs where the approach is different but equally applicable.

Another drawback of [8] is that the strategy computes the invariant sets according to the maximum value that the excitation signals can take, without exploiting the knowledge of their probabilistic distribution. This fact results in large target regions that conservatively contain the excited system evolution. This way, only large disturbances that take the system trajectories outside these unnecessary large sets are rejected by the controller, implying that the scheme achieves a relatively poor performance.

With the goal of computing invariant sets that take into account the probability distributions of the disturbance signals, [16] introduces the concept of *Probabilistic Invariant Sets* (PIS). PIS are sets where the state trajectories remain most of the time with a given probability (close to 1 for most practical problems). That way, PIS result significantly smaller than classic invariant sets and appear as a promising alternative for obtaining less conservative (smaller) invariant target regions for the robust MPC scheme proposed here.

The contribution of this work is then twofold: (i) it extends the Model Predictive Control suitable for re-identification presented in [8] to the parametric-uncertainty robust case; and (ii) it properly includes the novel concepts of Probabilistic Invariant Sets [16] in order to obtain a less conservative robust formulation. Besides, it is shown that – with a given probability – the system remains inside the target set during a period of time that is long enough to ensure a proper re-identification of the model, even when model uncertainty is considered. All these advantages, and a comparison with the deterministic formulation [8], are illustrated by means of numerical simulations.

Notation: The natural set is defined by $\mathbb{N} = \{1, 2, 3, \ldots\}$. A C-set is a convex and compact set that contains the origin. A proper C-set is a C-set that contains the origin as an interior point. Consider two sets $\mathcal{U} \subseteq \mathbb{R}^n$ and $\mathcal{V} \subseteq \mathbb{R}^n$, containing the origin, and a real number λ. The Minkowski sum $\mathcal{U} \oplus \mathcal{V} \subseteq \mathbb{R}^n$ is defined by $\mathcal{U} \oplus \mathcal{V} = \{(u + v) : u \in \mathcal{U}, v \in \mathcal{V}\}$; and the set $\lambda\mathcal{U} = \{\lambda u : u \in \mathcal{U}\}$ is a scaled set of $\mathcal{U}$. Furthermore, $\mathcal{U} \ominus \mathcal{V} \triangleq \{x : x + v \in \mathcal{U},\ \textit{for all}\ v \in \mathcal{V}\}$. Given $x, y \in \mathbb{R}^n$, $\|x - y\|_M^2 = (x - y)^T M(x - y)$, with $M \in \mathbb{R}^{n\times n}$ positive definite, while $\|x - y\| = \sqrt{(x - y)^T(x - y)}$. The open ball with center in $x \in \mathbb{R}^n$ and ratio $\varepsilon > 0$ is defined as $\mathcal{B}_\varepsilon(x) \triangleq \{y \in \mathbb{R}^n : \|x - y\| < \varepsilon\}$. x is an interior point of $\mathcal{V}$ if the there exist $\varepsilon > 0$ such that $\mathcal{B}_\varepsilon(x) \subset \mathcal{V}$. The interior of $\mathcal{V}$ is the set of all its interior points and it is denoted by $\mathcal{V}^\circ$. If $\mathcal{V}$ is closed, the boundary of $\mathcal{V}$ is denoted by $\partial\mathcal{V}$, and it is defined as $\partial\mathcal{V} \triangleq \mathcal{V} \setminus \mathcal{V}^\circ$. Let $\mathcal{V}$ be a proper C-set on $\mathbb{R}^n$. The distance from $x \in \mathbb{R}^n$ to $\mathcal{V}$ is defined as $d_\mathcal{V}(x) \triangleq \inf\{\|x - \bar{x}\|_M^2 : \bar{x} \in \mathcal{V}\}$. $d_\mathcal{V}(x)$ is a convex and continuous function, and $d_\mathcal{V}(x) \geq 0$ for all $x \in \mathbb{R}^n$, while $d_\mathcal{V}(x) = 0$ if and only if $x \in \mathcal{V}$.

2. Problem statement

We assume in this work a general control structure in which a non-robust (nominal model based) MPC is used to control a plant. Then, when there is a suspicion that the prediction model used by this MPC is significantly deteriorated, this nominal MPC is replaced by a MPC suitable for re-identification, and the excitation and data collecting procedure is made, to obtain a new (more accurate) model. Once the new model is obtained, the nominal MPC resumes the control of the plant. The idea behind this strategy is that a nominal MPC with an accurate model is the best option to control a plant, in comparison to robust MPC strategies (a lot of studies were made regarding the intrinsic high conservativeness and computational expense of most robust MPC's, [17,18]).

Regarding the MPC suitable for closed-loop re-identification of the general control structure, the goal of this note is to develop a **robust** MPC strategy that allows for a safe closed-loop re-identification of a whole family of linear systems that we will present in (1). This controller spatially divides the whole control/identification task in two parts: outside a given target set (TS), it robustly steers the uncertain system to it and, inside, it performs a system excitation that allows for a proper and safe identification. In this context, the TS must be understood in its dual role of a control objective set (when the system input is a manipulated control action) and that of excitation set (when the system input is a non-manipulated excitation signal). In the first role, the TS must be robust invariant for the parametric uncertainty described in the next subsection (to account for robust stability) while, in the second role, the TS must be not only robust invariant for the parametric uncertainty, but also invariant to the persistent excitation signal necessary for the identification, for the system to remain inside the TS set during the excitation.

2.1. Model description

Consider a family of discrete-time systems described by

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k, \quad \theta \in \Theta, \tag{1}$$

where $x_k \in \mathcal{X} \subset \mathbb{R}^n$ is the system state at the kth sample time, $u_k \in \mathcal{U} \subset \mathbb{R}^m$ is the current control input, $A(\theta)$ and $B(\theta)$ are Lipschitz functions of $\theta \in \Theta \subset \mathbb{R}^l$, and θ is an unknown fixed parameter that accounts for a constant plant-model mismatch. It is assumed that the nominal model is represented by $(\bar{A}, \bar{B})$, where $\bar{A} \triangleq A(0)$, and $\bar{B} \triangleq B(0)$. It is also assumed that set $\mathcal{X}$ is convex, closed and robust control invariant (see the definition below), and contains the origin; set $\mathcal{U}$ is convex and compact, and contains the origin; and set Θ is compact and contains the origin. Furthermore, $A(\theta)$ has all its eigenvalues strictly inside the unit circle, and the pair $(A(\theta), B(\theta))$ is assumed to be controllable, for all $\theta \in \Theta$.

It is assumed that the set Θ is convex. Thus, the set of possible models is given by

$$\mathcal{M} = co\{(A(\theta), B(\theta)) : \forall\, \theta \in \Theta\}.$$

Hence, the pair $(A(\theta), B(\theta))$ can take, at any time, any value in the convex set $\mathcal{M}$. Set $\mathcal{M}$ represents the family of possible models.

2.2. Control scheme

In the next section, the design of the Robust MPC controller suitable for closed-loop re-identification will be addressed. The general formulation under which the proposal will be developed is based on a cost function of the form:

$$V_N(x, \Omega; \mathbf{u}) = \sum_{j=0}^{N-1}[\alpha d_\Omega(x_j) + \beta d_\Pi(u_j)], \tag{2}$$

where, as usual in MPC literature, $\mathbf{u} = \{u_0, \ldots, u_{N-1}\}$ is a control input sequence, x_j and u_j represent the state and input prediction along a horizon N, respectively, $x = x_0$ is the current state, and Ω is a target set in the state space, with an associated input set Π. α and β are penalization constants. Furthermore, the general MPC optimization problem is given by:

$$V_N^*(x, \Omega) = \min_{\mathbf{u}}\{V_N(x, \Omega; \mathbf{u}) : \mathbf{u} \in \mathbb{U}_N(x, \Omega)\},$$

where the feasible set $\mathbb{U}_N(x, \Omega)$ is a general representation – depending on current state x and on the set Ω – that includes the state, input and terminal constraints. According to the robust and probabilistic properties of the proposed MPC, this general formulation will take different forms, to account for a safe excitation/identification procedure.

3. Robust MPC for closed-loop re-identification

3.1. Preliminaries

In this section, the MPC proposed in [8] is extended to the robust case, according to the tube-based approach strategy presented in [10]. To account for an additive uncertainty representation, which also covers the parametric one presented in (1),[1] the following model is considered:

$$x_{k+1} = \bar{A}x_k + \bar{B}u_k + w_k, \quad w_k \in \mathcal{W}, \tag{3}$$

with

$$\mathcal{W} = \{(A(\theta) - \bar{A})x + (B(\theta) - \bar{B})u : \theta \in \Theta, x \in \mathcal{X}, u \in \mathcal{U}\}. \tag{4}$$

Notice that $\mathcal{W} \subseteq \mathbb{R}^n$ is a bounded set containing the origin in its interior and, furthermore, if $w_k = (A(\theta) - \bar{A})x_k + (B(\theta) - \bar{B})u_k$, then $w_k \in \mathcal{W}$, and the family of models (3) contains (maybe conservatively) the entire family (1) [19].

Remark 1. Representing parametric uncertainties as additive uncertainties may be a conservative solution which may result in a conservative set $\mathcal{W}$ as the one defined in (4). However it is a practical way of treating parametric uncertainties, and it allows one to apply well known robust MPC strategies like the tube-MPC [19, Chapter 3]. Indeed, if the original system is perturbed only by additive uncertainties, the model of (3) can represent the family of models without conservatism.

The nominal system corresponding to (3) is given by

$$\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}u_k, \tag{5}$$

and so, the error $e_k = x_k - \bar{x}_k$ has the following uncertain dynamic: $e_{k+1} = \bar{A}e_k + w_k$. Based on this error system, the so-called tubes are introduced:

Proposition 1. *Suppose that Z is a robust invariant set*[2] *for $e_{k+1} = \bar{A}e_k + w_k$. If $x_k \in \bar{x}_k \oplus Z$, then $x_{k+1} \in \bar{x}_{k+1} \oplus Z$ for all $w_k \in \mathcal{W}$, where $x_{k+1} = \bar{A}x_k + \bar{B}u_k + w_k$ and $\bar{x}_{k+1} = \bar{A}\bar{x}_k + \bar{B}u_k$.*

Before introducing the target set of the MPC formulation, the definition of robust invariant set for system (1) is given:

Definition 1 (*Robust Control Invariant Set, RCIS*)**.** A proper C-set $\Omega \subseteq \mathcal{X}$ is a robust control invariant set for system (1) if $x \in \Omega$ implies that there exists $u \in \mathcal{U}$ such that $A(\theta)x + B(\theta)u \in \Omega$ for all $\theta \in \Theta$. The corresponding input set associated to Ω, is given by $\Pi \triangleq \{u \in \mathcal{U} : \exists x \in \Omega \text{ such that } A(\theta)x + B(\theta)u \in \Omega, \ \forall \theta \in \Theta\}$.

For the case of no uncertain systems (when $\Theta = \{0\}$, for instance), the latter set is named control invariant set (CIS).

Consider now a proper C-set $\Omega \subset \mathcal{X}$ (which will play the role of TS of the MPC controller), which is a RCIS for system (1), and let us define the following restricted sets:

$$\bar{\mathcal{X}} \triangleq \mathcal{X} \ominus Z, \quad \bar{\Omega} \triangleq \Omega \ominus Z. \tag{6}$$

[1] The parametric uncertainties of system (1) are transformed in additive uncertainties (using the ideas in [19, Chapter 3]) in order to treat them in an easier way.

[2] Z is a robust invariant set for $e_{k+1} = \bar{A}e_k + w_k$ if $e_k \in Z \Rightarrow e_{k+1} \in Z$, for all $w_k \in \mathcal{W}$.

It must be assumed that $Z \subset \mathcal{X}$ and $\Omega \subset Z$ (usual assumptions in this context), then the sets in (6) are not empty. The set $\bar{\Omega}$ is a control invariant set for the nominal system (5). To see that, note that Ω is in particular a control invariant set for system (5), since $0 \in \Theta$. Consider $x_k \in \bar{\Omega}$, which means that $x_k = y_k - z_k$ for some $y_k \in \Omega$, $z_k \in Z$, and let u_k be such that $\bar{A}y_k + \bar{B}u_k \in \Omega$. Then

$$\begin{aligned} x_{k+1} &= \bar{A}x_k + \bar{B}u_k \\ &= \bar{A}(y_k - z_k) + \bar{B}u_k \\ &= \bar{A}y_k + \bar{B}u_k - \bar{A}z_k \\ &\in \Omega \ominus Z = \bar{\Omega}. \end{aligned}$$

3.2. Robust MPC formulation

The idea in this subsection is to present a robust extension of the MPC formulation presented in [8], based on nominal predictions and on the restricted state and terminal constraint sets $\bar{\mathcal{X}}$ and $\bar{\Omega}$. Suppose that $\bar{\Pi}$ is the corresponding input set to $\bar{\Omega}$ (according to Definition 1). The proposed controller cost function (2), will be rewritten as:

$$V_N(x, \bar{\Omega}; \mathbf{u}) = \sum_{j=0}^{N-1} [\alpha d_{\bar{\Omega}}(\bar{x}_j) + \beta d_{\bar{\Pi}}(u_j)], \tag{7}$$

where $\mathbf{u} = \{u_0, \ldots, u_{N-1}\}$ is the sequence of control actions, and $\bar{x}_j$ is the nominal system state (i.e., $\bar{x}_{j+1} = \bar{A}\bar{x}_j + \bar{B}u_j$, with $\bar{x}_0 = x$). α and β are positive real numbers, and $N \in \mathbb{N}$ is the control horizon. In contrast to conventional MPC cost functions, $V_N(x, \bar{\Omega}; \cdot)$ can be zeroed at a given equilibrium point, but also at the whole set $\bar{\Omega}$, together with its corresponding input set $\bar{\Pi} \subset \mathcal{U}$.

For any state $x \in \bar{\mathcal{X}}_N \subset \bar{\mathcal{X}}$, where $\bar{\mathcal{X}}_N$ is the N-step controllable set to $\bar{\Omega}$ for the nominal system (i.e., the set of states that can be feasibly steered to $\bar{\Omega}$ in N steps, by means of the nominal system), the optimization problem $P_N(x, \bar{\Omega})$ to be solved at each time-step is given by:

$$\begin{aligned} V_N^0(x, \bar{\Omega}) = \min_{\mathbf{u}} \{ & V_N(x, \bar{\Omega}; \mathbf{u}) : \bar{x}_j \in \bar{\mathcal{X}}, \ u_j \in \mathcal{U}, \ \bar{x}_N \in \bar{\Omega}, \\ & \forall j \in \mathbb{I}_{0:N-1}\}, \end{aligned}$$

where $\mathbb{I}_{0:N-1} \triangleq \{0, 1, 2, \ldots, N-1\}$.

If no uncertainty is explicitly considered, from [8] we know the optimal solution to $P_N(x, \bar{\Omega})$ and the relative optimal control sequence $\mathbf{u}^0(x, \bar{\Omega}) \triangleq \{u_0^0(x, \bar{\Omega}), \ldots, u_{N-1}^0(x, \bar{\Omega})\}$, are such that

$$V_N^0(\bar{A}x + \bar{B}\kappa_N(x, \bar{\Omega}), \bar{\Omega}) - V_N^0(x, \bar{\Omega}) \leq -\alpha d_{\bar{\Omega}}(x). \tag{8}$$

where $\kappa_N(x, \bar{\Omega}) \triangleq u_0^0(x, \bar{\Omega})$, thus deriving asymptotic stability of set $\bar{\Omega}$.

If the uncertainty is explicitly considered, it is not necessarily true that, given any $x \in \bar{\mathcal{X}}_N \setminus (\bar{\Omega} \oplus Z)$, $V_N^0(\bar{A}x + \bar{B}\kappa_N(x, \bar{\Omega}) + w, \bar{\Omega})$ is smaller than $V_N^0(x, \bar{\Omega})$, for all $w \in \mathcal{W}$.

Thus, we need to establish the robust asymptotic stability of $\bar{\Omega} \oplus Z$, which, considering that $\bar{\Omega} \oplus Z = \Omega \ominus Z \oplus Z \subseteq \Omega$ [20], implies the robust asymptotic stability of the set Ω.

The proposed robust problem $P_N^{rob}(x, \bar{\Omega})$ is defined by

$$\begin{aligned} V_N^*(x, \bar{\Omega}) = \min_{\mathbf{u}, \bar{x}_0} \{ & V_N(\bar{x}_0, \bar{\Omega}; \mathbf{u}) : \bar{x}_j \in \bar{\mathcal{X}}, \ u_j \in \mathcal{U}, \ \bar{x}_N \in \bar{\Omega}, \\ & x \in \bar{x}_0 \oplus Z, \ \forall j \in \mathbb{I}_{0:N-1}\}. \end{aligned}$$

In problem $P_N^{rob}(x, \bar{\Omega})$, the function $V_N(x, \bar{\Omega}; \mathbf{u})$ and the state, input and terminal constraint are the same as the ones defined in problem $P_N(x, \bar{\Omega})$. However, to account for the desired robustness, an initial state constraint is necessary. Given that now x is an uncertain state, the initial state for nominal predictions, $\bar{x}_0$, is a new decision variable that must fulfill [10]:

$$x \in \bar{x}_0 \oplus Z.$$

The solution of $P_N^{rob}(x, \bar{\Omega})$ yields the optimal control sequence

$\mathbf{u}^*(x, \bar{\Omega}) \triangleq \{u_0^*(x, \bar{\Omega}), \dots, u_{N-1}^*(x, \bar{\Omega})\}$

and the associated optimal state sequence $\bar{\mathbf{x}}^*(x, \bar{\Omega}) \triangleq \{\bar{x}_0^*(x, \bar{\Omega}), \dots, \bar{x}_N^*(x, \bar{\Omega})\}$, where the optimal initial state $\bar{x}_0^*(x, \bar{\Omega})$ is not necessarily equal to the current state x of the controlled system. The (implicit) model predictive control law is, therefore,

$$\kappa_N^*(x, \bar{\Omega}) \triangleq u_0^*(x, \bar{\Omega}), \tag{9}$$

while the uncertain system – under the robust model predictive control – satisfies

$$x_{k+1} = \bar{A}x_k + \bar{B}\kappa_N^*(x_k, \bar{\Omega}) + w_k, \quad w_k \in \mathcal{W}. \tag{10}$$

3.3. Robust stability

The robust MPC control law $\kappa_N^*(x, \bar{\Omega})$ provides a non-increasing Lyapunov function along trajectories starting at any state $x \in \mathcal{X}_N = \{x : \exists \bar{x}_0 \in \bar{\mathcal{X}}_N \text{ such that } x \in \bar{x}_0 \oplus Z\}$, for all $w \in \mathcal{W}$, as it is shown in the next theorem:

Theorem 1. *Suppose $x \in \mathcal{X}_N$, so that $(\bar{x}_0^*(x, \bar{\Omega}), \mathbf{u}^*(x, \bar{\Omega}))$ exists and is optimal for $P_N^{rob}(x, \bar{\Omega})$, where $\mathbf{u}^*(x, \bar{\Omega}) = \{u_0^*(x, \bar{\Omega}), u_1^*(x, \bar{\Omega}), \dots, u_{N-1}^*(x, \bar{\Omega})\}$, and the associated trajectory (for the nominal system) is given by*
$\bar{\mathbf{x}}^*(x, \bar{\Omega}) = \{\bar{x}_0^*(x, \bar{\Omega}), \bar{x}_1^*(x, \bar{\Omega}), \dots, \bar{x}_N^*(x, \bar{\Omega})\}$. Therefore, for all $x^+ \in \bar{A}x + \bar{B}\kappa_N^*(x, \bar{\Omega}) \oplus \mathcal{W}$, $(\bar{x}_1^*(x, \bar{\Omega}), \tilde{\mathbf{u}}(x, \bar{\Omega}))$ is feasible for $P_N^{rob}(x^+, \bar{\Omega})$, with $\tilde{\mathbf{u}}(x, \bar{\Omega})$ defined by

$$\tilde{\mathbf{u}}(x, \bar{\Omega}) \triangleq \{u_1^*(x, \bar{\Omega}), \dots, u_{N-1}^*, \tilde{u}\},$$

such that $\tilde{x} \triangleq \bar{A}\bar{x}_N^*(x, \bar{\Omega}) + \bar{B}\tilde{u} \in \bar{\Omega}$, and

$$V_N^*(x^+, \bar{\Omega}) - V_N^*(x, \bar{\Omega}) \le -d_{\bar{\Omega}}(\bar{x}_0^*(x, \bar{\Omega})). \tag{11}$$

Proof. We follow similar steps of that in the Proof of Property 3 in [10], but considering that we now have target sets instead of target equilibrium points. In what follows, the dependence from $\bar{\Omega}$ will be omitted for the sake of clarity.

The state sequences associated with $\mathbf{u}^*(x, \bar{\Omega})$ and $\tilde{\mathbf{u}}^*(x, \bar{\Omega})$ are, respectively, $\bar{\mathbf{x}}^*(x, \bar{\Omega})$ and $\tilde{\mathbf{x}}(x, \bar{\Omega})$ where

$$\tilde{\mathbf{x}}(x, \bar{\Omega}) = \{\bar{x}_1^*(x, \bar{\Omega}), \dots, \bar{x}_N^*(x, \bar{\Omega}), \tilde{x}\}.$$

Because $x \in \bar{x}_0^*(x, \bar{\Omega}) \oplus Z$, it follows, from Proposition 1, that $x^+ \in \bar{x}_1^*(x, \bar{\Omega}) \oplus Z$.

Since $(\bar{x}_0^*(x, \bar{\Omega}), \mathbf{u}^*(x, \bar{\Omega}))$ is feasible for $P_N^{rob}(x, \bar{\Omega})$, constraints are satisfied by $\mathbf{u}^*(x, \bar{\Omega})$ and $\bar{\mathbf{x}}^*(x, \bar{\Omega})$, hence the input constraints are satisfied by the first $N-1$ elements of $\tilde{\mathbf{u}}(x, \bar{\Omega})$ and the state constraints are satisfied by the first N elements of $\tilde{\mathbf{x}}(x, \bar{\Omega})$. We take $\tilde{u} \in \mathcal{U}$ such that $\tilde{x} \in \bar{\Omega}$, so both, the input and final constraints, are satisfied. Thus, $\tilde{\mathbf{u}}(x, \bar{\Omega})$ is a feasible input sequence. Moreover, since $x^+ \in \bar{x}_1^*(x, \bar{\Omega}) \oplus Z$, the pair $(\bar{x}_1^*(x, \bar{\Omega}), \tilde{\mathbf{u}}(x, \bar{\Omega}))$ is feasible for $P_N^{rob}(x^+, \bar{\Omega})$ and $x^+ \in \mathcal{X}_N$.

To prove the decreasing of the cost, note that $x^+ \in \bar{x}_1^*(x, \bar{\Omega}) \oplus Z$ so that $(\bar{x}_1^*(x, \bar{\Omega}), \mathbf{u}^0(\bar{x}_1^*(x, \bar{\Omega}), \bar{\Omega}))$ is feasible for $P_N^{rob}(x^+, \bar{\Omega})$, hence $V_N^*(x^+) \le V_N^0(\bar{x}_1^*(x, \bar{\Omega}))$. But, from (8), $V_N^0(\bar{x}_1^*(x, \bar{\Omega})) \le V_N^0(\bar{x}_0^*(x, \bar{\Omega})) - \alpha d_{\bar{\Omega}}(\bar{x}_0^*(x, \bar{\Omega}))$, since $\bar{x}_1^*(x, \bar{\Omega})$ is the state of the nominal system at time 1 if at time 0 the state is $\bar{x}_0^*(x, \bar{\Omega})$ and the control is $\kappa_N(\bar{x}_0^*(x, \bar{\Omega}))$. Finally, since $V_N^*(x) = V_N^0(\bar{x}_0^*(x, \bar{\Omega}))$, the decreasing of the cost follows. ■

Based on the latter result, we can show that the TS Ω is asymptotically stable for the uncertain closed-loop system (10).

Theorem 2 (*Robust Stability*)**.** *The set Ω is robust asymptotically stable for the uncertain closed-loop system (10), with a domain of attraction given by $\mathcal{X}_N$.*

Proof. Let $x \in \mathcal{X}_N$. Taking the same notation of Theorem 1, we know that $x^+ \in x_0^*(x, \bar{\Omega}) \oplus Z$ and, from the fact that $d_{\bar{\Omega}}(x_0^*(x, \bar{\Omega})) \ge d_{\bar{\Omega}\oplus Z}(x_0^*(x, \bar{\Omega}) + z)$ for all $z \in Z$, we have that $d_{\bar{\Omega}}(x_0^*(x, \bar{\Omega})) \ge d_{\bar{\Omega}\oplus Z}(x^+)$. This implies, by means of (11), that

$$V_N^*(x^+, \bar{\Omega}) - V_N^*(x, \bar{\Omega}) \le -d_{\bar{\Omega}}(x_0^*(x, \bar{\Omega})) \le -d_{\bar{\Omega}\oplus Z}(x^+), \tag{12}$$

for all $x^+ \in x_0^*(x, \bar{\Omega}) \oplus Z$. So, by the classical Lyapunov theory, the set $\bar{\Omega} \oplus Z$ is robust asymptotically stable for the uncertain closed-loop system (10), and given that $\bar{\Omega} \oplus Z \subseteq \Omega$, the result follows. ■

The latter result shows that the uncertain closed-loop system (10) converges to the TS Ω, for all $w_k \in \mathcal{W}$. Furthermore, according to (4), it means that the uncertain closed-loop system

$$x_{k+1} = A(\theta)x_k + B(\theta)\kappa_N^*(x_k), \quad \theta \in \Theta, \tag{13}$$

also converges to Ω, for all $\theta \in \Theta$.

3.4. Final control formulation

According to the "*excitation set*" role of the TS, the following persistent excitation signal is defined, which account for the formal concept of "*persistent excitation*" presented in [21].

Definition 2 (*Persistent Excitation Signal*)**.** Given a compact non empty set $V \subset \mathbb{R}^m$, we say that a stationary process $v : \mathbb{N} \to V$ is a persistent excitation signal if it satisfies $\mathrm{E}[v_k] = 0$ and $\mathrm{cov}[v_k] > 0$ for all $k \in \mathbb{N}$, and, additionally, v_k is uncorrelated with v_j, for $k \neq j$.

As we mentioned earlier, once the robust MPC – derived from previous problem $P_N^{rob}(x, \bar{\Omega})$ – drives system (1) to the TS Ω, the idea is to persistently excite the system (by means of the persistent excitation signal) to perform a suitable re-identification procedure. To do that, the cost function of problem $P_N^{rob}(x, \bar{\Omega})$ is modified in order to include the excitation task.

Let $v_k \in V$ be a persistent excitation signal as the one defined in Definition 2, and let k be the actual sample time. Then, the proposed cost function is given by:

$$V_N^{exc}(x, \bar{\Omega}, v_k; \mathbf{u}) = [1 - \rho(x)]V_N(x, \bar{\Omega}; \mathbf{u}) + \rho(x)\|u(0) - v_k\|,$$

where $\rho(x) = 1$ if $x \in \Omega$, and $\rho(x) = 0$ otherwise.

For any initial state x in $\mathcal{X}_N \subset \mathcal{X}$, at a given time step k, the optimization problem $P_N^{exc}(x, \bar{\Omega}, v_k)$, to be solved at each time instant k, is given by:

$$\begin{aligned} V_N^*(x, \bar{\Omega}, v_k) = \min_{\mathbf{u}, \bar{x}_0}\{&V_N^{exc}(\bar{x}_0, \bar{\Omega}, v_k; \mathbf{u}) : \bar{x}_j \in \bar{\mathcal{X}}, u_j \in \mathcal{U}, \\ &\bar{x}_N \in \bar{\Omega}, x \in \bar{x}_0 \oplus Z, j \in J_{N-1}\}. \end{aligned} \tag{14}$$

Notice that $\rho(x)$ is a discontinuous function necessary to cancel the control law and apply the persistent excitation whenever the state enters Ω. The idea of this Optimization Problem is to spatially separate the controller actions. Inside the TS Ω, a persistent excitation signal is injected to the system according to the cost term $\|u(0) - v_k\|$. Outside Ω, the objective is exclusively to steer the system to Ω. So, the solution of Problem $P_N^{exc}(x, \bar{\Omega}, v_k)$ can be expressed as:

$$\kappa_N^{exc}(x) = \begin{cases} \kappa_N^*(x) & \text{if } x \in \mathcal{X}_N \setminus \Omega, \\ v_k & \text{if } x \in \Omega, \end{cases}$$

and the uncertain closed-loop system (13) becomes:

$$x_{j+1} = A(\theta)x_j + B(\theta)\kappa_N^{exc}(x_j), \quad \theta \in \Theta. \tag{15}$$

Under this proposed control schemes, it remains to define the additional conditions that the TS should satisfy to ensure a proper identification procedure, i.e. the conditions under which the probability that the excitation will not take the system out this set is considered low.

4. Target set design

In the MPC suitable for re-identification context, the TS has to satisfy two (opposite) requirements: to be robust stable for the family of models (13) – what was proved in Section 3.1 – and, to be *invariant in some sense*, to ensure that the persistent excitation of the system (with aim of proper re-identification) does not take the system out of it.

For this latter requirement, it is sufficient (although it is not necessary) for the TS to be invariant for all possible persistent excitations, i.e. invariant for the following system family:

$$z_{j+1} = A(\theta)z_j + B(\theta)v_j, \quad \theta \in \Theta, \tag{16}$$

where v_j is a persistent excitation signal on the compact set $V \subset \mathcal{U}$.

The ISI sets presented in [8] fulfill this condition, but paying the price of being too large to account for "all" possible excitation signals.[3] Besides, the uncertainty set Θ must be considered very small: depending on the size of the excitation set V, it may be so small that it does not cover all the models required for a proper uncertainty description.

To overcome these drawbacks, the TS proposed in this work will be the smallest one ensuring the robust stability of (13), but still large enough to ensure that the system can be persistently excited inside it. To simultaneously match these two conditions, we used the concept of probabilistic invariant sets for system (16) [16], which take advantages of the knowledge of the persistent excitation probability distribution.

4.1. One step probabilistic invariant set

In [16], the concept of probabilistic invariant sets (PIS) is introduced, to characterize sets that are invariant for a disturbed system with a given probability. The considered system is of the form $z_{k+1} = \bar{A}z_k + \bar{B}v_k$, where v_k is a persistent excitation signal and, if the state z_0 is inside the PIS, it will remain there, for all $k > 0$, with certain probability p. This kind of sets clearly are a possible TS candidate for the proposed robust MPC, since they take advantage of the knowledge of the probabilistic distribution of the excitation signal to reduce the set size and, at the same time, ensure that the system state will remain in the set – if not forever – for a large enough time to perform the identification.

In our context, however, it is not necessary (although it is sufficient to reduce the TS length) to consider a PIS as TS. In fact, we only need the probabilistic invariance of the TS for only one time step, given that, if the state leaves the set, the controller will abort the excitation to steer the state back to the set. Furthermore, the TS must be robust for the parametric uncertain system (16). The TS candidate for the proposed robust MPC is defined next:

Definition 3 (*Robust γ-One Step Probabilistic Invariant Set, Rγ-OSPIS*)**.** Let $p \in (0, 1]$ and $\gamma \in (0, 1]$. Let v be a persistent excitation signal in V. A proper C-set $\Omega \subseteq \mathcal{X}$ is a robust γ-one step probabilistic invariant set with probability p for system (16), if and only if $\Pr[(A(\theta)z + B(\theta)v) \in \gamma\Omega \mid z \in \Omega] \geq p$, for all $\theta \in \Theta$.

[3] Here, a large set means a (deterministic or probabilistic) invariant set with large volume. The invariance condition implies a set shape that captures the system dynamic.

Notice that the robust property of the above definition refers to the entire family of models determined by $\theta \in \Theta$, while the probabilistic property refers to the persistent excitation signal $v_j \in V$. A set Ω fulfilling Definition 3 for system (16), with $\Theta = \{0\}$ (nominal system), is named γ-OSPIS (if $\gamma = 1$, simply OSPIS). Furthermore, when $\gamma = 1$ a Rγ-OSPIS is simply an ROSPIS and, when $p = 1$, a Rγ-OSPIS is a deterministic robust γ-ISI set, as defined in [8].

Remark 2. The OSPIS is in general a concept that requires weaker conditions that the PIS from [16]. Every PIS is an OSPIS, although the opposite is not true. Furthermore, there is not a method to compute a robust PIS, while a robust OSPIS (necessary in the proposed MPC formulation) can be computed by means of a relatively simple procedure, as it is shown in the next section. Even more, in the closed-loop re-identification procedure it is enough to ensure invariance of the TS in one step only, since in case the state leaves the set, the excitation procedure will be suspended and the controller will steer the state back to the set.

Next, we will show that a Rγ-OSPIS with probability $p > 0$ for (16) is also a RCIS of system (1), which means that it can be used as TS for problem $P_N^{exc}(\cdot)$.

Property 1. *Let $p \in (0, 1]$ and $\gamma \in (0, 1]$. Let $\Omega \subseteq \mathcal{X}$ be a Rγ-OSPIS with probability p for system (16). Then, Ω is a RCIS for system (1).*

Proof. Let $x_k \in \Omega$. Then, $Pr[A(\theta)x + B(\theta)v \in \gamma\Omega] \geq p$, where $v \in V \subseteq \mathcal{U}$ is a persistent excitation signal. The fact that $p > 0$ means that some $\bar{v} \in V$ exists such that $A(\theta)x + B(\theta)\bar{v} \in \gamma\Omega$ since, otherwise, $Pr[A(\theta)x + B(\theta)v \in \gamma\Omega] = 0$. Furthermore, $V \subseteq \mathcal{U}$ and $\gamma\Omega \subseteq \Omega$, which means that Ω is a RCIS for system (1). ■

To collect enough input–output data for a proper identification (say a data vector of length q), the system under the excitation must remain inside the TS – for the next q time steps – with some high probability. The following property of the ROSPIS helps us to estimate this probability:

Property 2. *Let $p \in (0, 1]$. Let Ω be a ROSPIS with probability p for System (16). Then, provided that $z_k \in \Omega$, it results that $Pr[z_{k+1} \in \Omega \wedge z_{k+2} \in \Omega \wedge \cdots \wedge z_{k+q} \in \Omega] \geq p^q$, for all $\theta \in \Theta$.*

Proof. The fact that v_k is a persistent excitation signal and θ is an unknown fixed parameter implies that z_k has the Markov property, i.e., given z_k, the value of z_{k+1} does not depend on past values of the state prior to time k. This way, the ROSPIS property that ensures $Pr[z_{k+2} \in \Omega \mid z_{k+1} \in \Omega] > p$ is accomplished independently on the fact that $z_k \in \Omega$. Thus, $Pr[z_{k+2} \in \Omega \mid z_{k+1} \in \Omega \wedge z_k \in \Omega] > p$.

Then, given that $z_k \in \Omega$, it results that

$$\begin{aligned} &Pr[z_{k+2} \in \Omega \wedge z_{k+1} \in \Omega] \\ &\quad = Pr[z_{k+2} \in \Omega \mid z_{k+1} \in \Omega] \cdot Pr[z_{k+1} \in \Omega] \geq p^2, \end{aligned}$$

and the proof concludes by the recursive use of this reasoning. ■

4.2. Characterization of the OSPIS

In this section several propositions are introduced, in order to provide a method to compute one step probabilistic invariant sets.

4.2.1. Computation of the OSPIS

An easy way to characterize an OSPIS is presented in the following proposition.

Proposition 2. *Let $p \in (0, 1]$. Let $v \in V \subset \mathcal{U}$ the persistent excitation signal from system (16). Consider a set $\Upsilon \subseteq V$ such that $Pr[v \in \Upsilon] \geq p$. If $\Omega \subset \mathcal{X}$ is a set fulfilling the following condition*[4]

$$z \in \Omega \Rightarrow \bar{A}z + \bar{B}v \in \Omega \quad \forall v \in \Upsilon. \tag{17}$$

Then, Ω is an OSPIS with probability p for system (16).

Proof. Consider a state $z_k \in \Omega$. If v_k falls inside Υ then $z_{k+1} \in \Omega$. Therefore

$$Pr[z_{k+1} \in \Omega] \geq Pr[v_k \in \Upsilon] \geq p,$$

concluding the proof. ■

Notice that in this context, given a set $\Upsilon \subseteq V \subset \mathcal{U}$ such that $Pr[v_k \in \Upsilon] \geq p$, an OSPIS for system (16) can be computed as $\Omega = \bigoplus_{i=0}^{\infty} \bar{A}^i \bar{B} \Upsilon$ (provided that this computation is possible). This latter set is known as the minimal robust invariant set, if the persistent excitation is considered as a disturbance.

Remark 3. From Proposition 2, it is easy to see that an OSPIS with $p < 1$, is smaller than an OSPIS with $p = 1$ (one set contains the other), being this latter set the ISI set presented in [8]. If a smaller TS is desired, a smaller value for p must be selected.

Notice that having a smaller OSPIS implies that the state remains in a small region during the identification procedure improving the control performance. However, according to Property 2, a large value for p is required in order to ensure that the state remains inside the target set during the time required to finish the identification process. Thus, selecting a right value for p involves a trade-off between the control goal of reducing the size of the OSPIS and the requirements of the identification procedure.

4.2.2. Computation of the γ-OSPIS

Based on the OSPIS computed in the last section, a γ-OSPIS with $\gamma < 1$, is obtained in the following proposition.

Proposition 3. *Let $p \in (0, 1]$ and $\lambda \in (0, 1)$. Let a proper C-set $\Phi \subset \mathcal{X}$ be an OSPIS with probability p for system (16), and a proper C-set $\Psi \subset \mathcal{X}$ such that*

$$x \in \Psi \Rightarrow \bar{A}x \in \lambda\Psi. \tag{18}$$

Then, the set $\Omega = \Phi \oplus \Psi$ is a γ-OSPIS with probability p of system (16) with $\gamma < 1$.

Proof. Suppose that $x_k \in \Omega$. This means that there exists $z_k \in \Phi$ and $y_k \in \Psi$ such $x_k = z_k + y_k$. Then,

$$\begin{aligned} x_{k+1} &= \bar{A}x_k + \bar{B}v_k \\ &= \bar{A}z_k + \bar{B}v_k + \bar{A}y_k \\ &= z_{k+1} + y_{k+1} \end{aligned}$$

where $z_{k+1} = \bar{A}z_k + \bar{B}v_k$, $y_{k+1} = \bar{A}y_k$. Note that $Pr[z_{k+1} \in \Phi] > p$ and $y_{k+1} \in \lambda\Psi$.

Since $\lambda < 1$ it fulfills that $\Phi \oplus \lambda\Psi \subset \Phi \oplus \Psi$. So, we can find a constant $\gamma < 1$ such that $\Phi \oplus \lambda\Psi \subseteq \gamma\,(\Phi \oplus \Psi)$ (such γ exists provided that Ψ and Φ are proper C-sets). Then,

$$\begin{aligned} Pr[x_{k+1} \in \gamma\Omega] &\geq Pr[x_{k+1} \in \Phi \oplus \lambda\Psi] \\ &= Pr[z_{k+1} + y_{k+1} \in \Phi \oplus \lambda\Psi] \\ &\geq Pr[z_{k+1} \in \Phi] > p. \end{aligned}$$

Then, Ω is a γ-OSPIS with probability p and $\gamma < 1$ for system (16). ■

[4] In [22] a form to compute this type of set is provided.

4.2.3. Computation of the robust OSPIS

Finally, based on the latter computed γ-OSPIS, the Robust OSPIS that will be used as TS of problem $P_N^{exc}(\cdot)$, is presented in the following proposition.

Proposition 4. *Let a proper C-set $\Omega \subset \mathcal{X}$ be a γ-OSPIS with probability $p \in (0, 1]$ and $\gamma < 1$ for system (16). Then, there exists a proper C-set $\bar{\Theta} \subseteq \mathbb{R}^l$ such that the set Ω is a robust OSPIS with probability p for system (16), with the uncertainty set $\Theta \triangleq \bar{\Theta}$.*

Proof. Let $z_k \in \Omega$. Compute the nominal system $\bar{z}_{k+1} = \bar{A}z_k + \bar{B}v_k$, and the uncertain system $z_{k+1} = A(\theta)z_k + B(\theta)v_k$. Then, subtracting both future values of the state we obtain

$$z_{k+1} - \bar{z}_{k+1} = [A(\theta) - \bar{A}]z_k + [B(\theta) - \bar{B}]v_k$$

applying norms and triangular inequality, it results that

$$\begin{aligned} \|z_{k+1} - \bar{z}_{k+1}\| &= \|[A(\theta) - \bar{A}]z_k + [B(\theta) - \bar{B}]v_k\| \\ &\leq \|A(\theta) - \bar{A}\| \cdot \|z_k\| + \|B(\theta) - \bar{B}\| \cdot \|v_k\| \\ &\leq L_A \cdot \|\theta\| \cdot \|z_k\| + L_B \cdot \|\theta\| \cdot \|v_k\| \end{aligned}$$

where L_A and L_B are the Lipschitz constants of $A(\theta)$ and $B(\theta)$ on $\mathbb{R}$. Then,

$$\|z_{k+1} - \bar{z}_{k+1}\| \leq (L_A \cdot r_z + L_B \cdot r_v) \cdot \|\theta\| \triangleq \alpha \cdot \|\theta\| \tag{19}$$

where $r_z \triangleq \max_{z \in \Omega} \|z\|$ and $r_v \triangleq \max_{v \in V} \|v\|$.

Let $d \triangleq \inf_{z \notin \Omega} d_{\gamma\Omega}(z)$, i.e., the minimum distance from the border of Ω to set $\gamma\Omega$. Then, consider the set

$$\bar{\Theta} \triangleq \{\theta \in \mathbb{R}^l : \|\theta\| \leq \frac{d}{\alpha}\}. \tag{20}$$

Thus, $\theta \in \bar{\Theta}$ implies that $\alpha\|\theta\| \leq d$, and, from Eq. (19), we have

$$\theta \in \bar{\Theta} \Rightarrow \|z_{k+1} - \bar{z}_{k+1}\| \leq d.$$

Taking into account that d is the minimum distance from the border of Ω to the set $\gamma\Omega$, the latter condition establishes that $\bar{z}_{k+1} \in \gamma\Omega \Rightarrow z_{k+1} \in \Omega$. Then,

$$Pr[z_{k+1} \in \Omega] \geq Pr[\bar{z}_{k+1} \in \gamma\Omega] \geq p \quad \forall\, \theta \in \bar{\Theta}$$

which proves that Ω is a robust OSPIS with probability p for system (16) for the uncertain set $\bar{\Theta}$. ■

Remark 4. Note that the size of the set $\bar{\Theta}$ depends on d/α, according to (20). d depends on the contractivity of the TS Ω, but α is proportional to the size of the TS Ω, which means that the size of $\bar{\Theta}$ is inversely proportional to the size of the TS. This means that a smaller TS implies a larger uncertainty set $\bar{\Theta}$, which accounts for larger model families.[5] This is a crucial benefit of using OSPIS as TS, with $p < 1$, since in this case, a significant reduction is obtained compared to the case of $p = 1$ (the ISI set of [8]).

5. Summary of the proposed strategy

The proposed robust MPC suitable for re-identification is the one presented in (14), where the TS is a robust OSPIS for system (16). This controller has the following benefits in comparison with the nominal MPC based on the ISI set [8]: (i) The proposed MPC ensures the robust stability of the TS for the entire family (1). This is a crucial point in the context of a re-identification scenario, when there is a significant model deterioration. (ii) The new TS is now significantly smaller (Remark 3), depending on the probability p, which is selected *a priori*. Given that inside TS the system is in

[5] That is, if we want larger robustness for model family (1), we must sacrifice the probability of permanence.

open-loop (to perform the re-identification), this reduction derives in a safer control operation. (iii) The set Θ accounting for the system uncertainty is significantly larger, for $p < 1$, as stated in Remark 4. This implies that a bigger model family can be considered for robustness. (iv) The persistent excitation of the system is ensured – with certain probability – for a large enough time, in such a way that output–input uncorrelated data are obtained, which is a main advantage from the identification theory point of view. This property comes from the fact that inside the TS no control actions are taken by the robust MPC.

An algorithm of the proposed strategy can be resumed by the followings steps.

- Select a suitable persistent excitation signal, v_k, for re-identifying the model (see Definition 2).
- Select a value of the probability p for the target set (see Remarks 3 and 4).
- For the selected v_k and p, compute the robust OSPIS, Ω, and its corresponding input set, Π, for system (16) (see Proposition 4).
- Formulate Problem $P_N^{exc}(x, \bar{\Omega}, v_k)$ presented in (14).
- Provided the nominal MPC performance is deteriorated, replace it by $P_N^{exc}(x, \bar{\Omega}, v_k)$ until the parameters of the model are estimated and validated. Then return to the nominal MPC with the updated model.

6. Simulation results

In order to test the proposed methodology, an uncertain and disturbed second order continuous-time system is considered. The discrete-time version of the system for a sampling time of $Ts = 1$ reads:[6]

$$\begin{aligned} x_{k+1} &= A(\theta)x_k + B(\theta)u_k \\ y_k &= Cx_k + d_k, \end{aligned} \tag{21}$$

where

$$\begin{aligned} A(\theta) &= \begin{bmatrix} 0.42 & -0.28 \\ 0.02 & 0.6 \end{bmatrix} + \theta \begin{bmatrix} -0.13 & 0.12 \\ -0.1 & -0.11 \end{bmatrix}, \\ B(\theta) &= \begin{bmatrix} 0.3 \\ -0.4 \end{bmatrix} + \theta \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}, \end{aligned} \tag{22}$$

$C = [-0.3\ 0.6]$, $\theta \in \Theta = [-0.15, 0.15] \subseteq \mathbb{R}$ and d_k is an output white noise, with zero mean and variance $\sigma_d = 0.005^2$. The unknown *real system* is one of the latter family, corresponding to some $\theta \neq 0$, while the nominal model is given by $A(0)$ and $B(0)$. The constraints of the system are given by $\mathcal{X} = \{x \in \mathbb{R}^2 : \|x\|_\infty \leq 10\}$ and $\mathcal{U} = \{u \in \mathbb{R} : \|u\|_\infty \leq 4\}$.

6.1. Normal operation

We start the simulation with a Normal Operation, by controlling the output of the real system to three given operating zones, $\mathcal{Y}_1 = \{y \in \mathbb{R} : 2.6 \leq y \leq 2.8\}$ for time $0 \leq t \leq 20$, $\mathcal{Y}_2 = \{y \in \mathbb{R} : -1.2 \leq y \leq -1\}$ for time $20 < t \leq 35$ and $\mathcal{Y}_3 = \{y \in \mathbb{R} : -2.8 \leq y \leq -2.6\}$ for time $35 < t \leq 50$, by means of a nominal MPC based on the nominal model $(A(0), B(0))$. The nominal controller is a zone MPC (as the one shown in [8], subsection 4.1), with $N = 7$, $Q = diag([1000\ \ 1000])$, $R = 10$ and an equality terminal constraint. The selected simulation scenario consists in an initial state of [0, 0], and a simulation time of 50 time steps. Furthermore, a properly tuned Kalman filter is used to estimate the states from the output.

[6] This system is very similar to the one introduced in [8].

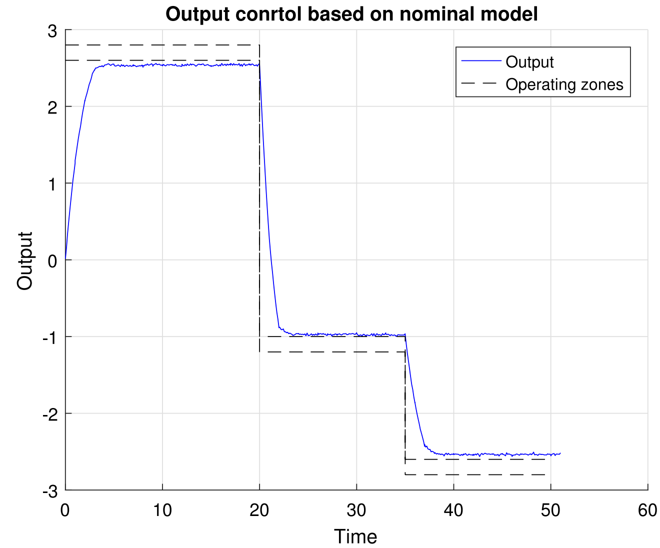

Fig. 1. Output evolution (in blue) to the operating zones (in black) of the nominal zone MPC controller based on the nominal model. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

To measure the performance of this controller, the following index is proposed:

$$I_{perf} \triangleq \sum_{t=0}^{50} [d_{\mathcal{Y}_i}(y_t) + d_{\mathcal{U}_i}(\kappa_{MPC}(x_t))]$$

where $\mathcal{Y}_i$ is the operating zone depending on the time t and $\mathcal{U}_i = (C(I - A)^{-1}B)^{-1}\mathcal{Y}_i$ is the input set corresponding to $\mathcal{Y}_i$, x_t is the measured state of the closed-loop system and $\kappa_{MPC}(\cdot)$ is the implicit nominal MPC control law. If we define I_{ref} as a reference index — and index computed just after a proper identification, and denoting the best possible value of the index I_{perf} for the given scenario, then the following standard index can be used to assess the closed-loop performance: $I_{std} \triangleq I_{ref}/I_{perf}$.

The closed-loop system evolution of the output can be seen in Fig. 1. The standard Index corresponding to the current scenario is given by $I_{std} = 0.706$. Although many other methods could be considered to diagnose a model deterioration, a standard Index value far from 1 can be interpreted as an alarm indicating that the nominal model parameters fail to describe those of the real system. As a consequence, not only a poor performance is achieved, but also neither recursive feasibility nor stability can be properly ensured, as it can be seen in Fig. 1 where the output fails to reach the operating zone.[7]

Under the suspicion of a model deterioration two main alternatives arise. The first – mainly to account for feasibility and stability issues – is to replace the nominal MPC by some robust formulation. This way, a tube-based RMPC (as the one presented in [10]) is used. The standard Index, however, is significantly smaller, as it is usually the case of robust MPC controllers: $I_{std} = 0.1771$. The reason why a poor performance is obtained is the conservativeness of the strategy, which is the price we have to pay to ensure feasibility and stability. Fig. 2 shows the state evolution in this case.

The second alternative – that we find a better option for real life applications and it is the main objective of this work – is to make a re-identification to obtain a new accurate model, and so continuing using the nominal zone MPC controller. In order to have

[7] This problem can be solved by means of an offset-free, even so, indicates a failure of the nominal model.

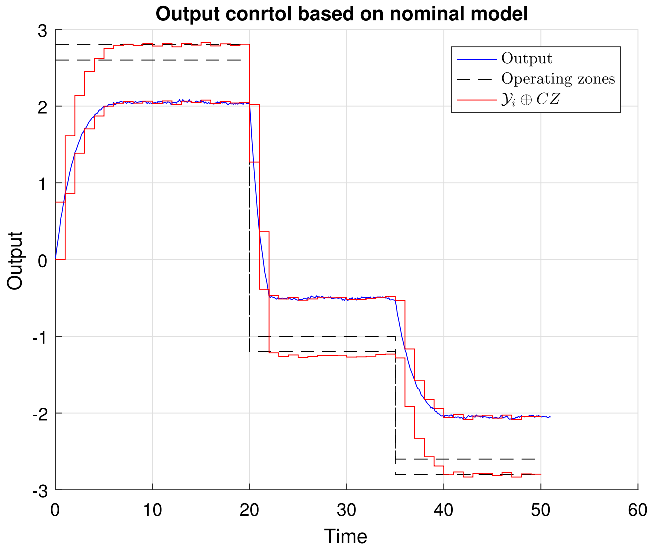

Fig. 2. Output evolution of the Robust tube-based zone MPC controller. The output (in blue) is steered to the set $\mathcal{Y}_i \oplus CZ$ (in red), depending on the time, where Z is a robust invariant set as the one in Proposition 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

proper input–output data in a safe closed-loop fashion and without stopping the operation of the plant, this will be made by using the proposed robust MPC for re-identification.

6.2. Re-identification operation

To perform the re-identification, the system is first steered to $y = 0$, and there, the controller of the latter section is replaced by the proposed Robust MPC for re-identification. In this case, the simulation time is extended to 500 time steps to have enough input–output data to identify and validate a new model and, furthermore, state disturbances ($x = [-9, 9]$ at time step $k = 0$ and $x = [2, -6]$ at time step $k = 30$) are included to test the regulatory benefits of the strategy.

The proposed RMPC for re-identification (derived from problem P_N^{exc} in Section 3.4) is strongly dependent on a proper target set (TS). The TS Ω is computed according to the procedure described in Section 4.2, with probability $p = 0.99$ (see Fig. 4), and taking into account the persistent excitation signal v_k, which is assumed to have a truncated normal distribution, and lies within $\mathcal{V} = \{v \in \mathbb{R} : \|v\|_\infty \leq 3.5\}$, with mean $\mathrm{E}[v_k] = 0$ and covariance $\mathrm{cov}[v_k] = 1.64^2$ (Fig. 3 shows the persistent excitation signal for the first 100 time steps). Furthermore, the terminal constraint is given by $\bar{\Omega} = \Omega \ominus Z$, where Z is a robust invariant set as the one defined in Proposition 1.

Fig. 4 shows the state evolution under the Robust MPC suitable for closed-loop re-identification (for first 50 time instances). As it can be seen, the real state evolves inside the tube (as stated in Proposition 1) with center on the nominal state. The real state converges to the TS Ω and, once inside it, the control switch to the persistent excitation of the system. Note that even though the TS Ω is not a robust deterministic invariant set, but only a robust probabilistic one, there is not a state leaving the TS. This occurs due to the fact that the TS is a probabilistic invariant set for all the models in the family, and the one been excited is the unknown real one. The first 300 excited states are collected for re-identification purpose, and the last 200 to validate the estimated system, excluding those data corresponding to the disturbance rejection, where the system is not excited. Fig. 5 shows the output signal – for the first 100 time steps – during the excitation procedure, which is assumed to be noisy.

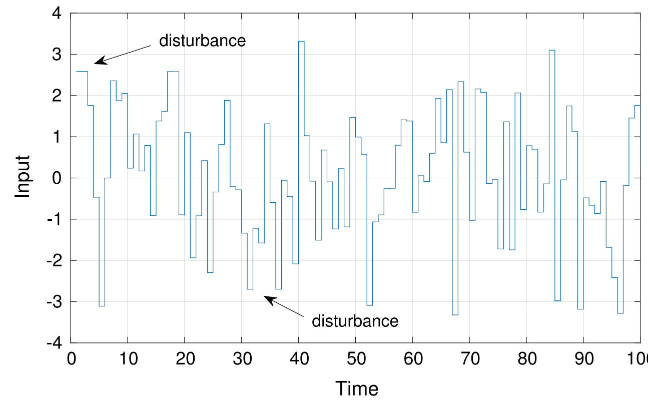

Fig. 3. Persistent excitation signal used for the identification.

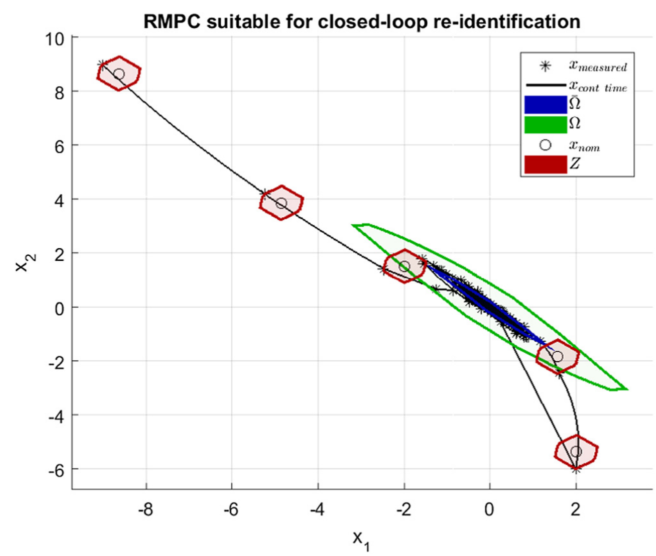

Fig. 4. State evolution of the Robust MPC for re-identification. The real system is steered to the TS Ω. Once the real system enters Ω, however, the excitation procedure is started. Note also that when the disturbance enters the system, the excitation is aborted, and the system is driven back to TS.

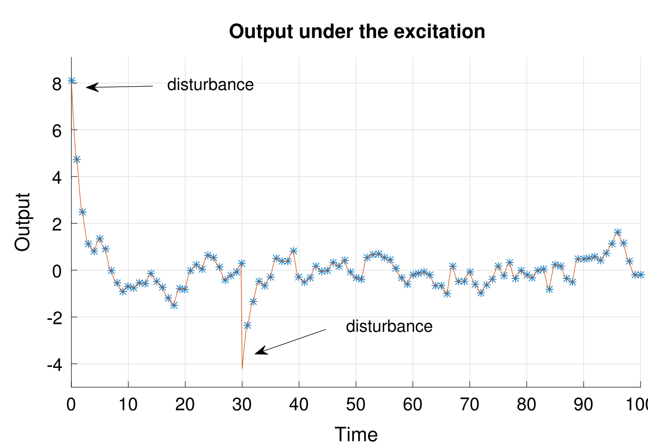

Fig. 5. Output evolution under the excitation procedure.

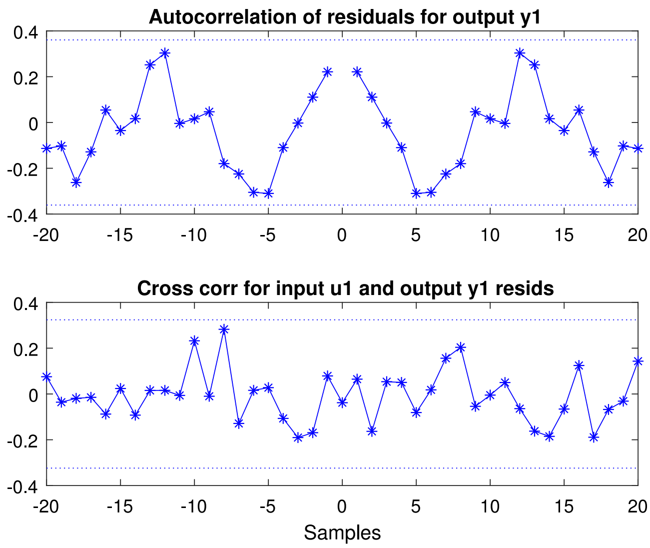

Fig. 6. Autocorrelation of the output residual and cross correlation for input and output residuals. Proposed strategy.

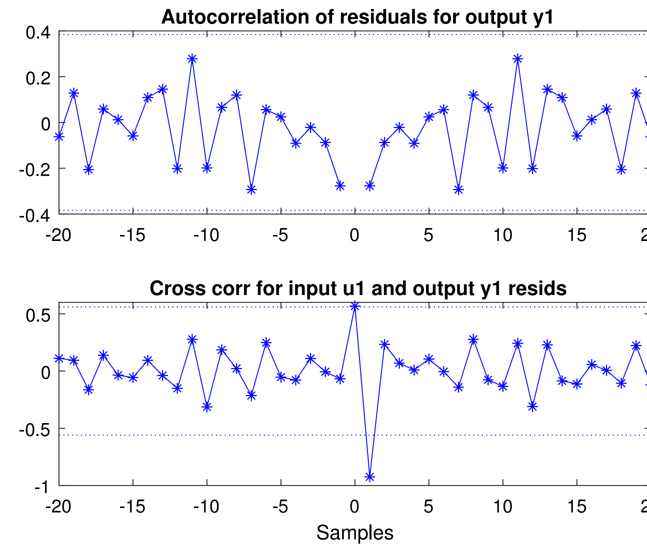

Fig. 7. Autocorrelation of the output residual and cross correlation for input and output residuals. Classical closed-loop identification.

6.3. Identification characteristics

The new model is obtained based on the uncorrelated input–output data. The selected method was the N4SID identification method, and the obtained model FIT is of 99.6%. Given that the input–output data are uncorrelated, the FIT is not 100% exclusive because of the output noise d_k. To analyze the re-identification characteristics, the Matlab function *ident* is used. Fig. 6 shows both, the autocorrelation of the output residual and cross correlation for input and output residuals. As it can be seen, the first plot approximates an impulsive signal (as desired) while the last one shows values inside the confidence region, which means there exists no input–output correlation (out of the one explained by the identified model).

To highlight the benefits of the proposal – in terms of the uncorrelated data – we simulated the same closed loop, but exciting the system by superposing the persistent excitation signal to the input computed by the controller (instead of separating the tasks of control and excitation of the system). As a result, a FIT of 99.37% is obtained when the N4SID identification method is used (the FIT is similar to the first one because of the simplicity of the model and the excitation scenario). Fig. 7 shows the autocorrelation of the output residual and cross correlation for input and output residuals, where it can be seen that now, two points of the cross correlation are out of the confidence region, denoting an input–output correlation.

6.4. Normal operation with estimated model

Once the new model is identified, the Normal Operation is performed with a nominal MPC based on the new model. The standard Index is now given by $I_{std} = 0.9795$, and Fig. 8 shows the output evolution under the same disturbances described in Section 6.1.

The performance of the MPC based on the identified model shows an improvement of 27% compared to the one based on the original nominal model, besides the recursive feasibility and stability assured; and an improvement of 80% with respect to the Robust MPC. As it was expected, a significant improvement in the performance was obtained regarding the two alternatives scenarios.

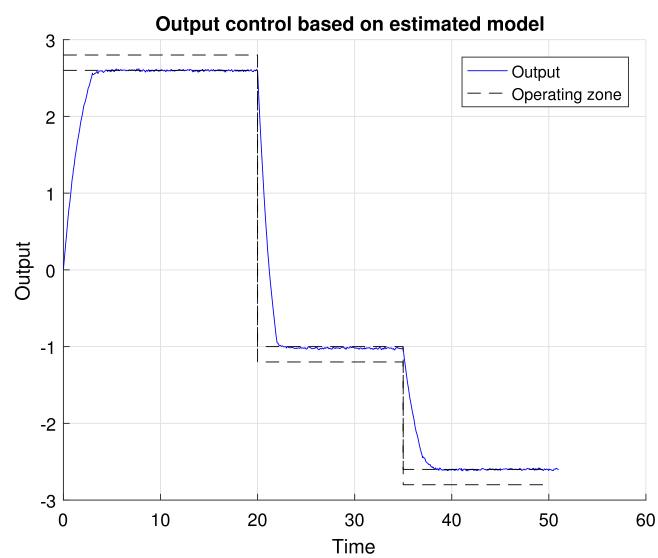

Fig. 8. Output evolution of the nominal zone MPC controller based on the identified model.

7. Conclusion

In this work a robust extension of an MPC suitable for closed-loop re-identification – which takes into account the knowledge of the persistent excitation probability distribution – is proposed. The main benefits of the strategy – out of the robust properties – come from the use of a reduced target set, that is computed taking into account the probabilistic invariance concepts. This way, the persistent excitation of the closed-loop system is ensured, and furthermore, output–input uncorrelated data can be obtained, using only a reduced state space region around the equilibrium. This represents – from the control point of view – a less conservative and more flexible formulation, which considerably increases the applicability of the proposed methodology.

References

[1] H. Genceli, M. Nikolaou, New approach to constrained predictive control with simultaneous model identification, AIChE J. 42 (1996) 2857–2868.

[2] E. Zacekova, S. Pravara, M. Pcolka, Persistent excitation condition within the dual control framework, J. Process Control 23 (2013) 359–371.
[3] T.A.N. Heirung, B.E. Ydstie, B. Foss, Towards dual MPC, in: 4th IFAC Nonlinear Model Predictive Control Conference, 2012, pp. 502–507.
[4] A.S. Potts, R.A. Romano, C. Garcia, Improving performance and stability of MPC relevant identification methods, Control Eng. Pract. 22 (2014) 20–33.
[5] R.S. Patwardhan, R.B. Gopaluni, A moving horizon approach to input design for closed loop identification, J. Process Control 24 (2014) 188–202.
[6] G. Marafioti, R.R. Bitmead, M. Hovd, Persistently exciting model predictive control, Internat. J. Adapt. Control Signal Process. 28 (2014) 536–552.
[7] C. Larsson, M. Annergren, H. Hjalmarsson, C.R. Rojas, X. Bombois, A. Mesbah, P.E. Modén, Model predictive control with integrated experiment design for output error systems, in: Control Conference (ECC), 2013 European, IEEE, 2013, pp. 3790–3795.
[8] A.H. González, A. Ferramosca, G.A. Bustos, J.L. Marchetti, M. Fiacchini, D. Odloak, Model predictive control suitable for closed-loop re-identification, Systems Control Lett. 69 (2014) 23–33.
[9] G. Bustos, A. Ferramosca, J.L. Godoy, A.H. González, Application of model predictive control suitable for closed-loop re-identification to a polymerization reactor, J. Process Control 44 (2016) 1–13.
[10] D.Q. Mayne, M.M. Seron, S.V. Rakovic, Robust model predictive control of constrained linear systems with bounded disturbances, Automatica 41 (2005) 219–224.
[11] Paul J. Goulart, Eric C. Kerrigan, Jan M. Maciejowski, Optimization over state feedback policies for robust control with constraints, Automatica 42 (4) (2006).
[12] Paul J. Goulart, Eric C. Kerrigan, Daniel Ralph, Efficient robust optimization for robust control with constraints, Math. Program. 114 (1) (2006) 115–147.
[13] D. Limon, T. Alamo, D.M. Raimondo, D. Muñoz de la Peña, J.M. Bravo, E.F. Camacho, Input-to-state stability: an unifying framework for robust model predictive control, in: Proceedings of the Int. Workshop on Assessment and Future Directions of Nonlinear Model Predictive Control (NMPC08). 5-9 Sept. Pavia, Italy, 2008.
[14] D.M. Raimondo, Daniel Limon, Mirce Lazar, Lalo Magni, Eduardo Camacho, Min-max model predictive control of nonlinear systems: A unifying overview on stability, Eur. J. Control 15 (1) (2009) 5–21.
[15] W. Langson, I. Chryssochoos, S.V. Rakovic, D.Q. Mayne, Robust model predictive control using tubes, Automatica 40 (2004) 125–133.
[16] E. Kofman, J.A. De Dona, M.M. Seron, Probabilistic set invariance and ultimate boundedness, Automatica 48 (2012) 2670–2676.
[17] D.Q. Mayne, Robust and stochastic MPC: Are we going in the right direction? in: Conference on Nonlinear Model Predictive Control, 2015.
[18] P. Bumroongsri, S. Kheawhom, Improving the performance of robust MPC using the perturbation on control input strategy based on nominal performance cost, Procedia Eng. 42 (2012) 1027–1037.
[19] J.B. Rawlings, D.Q. Mayne, Model Predictive Control: Theory and Design, first ed., Nob-Hill Publishing, 2009.
[20] F. Blanchini, S. Miani, Set-Theoretic Method in Control, Birkhauser, 2008.
[21] L. Ljung, System Identification-theory for the User, second ed., Prentice-Hall, 1999.
[22] K. Hirata, Y. Ohta, e-feasible approximation of the state reachable set for discrete time systems, in: Proceedings of 42nd IEEE Conf. Decision and Control, Maui, HI, 2003, pp. 5520–5525.