# Are There Many Inactive Jupiter-Family Comets among the Near-Earth Asteroid Population? 

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Received August 27, 2001; revised February 14, 2002

We analyze the dynamical evolution of Jupiter-family (JF) comets and near-Earth asteroids (NEAs) with aphelion distances $Q>3.5 \mathrm{AU}$, paying special attention to the problem of mixing of both populations, such that inactive comets may be disguised as NEAs. From numerical integrations for $2 \times 10^{6}$ years we find that the half lifetime (where the lifetime is defined against hyperbolic ejection or collision with the Sun or the planets) of near-Earth JF comets (perihelion distances $q<1.3 \mathrm{AU}$ ) is about $1.5 \times 10^{5}$ years but that they spend only a small fraction of this time ( $\sim$ a few $10^{3}$ years) with $q<1.3 \mathrm{AU}$. From numerical integrations for $5 \times 10^{6}$ years we find that the half lifetime of NEAs in "cometary" orbits (defined as those with aphelion distances $Q>4.5 \mathrm{AU}$, i.e., that approach or cross Jupiter's orbit) is $4.2 \times 10^{5}$ years, i.e., about three times longer than that for near-Earth JF comets. We also analyze the problem of decoupling JF comets from Jupiter to produce Encke-type comets. To this end we simulate the dynamical evolution of the sample of observed JF comets with the inclusion of nongravitational forces. While decoupling occurs very seldom when a purely gravitational motion is considered, the action of nongravitational forces (as strong as or greater than those acting on Encke) can produce a few Enckes. Furthermore, a few JF comets are transferred to low-eccentricity orbits entirely within the main asteroid belt ( $Q<4 \mathrm{AU}$ and $q>$ 2 AU ). The population of NEAs in cometary orbits is found to be adequately replenished with NEAs of smaller $Q$ 's diffusing outward, from which we can set an upper limit of $\sim \mathbf{2 0 \%}$ for the putative component of deactivated JF comets needed to maintain such a population in steady state. From this analysis, the upper limit for the average time that a JF comet in near-Earth orbit can spend as a dormant, asteroid-looking body can be estimated to be about $40 \%$ of the time spent as an active comet. More likely, JF comets in nearEarth orbits will disintegrate once (or shortly after) they end their active phases. © 2002 Elsevier Science (USA)

Key Words: asteroids, dynamics; comets, dynamics.

## 1. INTRODUCTION

There has been a long discussion on whether near-Earth asteroids (NEAs) are mainly bona fide asteroids coming from the main belt, or whether deactivated Jupiter-family (JF) comets (either extinct or dormant) have an important contribution. The hypothesis that most or at least a significant fraction of NEAs are of cometary origin (e.g., Öpik 1963, Wetherill 1988) has been neither confirmed nor rejected, and the different tests have provided conflicting conclusions. For instance, many objects cataloged as asteroids later showed activity which prompted their reclassification as comets. It is thus possible that some comets may go through periods of dormancy without showing perceptible activity, disguised as asteroids (Kresák 1987). In this regard, it has been argued that objects like 107P/Wilson-Harrington (also cataloged as asteroid 4015 Wilson-Harrington), 2201 Oljato, or 3200 Phaethon are deactivated comets. 107P/WilsonHarrington showed some activity when it was discovered in 1949, but it has remained inactive in following returns since its rediscovery in 1979. As regards to 2201 Oljato, McFadden et al. (1993) reported the observation of a high ultraviolet reflectance during its 1979 and 1983 apparitions, which they suggested might be related to fluorescent emission from neutral species found in comets, such as CN or OH . The orbit of 3200 Phaethon was found to match the mean orbit of the Geminid meteor stream (Williams and Wu 1993), which again suggests a cometary nature of the object (meteor streams are produced when the Earth encounters dust particles and meteoroids left by a comet along its orbit as a result of the sublimation of its volatiles). The claim of a cometary nature for some NEAs has never been proved and the search for residual activity in those bodies led to negative results, giving upper limits for the possible active areas
of $0.02 \%$ for Wilson-Harrington and $0.01 \%$ for 3200 Phaethon (Chamberlin et al. 1996).

The transport of bodies from the main belt to NEA-type orbits was regarded before as very slow and inefficient. It was also noted that perturbations by Mars on approaching asteroids were not efficient enough to produce the right number of Earth-crossers from Mars-crossers (Öpik 1963). However, it was shown later that there exist mechanisms that could provide efficient dynamical routes of escape from the main belt to NEA-type orbits (e.g., Wisdom 1983). Mutual collisions among main-belt asteroids can inject fragments into either mean motion resonances with Jupiter (e.g., $3: 1,5: 2$ ) or the $v_{6}$ secular resonance, from which they are quickly transferred to NEA-type orbits on a time scale of a few Myr (e.g., Gladman et al. 1997). Menichella et al. (1996) estimate that a few hundred kilometer-sized NEAs per Myr can be produced in this way, which seemingly makes the consideration of extra sources unnecessary (see also Rabinowitz 1997). Even the large NEAs with diameters $D \gtrsim 5 \mathrm{~km}$ may be satisfactorily explained as driven from the main asteroid belt by the previous mechanism, overcoming some previous objections pointing to the insufficient production rate of big fragments there (Migliorini et al. 1998). Gladman et al. (2000) also argue that there are several dynamical mechanisms able to force large eccentricities and inclinations on main-belt asteroids, so they do not see any dynamical reason to demand that any significant fraction of the NEA population must come from a comet source. On the other hand, Rickman et al. (2001) argue that NEAs in JF-type orbits are too numerous to have an origin in the main asteroid belt, so they might presumably be inactive comets, which should exceed the active comets in the ratio of two to one. This corresponds to the lower end of the range 2.0-6.7 derived by Levison and Duncan (1997) from massive numerical integrations of test bodies evolving from the Kuiper belt to JF orbits.

One may question whether comets are a suitable extra source of NEAs. Comets are observed to fade away, split, or disintegrate, which suggests an extreme friability of the nucleus material, making it difficult for their survival as inactive, asteroidlooking objects. Fernández et al. (2001) have recently argued that some extinct comet candidates among the asteroids show very low geometric albedos ( $p_{v} \sim 0.02-0.03$ ), similar to those found for comets, but significantly lower than the geometric albedos of NEAs with Tisserand constant $>3$ ( $p_{v} \sim 0.1-0.6$ ). Yet, among the asteroids of the outer belt ( $a \gtrsim 3.5 \mathrm{AU}$ ) there is a predominance of spectral types P and D (e.g., Hartmann et al. 1987). These have spectra that are dark, are red to very red, and are probably of very low albedo, so a low albedo does not necessarily mean a cometary origin, since the outer belt is also a possible source of dark objects.

Binzel et al. (1992) found that the distributions of spin periods of NEAs and JF comets are statistically distinct: While NEAs show a significant fraction of fast rotators, there are no fast rotators among the JF comets. Binzel et al. then concluded that no more than $40 \%$ (but it may as low as $0 \%$ ) of the NEA population can be derived from extinct or dormant comet nuclei.

The issue discussed before has relevance to understanding the physical processes affecting the evolution of a comet nucleus, such as dust-mantle buildup, outbursts, and splittings. These processes act in opposite sense: Dust-mantle buildup may favor the preservation of the object under an asteroidal appearance; outbursts and splittings will enhance sublimation and ultimate disintegration of the body. We shall further analyze the problem of the mixing of both populations-asteroidal and cometaryusing results from numerical integrations.

## 2. POPULATION AND DYNAMICAL PROPERTIES OF NEAs AND JF COMETS

The Tisserand invariant derived from the Jacobi integral of the circular, restricted three-body problem provides a useful criterion for distinction between NEAs and JF comets (e.g., Kresák 1979). For a body moving on an orbit with perihelion distance $q$, semimajor axis $a$, and inclination $i$, the Tisserand invariant $T$ is given by

$$
\begin{equation*}
T=\frac{1}{a}+2 \sqrt{2 q\left(1-\frac{q}{2 a}\right)} \cos i \tag{1}
\end{equation*}
$$

which is valid under the assumptions that the perturbing planet (Jupiter) has a circular orbit of unit radius and that other planets do not perturb the body. The encounter velocity $U$ of the body with respect to Jupiter's (circular) motion can be expressed in terms of $T$ as

$$
\begin{equation*}
U=(3-T)^{1 / 2} \tag{2}
\end{equation*}
$$

which shows that encounters with Jupiter are possible only if $T<3$.

Figure 1 plots the aphelion distance $Q$ versus the Tisserand parameter $T$ of all cataloged NEAs with $Q>3.5 \mathrm{AU}$ and


FIG. 1. Aphelion distance vs Tisserand parameter of NEAs with $Q>$ 3.5 AU and JF comets with $q<1.3 \mathrm{AU}$. Only two comets-2P/Encke and 107P/Wilson-Harrington-shared the NEA space ( $Q<4.5 \mathrm{AU}, T>3$ ) and are clearly detached from the rest of the JF comets.

Earth-approaching JF comets with $q<1.3 \mathrm{AU}$ taken from the sources described in Section 3. We can see that both populations tend to occupy different regions in the parametric plane ( $Q, T$ ): Most NEAs have $T>3$, so encounters with Jupiter are not possible at present, while most JF comets have $T<3$, indicating that they are subject to close interactions with Jupiter. Notwithstanding the segregation of both populations, a small fraction of NEAs extend and overlap the JF comet zone. These large- $Q$ NEAs, for which we set the rather arbitrary limit $Q=4.5 \mathrm{AU}$, are what we shall describe as NEAs in "cometary" orbits and the question is whether some of them are indeed inactive comets. Almost all NEAs in cometary orbits have $T<3$, which indicates that they are subject to close encounters with Jupiter like JF comets. It is interesting to note that two comets-2P/Encke and 107P/Wilson-Harrington-are clearly detached from the rest of the JF population and they are the bodies in our JF comets sample with $T>3$. We will analyze in the following the reason why such a detachment implies a shortening of their aphelion distances.

Fernández et al. (1999) have estimated the population of JF comets in different ranges of $q$. If we use their results, we derive a population of $30_{-5}^{+10}$ comets with $q<1.3 \mathrm{AU}$ down to an absolute nuclear magnitude $H_{\mathrm{N}}=18$ (which corresponds to a nucleus radius $R_{\mathrm{N}} \sim 0.8 \mathrm{~km}$ for an assumed geometric albedo $\left.p_{v}=0.04\right)$. The known population of NEAs in cometary orbits has shown a dramatic increase in the past few years, but most of the members are very faint $(H>18)$ (Fig. 2). On the other hand, the observed population of bright $(H<16)$ members ( 7 ) shows a very modest increase, suggesting that it is close to completeness. We can assume $N(H<16)=10$ and extrapolate it to fainter members, provided that we know the cumulative mass distribution of NEAs. Rabinowitz (1993) found an index of -0.66 , while Bottke et al. (2000) estimated a cumulative size distribution proportional to $D^{-1.8}$ for NEAs with diameters $D$ between 170 m and 4 km . Their result will translate into a cumulative mass distribution of index -0.60 , i.e., only slightly smaller than


FIG. 2. Absolute magnitude vs discovery year of NEAs in cometary orbits ( $Q>4.5 \mathrm{AU}$ ). As seen, nearly all discoveries in the past 10 years are for $H>16$.
the one obtained by Rabinowitz. Using the previous results to extrapolate the population of bright NEAs down to $H=17.1$, we can estimate it at about 30 members. The magnitude $H=17.1$ corresponds to the radius $R=0.8 \mathrm{~km}$ for a geometric albedo $p_{v}=0.1$, which is close to the mean albedo for main-belt asteroids (Rabinowitz et al. 2000). If we assume instead that NEAs in cometary orbits are as dark as JF comets ( $p_{v}=0.04$ ), we have to extrapolate down to $H=18$, as for JF comets, instead of $H=17.1$. We obtain in this case a population of about 70. The estimated population of NEAs in cometary orbits of $\sim 30-70$ is in fairly good agreement with the Bottke et al. (2000) estimate of $\sim 900$ NEAs with $H<18$ ( $\sim 400$ with $H<17.1$ ), from which $\sim 10 \%$ have $Q>4.5 \mathrm{AU}$.

## 3. NUMERICAL INTEGRATIONS

We use our numerical code EVORB, which consists of a second-order symplectic integrator based on Wisdom and Holman's (1991) algorithm, with a Bulirsch-Stoer routine that computes every close encounter between a test body and a planet within three Hill radii. The accuracy of the integration of the massive objects (planets) was checked by the evolution of its total energy, which kept nearly constant in all our numerical integrations (it showed oscillations of at most one part in $10^{8}$ ). The precision of the integration for the case of particles encountering a planet was evaluated by computing the evolution of the Jacobi constant in the frame of the circular, restricted, three-body problem. After several hundreds of encounters the particles can experience relative changes in the Jacobi constant at most on the order of $10^{-5}$ to $10^{-6}$ with a time step $P / 50$, where $P$ is the revolution period of the planet, and this holds even for orbits with very small perihelion distances. Maximum changes of an order of magnitude greater can occur but only if $e>0.96$. The integrator was also tested by computing the orbital evolution of objects already studied by other authors and also reproducing the circumstances of the next two or three encounters with Earth of some potential hazardous asteroids (PHA), as predicted in the JPL NEO Web site (neo.jpl.nasa.gov/neo/pha.html).

We have computed the orbits of 324 NEAs with aphelion distances $Q>3.5 \mathrm{AU}$ and perihelion distances $q<1.3 \mathrm{AU}$ taken from the Lowell Observatory data base of asteroid orbital elements by M. Murison at the Web site http://Arnold.usno.navy. $\mathrm{mil} /$ murison/asteroids/. This data base was updated through April 30, 2001, and all asteroids have the same epoch. The integrations were followed for $5 \times 10^{6}$ years.

We have also integrated the orbits of 202 JF comets taken from the Catalogue of Cometary Orbits (Marsden and Williams 1999) and the JPL Solar System Dynamic Web site (http://ssd.jpl.nasa. gov/). In this case the integrations were followed for $2 \times$ $10^{6}$ years.

Both samples were integrated including the planets from Venus to Neptune and adding the mass of Mercury to that of the Sun. The time step for the sympletic integration was 0.01 year, which is roughly $1 / 60$ of the orbital period of Venus (the planet with the smallest orbital period in our integrations).

We have included nongravitational (NG) forces in some of the computer runs of JF comets under the following assumption: We adopted Comet 2P/Encke as a model for the NG force. 2P/Encke has shown an average delay in its perihelion passage of $\Delta P \sim 1$ day during its first century after discovery. The time delay has been decreasing with time and it is now $\sim 1 / 10$ day per orbital revolution. The time delay can be normalized for different comets with different semimajor axes $a$ to a standard semimajor axis $a=3.5 \mathrm{AU}$ according to the relation (Rickman et al. 1991)

$$
\begin{equation*}
\Delta P^{\prime}=\Delta P\left(\frac{a}{3.5 \mathrm{AU}}\right)^{-5 / 2} \tag{3}
\end{equation*}
$$

which measures the absolute strength of the NG effect independent of the orbital period. Furthermore, we know that the NG effect can accelerate or decelerate the comet (i.e., to cause a delay or an advance in the perihelion passage) for different comets or even for the same comet. The change in $\Delta P$ for a given comet can be smooth (as Encke's) or large and fast (as in 5D/Brorsen and 21P/Giacobini-Zinner), where $\Delta P$ changes its sign in a few revolutions. Bearing these different comet behaviors in mind, we have modeled the variation of the NG force with time, both in modulus and in sign, by means of a sinusoidal law following Steel and Asher's (1996) model for the Taurid Complex asteroids, namely

$$
\begin{equation*}
\delta v=A \sin \left(\theta_{o}+\frac{2 \pi\left(t-t_{o}\right)}{T}\right), \tag{4}
\end{equation*}
$$

where $\delta v$ is the change in the comet's velocity $v$ during $(t, t+\delta t), \theta_{o}$ is the phase angle at the initial time $t_{o}$, and $T$ is the time scale for the variation of the NG effect from positive to negative values. Encke's example shows us that it may take several hundred years for the NG force to change sign, so we adopt $T=500$ years. Since most comets show random activity, such as outbursts and splittings, or even episodic activation of new emission regions or deactivation of existing ones, their NG forces can experience abrupt changes on time scales of a few revolutions (Sekanina 1993). We have taken this effect into consideration by introducing a random change $\theta_{o}$ in the phase angle of Eq. (4) on a time scale of 50 years. The amplitude of the NG force, is given by

$$
\begin{equation*}
A \simeq 10^{-6} \frac{\left(G M_{\odot}\right)^{2}}{v a^{3 / 2}} \delta t \tag{5}
\end{equation*}
$$

where $G$ is the gravitational constant and $M_{\odot}$ is the Sun's mass. Equation (5) is derived in the Appendix.

## 4. RESULTS

### 4.1. NEAs

From the initial sample of 324 NEAs, there were 78 survivors at the end of the studied period $\left(t=5 \times 10^{6}\right.$ years). The rest

TABLE I
End States of NEAs

| Range of $Q(\mathrm{AU})$ | Ejected/Sun-colliders |
| :---: | :---: |
| $>4.5$ | 3.7 |
| $4.25-4.50$ | 0.75 |
| $4.00-4.25$ | 0.51 |
| $3.75-4.00$ | 0.30 |
| $3.50-3.75$ | 0.35 |

were lost during the integration due to (1) hyperbolic ejection (99 NEAs), (2) collision with the Sun (144 NEAs), or (3) collision with Jupiter ( 3 NEAs). The ratio of ejected NEAs to those colliding with the Sun depends on $Q$, as shown in Table I. We find that NEAs whose aphelia are well below Jupiter's orbital radius tend to end their dynamical evolution colliding with the Sun rather than being ejected by Jupiter. For NEAs approaching Jupiter ( $Q>4.5 \mathrm{AU}$ ) there are about three that are ejected by every one colliding with the Sun, while the opposite holds for NEAs with $Q<4 \mathrm{AU}$. The dynamical lifetime $\tau_{\mathrm{dyn}}$ also depends on $Q$. The half lifetime of NEAs in cometary orbits is found to be $4.2 \times 10^{5}$ years, while it increases to $1.35 \times 10^{6}$ years for NEAs with $4.0<Q \leq 4.5 \mathrm{AU}$, and to $2.1 \times 10^{6}$ years for NEAs with $3.5<Q \leq 4.0 \mathrm{AU}$.

As the dynamical evolution of NEAs proceeds, there is a slow diffusion of their aphelia outward and inward. It is worthwhile to pay attention to those diffusing outward, since they may be a source of replenishment of NEAs in cometary orbits. We show in Fig. 3 the evolution of one such NEA: 2000 CO33, a


FIG. 3. Evolution of perihelion distance ( $q$ ), semimajor axis ( $a$ ), and aphelion distance $(Q)$ of NEA 2000 CO33. The object experiences frequent encounters with Venus, Earth, and Mars at less than a half Hill's radius (represented by open squares at the respective distances to the Sun of Venus, Earth, and Mars). At $t=1.08 \times 10^{6}$ years the object experiences an encounter with Earth and immediately afterward three encounters with Jupiter (open squares at Jupiter's heliocentric distance), which raise its perihelion to the Jupiter-Saturn region, and the body acquires a very eccentric orbit which allows close encounters with Saturn (open squares at Saturn's distance).


FIG. 4. Fraction of NEAs within the range of aphelion distances indicated at the upper left corner of each plot (in AU) as a function of time.
half-kilometer-sized body whose initial orbit is $q=1.016 \mathrm{AU}$, $Q=3.651 \mathrm{AU}, i=18.2$. The object experiences frequent close encounters (< half Hill's radius) with Venus, Earth, and Mars. Its semimajor axis stays more or less constant during $6 \times 10^{5}$ years, while its perihelion and aphelion random walk with a tendency to decrease the former and to increase the latter. Afterward, the object increases somewhat its semimajor axis and the average $Q$, becoming an "NEA in cometary orbit" for most of its remaining lifetime. At $t=1.08 \times 10^{6}$ years, the object raises its perihelion to Jupiter's region, becoming a long-period asteroid, until it is ejected at $t=1.47 \times 10^{6}$ years. In general, our numerical integrations show that perturbations by the terrestrial planets play a fundamental role in raising the aphelia of NEAs to Jupiter's region.

There is always a certain fraction of NEAs in cometary orbits (i.e., with $Q>4.5 \mathrm{AU}$ ), among the surviving NEAs with $Q>3.5 \mathrm{AU}$ (Fig. 4), that on average varies very little with time, despite some fluctuations probably due to the smallness of the sample. At the lower end, some NEAs are transferred to orbits with $Q<3.5 \mathrm{AU}$, reaching $\sim 34 \%$ of the survivors at $5 \times 10^{6}$ years. The $Q$-distribution of the remaining NEAs with $Q>3.5$ AU keeps more or less constant with time. The continuous presence of a certain fraction of NEAs in cometary orbits
can thus be explained as the result of the slow diffusion outward of NEAs with smaller $Q$, without the need of invoking an extra source (e.g., JF comets).

### 4.2. JF Comets

We have computed the sample of all JF comets (both active and extinct) discovered through December 31, 2000, with the exceptions of Shoemaker-Levy 9 (D/1993 F2) and Lexell (D/1770 L1) because of their well-known fate (collision with Jupiter for the former and ejection by Jupiter for the latter). As in Marsden and Williams's (1999) catalog, we have included the strange object 133P/Elst-Pizarro (also cataloged as asteroid 7968 Elst-Pizarro), which is entirely within the main asteroid belt ( $q=2.63 \mathrm{AU}, Q=3.68 \mathrm{AU}, i=1.38$ ) and showed some activity at discovery. This object remains stable during the studied period: Its semimajor axis keeps within $\pm 0.014 \mathrm{AU}$ of the mean value, while $q$ and $Q$ show oscillations of $\pm 0.15 \mathrm{AU}$ with a quasiperiod of $\sim 4 \times 10^{4}$ years.

From the initial sample of 202 JF comets, 187 were lost during the integration for $t=2 \times 10^{6}$ years. Their half lifetime is found to be $1.95 \times 10^{5}$ years for the whole sample, and it decreases to $1.45 \times 10^{5}$ years for the subsample of near-Earth JF comets with $q<1.3 \mathrm{AU}$, i.e., about three times smaller than that for NEAs in cometary orbits. The general end state is hyperbolic ejection (178); only 5 end up colliding with the Sun and 4 with Jupiter.

Figures 5 and 6 show two interesting examples of orbital evolution of JF comets: P/Korlevic (1997 WJ7) and 503D/Pigott. Both runs are for purely gravitational solutions. As shown, after an initial period of chaotic dynamics P/Korlevic evolves to an Encke-type orbit and ends up colliding with the Sun at $1.78 \times$ $10^{5}$ years. On the other hand, 503D/Pigott evolves to an Elst-Pizarro-type orbit, where it falls into ever deeper regions of the


FIG. 5. Evolution of perihelion distance ( $q$ ), semimajor axis (a), and aphelion distance ( $Q$ ) of JF Comet P/Korlevic (1999 WJ7) that ends up colliding with the Sun. It acquires an Encke-type orbit during a short time. The results are for numerical integration without nongravitational forces.


FIG. 6. Evolution of perihelion distance ( $q$ ), semimajor axis ( $a$ ), and aphelion distance $(Q)$ of JF Comet 503D/Pigott. Results are for numerical integration without nongravitational forces.

2:1 mean-motion resonance with Jupiter, remaining in such a state for most of the studied period. Such stability contrasts with the more typical comet behavior consisting of brief passages through resonant states and quick jumps or "hops" between different resonances (Belbruno and Marsden 1997).

The transfer of JF comets to orbits within the main asteroid belt is interesting, since it may provide clues to whether bodies that showed unexpected activity (e.g., 133P/Elst-Pizarro itself) might be comet interlopers. The fact that 503D/Pigott can evolve to such an orbit leaves open this possibility, even though it is very difficult to estimate the steady-state number of comet interlopers in the main belt. Our comet interlopers obtained from purely graviational solutions-503D/Pigott and 97P/Metcalf-Brewington-showed high inclinations ( $\sim 25^{\circ}-30^{\circ}$ ) in contrast to the observed low inclination of 133P/Elst-Pizarro, which may argue against a cometary origin for this particular body. Indeed, 133/Elst-Pizarro may be a bona fide asteroid whose activity arose by the collision with another asteroid or with collisional debris of a parent asteroid on a neighbor orbit (Toth 2000).

It is interesting to see that of the five JF comets that end up colliding with the Sun, one of them-96P/Machholz 1is presumed to be a Halley-type rather than a JF comet, since it has a Tisserand constant $T<2$ (Fernández 1994). The other two comets-2P/Encke and 107P/Wilson-Harrington-are very special; the first one is thought to have been decoupled by Jupiter by NG forces, while the actual nature of the latter is still controversial. We have tested the end states of three of the five comets that collide with the Sun with samples of 10 clones. All our clones were generated keeping the same orbital elements as the original body and varying the initial mean anomaly by a random quantity within a range of $\pm 20^{\circ}$. In the case of 66P/du Toit we find 3 collisions with the Sun, 6 ejections, and 1 collision with Jupiter. For 107P/Wilson-Harrington, there are 3 collisions with the Sun and 7 ejections, and for $\mathrm{P} /$ Korlevic all of the clones
were ejected, so our first run was a very peculiar case in which the comet is decoupled from Jupiter by a strong perturbation by Earth. We also integrated 2P/Encke several times and in all cases the comet ended up colliding with the Sun on a time scale of $\sim 10^{5}$ years, being that its dynamical evolution is dominated mainly by the secular resonance with the apsidal motion of Saturn, $v_{6}$, in agreement with previous results from other authors (Valsecchi et al. 1995).

The perihelion distances $q$ of JF comets evolve very quickly and generally leave the region with $q<1.3$ AU after a few $10^{4}$ years. At $10^{5}$ years essentially no JF comets remain with $q<1.3 \mathrm{AU}$; the only exceptions are 2P/Encke and 107P/ Wilson-Harrington.

We have integrated the orbits of the 202 discovered JF comets plus four clones for each comet (generated in the same way as describe in the preceding), giving a total sample of 1010 bodies, adding the NG force as described in Section 3. We have adopted two values for the NG force: (1) a value similar to the largest value found for $2 \mathrm{P} /$ Encke acting during $10^{5}$ years and (2) a value 10 times greater than the previous one acting during $10^{4}$ years. The integrations were carried out for $2 \times 10^{5}$ years and for $2 \times 10^{4}$ years, respectively; i.e., we allowed the test comets to be under the action of NG forces during the first half of the integration period. Our intention for continuing the orbital integration after the NG forces ceased was to analyze whether there were lasting influences of their action on the later purely gravitational motion.

We paid special attention to the decoupling of comets from Jupiter to Encke-type orbits (namely, in near-Earth orbits with $q<1.3 \mathrm{AU}$ and aphelion distances $Q<4.2 \mathrm{AU}$ ). Figure 7 shows the example of P/Jager (1998 U3) where the integration was carried out with an Encke-type NG force. We can see that NG forces are efficient in bringing the comet to an Encke-type orbit after 45,000 years. The comet ends up colliding with the Sun.


FIG. 7. Evolution of perihelion distance ( $q$ ), semimajor axis (a), and aphelion distance ( $Q$ ) of JF Comet P/Jager (1998 U3). Results are for numerical integration with Encke-type nongravitational forces.

A much more efficient transfer to Encke-type orbits is obtained with a NG force equal to 10 times that of Encke, in some cases on time scales as short as a few thousand years. Presumably such strong NG forces acting during several thousand years are unrealistic, but nevertheless they are useful for illustrating what would happen in very extreme cases. We will analyze this point in greater detail in the following.

## 5. THE DECOUPLING OF JF COMETS FROM JUPITER: EVALUATION OF ITS EFFICIENCY

By contrast to our definition of "cometary" orbits, typical NEA orbits will be understood as those with aphelion distances $Q<4.5 \mathrm{AU}$. According to our definition, the only JF comets that currently penetrate within the NEA domain are 2P/Encke and 107P/Wilson-Harrington. As mentioned, there are some doubts as to whether 107P/Wilson-Harrington is a mildly active or inactive comet or a bona fide asteroid. Furthermore, we will consider that a body is "decoupled" from Jupiter when $Q<4.2$ AU; i.e., it is far enough to be relatively safe from Jupiter's strong perturbations. In other words, it will always stay at more than two Hill's radii from Jupiter.

It is still not very clear how some JF comets can decrease their aphelia while simultaneously keeping small perihelion distances. The most striking example is of course 2P/Encke. Wetherill (1991) argues that several dynamical mechanisms can decouple JF comets from Jupiter into Encke-type orbits on time scales of $10^{5}-10^{6}$ years. He quotes (a) secular and resonant perturbations by Jupiter and the other giant planets, (b) perturbations by the terrestrial planets, and (c) nongravitational forces.

From our orbit computations we have analyzed, among other things, how often JF comets penetrate within the NEA domain. For the case of purely gravitational solutions, there were a few cases in which the aphelion distances decreased to values $Q<4.5 \mathrm{AU}$. These were limited to 11 comets (other than 2P/ Encke and 107P/Wilson-Harrington). It is interesting to note that most comets that drastically decrease their aphelion distances simultaneously increase their perihelion distances; i.e., they essentially change their eccentricity, keeping their semimajor axes nearly constant. Only two comets-10P/Tempel 2 and $\mathrm{P} /$ Korlevic-acquired $Q<4.5 \mathrm{AU}$ while keeping $q<1.3 \mathrm{AU}$.

As mentioned above (cf. Section 4.2), only P/Korlevic evolves under a purely gravitational integration to an Encke-type orbit. The average probability that a JF comet will be on an Encke-type orbit at any time will be approximately given by

$$
\begin{equation*}
p=\frac{\sum_{i=1}^{N_{\mathrm{FF}}}\left(t_{\mathrm{ET}}\right)_{i}}{\sum_{i=1}^{N_{\mathrm{F}}}\left(t_{\mathrm{JF}}\right)_{i}} \simeq 4 \times 10^{-5}, \tag{6}
\end{equation*}
$$

where $\left(t_{\mathrm{ET}}\right)_{i}$ is the time spent by comet $i$ on an Encke-type orbit, and $\left(t_{\mathrm{JF}}\right)_{i}$ is its lifetime as a JF comet. We can compare our result with that of Harris and Bailey (1996), who found a transition probability $<0.0028$ to an orbit with $q<1.4 \mathrm{AU}$ and $Q<$ 4.2 AU. If we assume that the total JF population is about $6 \times 10^{3}$
comets with $H_{\mathrm{N}}<18$ (cf. Fernández et al. 1999), we find that for steady-state comets in Encke-type orbits $p \times n \simeq 0.24$; i.e., there is a $25 \%$ chance of having an Encke at any time if we consider purely gravitational solutions. We stress that this result is very uncertain, since it is based on a single case of a JF comet transferred to an Encke-type orbit, but it nevertheless suggests that a purely gravitational motion very seldom produces such an orbit.

We will now analyze whether nongravitational forces may account for the shortening of the aphelion distances of some inactive JF comets now disguised as NEAs. The observed sample of JF comets shows variations $\Delta P$ in the orbital periods of JF comets of the order $\Delta P \sim 0.001-0.1$ days/revolution for a JF comet with a standard semimajor axis $a=3.5 \mathrm{AU}$ (orbital period $P=6.55$ years) (Rickman et al. 1991). Since the change in the period $\Delta P$ tends to vary randomly (sometimes being positive and sometimes being negative in the same comet), the cumulative change due to NG forces will be best represented by the quadratic sum of the $\Delta P$ terms per revolution rather than the lineal sum; i.e., after $N$ revolutions the cumulative change will be approximately given by

$$
\begin{equation*}
\Delta P_{\text {cum }} \sim N^{1 / 2} \times \Delta P \tag{7}
\end{equation*}
$$

Significant changes in the period (and thus in $a$ ) will occur when $\Delta P \sim P$, which gives $N \sim 10^{8}-10^{12}$ revolutions; i.e., this range of $N$ turns out to be much longer than the typical dynamical lifetimes of JF comets.

We can ask the question whether JF comets may transit through very active phases with strong NG perturbations. The change $\Delta P$ due to NG forces is (Rickman et al. 1991)

$$
\begin{equation*}
\Delta P \propto \frac{f_{\mathrm{m}} Z_{\mathrm{m}}}{\rho\langle R\rangle} \tag{8}
\end{equation*}
$$

where $f_{\mathrm{m}}$ is the maximum fraction of free-sublimating area reached during a revolution, $Z_{\mathrm{m}}$ is the free-sublimation gas flux at that moment, $\rho$ is the nucleus density, and $\langle R\rangle$ is its mean radius. But the sublimation lifetime $L_{\mathrm{s}}$ is approximately proportional to $\rho\langle R\rangle / f_{\mathrm{m}} Z_{\mathrm{m}}$, so that

$$
\begin{equation*}
\Delta P \propto \frac{1}{L_{\mathrm{s}}} . \tag{9}
\end{equation*}
$$

Therefore, comets with small $L_{\mathrm{s}}$ can experience large $\Delta P$, but then it is very doubtful that such comets can survive long enough to see their periods (and thus their $Q$ 's) significantly shortened. On the other hand, large bodies can have long $L_{\mathrm{s}}$, but the NG effect is accordingly reduced, so it is hard to see how they can decrease their aphelia to values $Q<4 \mathrm{AU}$ before being ejected.

Our previous theoretical analysis was complemented with numerical integrations with NG forces as described in Section 3 for the samples of JF comets described in Section 4.2. A NG force similar to that of Encke acting during $10^{5}$ years $\left(\sim 10^{4}\right.$ revolutions) decreases the aphelion distances of a few comets while


FIG. 8. Distribution of JF comets in the plane ( $q, Q$ ). (a) Current orbits of the observed sample. (b) Location of the surviving comets every $10^{3}$ years, where their orbits were integrated without NG forces for $2 \times 10^{6}$ years ( $2 \mathrm{P} /$ /Encke and 107P/Wilson-Harrington were removed from the sample here and in the following plots). (c) Same as (b) but for integrations for $2 \times 10^{5}$ years with NG forces $=$ Encke's NG forces acting during the first $10^{5}$ years. (d) Same as (b) but for integrations for $2 \times 10^{4}$ years with NG forces $=10 \times$ Encke's NG forces acting during the first $10^{4}$ years. The Encke zone is enclosed by the rectangle at the lower left corner, where $q<1.3 \mathrm{AU}$ and $Q<4.2 \mathrm{AU}$.
keeping their perihelion distances small, thus producing Encketype objects. We can compute the probability that a JF comet be on an Encke-type orbit by means of Eq. (6), obtaining $2 \times 10^{-3}$, so the steady-state number of Enckes for a JF population of 6000 comets with $H<18$ will be $2 \times 10^{-3} \times 6000=12$. Therefore, NG forces might help to produce a handful of Enckes at any time. When we add a NG force equal to 10 times that of Encke acting during $10^{4}$ years ( $\sim 10^{3}$ revolutions), the number and time span of comets reaching Encke-type orbits increase significantly.

Figure 8 shows a plot of the observed sample of JF comets in the plane $(q, Q)$ at the time of their discovery, compared with the distribution in the same plane of the survivors plotted at intervals of $10^{3}$ years, considering their dynamical evolution without and with NG forces equal to Encke's NG force and 10 times Encke's NG force. The efficiency of producing Encke-type orbits is represented by the ingress of comets within Encke's zone (defined by the rectangle of sides $q=1.3 \mathrm{AU}$ and $Q=4.2 \mathrm{AU}$ ). As seen, the sample has only one case at present, 2P/Encke itself, and another very close, 107P/Wilson-Harrington. In the case without NG forces there is only one comet that invades Encke's zone during the studied period of $2 \times 10^{6}$ years: P/Korlevic (1999

WJ7). (In this and the following plots we have eliminated Encke and Wilson-Harrington.) We see that the degree of penetration within Encke's zone increases significantly when we increase the strength of the NG force. Similar results were obtained by Harris and Bailey (1996), but under different assumptions and considering a somewhat enlarged Encke zone (same boundary for $Q$, but a periheion distance $q=1.4 \mathrm{AU}$ ). The NG model of Harris and Bailey only considered the transverse component of the NG force: $A_{2}$ with a value $\sim 10$ times that for Encke. On the other hand, they neglected the radial component of the NG force, $A_{1}$, even though it may play a fundamental role in the case of asymmetric lightcurves (with respect to the perihelion passage), so it seems more apropriate to choose $\Delta P$ to model the NG force (Rickman et al. 1991).

The results presented in Fig. 8 should be interpreted with care. First, one may wonder whether a comet can be so active during $\sim 10^{3}$ revolutions, such that its NG force can equal $10 \times$ Encke's NG force. As discussed before, if the comet is too small, it will probably disappear on a much shorter time scale. If the comet is too large, the NG effect will be much smaller since $\Delta P$ is inversely proportional to its size (cf. Eq. (8)). Furthermore, such
big comets will very likely build dust mantles that will choke off the sublimation of volatiles (e.g., Rickman et al. 1990). This suggests that large NEAs must have an origin in the main asteroid belt, despite the seeming difficulty in explaining the supply of multikilometer bodies from this region (Migliorini et al. 1998). It is difficult to see how bodies like 2201 Oljato on a typical NEA orbit ( $q=0.62 \mathrm{AU}, Q=3.72 \mathrm{AU}, i=2^{\circ} .52$ ) may be evolved JF comets that have become inactive. The dynamical routes from JF comets to Oljato-type orbits are scarce and mainly depend on very rare, strong perturbations from the terrestrial planets. Actually, the detachment of 2P/Encke from the rest of the JF population (cf. Fig. 1) suggests that NG forces are not very effective in shortening their aphelion distances; otherwise the gap in the range of aphelion distances $Q \sim 4.2-4.6$ AU would be filled with other JF comets.

## 6. PHYSICAL END STATES OF JF COMETS: DEACTIVATION VERSUS DISINTEGRATION

The question of whether some NEAs may be inactive comets can be analyzed from the point of view of the physical nature of comets rather than their dynamical properties. It is well known that some periodic comets suffer frequent splittings (e.g., Chen and Jewitt 1994) and some of them ceased to be observed. There are good reasons to suspect that they disintegrated, generally after suffering several splittings. This seems to have been the case of 3D/Biela and 18D/Perrine-Mrkos. The discovery of faint, near-Earth JF comets may be a consequence of the fragmentation of worn-out comet nuclei, leading to a copious release of dust that favors their detection, as happened with 141P/Machholz 2 (Sekanina 1999). Comet C/1999 S4 (LINEAR) may probably be the most recent case of the demise of a comet. It was presumably disrupted in innumerable fragments ranging in size from millimeters to $\sim 50 \mathrm{~m}$ (Farnham et al. 2001). Table II shows the list of JF comets with $q<1.3 \mathrm{AU}$ that ceased to be observed and the number of apparitions $N$ in which they were observed.

The orbital evolution of the JF comets in Table II shows that no dynamical reason, either ejection or a large increase in their $q$ (Tancredi and Rickman 1992), can explain their negative de-

TABLE II
JF Comets with $q<$ 1.3 AU That Have Disappeared

| Comet | Discovery year | Last apparition | $q(\mathrm{AU})$ | $N$ |
| :--- | :---: | :---: | :---: | :---: |
| Helfenzrieder (D/1766 G1) | 1766 | 1766 | 0.41 | 1 |
| Lexell (D/1770 L1) | 1770 | 1770 | 0.67 | 1 |
| 3D/Biela | 1772 | 1852 | 0.99 | 6 |
| Blanpain (D/1819 W1) | 1819 | 1819 | 0.89 | 1 |
| 5D/Brorsen | 1846 | 1879 | 0.65 | 5 |
| Barnard 1 (D/1884 O1) | 1884 | 1884 | 1.28 | 1 |
| Denning (D/1894 F1) | 1894 | 1894 | 1.15 | 1 |
| Swift (D/1895 Q1) | 1895 | 1895 | 1.30 | 1 |
| 18D/Perrine-Mrkos | 1896 | 1968 | 1.11 | 5 |
| 34D/Gale | 1927 | 1938 | 1.21 | 2 |
| Haneda-Campos (D/1978 R1) | 1978 | 1978 | 1.10 | 1 |

tection in subsequent returns, with the exception of $\mathrm{D} / 1770 \mathrm{~L} 1$ Lexell, observed in 1770, which ceased to be observed in subsequent apparitions because it drastically changed its orbit after a close encounter with Jupiter, increasing its perihelion distance to about 4.8 AU .

If all the comets in Table II-except Lexell—faded away, we can estimate the rate of physical destruction at about 4/century. If there are about 100 JF comets with $q<1.3 \mathrm{AU}$ and $H_{\mathrm{N}}<19$, where $H_{\mathrm{N}}=19$ corresponds to a nucleus radius $R \sim 0.5 \mathrm{~km}$, thought to be about the minimum radius for an active comet (Fernández et al. 1999), the mean physical lifetime will approximately be

$$
\begin{equation*}
L_{\mathrm{ph}} \sim \frac{100}{0.04}=2500 \text { years } \tag{10}
\end{equation*}
$$

which is on the order of the time scale for removal of perihelia from the region $q<1.3 \mathrm{AU}$. Consequently, a significant fraction of JF comets can disintegrate before dynamical removal. The large meteoroids associated with the Leonid stream (Beech 1998) may be an indication of the progressive release of large chuncks of material from the nucleus of the associated comet, leading to its final disintegration.

As mentioned, the fraction of NEAs in cometary orbits with respect to the surviving population of NEAs with $Q>3.5 \mathrm{AU}$ remains more or less constant or at most shows a slight decrease through $5 \times 10^{6}$ years. Therefore, the population of NEAs in cometary orbits can be steadily maintained by the orbital diffusion of NEAs of smaller Q's (presumably of main-belt origin), so it does not seem necessary to invoke a significant extra source (JF comets) to replenish the dynamical losses. A strong contribution of inactive JF comets would be reflected by a marked excess of NEAs in cometary orbits at the beginning of the integration, which does not show up. As shown in Fig. 4, the decrease in the fraction of computed NEAs in cometary orbits is at most $\sim 0.04$ (against a background noise of $\sim 0.15$ ). If we accept that such a decrease is real and corresponds to an initial contribution of inactive comets to the population of NEAs in cometary orbits (that at the beginning makes up $\sim 0.18$ of NEAs with $Q>3.5 \mathrm{AU}$ ), it will then follow that $\sim 20 \%$ of the observed NEAs in cometary orbits would have cometary origin. Let $n_{\mathrm{CO}}$ be the total number of NEAs in cometary orbits with radius $R>0.8 \mathrm{~km}$ (or $H<18$ for albedo $p_{v}=0.04$ ) and assume it to be given by

$$
\begin{equation*}
n_{\mathrm{CO}}=n_{\text {inac }}+n_{\mathrm{a}} \tag{11}
\end{equation*}
$$

where $n_{\text {inac }}$ is the number of inactive JF comets with $q<1.3 \mathrm{AU}$ disguised as asteroids, and $n_{\mathrm{a}}$ is the number of bona fide NEAs coming from the main belt.

Let $n_{\mathrm{ac}}$ be the number of active JF comets with $q<1.3 \mathrm{AU}$, for which we found $n_{\mathrm{ac}} \sim 30$ brighter than $H_{\mathrm{N}}=18$ (cf. Section 2), and let $\Delta T_{\text {ac }}$ and $\Delta T_{\text {inac }}$ be the fractions of time that a JF comet with $q<1.3$ AU remains active and inactive, respectively. Thus we have $n_{\text {inac }}=n_{\text {ac }} \times \Delta T_{\text {inac }} / \Delta T_{\text {ac }}$ and $n_{\text {inac }} \sim 0.2 \times n_{\text {CO }}$ if we assume that $20 \%$ of NEAs in cometary orbits are of cometary
origin. Therefore, by combining these two expressions we get

$$
\begin{equation*}
\Delta T_{\mathrm{inac}} / \Delta T_{\mathrm{ac}}=\frac{0.2 \times n_{\mathrm{CO}}}{n_{\mathrm{ac}}} \tag{12}
\end{equation*}
$$

Introducing the numerical values $n_{\mathrm{ac}}=30$ and $n_{\mathrm{CO}}=50$ (cf. Section 2) we obtain $\Delta T_{\text {inac }} / \Delta T_{\mathrm{ac}} \sim 1 / 3$. Thus, the average time that JF comets can stay as inactive bodies (dormant or defunct) is at most about one third the time they stay as active. This is a rather short time. The conclusion drawn from this analysis is that progressive disintegration quickly reduces exhausted comet nuclei to meteoritic dust, in agreement with our previous analysis based on physical considerations.

One can argue that the observed sample of NEAs that we have integrated contains several biases that affect our previous conclusion. The biases in the observed population of NEAs have been analyzed by Bottke et al. (2000). There is an obvious bias against the discovery of faint NEAs, so the completeness factor decreases very quickly for absolute magnitudes $H>17$. The Bottke et al. results do not show any significant variation in the degree of completeness for the range of semimajor axes $2<a<3 \mathrm{AU}$ to which most of our test bodies belong. Yet, NEAs with $a>3$ AU are on average brighter in our sample, which indicates a greater degree of incompleteness. Thus, the average diameter of the discovered NEAs with $a>3$ AU is $\bar{D}=2.74 \mathrm{~km}$, compared with $\bar{D}=2.30 \mathrm{~km}$ for $2.5<a<3 \mathrm{AU}$ and $\bar{D}=2.16 \mathrm{~km}$ for $2<a<2.5 \mathrm{AU}$. If we assume, following Bottke et al., that the cumulative size distribution of NEAs is proportional to $D^{-1.8}$, the observed population of NEAs with $a>3 \mathrm{AU}$ will have to be multiplied by a factor of about 1.5 to match the degree of completeness of the remaining NEAs with $a<3 \mathrm{AU}$. If we make allowance for the missed NEAs with $a>3 \mathrm{AU}$ to bring this population to the same degree of completeness as the NEAs with $a<3 \mathrm{AU}$, the population of NEAs with $Q>4.5 \mathrm{AU}$ will have to be increased by about $18 \%$ (note that $36 \%$ of NEAs with $Q>4.5$ AU have $a>3 \mathrm{AU}$ ). The ratio $\Delta T_{\text {inac }} / \Delta T_{\text {ac }}$ will rise accordingly to about $40 \%$, which does not significantly change the above numerical result from Eq. (12).

As discussed by Bottke et al. (2000), there is also a bias against the detection of NEAs in high-inclination orbits $\left(i \gtrsim 30^{\circ}\right)$, though this bias does not seem to have any consequence in our previous analysis, in particular because these high-inclination NEAs are a small minority within the whole NEA population.

## 7. CONCLUDING REMARKS

Neither dynamical nor physical reasons seem to support the presence of a substantial fraction of inactive comets among the population of NEAs in cometary orbits. On the contrary, the dynamical analysis shows that the population of NEAs in cometary orbits can be explained as a diffusion from NEAs with shorter aphelion distances. In this process perturbations by the terrestrial planets, particularly Venus and Earth, seem to play a very important role in raising their aphelia. This is also consistent with other dynamical studies showing that a few hundred kilometersized bodies in the main asteroid belt can be injected into mean-
motion resonances (especially $3: 1$ and $5: 2$ ) and the $\nu_{6}$ secular resonance from where they quickly evolve to NEAs. Our study of the orbital evolution of the JF population shows that comet interlopers among NEAs, may be rare. It is also very difficult to produce Encke-type comets, since their formation mainly depends on the action of very strong NG forces for at least several tens of revolutions.

Summing up, we highlight the following points:

1. The population of NEAs in cometary orbits can be explained by the orbital diffusion of NEAs with smaller aphelion distances with little contribution (or no contribution at all) from an extra source (JF comets).
2. The average time that near-Earth JF comets $(q<1.3 \mathrm{AU})$ can stay as inactive bodies (dormant or defunct) is at most $40 \%$ of the time they stay as active, but it can be as low as zero.
3. It is very difficult to produce Encke-type objects by purely gravitational mechanisms ( $\sim 25 \%$ probability to have an object at any time).
4. It is possible to produce $\sim 10$ Encke-type objects $\left(H_{\mathrm{N}}<18\right)$ at any time under the action of a NG force similar to the one acting on Encke, but only if the force is exerted for $10^{5}$ years. This should be taken as an upper limit since it is very unlikely that a JF comet can remain so active for such a long time. More likely, bursts of strong activity combined with a moderate activity for a shorter time might be the way to produce a few Enckes.

## APPENDIX

The amplitude $A$ in Eq. (5) is derived as follows: We assume that the NG force acts continuously along the comet's orbit under the same law given by Eq. (4), which is independent of the heliocentric distance $r$. This of course is not true: NG forces are stronger close to perihelion, but since we are interested in the global effect of the NG force during a revolution, for the numerical integration it is better to consider that the NG force acts under the same law along the orbit, with the condition of producing the desired change $\Delta P$ per orbital revolution. Therefore, the change $\delta P$ in the orbital period $P$ during the time step $\delta t$ will be given by

$$
\begin{equation*}
\delta P=\Delta P^{\prime}\left(\frac{a}{3.5 \mathrm{AU}}\right)^{5 / 2} \frac{\delta t}{P} \tag{13}
\end{equation*}
$$

Since $P^{2}=4 \pi^{2} a^{3} / G M_{\odot}$ we obtain

$$
\begin{equation*}
\delta a=\frac{2}{3} \frac{a}{P} \delta P \tag{14}
\end{equation*}
$$

The orbital velocity is given by

$$
\begin{equation*}
v^{2}=G M_{\odot}\left(\frac{2}{r}-\frac{1}{a}\right) \tag{15}
\end{equation*}
$$

by differentiating Eq. (15) we get

$$
\begin{equation*}
\delta v=\frac{G M_{\odot}}{2 v a^{2}} \delta a \tag{16}
\end{equation*}
$$

By introducing Eqs. (13) and (14) into Eq. (16) we obtain

$$
\begin{equation*}
\delta v=\frac{1}{3} \frac{G M_{\odot}}{v a^{2}} \frac{a}{P} \Delta P^{\prime}\left(\frac{a}{3.5 \mathrm{AU}}\right)^{5 / 2} \frac{\delta t}{P} \tag{17}
\end{equation*}
$$

which leads to

$$
\begin{equation*}
\delta v=\frac{1}{275 \pi^{2}} \frac{\left(G M_{\odot}\right)^{2}}{a^{3 / 2}} \frac{1}{v} \Delta P^{\prime} \delta t \tag{18}
\end{equation*}
$$

where, according to our model of sinusoidal law for the NG effect, we set $\Delta P^{\prime}=\Delta P_{o}^{\prime} \sin \left(\theta_{o}+\frac{2 \pi\left(t-t_{o}\right)}{T}\right)$ and adopt $\Delta P_{o}^{\prime}=\frac{1}{365}$ year, which is of the order of the maximum NG perturbation in $P$ observed in 2P/Encke (see Section 3). Substituting $\Delta P^{\prime}$ into Eq. (18) and writing it as an equation of the form given by Eq. (4) we get

$$
\begin{equation*}
A=\frac{1}{275 \pi^{2}} \frac{\left(G M_{\odot}\right)^{2}}{a^{3 / 2}} \frac{1}{v} \frac{1}{365} \delta t \simeq 10^{-6} \frac{\left(G M_{\odot}\right)^{2}}{v a^{3 / 2}} \delta t . \tag{19}
\end{equation*}
$$

## ACKNOWLEDGMENTS

We gratefully acknowledge financial support from the Comisión Sectorial de Investigación Cientifica (CSIC) of the Universidad de la República and the organizing committee of the Asteroids 2001 meeting that allowed us to present these results there. We also thank Gonzalo Tancredi and Andrea Sánchez for helpful discussions on the integration methods.

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