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On control of chaos in the Kerr lens mode locked Ti:Sapphire laser

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Abstract

In this paper, I present the numerical demonstration of a simple way to reduce the pulse duration of a Kerr lens mode locked Ti:Sapphire laser. The core of the method lies in the low dimensional deterministic chaos that displays the laser. By adding an acousto optic modulator and applying an adequate control algorithm, it is possible to reduce the pulse duration by a factor of two with a laser that otherwise operates in the range of 40 fs.

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1. Introduction

Ultrashort pulses are becoming increasingly important in almost all fields of science and technology. Femtosecond pulses are generated by several passive mode locking techniques in solid state materials ranging from Nd:Glass to Cr:Forsterite, and external cavity semiconductor lasers. However, the most widely used source of femtosecond pulses since its first demonstration by Spence et al. in 1991 [1] is the Ti:Sapphire laser. In particular Kerr Lens Mode Locking technology is ultimately the way to produce the shortest pulses and broadest spectra. In spite of its popularity, a major drawback of this kind of lasers is its tendency to instabilities in the pulse train. An important issue is, then, to understand and control these instabilities.

Besides the non-linearities that shares with all passive mode locked lasers, the KLM Ti:Sapphire laser has an intrinsically complex physics due to the coupling between spatial and temporal variables. In the temporal domain the group velocity dispersion (GVD) in all optical components located inside the cavity and the intensity dependent self phase modulation (SPM), mostly in the laser rod, are balanced by the dispersion produced by a pair of prisms. In the spatial domain, the relevant features are the cavity laser configuration and the intensity dependent self focusing. The amplification is an additional source of non-linearity, through gain saturation. As a result this kind of lasers displays a variety of hidden instabilities [2] in the pulse train that have undesirable consequences in any practical application. Different groups have shown self Q-switching, period doubling and tripling as well as chaotic behavior in KLM lasers [3]. In a recent work [4] the two coexistent pulsed modes of operation of the KLM Ti:Sapphire laser, named P_1 (transform limited output pulses) and P_2 (chirped output pulses) were described both theoretically and experimentally. It was experimentally found that each mode follows its own, clearly distinguishable, route to chaos: P_1 through quasiperiodicity, P_2 through a bifurcations' cascade and intermittencies. It was also found that the dimensionality of the attractors in the fully developed chaotic regime is between 3 and 4. What is important is

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that, in spite of the complexity of the non-linearities, the whole laser dynamics can be described by at most 4 independent observables of the system. The reached understanding of the Ti:Sapphire dynamics opens a door to new ways to stabilize and reduce the pulse duration based on the demonstration that this laser displays a low dimensional chaotic behavior.

In this paper, I present a control scheme that heavily relies on the deterministic low dimensional chaos of the KLM Ti:Sapphire laser. By using the well known OGY method plus the peak to peak dynamics to obtain a reduced model, it is possible, by adding an intracavity element that allows a fast modulation of the losses, to reduce the pulse duration by a factor of two. It is worth to mention that the map model assume a pulse duration greater than 10 fs in order to avoid the third order dispersion. So, in all the numerical simulations that follows, I check that the resulting pulse duration reached do not enter the sub 10 fs region. Even if this fact put a barrier to the shortest pulse duration, the whole laser that make possible a sub 10 fs pulse regime is different to the typical laser – hundred of femtosecond – of this paper. To make possible a sub 10 fs pulse it is mandatory to take into account higher order dispersion and use chirped mirrors in the cavity [5]. In this work, I develop a numerical model that is easy to implement experimentally, allowing a better performance of a Ti:Sapphire laser.

For example, working in the transform limited mode, the shortest measured pulse duration was 40 fs, and the model predicts, in the same situation, with a net GVD of $-150\ \mathrm{fs}^2$, a pulse of 49 fs. It is well known that decreasing, in modulus, the total intracavity – negative – GVD, the pulse duration is also shortened. However, when we approach the zero GVD region the laser becomes chaotic and the validity of linear relation between GVD and pulse duration is no longer valid. For our laser this region begins in the neighborhood of the $-50\ \mathrm{fs}^2$. So, between this value and $-150\ \mathrm{fs}^2$ the model is already valid and predicts an unstable behavior that can be stabilized by the proposed scheme of chaos control. In this case, and by only adding the AOM modulator and extracting a pair of millimeters the intracavity prisms, it is easily attainable a pulse duration of 25 fs.

In what follows, I will make use of the peak to peak dynamics to construct the control scheme, and then I will test it with a numerical simulation of the system employing the iterative map that has been proven to reliably reproduce the dynamics of the laser.

2. Chaos control

The major key ingredient for the chaos control scheme is the observation that a chaotic set, on which the trajectory of the chaotic process lives, has embedded within it a large number of unstable low-period orbits. Besides, because of ergodicity, the trajectory visits or accesses the neighborhood of each one of these periodic orbits. Some of these orbits may correspond to a desired system's performance according to some criterion, for instance an orbit in with shorter pulse duration. The other ingredient is the realization that these orbits can be altered by applying a small perturbation, just because the chaotic nature of the system. Then, the accessibility of the chaotic systems to many different periodic orbits combined with its sensitivity to small perturbations allows for the control of the chaotic process.

One of the simplest ways to achieve this control is the OGY method [6]. Roughly speaking, the OGY approach is as follows. The first step is the determination of the specific unstable periodic orbit to be stabilized, embedded in the chaotic set. Secondly, one examines the location and the stability of these orbits and chooses one which yields the desired system performance. Finally, one applies some clever small signal in the system that stabilizes the desired orbit.

The main objection to the OGY method is that one must wait until the system reaches, ergodically, the desired unstable periodic orbit. In a general case this time can be too large, making the complete scheme useless. Fortunately, in the case of Ti:Sapphire laser this argument is no longer valid, because the characteristic time evolution is given by the round trip time (typically in the order of 10 ns), so that in 1 s we have about 90 millions of pulses, enough to evolve close to every desirable point of the phase space. On the other hand, this same property makes the experimental realization of the control signal harder. In many cases this can be an excessively large quantity of information to handle. Therefore, methods or techniques for reducing the amount of information or the number of degrees of freedom, in such a way that the features of the dynamical evolution be retained are very useful.

It is well known that in certain chaotic systems the values of the peaks (relative maxima) of a given observable and its occurrence time can be predicted, with very good accuracy, knowing the previous peak values. This property, known as *peak to peak dynamics* will allow us to generate the control function as one variable map reduced model to be applied to a parameter of the system.

A word of warning is in order here. To the reader unfamiliar with the dynamics language it is easy to confuse the mode locking peaks with the peak to peak dynamics. It is important to distinguish between the maximum of each optical pulse and the relative maxima of a general observable (for example the pulse duration, the spot size, etc.) of the PPD dynamics.

In general, a chaotic system with a single output variable $y(t)$ is said to display peak to peak dynamics (PPD) when the set $\{(y_k,\ y_{k+1})\}$ of the pairs of consecutive peaks y_k can be approximated by one or more curves in the plane $(y_k,\ y_{k+1})$ [7,8]. This property is strictly related to the low dimensionality of the chaotic attractor. In the Kerr lens mode locking Ti:Sapphire laser, operating in the negative GVD zone, it has been recently demonstrated that the laser has a chaotic attractor, and that its embedding dimension is 4 [4,9].

3. System control design

Although the whole method has been previously described in great detail [7,8,10], I think it is useful to the "optical" reader to have a resume of the control scheme. This is done in the two following sections.

We name u as the control parameter, and assume that it can move in a certain range $U = [u_{\min},\ u_{\max}]$. For each value of u and, employing the map equations, it is possible to calculate the peak value of the observable y, and then obtain the peak to peak map, that is:

$$y_{k+1} = F(y_k, u), \tag{1}$$

where u is a fixed value of the control parameter.

Let suppose that u varies but that it is constant between two consecutive peaks:

$$u(t) = u_k \quad \forall t \in (t_k, t_{k+1}), \tag{2}$$

where t_k,t_{k+1} are the times of the peaks $y_{k,k+1}$.

Since Eq. (1) describes the dynamics of the system for a fixed value of u only after a transitory time (when the attractor $X(u)$ is reached), it is not correct to employ (1) and replace u by u_k to predict the value of the next peak, y_{k+1}. Nevertheless, it is reasonable to assume that, if the transient time is shorter that τ_k (the time between peak k and peak $k+1$), and if the difference between u_{k-1} and u_k is not too large, then the peak will remain well approximated by

$$y_{k+1} = F(y_k, u_k). \tag{3}$$

The validity of (3) is, definitely, hard to evaluate. However, a possible good estimation is given by Piccardi and Rinaldi [7], and is based on the Lyapunov exponents. Concretely, one must calculate the Lyapunov exponents of the system, particularly the most negative (λ^-), and compare the temporal constant, $\tau^- = |\lambda^-|^{-1}$, with the average time between consecutive peaks, τ_p. Allowing a typical transient time of $5\tau^-$, we will say that if

$$5\tau^- < \tau_p \tag{4}$$

the approximation is valid and the control scheme will be adequate.

4. Stabilization of the unstable periodic orbit

By employing the OGY method it is possible to stabilize, varying the control parameter, an unstable periodic orbit (UPO) embedded in the chaotic attractor.

Although the research has been very successful for slow systems (characteristic time scale >1 μs) [11,12], applying feedback control to fast chaotic systems is challenging because the controller requires a finite time to sense the current state of the system, determine the appropriate perturbation, and apply it to the system. This finite time interval, often called the control-loop latency, can be problematic if the state of the system is no longer correlated with its measured state at the time when the perturbation is applied. Typically, chaos control fails when the latency is on the order of the period of the UPO to be stabilized [13–15]. From the numerical simulation of the map equation of the Ti:Sapphire laser, the period of the UPO is about 10 round trip, which implies a speed limit in order of the 10 MHz. The control of very fast chaotic systems is an outstanding problem because of two challenges that arise: control-loop latencies are unavoidable, and complex high-dimensional behavior of systems is common due to inherent time-delays.

The problem becomes much easier if the dynamics is deduced from a reduced order model, such as the peak to peak map. In that case, the fixed point $y = F(\bar{y}, \bar{u})$ identifies the UPO of the trajectory. The linear approximation of the F map around $\bar{y}$ is given by

$$y_{k+1} - \bar{y}_k = A(y_k - \bar{y}) + b(u_k - \bar{u}), \tag{5}$$

where

$$A = \left[\frac{\partial F}{\partial y}\right]_{\bar{y},\bar{u}} \quad \text{and} \tag{6}$$

$$b = \left[\frac{\partial F}{\partial u}\right]_{\bar{y},\bar{u}} \tag{7}$$

are non-zero real numbers. The control law will be

$$u_k = \begin{cases} \bar{u} + h(y_k - \bar{y}) & \text{if } \|y_k - \bar{y}\| \leqslant \varepsilon, \\ \bar{u} & \text{if } \|y_k - \bar{y}\| > \varepsilon, \end{cases} \tag{8}$$

where h is now a scalar that must satisfy

$$|A + bh| < 1. \tag{9}$$

Note that the control u_k is allowed to vary between $(\bar{u} - h\varepsilon)$ and $(\bar{u} + h\varepsilon)$. Among those infinite values for h, we take

$$h = -\frac{A}{b}. \tag{10}$$

With this choice, $A + bh = 0$, and the control is optimal, in the sense that the response time of the linear approximation is minimum (i.e., equal to one: $y_{k+1} = \bar{y}$).

5. Implementation in the Ti:Sapphire laser

Prior to the development of the control scheme, I will describe both the map equation and the real laser over which the model is tested.

An iterative map is a set of recursive equations that relate the values of each of the pulse variables in the n round trip of the pulse into the laser cavity to the values that these variables had in the previous $n-1$ round trip. To describe the pulsed modes of operation of the Ti:Sapphire laser, I choose five variables: two spacial: spot size and beam curvature radius, two temporal: pulse duration and chirp and the pulse energy. As the pulse propagates through an optical element or a distance, the variables change according to the map equation. Besides the map's variables there are also a number of parameters that affect the variable's values. The geometrical ones are set to match those of the Ti:Sapphire laser of the [4]. It is a typical seven elements, constructed in X configuration with a flat rear mirror and a 12% output coupler. With a total length of 172.4 cm the repetition rate is 86.94 MHz. A pair of prisms of fused silica, separated for 60 cm, provides the GVD compensation. With the geometrical parameter's values that are summarized in Appendix 2, and depending on the prism insertion in the beam path, the laser can be operated with values of GVD ranging from positive to $-300\ \mathrm{fs}^2$.

The map model not only predicts the different dynamical regimes for P_1 and P_2 and the embedding and correlation dimensions of the attractors, but also the approximate size of the transition regions and the Fourier spectra. In what follows I use the map equation as a "virtual Ti:Sapphire" in order to evaluate the control scheme.

As a first step it is necessary that Eq. (4) holds for the Ti:Sapphire laser. Then, I construct, using the complete map equations, with the numerical values of the parameters as in Appendix 2, the temporal series belonging to embedding dimension 4 for both P_1 and P_2 modes of operation. In this condition for P_1 mode the net GVD is about $150\ \mathrm{fs}^2$ and the pulse duration is 40 fs in the P_1 mode and 74 fs in the P_2 mode. With these series, and employing an algorithm based on [9] and implemented by the TISEAN software [16], it is possible to determine the Lyapunov exponents of the series. In Table 1 the most negative one as well as the typical time between peaks are shown. It is clear that the approximation is valid for both P_1 and P_2.

Another important issue is the adequate election of the control parameter, in view of a future experimental implementation. The apparent choice for a KLM laser is the GVD. By only moving the micrometric screw that holds one of the intracavity prisms, it is possible to modify the net GVD of the laser. However, in a control scheme, peak to peak modifications of the parameter are needed. This means adjusting the control parameter with a typical frequency of the order of 10 MHz. This rules out the GVD as the control parameter, for it is not possible to modulate mechanical devices at such frequencies. A magnitude suitable for fast modifications is the cavity loss μ, which can be easily controlled by an intracavity acousto optic modulator. It is important to mention that from the point of view of the numerical simulations, the final result is independent of the control parameter chosen (see Appendix 1).

As to whether is the best parameter for the control implementation, it is well known [17] that the spatial variables (spot size and beam curvature) are the first ones to lose stability. The temporal variables (pulse duration and chirp) become unstable only after the spatial ones. This means that if we guarantee stability in the spatial variables the whole set of variables will remain stable. Moreover, taking into account that the spot size is easily monitored with a small area fast photodiode, I decide to pick up that variable to implement the control scheme.

Regarding the experimental implementation of the control scheme, the best way to fast control the intracavity loss is to add an acousto optic modulator (AOM) into the laser cavity. The device basically comprise a crystal where the traveling light is diffracted or transmitted depending on an applied electrical signal. The amount of the diffracted light is proportional to the amplitude of the applied signal. If there is no signal applied, the light goes through the modulator unaltered. Of course, adding any element into the cavity signify to add positive GVD. A typical fused silica glass AOM can increase the GVD by $70\ \mathrm{fs}^2$. Then, it is necessary to extract some amount of prism to compensate the additional GVD provided by the AOM. Taken into account that 0.5 mm of transversal to the beam path insertion of a prism make a change of $20.72\ \mathrm{fs}^2$ in the total GVD, the only additional change that the cavity needs to recover its previous GVD value is to extract less than 1 mm each prism.

The modulator input signal must be, of course (11). At this point, we are finally ready to run the control process, so let control the spot size (which is associated with the variable S in the five equation map model) of the chirped pulses mode (P_2) by employing the cavity losses, μ, as parameter.

We start with the values:

$$\bar{S} = 120,000\ \mathrm{cm}^{-1} \quad (\text{i.e., spot size}, \sigma = 28\ \mu\mathrm{m}),$$
$$\bar{\mu} = 3.15,$$

where the laser is known to be unstable in all its five variables. If it were stable, the pulse duration corresponding would be as short as 18 fs. It is adequate to remark that the shortest stable pulse duration predicted by the model for the P_2 mode of operation is 40 fs, and that the shortest observed value (for this mode, P_2) was 66 fs [9].

For the construction of the control function u_k(8), I need to determine the derivative values (6) and (7), where y now represents to S and u means μ. The calculus is made numerically with the map F, and the results are:

$$A = 42.124,$$
$$b = 41.74\ \mathrm{cm}^{-1},$$
$$h = -1.009\ \mathrm{cm}.$$

In this way the control function is written as

$$\mu_k = \begin{cases} \bar{\mu} + h(s_k - \bar{s}) & \text{if } \|s_k - \bar{s}\| \leqslant \varepsilon, \\ \bar{\mu} & \text{if } \|s_k - \bar{s}\| > \varepsilon. \end{cases} \tag{11}$$

Allowing variations up to 4%, that is, $\varepsilon = 0.04$, variations in the control parameter μ are restricted to the interval (3.1083; 3.1917).

In Fig. 1, it is shown the control of chaos obtained from a simulation of the complete map. At first the control is off,

Table 1
For the two pulsed modes of operation of the Ti:Sapphire laser it is shown the most negative Lyapunov exponent, λ^-; characteristic time between peaks, τ_p and the typical time

Mode	λ^-	τ_p	$\tau_p\lvert\lambda^-\rvert/5$
P_1	−0.572	18	5.148
P_2	−0.800	15	2.4

If this time is greater than one, the control is possible. Time unit: round trip time.

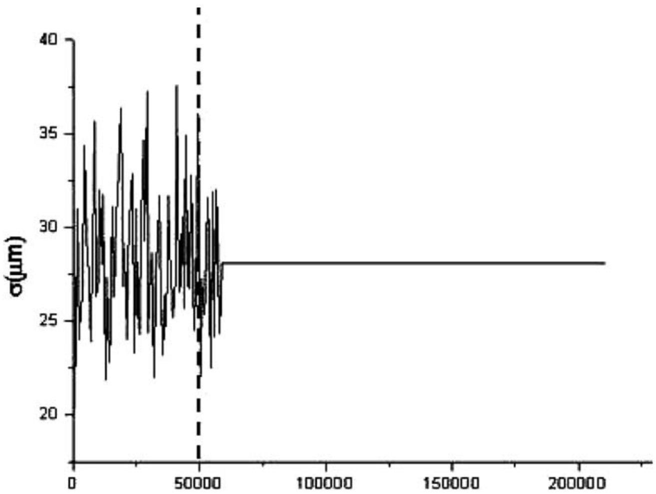

Fig. 1. Spot size as time function (in round trip units). The dotted line in $t = 50{,}000$ shows the start of the control. Note the transient time until the control is reached.

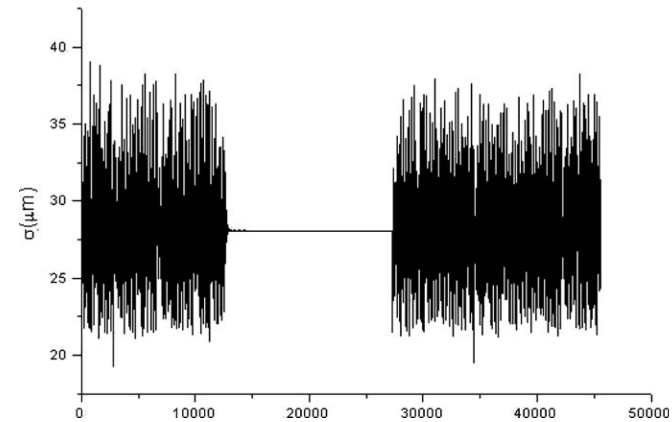

Fig. 2. The same as in Fig. 1 but with intracavity GVD as control parameter. The dotted line in $t = 10{,}000$ shows the start of the control. After a short transient time, the variable stabilizes in the fixed-point value of the UPO. In $t = 27{,}170$ the control is switched off and the chaotic behavior is restored.

so the signal is chaotic (embedding dimension 4); in $t = 50{,}000$, that is 500 μs later, the algorithm turns on the control, changing the value of the cavity loss μ according to the function (11). After a brief transient (of about 8000 round trips) the signal stabilizes. The stable behavior in the spot size ensures us stability in the temporal variables, in particular, the pulse duration.

The same algorithm applied to the P_1 mode (i.e, transform limited pulses mode) allows reaching a stable pulse duration close to 25 fs.

In summary, the operability of the control scheme based on the peak to peak map, tested with the complete iterative map, was demonstrated for the fs Ti:Sapphire laser. The scheme is not only interesting from the point of view of the studies on non-linear dynamics, but it also would allow the stabilization of the laser to reach pulse duration close to half the standard duration by just adding an acousto optic modulator and a very simple control circuit. However, and due to the very fast time involved in an on line control, the actual experimental realization can be difficult. An intermediate or hybrid solution would be to use the numerical generated control signal to feed the AOM.

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Appendix 1. Algorithm employing the intracavity GVD as control parameter

To verify that the same result as in Fig. 1 is obtained independently of the control parameter, I repeat the algorithm changing the control parameter to δ, the net intracavity group velocity dispersion (GVD). Even though, as it was discussed in the text, a real implementation with δ as a control parameter is impractical because there is no way to modulate δ in the range of MHz, the numerical results are of interest. The values of the variables and the parameter of the UPO will be in this case:

$$\bar{S} = 12,0000\ \mathrm{cm}^{-1}(\text{i.e., } \sigma = 28\ \mu\mathrm{m}),$$
$$\bar{\delta} = 30\ \mathrm{fs}^2.$$

These values, once stability is reached, give a pulse duration of 18 fs.

The derivatives and the control function are:

$$A = 12.35,$$
$$b = 3.04\ \mathrm{cm}^{-1}\ \mathrm{fs}^{-2},$$
$$h = -4.0625\ \mathrm{cm}\ \mathrm{fs}^{-2},$$
$$\delta_k = \begin{cases} \bar{\delta} + h(s_k - \bar{s}) & \text{if } \|s_k - \bar{s}\| \leqslant \varepsilon, \\ \bar{\delta} & \text{if } \|s_k - \bar{s}\| > \varepsilon. \end{cases} \tag{1.1}$$

By allowing variations up to 4%, that is $\varepsilon = 0.04$, variations in the parameter δ will be restricted to the interval (29.8; 30.17) fs^2.

If we apply the control function (11) we will obtain a similar result as shown in Fig. 1, but now with δ as control parameter, as we can see in Fig. 2.

In this way, it is shown that the control is independent of the parameter chosen and that it allows to decrease the pulse duration of the stable operation of the KLM Ti:Sapphire laser. However, it is the parameter μ (losses) the one that can be switched in a real setup with the adequate speed to adequately achieve the control.

Appendix 2. Ti:Sapphire parameters

Beyond the geometrical dimension of the cavity, there are other relevant parameters that we need to take into

Table 2
Matrix elements

	Mode P_1	Mode P_2
A_0	4.138138	3.306214
B_0	−2.304786	−1.438793
C_0	8.327616	8.600394
D_0	−4.3965	−3.44072
A_γ	3.54758	−0.488072
B_γ	1.315279	0.195234
C_γ	−3.246724	−14.5647
D_γ	8.224233	6.338294

$[A_{0,\gamma}] = [D_{0,\gamma}] =$ adimensional. $[B_{0,\gamma}] =$ cm, $[C_{0,\gamma}] = \mathrm{cm}^{-1}$.

account when we run a numerical simulation of the map equation. It is worth mentioning that not always the parameter's value are accessible via direct measurement. Here I present a resume of these parameters (see Table 2).

- Non-linear constants spatial and temporal c_γ,c_β

$c_\gamma = 1.3799 \times 10^{-11}\ \mathrm{cm}^4\ \mathrm{fs}\ \mathrm{nJ}^{-1}$,

$c_\beta = 2.179 \times 10^{-7}\ \mathrm{cm}^2\ \mathrm{fs}\ \mathrm{nJ}^{-1}$.

- Total group velocity dispersion (GVD) into the cavity.Variable, depending on the prism insertion in the laser path, between positive and $-300\ \mathrm{fs}^2$.
- Saturation energy flux, that is the saturation energy multiplied by the cavity round trip time

$D_S : 1.22\ \mathrm{mJ}\ \mathrm{cm}^{-2}$.

- Coefficients for the ABCD round trip spatial matrix (A_0, B_0, C_0, D_0) and its corresponding non-linearities (A_γ, B_γ, C_γ, D_γ).

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