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The primordial explosion of a false white hole from a 5D vacuum ☆

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ABSTRACT

We explore the cosmological consequences of some possible big bang produced by a black-hole with mass M in a 5D extended SdS. Under these particular circumstances, the effective 4D metric obtained by the use of a constant foliation on the extra coordinate is comported as a false white hole (FWH), which evaporates for all unstable modes that have wavelengths bigger than the size of the FWH. Outside the white hole the repulsive gravitational field can be considered as weak, so that the dynamics for fluctuations of the inflaton field and the scalar perturbations of the metric can be linearized.

1. Introduction and motivation

In the last two decades, inflationary cosmology has become the strongest candidate to explain the observed properties of the universe on cosmological scales [1–6]. This fact is supported by experimental evidence [7]. During this epoch the energy density of the universe was dominated by some scalar field (the inflaton), with negligible kinetic energy density, in such a way that its corresponding vacuum energy density is responsible for the exponential growth of the scale factor of the universe. Along this second order phase transition a small and smooth region of the order of size of the Hubble radius, grew so large that it easily encompassed the comoving volume of the entire presently observed universe, and consequently the observable universe became so spatially homogeneous and isotropic on scales today of the range $[10^8–10^{10}]$ ly. There are plenty of inflationary models [8] the majority of them in good concordance with observations, but none free of problems, as for example the transplanckian problem, the hierarchy problem, etc. [9]. This has led cosmologists to look for some new theoretical alternatives [10].

Recently, we have suggested a new inflationary perspective [11] from a 5D Extended Theory of General Relativity (ETGR) [12] in which we use the Ricci-flat metric [12] represented by a 5D line element[1]

$$dS_5^2 = \left(\frac{\psi}{\psi_0}\right)^2\left[c^2 f(R)\,dT^2 - \frac{dR^2}{f(R)} - R^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] - d\psi^2. \tag{1}$$

Here, $f(R) = 1 - [(2G\zeta\psi_0)/(Rc^2)] - (R/\psi_0)^2$ is a dimensionless metric function, ψ is the space-like and non-compact fifth extra coordinate.[2] The metric (1) is Ricci flat. Furthermore, this is an extension to 5D spaces of the 4D SdS metric. The coordinate T is a time-like, c is denoting the speed of light, R, θ, ϕ are the usual spherical polar coordinates, ψ_0 is an arbitrary constant with length units and the constant parameter ζ has units of $(mass)(length)^{-1}$. As was shown in [12], for certain values of ζ and ψ_0, the metric in (1) has two natural horizons. The inner one is the analogous to the Schwarzschild horizon and the external one is the analogous to the Hubble horizon. A particular case of the metric (1) is such that $\zeta = 1/(3\sqrt{3}G)$. For this case there is a unique horizon at $R_* = \psi_0/\sqrt{3}$, which is false, because $f(R) \leqslant 0$ retains its sign for $R \lessgtr R_*$. However the metric (1) really fails at $R = 0$. It can be seen more clear by writing $f(R)$ in this special case as

$$f(R) = \left(-\frac{1}{\psi_0^2 R}\right)\left[\left(R + \frac{2\psi_0}{\sqrt{3}}\right)\left(R - \frac{\psi_0}{\sqrt{3}}\right)^2\right]. \tag{2}$$

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[1] Some related metrics were studied in [13].

[2] In our notation conventions henceforth, latin indices $a, b =$ run from 0 to 4, whereas the rest of latin indices $i, j, n, l, \ldots =$ run from 1 to 3.

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Of course the physical domain of interest is $R > 0$. It is easy to see that $F(R_*) = 0$, but however $dS_5^2 \leqslant 0$ has the same signature for $R \lessgtr R_*$. As was demonstrated in [12], the Newtonian induced acceleration in absence of angular moment reads

$$a_c = -\frac{\psi_0}{3\sqrt{3}R^2} + \frac{Rc^2}{\psi_0^2}. \tag{3}$$

This means that, for the special case $\zeta = 1/(3\sqrt{3}G)$, the acceleration (3) becomes zero at $R = R_*$. Moreover, this acceleration is negative for $0 < R < R_*$ and positive for $R > R_*$. In other words, the metric (1) predicts that for $\zeta = 1/(3\sqrt{3}G)$, gravitation is repulsive for $R > R_*$ and attractive for $R < R_*$. As we shall see, this has important cosmological consequences. In physical terms, the case we are dealing to study in this work is very important because describes the greatest possible mass which can have a black-hole, $M = 1/(3\sqrt{3}G)\psi_0$, in a universe with cosmological constant $3/\psi_0^2$. The metric (1) is static, however, it can be written on a dynamical coordinate chart $\{t, r, \theta, \phi, \psi\}$ by implementing the planar coordinate transformation [14]. For the case $M = 1/(3\sqrt{3}G)\psi_0$, the transformations are

$$R = ar\left[1 + \frac{\psi_0}{6\sqrt{3}ar}\right]^2,$$
$$T = t + H\int^r dR\,\frac{R}{f(R)}\left(1 - \frac{2\psi_0}{3\sqrt{3}R}\right)^{-1/2}, \tag{4}$$

$a(t) = a_0 e^{t/\psi_0}$ being the scale factor. With this transformation the line element (1) reads

$$dS_5^2 = \left(\frac{\psi}{\psi_0}\right)^2\left[F(\tau, r)\,d\tau^2 - J(\tau, r)\left(dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right)\right] - d\psi^2, \tag{5}$$

where τ is the conformal time, $d\tau = a^{-1}(\tau)\,dt$ and $a(\tau) = -\psi_0/\tau$. Furthermore, the metric functions $F(\tau, r)$ and $J(\tau, r)$ are given by

$$F(\tau, r) = a^2(\tau)\left[1 - \frac{\psi_0}{6\sqrt{3}a(\tau)r}\right]^2\left[1 + \frac{\psi_0}{6\sqrt{3}a(\tau)r}\right]^{-2},$$
$$J(\tau, r) = a^2(\tau)\left[1 + \frac{\psi_0}{6\sqrt{3}a(\tau)r}\right]^4. \tag{6}$$

The constant Hubble parameter satisfies

$$H = \frac{1}{\psi_0} = a^{-2}\frac{da}{d\tau}. \tag{7}$$

The acceleration a_c can be written in terms of the new coordinates r and τ

$$a_c(r, \tau) = \frac{324\sqrt{3}r^2\tau - 1944r^3 - 215\sqrt{3}\tau^3 - 54r\tau^2}{6\psi_0\tau(\sqrt{3}\tau - 18r)^2}. \tag{8}$$

The physical distance for which the acceleration (8) becomes null, which provides us the size of the FWH in a comoving frame, is

$$D_{wh} = a(\tau_{wh})r_{wh} = \frac{5\sqrt{3}}{18}\psi_0, \tag{9}$$

so that $a_c(D_{wh}) = 0$. The wavenumber corresponding to the scales D_{wh} is

$$k_{wh} = \left[\frac{2\pi}{D_{wh}}\right]\left[\frac{2D_{wh}}{2D_{wh} + \frac{\psi_0}{3\sqrt{3}}}\right]^2, \tag{10}$$

giving us the wavelength related to k_{wh}

$$\lambda_{wh} = \frac{2\pi}{k_{wh}} \simeq 0.6928\psi_0 > R_* \simeq 0.577\psi_0. \tag{11}$$

Notice that for the largest mass M the modes with wavelength bigger than λ_{wh} are unstable and these modes are larger than the horizon radius $R_* = \psi_0/\sqrt{3}$. However, R_* is not a true causal horizon [i.e., $f(R \lessgtr R_*) < 0$ has the same signature at both sizes of this horizon R_*].

From the gravitational point of view we see that for physical wavelengths: $\lambda > \lambda_{wh}$ the universe is repulsive and suffers an accelerated expansion, but for $\lambda < \lambda_{wh}$ it is collapsing. Physical distances with wavelengths in the range $\lambda_{wh} < \lambda < \psi_0$ are commonly known as cosmological scales and distances with wavelengths in the range $0 < \lambda < \lambda_{wh}$ are known as astrophysical scales. These scales we shall refer as the inner of the FWH.

In this work we shall study the cosmological consequences of some possible big bang produced by a black-hole with mass M in a 5D extended SdS metric (5). Under these particular circumstances, the effective 4D metric obtained by the use of a foliation $\psi = \psi_0 = 1/H$ is comported as a false white hole, which is repulsing all matter which is with $\lambda > \lambda_{wh}$ and attract modes with $\lambda < \lambda_{wh}$. The Letter is organized as follows: in Section 2 we develop the 5D dynamics of the scalar field fluctuations of the metric and the inflaton field in the weak-field approximation. In Section 3 we explore the effective 4D dynamics of these fields, by using a semi-classical approximation for the inflaton field. Furthermore, we calculate the squared field fluctuations for the quantum inflaton fluctuations and metric fluctuations. These spectra are valid outside the FWH. Finally, in Section 4 we give some final comments.

2. 5D weak-field limit of the inflation and metric fluctuations

In order to consider a 5D vacuum on the 5D Ricci flat metric (5), we shall consider a non-massive scalar field $\varphi(x^a)$ which is free of interactions. Its dynamics can be derived from the action [12]

$$^{(5)}\mathcal{S} = \int \sqrt{g_5}\left[\frac{^{(5)}R}{16\pi G} - \frac{1}{2}g^{ab}\varphi_{,a}\varphi_{,b}\right]d^4x\,d\psi, \tag{12}$$

where $^{(5)}\mathcal{R}$ is the Ricci scalar, g_5 is the determinant of the metric (5) and G is the gravitational constant. The energy–momentum tensor: $^{(5)}T_{ab} = 2\frac{\delta\mathcal{L}}{\delta g^{ab}} - g_{ab}\mathcal{L}$, derived from the action (12), reads

$$^{(5)}T_{ab} = \varphi_{,a}\varphi_{,b} - \frac{1}{2}g_{ab}\varphi_{,c}\varphi^{,c}, \tag{13}$$

which is obviously symmetric. The dynamics of the scalar field φ derived from the action (12).

We consider the scalar metric fluctuations $\Phi(\tau, r, \theta, \phi, \psi)$. In cartesian coordinates this perturbed line element in the weak-field limit is

$$dS_5^2\big|_{pert} = \left(\frac{\psi}{\psi_0}\right)^2\left[F(\tau, x, y, z)[1 + 2\Phi]\,d\tau^2 - J(\tau, x, y, z)[1 - 2\Phi]\delta_{ij}\,dx^i\,dx^j\right] - d\psi^2, \tag{14}$$

being now $\Phi = \Phi(\tau, x, y, z, \psi)$. On sufficiently large scales the following condition is satisfied:

$$\frac{\psi_0}{6\sqrt{3}a(\tau)r} \ll 1, \tag{15}$$

where r_H denotes the value of the radial coordinate at the horizon entry and the conformal time τ is related to the scale factor by the expression $a(\tau) = -\psi_0/\tau$, so that the constant Hubble parameter satisfies

$$H = \frac{1}{\psi_0} = a^{-2}\frac{da}{d\tau}. \tag{16}$$

On the other hand, one can define the function $\mathcal{H} = \dot{a}/a = -1/\tau$. On very large scales, when the weak-field limit holds, the functions $F(\tau, r)$ and $J(\tau, r)$ become independent of spatial coordinates [see Eqs. (6)], so that

$$F(\tau, r)\Big|_{\frac{\psi_0}{6\sqrt{3}a(\tau)r} \ll 1} \to a^2(\tau), \qquad J(\tau, r)\Big|_{\frac{\psi_0}{6\sqrt{3}a(\tau)r} \ll 1} \to a^2(\tau), \tag{17}$$

and the metric (5) describes a universe which is nearly 3D spatially homogeneous and isotropic. In this limit case, the equation of motion for φ can be linearized with respect to Φ

$$\ddot{\varphi} - (2\mathcal{H} - 4\dot{\Phi})\dot{\varphi} - (1 + 4\Phi)\nabla^2\varphi + \left(\frac{\psi}{\psi_0}\right)^2 a^2\left[\left(2\overset{\star}{\Phi} - \frac{4}{\psi}\right)\overset{\star}{\varphi} - (1 + 2\Phi)\overset{\star\star}{\varphi} - \frac{8}{\psi}\Phi\overset{\star}{\varphi}\right] = 0. \tag{18}$$

Since now we are dealing with lineal equations of motion for Φ and φ one can use a semi-classical approximation for φ: $\varphi(\tau, x, y, z, \psi) = \varphi_b(\tau, \psi) + \delta\varphi(\tau, x, y, z, \psi)$. From Eq. (18), we obtain separately the dynamics for both, the background part of the field φ_b and the quantum fluctuations $\delta\varphi$

$$\ddot{\varphi}_b + 2\mathcal{H}\dot{\varphi}_b - \left(\frac{\psi}{\psi_0}\right)^2 a^2\left[\frac{4}{\psi}\overset{\star}{\varphi}_b + \overset{\star\star}{\varphi}_b\right] = 0, \tag{19}$$

$$\delta\ddot{\varphi} + 2\mathcal{H}\delta\dot{\varphi} - \nabla^2\delta\varphi - \left(\frac{\psi}{\psi_0}\right)^2 a^2\left[\frac{4}{\psi}\delta\overset{\star}{\varphi} + \delta\overset{\star\star}{\varphi}\right] - 4\dot{\varphi}_b\dot{\Phi} + \left(\frac{\psi}{\psi_0}\right)^2 a^2\left[2\overset{\star}{\varphi}_b\overset{\star}{\Phi} - \left(\frac{8}{\psi}\overset{\star}{\varphi}_b + 2\overset{\star\star}{\varphi}_b\right)\Phi\right] = 0. \tag{20}$$

On the other hand, the background (diagonal) Einstein equations are

$$3\mathcal{H}^2 - \frac{3a^2}{\psi_0^2} = \frac{\kappa_5}{2}\left[\dot{\varphi}_b^2 + \left(\frac{\psi}{\psi_0}\right)^2 a^2\overset{\star}{\varphi}{}_b^2\right], \tag{21}$$

$$-\mathcal{H}^2 - 2\dot{\mathcal{H}} + \frac{3a^2}{\psi_0^2} = \frac{\kappa_5}{2}\left[\dot{\varphi}_b^2 - \left(\frac{\psi}{\psi_0}\right)^2 a^2\overset{\star}{\varphi}{}_b^2\right], \tag{22}$$

$$-3\left(\mathcal{H}^2 + \dot{\mathcal{H}}\right) + \frac{6a^2}{\psi_0^2} = \frac{\kappa_5}{2}\left[\dot{\varphi}_b^2 + \left(\frac{\psi}{\psi_0}\right)^2 a^2\overset{\star}{\varphi}{}_b^2\right], \tag{23}$$

while that the linearized fluctuated diagonal Einstein equations are

$$-6\mathcal{H}\dot{\Phi} + 2\nabla^2\Phi + 3a^2\left[\left(\frac{\psi}{\psi_0}\right)^2\overset{\star\star}{\Phi} + \frac{4\psi}{\psi_0^2}\overset{\star}{\Phi} - \frac{2}{\psi_0^2}\Phi\right] = \kappa_5\left[\dot{\varphi}_b\delta\dot{\varphi} + \left(\frac{\psi}{\psi_0}\right)^2 a^2\left(\overset{\star}{\varphi}_b\delta\overset{\star}{\varphi} + \overset{\star}{\varphi}{}_b^2\Phi\right)\right], \tag{24}$$

$$2\ddot{\Phi} + 6\mathcal{H}\dot{\Phi} + a^2\left[\frac{6}{\psi_0^2}\Phi - \left(\frac{\psi}{\psi_0}\right)^2\overset{\star\star}{\Phi} - \frac{4\psi}{\psi_0^2}\overset{\star}{\Phi}\right] = \kappa_5\left[\dot{\varphi}_b\delta\dot{\varphi} - \left(\frac{\psi}{\psi_0}\right)^2 a^2\left(\overset{\star}{\varphi}_b\delta\overset{\star}{\varphi} + \overset{\star}{\varphi}{}_b^2\Phi\right)\right], \tag{25}$$

$$3(\ddot{\Phi} + 4\mathcal{H}\dot{\Phi}) - \nabla^2\Phi + \frac{6a^2}{\psi_0^2}\left(2\Phi - \psi\overset{\star}{\Phi}\right) = \kappa_5\left[\dot{\varphi}_b\delta\dot{\varphi} + \left(\frac{\psi}{\psi_0}\right)^2 a^2\left(\overset{\star}{\varphi}_b\delta\overset{\star}{\varphi} + \overset{\star}{\varphi}{}_b^2\Phi\right)\right]. \tag{26}$$

On the other hand, linearizing the non-diagonal field Einstein equations, we obtain

$$2(\mathcal{H}\Phi_{,i} + \dot{\Phi}_{,i}) = \kappa_5\dot{\varphi}_b\delta\varphi_{,i}, \tag{27}$$

$$6\mathcal{H}\overset{\star}{\Phi} + 3\dot{\overset{\star}{\Phi}} = \kappa_5\left(\dot{\varphi}_b\delta\overset{\star}{\varphi} + \delta\dot{\varphi}\overset{\star}{\varphi}_b\right), \tag{28}$$

$$\overset{\star}{\Phi}_{,i} = \kappa_5\overset{\star}{\varphi}_b\delta\varphi_{,i}. \tag{29}$$

The dynamics of the field Φ can be described in terms of the scalar field fluctuations $\delta\varphi$ using a linear combination of Eqs. (24)–(26)

$$\ddot{\Phi} + \frac{9}{2}\mathcal{H}\dot{\Phi} - \frac{1}{2}\nabla^2\Phi + a^2\left[\frac{9}{2\psi_0^2}\Phi + \frac{\psi}{\psi_0^2}\overset{\star}{\Phi} + \frac{7}{4}\left(\frac{\psi}{\psi_0}\right)^2\overset{\star\star}{\Phi}\right] = \frac{\kappa_5}{4}\left[\dot{\varphi}_b\delta\dot{\varphi} + 9\left(\frac{\psi}{\psi_0}\right)^2 a^2\left(\overset{\star}{\varphi}_b\delta\overset{\star}{\varphi} + \overset{\star}{\varphi}{}_b^2\Phi\right)\right]. \tag{30}$$

Notice that in general the quantum fluctuations $\delta\varphi$ act as a source of scalar metric fluctuations Φ.

3. Induced 4D dynamics outside the FWH in the weak-field limit

In order to obtain the dynamics for both, Φ and φ on the effective 4D universe, we shall assume that the 5D spacetime can be foliated by a family of hypersurfaces $\Sigma : \psi = constant$. Our 4D universe will be here represented by a generic hypersurface $\Sigma_0 : \psi = \psi_0$. Thus, on every leaf member of the family, the line element induced by (5) has the form

$$ds_4^2 = F(\tau, r)\,d\tau^2 - J(\tau, r)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right], \tag{31}$$

with $F(\tau, r)$ and $J(\tau, r)$ given by (6).

In the weak-field limit, the 5D perturbed line element in cartesian coordinates (14), induces on the hypersurface Σ_0 the effective 4D line element

$$ds_4^2\big|_{pert} = F(\tau, \bar{x})\left[1 + 2\Omega(\tau, \bar{x})\right]d\tau^2 - J(\tau, \bar{x})\left[1 - 2\Omega(\tau, \bar{x})\right]\delta_{ij}\,dx^i\,dx^j, \tag{32}$$

where $\Omega(\tau, \bar{x}) \equiv \Phi(\tau, x, y, z, \psi_0)$ describes the 4D scalar metric fluctuations induced on Σ_0.

The 5D action (12) induces on our 4D spacetime the effective action

$${}^{(4)}\mathcal{S}_{eff} = \int d^4x\sqrt{g_4}\left[\frac{{}^{(4)}R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\bar{\varphi}_{,\mu}\bar{\varphi}_{,\nu} + V(\bar{\varphi})\right], \tag{33}$$

where g_4 is the determinant of the 4D induced metric, which for the background reads $\bar{g}_4 = -FJ^3$ while for the perturbed metric $g_4^{(p)} = -FJ^3(1 + 2\Omega)(1 - 2\Omega)^3$. In the linear approximation, the 4D Ricci scalar curvature ${}^{(4)}R$ is given by

$${}^{(4)}R = \frac{2}{a^2}\left[3\left(\mathcal{H}^2 + \dot{\mathcal{H}}\right) + \nabla^2\Omega - 6\Omega\left(\mathcal{H}^2 + \dot{\mathcal{H}}\right) - 3\left(4\mathcal{H}\dot{\Omega} + \ddot{\Omega}\right)\right], \tag{34}$$

and the induced 4D effective potential V has the form

$$V(\bar{\varphi}) = -\frac{1}{2}g^{\psi\psi}\left(\frac{\partial\varphi}{\partial\psi}\right)^2\bigg|_{\psi=\psi_0}. \tag{35}$$

Thus, let us use the semi-classical approximation: $\bar{\varphi}(\tau, \vec{x}) = \bar{\varphi}_b(\tau) + \delta\bar{\varphi}(\tau, \vec{x})$, where $\bar{\varphi}_b(\tau) \equiv \bar{\varphi}_b(\tau, \psi)|_{\psi=\psi_0}$ is the background 4D inflaton field and $\delta\bar{\varphi}(\tau, \vec{x}) \equiv \delta\bar{\varphi}(\tau, \vec{x}, \psi)|_{\psi=\psi_0}$ stands for the 4D inflaton field quantum fluctuations. In our analysis the fields Ω and $\bar{\varphi}$ are semi-classical fields, so they are constituted by a classical part plus a quantum part. To study the dynamics of the former, a standard quantization procedure will be implemented. To do it, we shall impose the commutation relations

$$\begin{aligned}&\big[\bar{\varphi}(\tau,\vec{x}),\Pi^0_{(\bar{\varphi})}(\tau,\vec{x}')\big]=i\delta^{(3)}(\vec{x}-\vec{x}'),\\&\big[\Omega(\tau,\vec{x}),\Pi^0_{(\Omega)}(\tau,\vec{x}')\big]=i\delta^{(3)}(\vec{x}-\vec{x}'),\end{aligned}\tag{36}$$

where $\vec{x}$ is denoting the 3D vector position in cartesian coordinates. Due to the fact that the conjugate momenta to $\bar{\varphi}$ and Ω [calculated on the background metric with determinant $\bar{g}_4=-FJ^3$], are respectively given by $\Pi^0_{(\bar{\varphi})}=\sqrt{-\bar{g}_4}F^{-1}\dot{\bar{\varphi}}$ and $\Pi^0_{(\Omega)}=[12/(16\pi G)]a^{-2}(3\dot{\Omega}-2\mathcal{H})\sqrt{-\bar{g}_4}$, the expressions (36) yield

$$\begin{aligned}&\big[\bar{\varphi}(\tau,\vec{x}),\dot{\bar{\varphi}}(\tau,\vec{x}')\big]=\frac{iF}{\sqrt{-\bar{g}_4}}\delta^{(3)}(\vec{x}-\vec{x}'),\\&\big[\Omega(\tau,\vec{x}),\dot{\Omega}(\tau,\vec{x}')\big]=i\frac{4\pi Ga^2}{9\sqrt{-\bar{g}_4}}\delta^{(3)}(\vec{x}-\vec{x}').\end{aligned}\tag{37}$$

3.1. 4D classical dynamics of the inflaton field

Taking this into account, the evaluation of Eq. (19) on Σ_0 yields

$$\ddot{\bar{\varphi}}_b+2\mathcal{H}\dot{\bar{\varphi}}_b+a^2m^2\bar{\varphi}_b=0,\tag{38}$$

where we have used the relation: $[(4/\psi)\overset{\star}{\bar{\varphi}}_b+\overset{\star\star}{\bar{\varphi}}_b]|_{\psi=\psi_0}=-m^2\bar{\varphi}_b$, such that m is a separation constant. The background field $\bar{\varphi}$ must obey the Friedmann-like equation

$$\left(\frac{\partial\bar{\varphi}_b}{\partial\tau}\right)^2+a^2\left(\frac{\partial\varphi_b}{\partial\psi}\right)^2_{\psi=\psi_0}=0.\tag{39}$$

A particular solution of (38), which also is satisfied when inflation begins, are the slow rolling conditions: $\partial\bar{\varphi}_b/\partial\tau=0$, where necessarily $m=0$. Using this solution in (39), it yields $\bar{\varphi}_b=0$. It means that all the energy density on the 4D hypersurface is induced geometrically by the foliation $\psi=\psi_0=1/H$, because the background energy density related to the background inflaton field is null. This is an important difference with respect to de Sitter models in standard 4D inflationary models [6] in which the background energy density is given by the potential. In our case, as can be seen from Eqs. (21), (22), (23), the right sides of these equations are zero. Hence, the pressure and energy density being geometrically induced by the foliation:

$$\mathcal{H}^2=\frac{a^2}{\psi_0^2},\tag{40}$$

$$\mathcal{H}^2+2\dot{\mathcal{H}}=\frac{3a^2}{\psi_0^2},\tag{41}$$

$$\big(\mathcal{H}^2+\dot{\mathcal{H}}\big)=\frac{2a^2}{\psi_0^2},\tag{42}$$

from which we obtain

$$\left(\frac{\mathcal{H}}{a}\right)^2=\frac{1}{\psi_0^2}=\frac{\dot{\mathcal{H}}}{a}.\tag{43}$$

Furthermore, the background induced 4D scalar curvature ${}^{(4)}\bar{\mathcal{R}}$ is

$${}^{(4)}\bar{\mathcal{R}}=\frac{2}{a^2}\big[3\big(\mathcal{H}^2+\dot{\mathcal{H}}\big)\big].\tag{44}$$

Using Eq. (43) in (44), we obtain the background induced scalar curvature

$${}^{(4)}\bar{\mathcal{R}}=\frac{12}{\psi_0^2},\tag{45}$$

which is the exact expression that can be obtained in STM [15] theory for a de Sitter expansion from the expression

$${}^{(4)}\bar{\mathcal{R}}=-\frac{1}{4}\big[\bar{g}^{\mu\nu}_{,4}\bar{g}_{\mu\nu,4}+\big(\bar{g}^{\mu\nu}\bar{g}_{\mu\nu,4}\big)^2\big],\tag{46}$$

where $\bar{g}_{\mu\nu}$ denotes the background components of the tensor metric.

3.2. 4D scalar metric fluctuations spectrum

After working with Eqs. (28), (29) and (30), we find that the scalar metric fluctuations Ω on Σ_0 obey the equation

$$\ddot{\Omega}+\frac{9}{2}\mathcal{H}\dot{\Omega}-\frac{1}{2}\nabla^2\Omega+\left[\frac{9}{2}a^2H^2+\lambda^2\right]\Omega=0.\tag{47}$$

Here, we have used $\{(\psi/\psi_0^2-9/\psi)\overset{\star}{\Phi}+[(7/4)(\psi/\psi_0)^2-9/4]\overset{\star\star}{\Phi}\}_{\psi=\psi_0}=\lambda^2\Omega$, where λ is a separation constant with mass units. Now it can be shown that Eq. (27), evaluated on Σ_0, leads to the condition $\dot{\Omega}=-\mathcal{H}\Omega$. Using this last condition in Eq. (47), we obtain

$$\ddot{\Omega}+2\mathcal{H}\dot{\Omega}-\frac{1}{2}\nabla^2\Omega+\big[a^2H^2+\lambda^2\big]\Omega=0.\tag{48}$$

If we introduce the auxiliary field $\chi(\tau,\bar{r})$, through the formula $\Omega(\tau,\bar{r})=e^{-\int\mathcal{H}(\tau)\,d\tau}\chi(\tau,\bar{r})$, Eq. (48) becomes

$$\ddot{\chi}-\frac{1}{2}\nabla^2\chi+\big(\lambda^2-\dot{\mathcal{H}}\big)\chi=0.\tag{49}$$

The field χ can be expanded in terms of the Fourier modes

$$\chi(\tau,\bar{r})=\frac{1}{(2\pi)^{3/2}}\int d^3k\,\big[a_ke^{i\bar{k}\cdot\bar{r}}\xi_k(\tau)+a_k^\dagger\xi_k^*(\tau)\big],\tag{50}$$

where the annihilation and creation operators a_k and $a_k^\dagger$ satisfy

$$\big[a_k,a_{k'}^\dagger\big]=\delta^{(3)}\big(\bar{k}-\bar{k}'\big),\qquad[a_k,a_{k'}]=\big[a_k^\dagger,a_{k'}^\dagger\big]=0.\tag{51}$$

Inserting (50) in (49) and using (51), we find

$$\ddot{\xi}_k+\left(k_{eff}^2-\frac{2}{\tau^2}+\lambda^2\right)\xi_k=0,\tag{52}$$

where $k_{eff}^2=k^2/2$. If we require that the modes ξ_k to be normalized, they must satisfy the following expression on the UV-sector:

$$\xi_k\dot{\xi}_k^*-\xi_k^*\dot{\xi}_k=i\frac{4\pi G}{9a_0^2}.\tag{53}$$

Thus, choosing the Bunch–Davies vacuum condition, the normalized solution of (52) reads

$$\xi_k(\tau)\simeq\frac{i\pi}{3a_0}\sqrt{G}\mathcal{H}_\nu^{(2)}\big[z(\tau)\big],\tag{54}$$

where $\mathcal{H}_\nu^{(2)}[z(\tau)]$ is the second kind Hankel function, $\nu=(1/2)\sqrt{1+4\beta}$ and $z(\tau)=k_{eff}\tau$. Notice that $\beta=2-\lambda^2\tau^2>0$, such that the conformal time at the beginning of inflation $\tau_0=-\frac{\sqrt{2}}{\lambda}$ is defined such that $\beta_0=2-\lambda^2\tau_0^2=0$ and the conformal time at the end of inflation complies with $\beta_e=2-\lambda^2\tau_e^2\simeq 2$. Using the definition of β, it is easy to show that the parameter $\nu\simeq 3/2$ for $\tau_e^2\ll 2/\lambda^2$, which assures that the spectral index $1>n_s>0.96$ [16]. Using the fact that $1-n_s=3-2\nu$, we obtain for $\lambda=10^{-10}G^{-1/2}$, that the range of acceptable values for τ_e is

$$0<(-\tau_e)<0.245\times 10^{10}G^{1/2}.\tag{55}$$

It is important to see how the universe becomes scale invariant with the expansion of the universe because $\beta(\tau)$ evolves from $\beta_0 = 0$ to $\beta_e \simeq 2$ along the inflationary expansion. The temporal evolution of β, and hence of the spectral index n_s, is due to the existence of λ, which has a clear origin in the extra space-like coordinate ψ [see Eqs. (47) and below]. This result cannot be found in standard 4D inflationary models.

The amplitude of the 4D gauge-invariant metric fluctuations $\langle\Omega^2\rangle_{IR}$ on the IR-sector ($k_{eff}\tau \ll 1$) is given by

$$\langle\Omega^2\rangle_{IR} = \frac{1}{2\pi^2}\left(\frac{a_0}{a}\right)^2 \int_{k=0}^{\epsilon_1 k_{wh}} dk\, k^2 \left(\xi_k \xi_k^*\right)\big|_{IR} \tag{56}$$

where $\epsilon_1 = k_{max}^{IR}/k_p \ll 1$ is a dimensionless parameter, being $k_{max}^{IR} = k_{wh}(\tau_i) = \sqrt{(4/\tau_i^2) - 2\lambda^2}$ the wave number related to the Hubble radius by the time when the modes re-enter to the horizon τ_i. The Planckian wave number is here denoted by k_p. Now, considering the asymptotic expansion for the Hankel function $\mathcal{H}_\nu^{(2)}[x] \simeq -(i/\pi)\Gamma(\nu)(x/2)^{-\nu}$ in the expression (54), Eq. (56) yields

$$\langle\Omega^2\rangle_{IR} = \frac{2^{1+3\nu}}{3^2}\Gamma^2(\nu)\left(\frac{H\tau}{2\pi}\right)^2\left(\frac{1}{aH}\right)^{2-2\nu} \int_0^{\epsilon_1 k_{wh}} \frac{dk}{k} k^{3-2\nu}. \tag{57}$$

It can be easily seen from this equation that the corresponding spectrum for scalar metric fluctuations reads

$$\mathcal{P}_\Omega(k) = \frac{2^{1+3\nu}}{3^2}\Gamma^2(\nu)\left(\frac{H\tau}{2\pi}\right)^2\left(\frac{1}{aH}\right)^{2-2\nu} k^{3-2\nu}. \tag{58}$$

This spectrum results nearly scale invariant for $\nu \simeq 3/2$, value that may be achieved when $\lambda^2\tau^2 \ll 1$. Performing the integration in (57), the amplitude of the scalar metric fluctuations is given finally by

$$\langle\Omega^2\rangle_{IR} = \frac{2^{1+3\nu}}{3^2}\Gamma^2(\nu)\left(\frac{H\tau}{2\pi}\right)^2\left(\frac{1}{aH}\right)^{2-2\nu} \epsilon_1^{3-2\nu} k_{wh}^{3-2\nu}, \tag{59}$$

which tends to zero as $\tau \to 0$.

3.3. 4D inflaton field fluctuations

Since the background inflaton field φ_b is a constant, the dynamics of the inflaton field fluctuations $\delta\varphi$ are given by the first row of the equation of motion (20) on the hypersurface $\psi = \psi_0$. The equation of motion for the time dependent modes $\zeta_k(\tau)$ is

$$\ddot{\zeta}_k + 2\mathcal{H}\dot{\zeta}_k + \left[k^2 - a^2\lambda^2\right]\zeta_k(\tau) = 0, \tag{60}$$

which has the general solution

$$\zeta_k(\tau) = A_1(-\tau)^{3/2}\mathcal{H}_\mu^{(1)}[-k\tau] + A_2(-\tau)^{3/2}\mathcal{H}_\mu^{(2)}[-k\tau]. \tag{61}$$

Here, $\mathcal{H}_\mu^{(1,2)}[-k\tau]$ are respectively the first and second kind Hankel functions, $\mu = \sqrt{9 + 4\lambda^2\psi_0^2}/2$. One can define $\zeta_k(\tau) = \tau\sigma_k(\tau)$, such that $\sigma_k(\tau)$ are the time dependent modes for the redefined field fluctuations $\sigma(\vec{x},\tau)$, that can be expanded in terms of Fourier modes

$$\sigma(\tau,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3k \left[A_k e^{i\vec{k}\cdot\vec{x}}\sigma_k(\tau) + A_k^\dagger e^{-i\vec{k}\cdot\vec{x}}\sigma_k^*(\tau)\right], \tag{62}$$

where the annihilation and creation operators A_k and $A_k^\dagger$ satisfy the commutation algebra

$$\left[A_k, A_{k'}^\dagger\right] = \delta^{(3)}\left(\vec{k} - \vec{k}'\right), \qquad \left[A_k, A_{k'}\right] = \left[A_k^\dagger, A_{k'}^\dagger\right] = 0, \tag{63}$$

and to be fulfilled the algebra

$$\left[\sigma(\tau,\vec{x}), \dot{\sigma}\left(\tau,\vec{x}'\right)\right] = i\delta^{(3)}\left(\vec{x} - \vec{x}'\right), \tag{64}$$

must require the normalization condition $\sigma_k\dot{\sigma}_k^* - \dot{\sigma}_k\sigma^* = i$. Therefore, the normalization constants are given by

$$A_2 = -\frac{\sqrt{\pi}}{2\psi_0}e^{-i\nu\pi/2}, \qquad A_1 = 0. \tag{65}$$

The amplitude of the fluctuations of the inflaton field on large scales ($k\tau \ll 1$) is obtained by means of the formula

$$\langle\delta\varphi^2\rangle_{IR} = \frac{1}{2\pi^2}\left(\frac{a_0}{a}\right)^2 \int_{k=0}^{\epsilon_2 k_{wh}} dk\, k^2\left(\sigma_k\sigma_k^*\right)\big|_{IR}, \tag{66}$$

where $\epsilon_2 = k_{max}^{IR}/k_p$, being in this case $k_{max}^{IR} = \sqrt{(2/\tau_i) + a_i^2\lambda^2}$, with $a_i = a(\tau_i)$. Thus, making use of the asymptotic expansion for the Hankel function $\mathcal{H}_\mu^{(2)}[x] \simeq -(i/\pi)\Gamma(\mu)(x/2)^{-\mu}$, the expression (66) becomes

$$\langle\delta\varphi\rangle_{IR} = \frac{a_0^2 2^{2\mu-1}}{\psi_0^2}\frac{\Gamma^2(\mu)}{\pi}\left(\frac{H}{2\pi}\right)^2\left(\frac{1}{aH}\right)^{3-2\mu} \times \int_0^{\epsilon_2 k_{wh}} \frac{dk}{k}k^{3-2\mu}. \tag{67}$$

After straightforward calculations, the mean squared fluctuations for the inflaton field read

$$\langle\delta\varphi^2\rangle_{IR} = \frac{a_0^2 2^{2\mu-1}}{\psi_0^2}\frac{\Gamma^2(\mu)}{\pi}\left(\frac{H}{2\pi}\right)^2 \epsilon_2^{3-2\mu}\left(\frac{k_{wh}}{aH}\right)^{3-2\mu}. \tag{68}$$

In this manner, the spectrum for the inflaton fluctuations on 4D is given by

$$\mathcal{P}_{\delta\varphi}(k) = \frac{a_0^2 2^{2\mu-1}}{\psi_0^2}\frac{\Gamma^2(\mu)}{\pi}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{3-2\mu}. \tag{69}$$

This spectrum shows that its nearly scale invariance is achieved for $\mu \simeq 3/2$, and in turn this value is obtained when $\lambda^2\psi_0^2 \ll 1/4$.

4. Final comments

We have studied an interesting case in which the universe expands on an effective 4D hypersurface obtained after take a constant foliation on the extra space-like coordinate of an extended SdS Ricci-flat metric. The 4D effective spacetime describes a universe that expands on cosmological scales but collapses on astrophysical ones because it has a black-hole in its center with mass $M = \psi_0/(3\sqrt{3}G)$. Under these circumstances, the universe behaves as a white hole that evaporates on scales greater its radius. The behavior of the universe on these large scales is similar to that of white hole, so that we have called it a false white hole (FWH). We have studied the large scales evolution of the scalar metric fluctuations and the quantum fluctuations of the metric. In both cases the amplitude of the fluctuations decreases during inflation for power spectra that are nearly scale invariant on cosmological scales.

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