ON THE QUIVER WITH RELATIONS OF A QUASITILTED ALGEBRA AND APPLICATIONS

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Abstract. In this paper we discuss, in terms of quiver with relations, sufficient and necessary conditions for an algebra to be a quasitilted algebra. We start with an algebra with global dimension at most two and we give a sufficient condition to be a quasitilted algebra. We show that this condition is not necessary. In the case of a strongly simply connected schurian algebra, we discuss necessary conditions, and combining both type of conditions, we are able to analyze if some given algebra is quasitilted. As an application we obtain the quiver with relations of all the tilted and cluster tilted algebras of Dynkin type E_p.

1. Introduction

An interesting problem in the theory of representation of algebras is to know if an algebra given by a quiver with relations is a tilted algebra. This problem was study for several authors and has been solved in particular cases, see [1], [10], [11], [12], [20], [21], [22], [17]. In this work, we consider a bigger class of algebras, that is, the quasitilted algebras introduced by Happel, Reiten and Smalo, see [18].

We introduced a notion of bound consecutive relations inspired by the one given by Assem and Redondo in [3]. Our first result states that, an algebra with global dimension at most two having non bound consecutive relations, is a quasitilted algebra. In general, we show that the converse of this result does not hold. On the other hand, if a quasitilted algebra has bound consecutive relations, we give a necessary condition over this kind of relations.

Finally, we obtain a sufficient condition for a strongly simply connected schurian algebra to be a quasitilted algebra. We arrive to this result combining the criterion for global dimension two developed in [6] with the previous results.

As applications of our main result, and using the results given in [13], we are able to give the quivers with relations of all the tilted and cluster tilted

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2000 *Mathematics Subject Classification.* Primary: 16G20, Secondary: 16G70,

We thank M. I. Platzeck for useful discussions about Theorem 4.3. This work is part of the Ph.D Thesis of Natalia Bordino, under the supervision of her advisers, Sonia Trepode and Elsa Fernández.

algebras of Dynkin type. In this work we consider tilted an cluster tilted algebras of type E_p.

2. Preliminaries

In this paper, by an algebra, we always mean a basic and connected finite dimensional algebra over an algebraically closed field k. Given a quiver Q, we denote by Q_0 its set of vertices and by Q_1 its set of arrows. A *relation* in Q from a vertex x to a vertex y is a linear combination $\rho = \sum_{i=1}^{m} \lambda_i w_i$ where, for each i, $\lambda_i \in k$ is non-zero and w_i is a path of length at least two from x to y. A relation ρ is called *minimal* if whenever $\rho = \sum_i \beta_i \rho_i \gamma_i$ where ρ_i is a relation for every i, then β_i and γ_i are scalars for some index i (see [8]). In this work, the term relation means minimal relation in this sense. For each ralation ρ, we denote by $s(\rho) \in Q_0$ its souce and $t(\rho) \in Q_0$ its target.

We denote by kQ the path algebra of Q and by $kQ(x, y)$ the k-vector space generated by all paths in Q from x to y. For an algebra A, we denote by Q_A its ordinary quiver. For every algebra A, there exists an ideal I in kQ_A, generated by a set of relations, such that $A \simeq kQ_A/I$. The pair (Q_A, I) is called a *presentation* of A and A is said to be given by the *bound quiver* (Q_A, I).

An algebra A is called *triangular* if Q_A has no oriented cycles, and it is called *schurian* if, for all $x, y \in A_0$, we have $\dim_k A(x, y) \leq 1$. A triangular algebra A is called *simply connected* if, for any presentation (Q_A, I) of A, the group $\pi_1(Q_A, I)$ is trivial, see [5]. It is called *strongly simply connected* if every full convex subcategory of A is simply connected, [26].

In this work, we always deal with schurian triangular algebras. For a vertex x in the quiver Q_A, we denote by e_x the corsponding primitive idempotent, S_x the corresponding simple A-module, and by P_x and I_x the corresponding indecomposable projective and injective A-module, respectively.

Let A be an algebra. A module T_A is called a *tilting module* [17] if $pdT_A \leq 1$; $\mathrm{Ext}^1_A(T;T) = 0$ and the number of isomorphism classes of indecomposable summands of T equals the rank of the Grothendieck group $K_0(A)$. An algebra A is called *tilted of type* Q if it is the endomorphism algebra of a tilting kQ-module.

An algebra A is called *quasitilted* if gl.dim. $A \leq 2$ and, for each indecomposable module M_A, we have $pd\, M \leq 1$ or $id\, M \leq 1$ (see [18]). It follows from [18], that tilted algebras are a subclass of quasitilted algebras, and if a representation finite algebra is quasitilted then it is tilted.

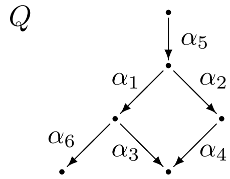
3. On the relations of a quasitilted algebra

The main objective of this section is the interaction between the relations of a quasitilted algebra. More precisely, for this class of algebras we going to study how the relations can interact.

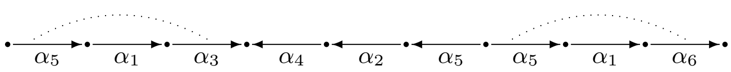
We begin by introducing the concept of bound consecutive relations inspired by the ones introduced in [3]. Observe that pair of relations defined in [3] are a subclass of the bound consecutive relations defined in the following.

Definition 3.1. Two relations ρ_1 and ρ_2 are called *bound consecutive relations* if there is a walk ϖ between $t(\rho_1)$ and $s(\rho_2)$ such that ϖ does not contain zero subpaths.

Example 3.2. If $A = kQ/I$ is the algebra given by quiver



bound by $\alpha_5\alpha_1\alpha_3 = 0$ y $\alpha_5\alpha_1\alpha_6 = 0$. These relations are bound consecutive for the walk $\varpi = \alpha_4\alpha_2\alpha_5$:



The following is the main theorem in this section.

Theorem 3.3. *Let A be an algebra with global dimension at most two. If there are no bound consecutive relations in A, then A is quasitilted.*

Proof. Suppose A is not quasitilted algebra. Then, there is a indecomposable A-module M such that $pd\, M = id\, M = 2$.

Let

$$0 \longrightarrow P_2 \xrightarrow{f_2} P_1 \xrightarrow{f_1} P_0 \xrightarrow{f_0} M \longrightarrow 0$$

and

$$0 \longrightarrow M \longrightarrow I_0 \longrightarrow I_1 \longrightarrow I_2 \longrightarrow 0$$

be the minimal projective and injective resolutions of M, respectively.

Any indecomposable sum $P_2(S_b)$ of P_2 gives rise to a diagram $P_2(S_b) \longrightarrow P_1 \longrightarrow P_0$ with zero composition. Then there is a sum $P_0(S_j)$ of P_0 and a relation $P_2(S_b) \longrightarrow P_1 \longrightarrow P_0(S_j)$. Consequently, there is a relation $\rho_2 : j \rightsquigarrow b$ starting at the vertex j, where $S_j \in \operatorname{Top} M$.

Dually, using the minimal injective resolution of M, there is $S_i \in \operatorname{Soc} M$ and a relation ρ_1 ending at the vertex i,

Let $S_k \in$ Top M such that $S_i \in f_0(P_0(S_k))$. Then there exists a nonzero morphism $h : P_0(S_i) \longrightarrow f_0(P_0(S_k))$ and an epimorphism $g : P_0(S_k) \longrightarrow f_0(P_0(S_k))$, i.e. the situation is as follows:

$$\begin{tikzcd} P_0(S_i) \arrow[r, "h"] \arrow[dr, dotted, "f"'] & f_0(P_0(S_k)) \\ & P_0(S_k) \arrow[u, "g"'] \end{tikzcd}$$

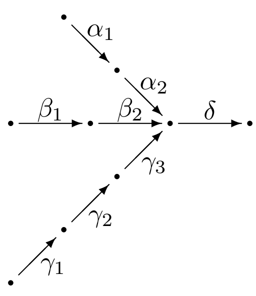
Then there is a nonzero morphism $f : P_0(S_i) \longrightarrow P_0(S_k)$ such that $gf = h$. Therefore, there is a nonzero path $\gamma : k \rightsquigarrow i$.

Since M is an indecomposable module, then sop$M$ is connected. We consider $Q'_0 = (\text{sop } M)_0$ and $Q'_1 = \{\alpha \in Q_1 \text{ such thate } M(\alpha) \neq 0\}$. Consequently, $Q' = (Q'_0, Q'_1)$ is a connected quiver and the vertices associated with the simple A-modules of Top M are sources of Q'. Given two sources j and k, there is a walk of nonzero paths that unites them, ([4], pp. 45). Therefore, ρ_1 y ρ_2 are bound consecutive relations □

We remark that the algebra A, given in the Example 3.2, is not quasitilted.

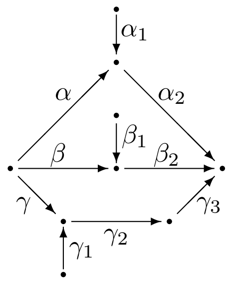
Now, we give an example of an algebra which is not tilted and satisfies the conditions of Theorem 3.3 .

Example 3.4. Let A be an algebra given by quiver



bound by $\alpha_1\alpha_2\delta = 0$, $\beta_1\beta_2\delta = 0$, $\gamma_1\gamma_2\gamma_3\delta = 0$. The algebra A has gl.dim.$A = 2$ and A does not contain bound consecutive relations, then, by Theorem 3.3, A is quasitilted.

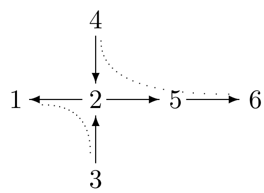
Furthermore A is not tilted. In fact $A = S_i^+ B$ where S_i^+ is the usual reflection, at the sink i, due to Hughes and Waschbüsch , see [23], B is the algebra given by the quiver



bound by $\alpha\alpha_2 = \beta\beta_2 = \gamma\gamma_2\gamma_3$. From ([24], pp. 464), this algebra B is concealed canonical type (3 3 4). We observe that, for [23], the repetitive categories coincide. Then, from ([16], Theorem 4.9) it follows that A and B are derived equivalent. Therefore, A is quasitilted of type (3 3 4). We recall that a tilted algebra is of canonical type if and only if it is derived equivalent to a hereditary algebra of Euclidean type. Then we get that A is not tilted.

We illustrated with the following example that the converse of Theorem 3.3 is in general not true.

Example 3.5. The algebra B given by the following quiver with relations:



is a tilted algebra with bound consecutive relations. The given relations are bound consecutive relations, where ϖ is given by the path $1 \longleftarrow 2 \longleftarrow 4$.

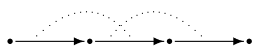
4. Strongly simply connected schurian algebras

Combining the previous result with some results from [6] we are able to state a sufficient condition for a strongly simply connected schurian algebra to be quasitilted. We have shown with examples that a quasitilted algebra can have bound consecutive relations. In the case of strongly simply connected quasitilted algebras with bound consecutive relations we give a necessary condition over these relations. None of this conditions is a necessary and sufficient condition.

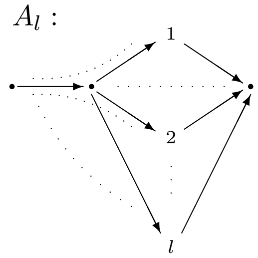
In [6] we gave the notion of critical algebra and we gave a list of these algebras.

Let Γ be a strongly simply connected schurian algebra, then Γ is a critical algebra if either Γ or Γ^{op} is one of the following algebras.

A_1 :

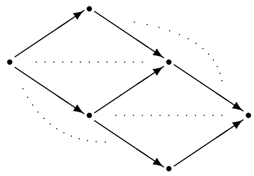


A_l :

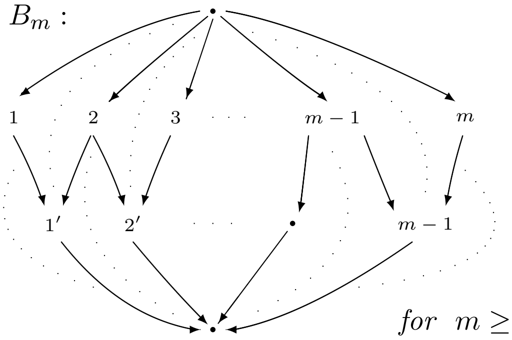


for $l \geq 2$

B_1 :



B_m :



for $m \geq 3$

Critical algebras are useful to decide when the global dimension of a given algebra is at most two. Next, we quote the result from [6] we need.

Theorem 4.1. [6] *Let $A = kQ/I$ be a strongly simply connected schurian algebra with global dimension at least three. Then there exists a full subcategory B of A such that B is critical.*

Then, we have the following corollary.

Corollary 4.2. *Let A be a strongly simply connected schurian algebra. If the following conditions hold:*

a) *A does not contain a proper full subcategory B such that B is a critical algebra.*
b) *A has not bounded consecutive relations.*

Then A is a quasitilted algebra.

Proof. It follows from a) and Theorem 4.1 that global dimension of A is at most two. From b) and appling Theorem 3.3 we get A is quasitilted algebra. □

Next, we give necessary conditions for an algebra of global dimension two to be quasitilted. To this end, we introduce the following notations.

Let $A = kQ/I$ be an algebra and $\rho = \sum_{i=1}^{m} \lambda_i \alpha_{i_1} \alpha_{i_2} \dots \alpha_{i_{r_i}}$ be a relation of A, with $\alpha_{i_j} \in Q_1$. We will note with $\mathbf{Q}_\rho$ the subquiver of Q induced by the arrows of ρ, i.e. $(\mathbf{Q}_\rho)_1 = \{\alpha_{i_j} : 1 \leq i \leq m, 1 \leq j \leq r_i\}$ and $(\mathbf{Q}_\rho)_0 = \{s(\alpha_{i_j}) : \alpha_{i_j} \in (\mathbf{Q}_\rho)_1\} \cup \{t(\alpha_{i_j}) : \alpha_{i_j} \in (\mathbf{Q}_\rho)_1\}$.

If $A = kQ/I$ is a schurian strongly simply connected algebra and ρ_1, ρ_2 are bound consecutive relations for the walk

$$\varpi = \gamma_1\gamma_2\cdots\gamma_r : t(\rho_1) = i \rightsquigarrow s(\rho_2) = j$$

with nonzero maximal paths γ_h and $\operatorname{long}(\gamma_h) \neq 0$, for $h = 2, \ldots, r-1.$, then we note with $\overline{\operatorname{sop}}\,\varpi$ the support of the union of all the parallel paths of γ_h (which are all equal and nonzero in A).

The following result has a strong relation with the Lemma 2.1 given by I. Assem and M. J. Redondo in [3]. They proved that a schurian tilted algebra A have not contain a certain class of bound consecutive relations. In the case of schurian strongly simply connected quasitilted algebras, we show that these algebras do not have the bounded consecutive relations defined in this work.

Proposition 4.3. *Let $A = kQ/I$ be a schurian strongly simply connected algebra with global dimension at most two. If A is quasitilted, then there is no bound consecutive relations ρ_1 y ρ_2 with walk ϖ such that $\overline{\operatorname{sop}}\,\varpi \cap \mathbf{Q}_{\rho_1} = \{t(\rho_1) = i\}$ and $\overline{\operatorname{sop}}\,\varpi \cap \mathbf{Q}_{\rho_2} = \{s(\rho_2) = j\}$.*

Proof. Since A is schurian strongly simply connected, we can define a representation M whose support is $\overline{\operatorname{sop}}\,\varpi$ and such that $M(x) = k$, for all $x \in (\operatorname{sop} M)_0$ and $M(\varepsilon) = Id_k$, for all $\varepsilon \in (\operatorname{sop} M)_1$. We will show that the projective dimension of M is 2.

Consider

$$P_1(M) \xrightarrow{f_1} P_0(M) \xrightarrow{f_0} M \longrightarrow 0.$$

The module M so defined is not projective because $P_j \in \operatorname{add} P_0(M)$ and $\operatorname{sop} P_j \nsubseteq \operatorname{sop} M$.

If $\rho_2 = \beta\delta_2$, where $\beta \in Q_1$ and δ_2 is a nonzero path in A, then $f_0(\beta) = 0$ because $\overline{\operatorname{sop}}\,\varpi \cap \mathbf{Q}_{\rho_2} = \{s(\rho_2) = j\}$. Then, $P_{t(\beta)}$ is a direct summand of $P_1(M)$ because the composition $P_{t(\beta)} \xrightarrow{\beta\cdot} P_j \xrightarrow{f_0} M$ is zero. Therefore, $Im\,\beta\cdot \subseteq \operatorname{Ker} f_0$.

As $\beta \notin \operatorname{rad} \operatorname{Ker} f_0$, we have that $Im\,\beta\cdot \nsubseteq \operatorname{rad} \operatorname{Ker} f_0$. This proves that $P_{t(\beta)} \xrightarrow{\beta\cdot} P_j$ is a sum of the projective presentation of M.

Finally, as $0 = \rho_2 = \beta\delta_2 = \beta \cdot (\delta_2)$, it follows that $0 \neq \delta_2 \in \operatorname{Ker} \beta\cdot$. Consequently, $P_2(M) \neq 0$.

If $\rho_2 = \beta\delta_2 + \beta'\delta_2'$, where $\beta, \beta' \in Q_1$ and δ_2, δ_2' are nonzero paths in A. Then, analogously to the previous case proof that $P_{t(\beta)} \oplus P_{t(\beta')} \xrightarrow{(\beta\cdot,\beta'\cdot)} P_j)$ is a sum of projective presentation of M. Then, $0 = \rho_2 = \beta\delta_2 + \beta'\delta_2' = (\beta\cdot, \beta'\cdot)\begin{pmatrix} \delta_2 \\ \delta_2' \end{pmatrix}$. Therefore, $P_2(M) \neq 0$.

By duality, considering the minimal injective co-resolution of M, it follows that $id\,M = 2$. $\square$

The next question is to study what happens when a quasitilted algebra has a bound consecutive relation by a walk ϖ such that $\overline{\text{sop}}\ \varpi$ intersects the quiver of one of relations in at least one arrow. In the next theorem we answer the question posed above, to schurian strongly simply connected algebras.

Theorem 4.4. *Let $A = kQ/I$ be a schurian strongly simply connected algebra and quasitilted such that there are ρ_1 and ρ_2 bound consecutive relations by the walk ϖ. Then, either $\overline{\text{sop}}\ \varpi \cap Q_{\rho_1}$ or $\overline{\text{sop}}\ \varpi \cap Q_{\rho_2}$ is a maximal subpath of ρ_1 or ρ_2, respectively.*

Proof. Suppose that for $l = 1, 2$, $\overline{\text{sop}}\ \varpi$ and Q_{ρ_l} do not intersect in a maximal subpath of ρ_l. Let M the representation whose support is $\overline{\text{sop}}\ \varpi$ and such that $M(x) = k$, for all $x \in (\text{sop}\, M)_0$ and $M(\varepsilon) = Id_k$, for all $\varepsilon \in (\text{sop}\, M)_1$. If we prove that the projective dimension of M is 2, then by duality result that $di\, M = 2$.

As M is not a projective module, we can consider

$$0 \neq P_1(M) \xrightarrow{f_1} P_0(M) \xrightarrow{f_0} M \longrightarrow 0.$$

For $\rho_2 = \delta_2\beta$, where $\beta \in Q_1$ and δ_2 is a nonzero path in A, must be $f_0(\delta_2) = 0$. Writing $\delta_2 = \psi_2\varphi_2$, with ψ_2 minimal path in δ_2 such that $f_0(\psi_2) = 0$, results that the composition $P_{t(\psi_2)} \xrightarrow{\psi_2 \cdot} P_j \xrightarrow{f_0} M$ is zero. Therefore, $Im\, \psi_2 \cdot \subseteq \text{Ker}\, f_0$.

Suppose that $\psi_2 \in \text{rad Ker} f_0$. Then there exist non-trivial paths λ_l such that $\psi_2 = \sum \mu_l \lambda_l$, with $\mu_l \in \text{Ker}\, f_0$. As A schurian strongly simply connected, $\psi_2 = a\lambda_1\mu_1$, with a scalar. But $f_0(\mu_1) = 0$ and $l(\mu_1) < l(\psi_2)$, which contradicts the minimality of ψ_2. Then, $Im\, \psi_2 \cdot \nsubseteq \text{rad Ker}\, f_0$. Consequently, $P_{t(\psi_2)} \xrightarrow{\psi_2 \cdot} P_j$ is a sum of projective presentation of M.

Since $0 = \rho_2 = \psi_2\varphi_2\beta = \psi_2 \cdot (\varphi_2\beta)$, Then $0 \neq \varphi_2\beta \in \text{Ker}\ \psi_2 \cdot$. Therefore, $P_2(M) \neq 0$.

For $\rho_2 = \delta_2 + \delta_2'$, where δ_2, δ_2' are nonzero paths in A, it follows that $f_0(\delta_2) = f_0(\delta_2') = 0$. If we write $\delta_h = \psi_h\varphi_h$, with ψ_h minimal path in δ_h such that $f_0(\psi_h) = 0$, for $h = 1, 2$, is proved similarly to the previous case that $P_{t(\psi_2)} \oplus P_{t(\psi_2')} \xrightarrow{(\psi_2 \cdot, \psi_2' \cdot)} P_j$ is a sum of projective presentation of M.

Finally, since $0 = \rho_2 = \delta_2 + \delta_2' = \psi_2\varphi_2 + \psi_2'\varphi_2' = (\psi_2 \cdot, \psi_2' \cdot)\begin{pmatrix} \varphi_2 \\ \varphi_2' \end{pmatrix}$ we have that $P_2(M) \neq 0$. □

Note that the Theoren 4.4 implies the Proposition 4.3.

Example 4.5. Let $A = kQ/I$ be the tilted algebra given by the following quiver with relations:

$$\begin{tikzcd}
 & & & 3 \arrow[d] & \\
6 \arrow[r] & 5 \arrow[r] & 4 & 2 \arrow[l] \arrow[r] & 1
\end{tikzcd}$$

Note that the two monomial relations having A are bound consecutive by the path $\varpi : 4 \longleftarrow 2 \longleftarrow 3$, that is:

$$\begin{tikzcd}
6 \arrow[r] & 5 \arrow[r] & 4 & 2 \arrow[l] & 3 \arrow[l] \arrow[r] & 2 \arrow[r] & 1
\end{tikzcd}$$

However, $\overline{\mathrm{sop}}\,\varpi$ intersects the relation $\rho : 3 \longrightarrow 2 \longrightarrow 1$ in the path $3 \longrightarrow 2$, which is maximal in ρ.

The condition of Theorem 4.4 is not a sufficient condition to ensure that an algebra is quasitilted algebra.

Example 4.6. Let A be the algebra given by the following quiver with relations:

$$\begin{tikzcd}
 & 1 & & \\
3 \arrow[r] & 2 \arrow[u] & 4 \arrow[l] \arrow[r] & 5 \\
 & 6 \arrow[u] & 7 \arrow[l] \arrow[u] &
\end{tikzcd}$$

and we will consider the following monomial relations: $\rho_1 = 7 \longrightarrow 4 \longrightarrow 5$ and $\rho_2 = 3 \longrightarrow 2 \longrightarrow 1$. Then, ρ_1 and ρ_2 are bound consecutive relations by the walk $\varpi : 5 \longleftarrow 4 \longrightarrow 2 \longleftarrow 3$:

$$\begin{tikzcd}
7 \arrow[r] & 4 \arrow[r] & 5 & 4 \arrow[l] \arrow[r] & 2 & 3 \arrow[l] \arrow[r] & 2 \arrow[r] & 1
\end{tikzcd}$$

It is clear that $\overline{\mathrm{sop}}\,\varpi$ intersects Q_{ρ_1} and Q_{ρ_2} in maximal subpaths of ρ_1 and ρ_2, respectively.

However, the A-module M given by the following representation

$$\begin{tikzcd}
 & 0 & & \\
k \arrow[r, "Id"] & k \arrow[u, "0"] & k \arrow[l, "Id"'] \arrow[r, "Id"] & k \\
 & 0 \arrow[u, "0"] & 0 \arrow[l, "0"] \arrow[u, "0"'] &
\end{tikzcd}$$

is indecomposable and $pd\, M = id\, M = 2$. Therefore, A is not quasitilted algebra.

5. Applications

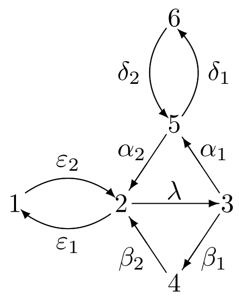
5.1. **Tilted algebras of Dynkin type.** In this section we provide a method to obtain a complete classification of tilted algebras of Dynkin type. In particular we give the classification of tilted algebras of type E_6.

The combinatorial description of how all the iterated tilted algebras of Dynkin type $\overline{Q}$ can be obtained from trivial extensions of finite representation type of Cartan class $\overline{Q}$ was studied in [13]. One of the essential tools in this description is the notion of admissible cuts of trivial extensions (see [13] and [14]). Given a quiver Q, a subset Δ of the set of arrows is called an *admissible cut* if Δ contains exactly one arrow of each chordless cycle in Q which is oriented.

The next step is to select the iterated tilted algebras with global dimension at most two, using [6]. Finally we use the Theorems 3.3, Proposition 4.3 and Theorem 4.4 to select the tilted algebras and discard the iterated tilted algebras not tilted. Recall that for algebras of finite representation type, the concepts of tilted and quasitilted are coincident.

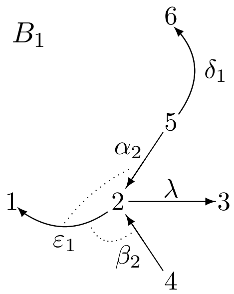
We illustrate this procedure in the following example.

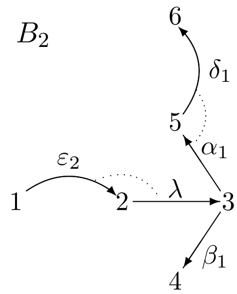
Example 5.1. Let $A = kQ/I$ be the algebra given by the quiver



bound by $\alpha_1\alpha_2 = \beta_1\beta_2$, $\beta_2\lambda\alpha_1 = 0$, $\alpha_2\lambda\beta_1 = 0$, $\varepsilon_2\lambda = 0$, $\alpha_2\varepsilon_1 = 0$, $\beta_2\varepsilon_1 = 0$, $\delta_2\alpha_2 = 0$ and $\alpha_1\delta_1 = 0$. Then the algebra A is a trivial extensions of finite representation type of class E_6. The algebra A has 14 possible admissible cuts, we look at some of them.

The admissible cuts $\Delta_1 = \{\alpha_1, \beta_1.\delta_2, \varepsilon_2\}$ and $\Delta_2 = \{\alpha_2, \beta_2.\delta_2, \varepsilon_1\}$ provides the iterated tilted algebras $B_1 = kQ/\langle I \cup \Delta_1\rangle$ and $B_2 = kQ/\langle I \cup \Delta_2\rangle$ respectively, given by the following quivers with relations





The algebra B_1 does not contain a critical algebra as a proper full subcategory, and has no bound consecutive relations. For Corollary 4.2, B_1 is a tilted algebra.

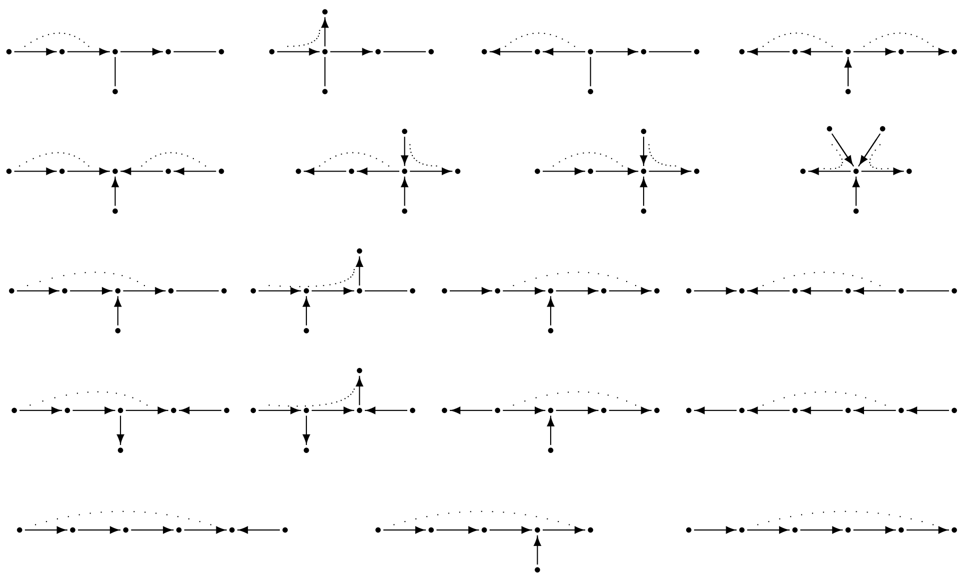
The relations of the algebra B_2 are bound consecutive whose walk ϖ is the stationary path in the vertex 3, and $\overline{\text{sop}}\ \varpi \cap \mathbf{Q}_{\varepsilon_2\lambda=0} = \overline{\text{sop}}\ \varpi \cap \mathbf{Q}_{\alpha_1\delta_1=0} = \{3\}$. For the Proposition 4.3, it follows that B_2 is not tilted.

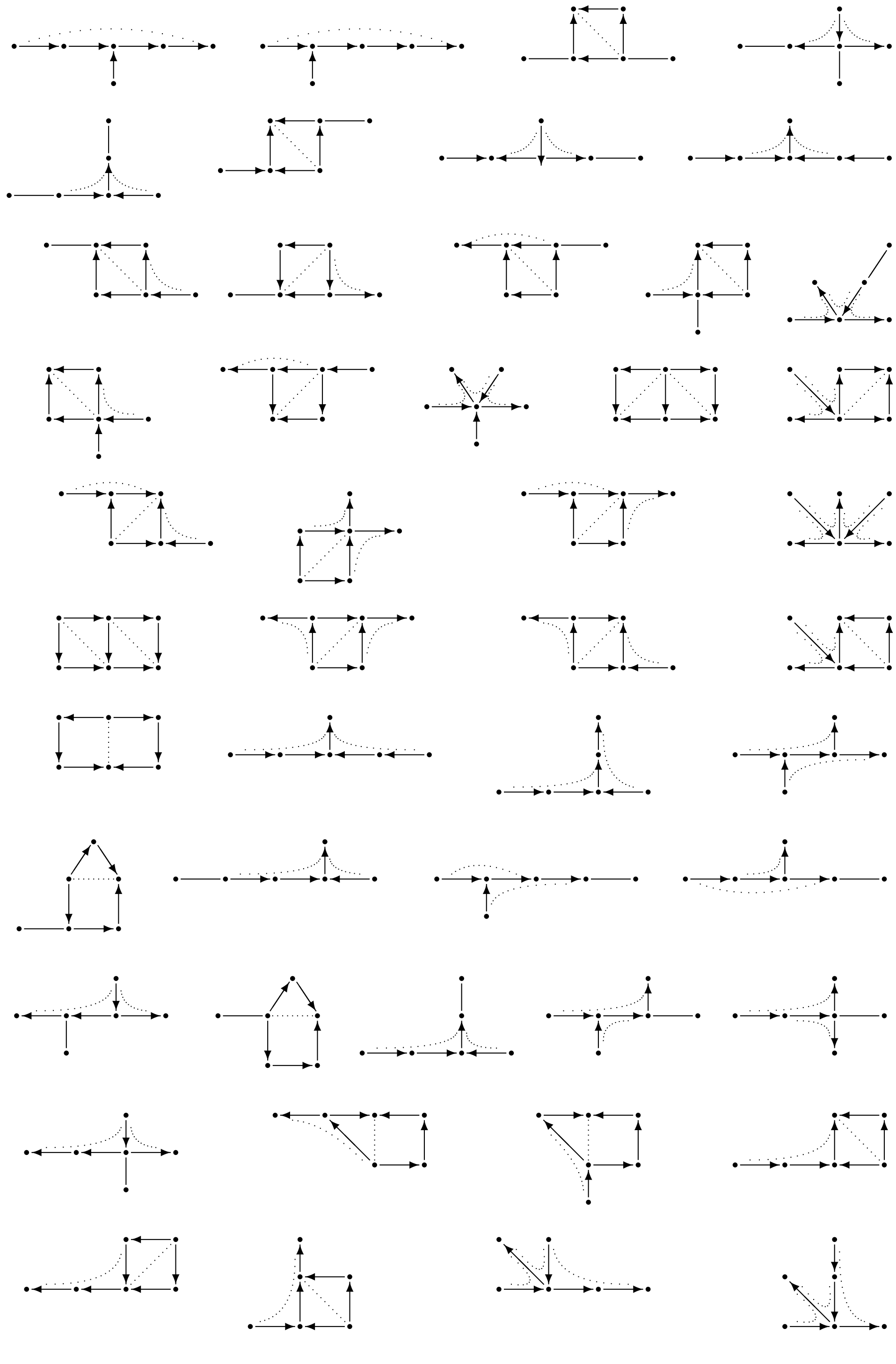
In this way, we could obtain a classification of all tilted algebras of Dynkin type. We now give the classification of Dynkin type E_6.

In the following, we will consider figures obtained from a quiver considering some arrows without orientation. Moreover, it is said that a quiver is associated with a figure if it is obtained by given an orientation to these arcs.

Theorem 5.2. *Let B be an algebra. Then B is tilted of Dynkin type E_6 if and only if*

i. *B is hereditary, or*
ii. *B or B^{op} is isomorphic to one of the following:*

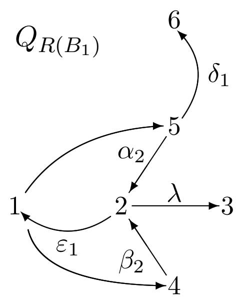




This classification is obtained from the combinatorial description illustrated in the Example 5.1.

The characterization of tilted algebras of type A_n in terms of their quiver with relations has been done by I. Assem in [1]. F. Huard in [19], has characterized the tilted algebras kQ/I where the underlying graph of Q is D_n. An alternative proof of this result can be made using the method described in this work. A complete classification of tilted algebras of type $E_p, p = 7, 8$, was done in [7].

5.2. **Cluster tilted algebras of finite representation type.** The relationship between tilted algebras and cluster-tilted algebras was given in [2]. For this, the authors introduced the concept of relation extension $R(B)$ of an algebra B with gl.dim.$B = 2$. We recall that the algebra $R(B) = B \propto \operatorname{Ext}^2_B(DB, B)$, where $DB = \operatorname{Hom}_k(B, k)$. In case the quiver of B has no oriented cycles, a construction of the quiver of the relation extension algebra is given in [2]. As an example, the quiver of the relation extension of tilted algebra B_1 given in Example 5.1, is



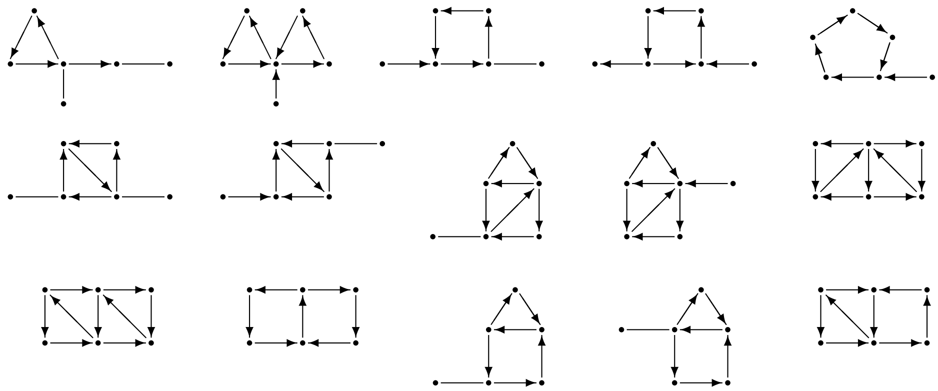
The main result in [2] characterizes the cluster tilted algebras as the relation extension of tilted algebras. More precisely, the authors prove that an algebra C is cluster tilted if and only if there exists a tilted algebra B such that C is isomorphic to the relation extension of B. Therefore if we make the relation extension of all algebras given in Theorem 5.2 and we analyze the non-isomorphic quivers, we obtain all cluster tilted algebras of Dynkin type E_6.

On the other hand, A. Buan, R. Marsh and I.Reiten in [8] proved that any cluster tilted algebra of finite representation type is uniquely determined by its ordinary quiver (up to isomorphism).

Using Theorem 5.2 and the two previous results, we deduce the following Theorem.

Theorem 5.3. *Let C be an algebra. Them C is cluster tilted of Dynkin type E_6 if and only if*

i. *C is an hereditary algebra of Dynkin type E_6, or*
ii. *the quiver Q_C or $Q_{C^{op}}$ is isomorphic to one of the following:*



It is interesting to point out that, the classification of the cluster tilted algebras of the remaining Dynkin types, can be done in a similar way. Since the cluster tilted algebras of type A_n and D_n were studied in [9] and [25] respectively, by geometric methods, and in [15] was showed that it is not possible to obtain the E_p type using geometric methods, we are mainly interested in the cluster tilted algebras of type E_p, $p = 7, 8$. Fort classification of these two classes of algebras see [7].

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