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Real polarizers and faster than light signaling

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Abstract

An imperfection of real polarizers (as opposite to ideal ones) seems to provide a method for faster than light signaling, threatening the structural stability of quantum field theory. The cause of the unphysical result is explained, and the problem of structural stability is solved.

A property required to any acceptable description of the physical world is that the unavoidable imperfections in the realization of a *gedankenexperiment* do not affect essential results. More precisely, what we expect is: if an experiment is performed in conditions having a small deviation from the ideal case, the observed results will also show a small deviation from the predictions obtained for that ideal case. This property is generally named "structural stability". The name comes from the theory of bifurcations [1], and it refers to the invariance of the configuration of critical points of a family of functions, against changes on their parameters. In the same context, the term "unfolding" names the expression of the family of functions which results to be structurally stable. The unfolding is reached by adding extra terms to the expression of the family of functions. It is said that the unfolding *stabilizes* the family of functions at the particular critical point under study.

In elementary quantum mechanics (QM), polarizers of light are represented as projectors. Hence, if two ideal, parallel polarizers are successively applied to an arbitrary input state, the probability of detecting a photon on the deflected channel of the second polarizer is zero. With real polarizers, instead, this probability is small but not identically equal to zero (see Fig. 1). As the state leaving the first polarizer is, *by definition*, $|o\rangle$, it follows that the output states of a real polarizer, $\{|o\rangle, |e\rangle\}$, are such that $|\langle o|e\rangle|^2 = \varepsilon^2$ (we will assume that ε is a real number for simplicity), where ε^2 is the probability of observing a photon in the deflected channel of the second polarizer. Note that this result arises from elementary QM notation and properties only.

If ε^2 is small enough to be undetectable $(1+\varepsilon^2 \approx 1)$ the imperfection is irrelevant. But let us consider now the fully symmetrical Bell state of two photons:

$$|\varphi^+_{\text{input}}\rangle = (1/\sqrt{2})(|x_a\rangle \otimes |x_b\rangle + |y_a\rangle \otimes |y_b\rangle) \tag{1}$$

which is emitted towards two remote stations, named as usual, "Alice" and "Bob". Bob performs a measurement on his photon by using an ideal polarizer oriented at an angle θ from the x-axis. The output states are named $\{|+\rangle, |-\rangle\}$, and $\langle -|+\rangle = 0$ as usual. Alice, instead, has a real, imperfect polarizer oriented parallel to the x-axis (see Fig. 2). The two-photon state after the polarizers is:

$$|\varphi^+_{\text{output}}\rangle = (1/\sqrt{2})\{|o\rangle \otimes (\cos\theta|+\rangle - \sin\theta|-\rangle) + |e\rangle \otimes (\sin\theta|+\rangle + \cos\theta|-\rangle)\} \tag{2}$$

Note that this state is normalized. The probability that Bob detects a photon in the detector "+" is:

$$P^+ = |\langle + |\varphi^+_{\text{output}}\rangle|^2 = |(1/\sqrt{2})(|o\rangle\cos\theta + |e\rangle\sin\theta)|^2 = (1/2)\cdot(1 + 2\varepsilon\cos\theta\sin\theta) \tag{3}$$

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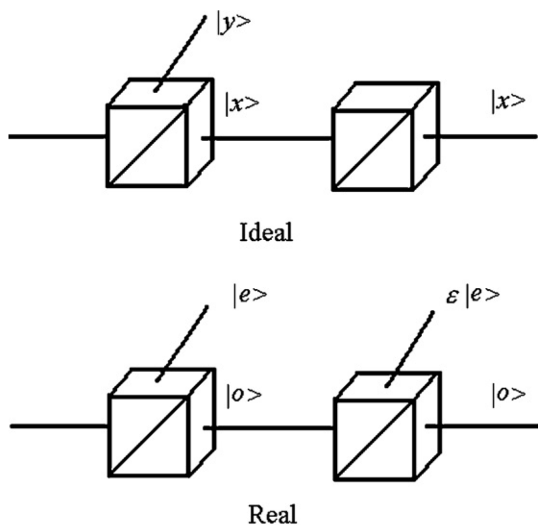

Fig. 1. Difference between ideal and real polarizers. In the real case, the amplitude probability of the deflected state after the successive application of two identical and parallel polarizers is not *strictly* equal to zero, but proportional to ε. It is assumed that $1+\varepsilon^2 \approx 1$. Note that, in this context, the first polarizer *defines* the states $\{|o\rangle, |e\rangle\}$.

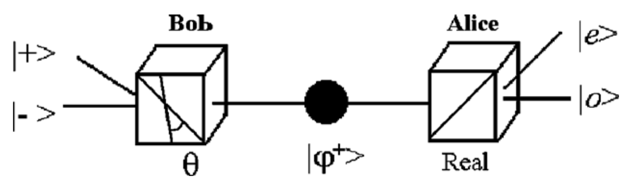

Fig. 2. Scheme of the setup that seemingly performs faster than light signaling (non-symmetrical setup). The ideal polarizer (Bob's) is rotated an angle θ respect of the real polarizer (Alice's).

In the same way, the probability of detection in the detector "−" is:

$$P^- = |\langle -|\varphi^+_{\mathrm{output}}\rangle|^2 = (1/2)\cdot(1-2\varepsilon\cos\theta\sin\theta) \tag{4}$$

Note that $P^+ + P^- = 1$ as it must be. But the perfect analyzer at Bob's station shows an unbalance, *at first order* in ε:

$$R \equiv P^+/P^- \approx 1 + 2\varepsilon\sin(2\theta) \tag{5}$$

So that Alice would be seemingly able to send to Bob messages instantaneously, by modulating the value of ε or by rotating her polarizer. The measurable unbalance of Alice's real polarizer is of order ε^2, which is not observable by hypothesis. In a symmetrical setup with two real polarizers at 45° of each other, the unbalance at Alice's side would be:

$$R|_{\mathrm{Alice}} \approx 1 + 2\varepsilon_{\mathrm{Bob}} + \text{higher orders in}(\varepsilon_{\mathrm{Alice}}, \varepsilon_{\mathrm{Bob}}), \tag{6}$$

And symmetrically for Bob, so that they would be able to send messages in both directions instantaneously, and simultaneously, using the same set of entangled states!

In many real world polarizers the value of ε^2 is far from being negligible, but this is not relevant. The relevant point is: an infinitesimally small imperfection does not affect the QM results, but it does affect instead, and in a non-infinitesimal way, the compatibility of QM with the special theory of relativity (SR).

The existence of faster than light signaling obviously is a wrong result. If it were correct, the compatibility between QM and SR would be broken. As a consequence, quantum field theory (which is based both in QM and SR) would be at stake.

Even though QM have always been suspected to be in contradiction with SR because of the nonlocal properties of entangled states, all attempts to exploit these nonlocal properties to send signals have failed, and SR and QM have enjoyed what Shimony has called a "peaceful coexistence" [2]. These words suggest a precarious situation, but there are good reasons to believe that the "peaceful coexistence" is based on firm grounds. In our opinion, the most sounding reason that ensures compatibility between QM and SR is the equivalence between the gauge invariance of the electromagnetic field and the phase invariance of the wavefunction (see, for example, Ref. [3]). Besides, explicit proofs that the non-locality of entangled states cannot be used to achieve signaling can be found in Refs. [4,5], and they are briefly reviewed in the Appendix.

We can rely on the existing demonstrations of compatibility between QM and SR for ideal realizations: perfect polarizers, perfect principle of superposition (see below), perfect phase invariance, etc. But what if there is an *infinitesimal* deviation from ideal conditions? A physically acceptable theory must be "robust" (i.e., structurally stable) against these deviations. As we have just seen, the compatibility seems to be broken if ε is not *strictly* equal to zero. It can be immediately shown that also the proofs in Refs. [4,5] fail if $\varepsilon \neq 0$ (see Appendix).

Therefore, the compatibility between QM and SR, and hence quantum field theory, appear to be structurally unstable at the point (in parameters' space) $\varepsilon = 0$. This is an unacceptable result. The problem now is to demonstrate that QM and SR remain compatible even if $\varepsilon \neq 0$.

A solution, that comes immediately into the mind, is that the effect of signaling arises from an erroneous notation of the output states of the imperfect polarizer. An approach following the quantum description of the beam splitter [6] writes the output states in terms of four orthogonal output modes. In this way, one always gets $\varepsilon = 0$. For several reasons [7], we do not find this explanation fully satisfactory. The main reason is that it does not solve the general problem (i.e. regardless the cause of the imperfection) of stabilizing the theory at $\varepsilon = 0$.

It has also been argued that the error in the seeming mechanism of signaling presented here is that only strictly unitary transformations (that is, that ε must be always strictly equal to zero) are acceptable in QM. But this argument is not really a solution. Banning even an infinitesimal deviation from $\varepsilon = 0$ is equivalent to accepting that there is no way to stabilize the theory at that critical point. Giving up the requirement of structural stability is not only dangerous but, as we will see, unnecessary, for the theory can indeed be stabilized, and at a negligible cost.

It is worth mentioning here, as an illustration, another example of structural instability of the compatibility between QM and SR. An essential property of QM is that it is a linear theory. But, some nonlinear extensions of QM appear unavoidable. The theories of decoherence often assume the interaction with an environment. This interaction naturally leads to a nonlinear Schrödinger equation, which does allow signaling. This problem has been solved by the addition of a noise term, which is a natural consequence of an interacting environment [8]. Hence, using the wording of bifurcation theory, we may say that quantum field theory has been stabilized by the noise term, at the critical point (in the parameters' space) where that nonlinearity vanishes. Also, we may say that (QM + SR + noise term) is the unfolding of that critical point. In short, structural stability imposes noise (fluctuations) to be introduced whenever decoherence (dissipation) is present. This sounds most reasonable. However, it should be remembered that, in general, stabilization of a theory might imply its complete reformulation.

Let us summarize the situation at this point. If one hypothesizes a small (actually infinitesimal) non-orthogonality of the output states of a polarizer, even if it is otherwise undetectable ($\propto\varepsilon^2$), the compatibility between QM and SR collapses (signal $\propto \varepsilon$). This implies that quantum field theory is structurally unstable, that is, physically untenable. This is a wrong result, no question about this. The right questions are: what does stabilize the theory, and why?

The acceptable answers cannot use ideas or properties belonging to a field theory. Otherwise, one may fall into a logical loop. Electromagnetism (and Optics) imply or include SR, because of the Lorentz invariance of the Maxwell's equations. One may believe that is demonstrating something that one has, actually, assumed. That is why a safe demonstration of the compatibility between QM and SR must use elementary QM concepts *only*. This is what has been achieved in the Refs. [4,5] for the case $\varepsilon = 0$. As we show, this is also possible even if $\varepsilon \neq 0$.

We present the answer we found below.

Note that a real polarizer transforms an input state $|\psi_i\rangle = \alpha|x\rangle + \beta|y\rangle$ into an output state $|\psi_o\rangle = \alpha|o\rangle + \beta|e\rangle$. This output state is, in general, not normalized:

$$\langle\psi_o|\psi_o\rangle = |\alpha|^2 + |\beta|^2 + \alpha^*\beta\langle o|e\rangle + \alpha\beta^*\langle e|o\rangle \tag{7}$$

For some particular case (as the state $|\varphi^+_{\text{output}}\rangle$ is) the output state is normalized, but its modulus changes if the axes are rotated, because in (7) the modulus is a function of the *product* of the original components. Regardless any consideration about compatibility with SR, this feature is unacceptable. It is unacceptable even *inside* the framework of elementary QM. In order to get the modulus of the output state independent of the axes' choice, it must be "renormalized". From elementary geometry, the correction for an arbitrary input state is:

$$|\psi_i\rangle = (\alpha|x\rangle + \beta|y\rangle) \to |\psi_o\rangle_{\text{corrected}} \\ = |\psi_o\rangle - \alpha^*\beta\langle o|e\rangle|\psi_o\rangle/(|\alpha|^2 + |\beta|^2) \tag{8}$$

The modulus of $|\psi_o\rangle_{\text{corrected}}$ is independent of the axes' orientation, as required. This equation can be regarded as a correction to the standard QM formalism, to include infinitesimal non-unitary components. Now, we show that this correction term *also* eliminates signaling.

Applying Eq. (8) to the state $\langle +|\varphi^+_{\text{input}}\rangle$:

$$\langle +|\varphi^+_{\text{input}}\rangle \to \langle +|\varphi^+_{\text{output}}\rangle_{\text{corrected}} \\ = \langle +|\varphi^+_{\text{output}}\rangle - \varepsilon\cos\theta\sin\theta\langle +|\varphi^+_{\text{output}}\rangle \tag{9}$$

Then:

$$P^+ = |\langle +|\varphi^+_{\text{output}}\rangle_{\text{corrected}}|^2 \\ = (1/2)\cdot(1+\varepsilon\cos\theta\sin\theta)^2(1-\varepsilon\cos\theta\sin\theta)^2 \approx 1/2 \tag{10}$$

The same happens with P^-, and the effect of signaling disappears. A description in terms of $\sin(\varepsilon)$ and $\cos(\varepsilon)$, which is valid at all orders in ε, leads to the same result. Also, the demonstrations of impossibility of signaling detailed in the Refs. [4,5] become valid again (see the Appendix). Note that we have not used ideas or arguments other than elementary QM properties and needs, as required.

We stress that the origin of the unfolding term in Eq. (8) is the renormalization of the output vector $|\psi_o\rangle$. Therefore, it is not an *ad hoc* recipe to avoid signaling but an operation of normalization, a necessary step previous to calculate probabilities. Note that it must be included every time $\langle o|e\rangle$ is not *strictly* equal to zero, even if the imperfection is not detectable, or if $|\psi_o\rangle$ is normalized for the particular axes' orientation being used, or if it is obtained as the projection of a normalized state of a higher dimension. As far as we know, this connection between the conservation of the state vector's norm and the impossibility of signaling has not been indicated before, and it may put some new light on this question.

In summary: a usual imperfection of real polarizers seemingly allows faster than light signaling. This imperfection involves a slightly non-orthogonal basis, or in other words, an infinitesimal deviation from a strict unitary transformation. Regardless of its cause, this deviation must be taken into account for the theory to be structurally stable (at $\varepsilon = 0$ in parameters' space). It cannot be simply "forbidden". We have found that an elementary QM requirement (i.e. that a state vector must have the same modulus for every axes' choice) forces the addition of a correction term, and that this term *also* cancels the signaling effect. This term is, in consequence, the "unfolding" that stabilizes the compatibility between QM and SR at $\varepsilon = 0$.

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Appendix. On the demonstrations of the impossibility of signaling

There are several proofs of the impossibility of signaling. We discuss here two of the main ones, and show why their conclusions do not apply if $\varepsilon \neq 0$. Let us begin by summarizing Bussey's [4] proof. Following his notation, consider the entangled state:

$$|\psi\rangle^{(1,2)} = \Sigma a_{ij}|i\rangle^{(1)}|j\rangle^{(2)} \tag{A.1}$$

Let $\{|i\rangle^{(1)}\}$ be the eigenstates of one of the quantities that Alice (or "1") can choose to measure. Then, the probability that Bob (or "2") observes the state $|j\rangle^{(2)}$ is:

$$W_j^{(2)}|_i = \Sigma a_{ij}^* a_{ij} \tag{A.2}$$

It is assumed that the basis $\{|i\rangle^{(1)}\}$ is orthonormal. Now, we assume that Alice chooses to measure on a different basis $\{|\xi\rangle^{(1)}\}$, which is linked to the previous basis through:

$$|i\rangle^{(1)} = \Sigma \beta_{i\xi}|\xi\rangle^{(1)} \tag{A.3}$$

Calling $b_\xi j = \Sigma a_{ij}\beta_{i\xi}$, the entangled state is now:

$$|\psi\rangle^{(1,2)} = \Sigma b_{\xi j}|\xi\rangle^{(1)}|j\rangle^{(2)} \tag{A.4}$$

And therefore:

$$W_j^{(2)}|_\xi = \Sigma b_{\xi' j}^* b_{\xi j}\langle\xi'|\xi\rangle \tag{A.5}$$

If the basis $\{|\xi\rangle^{(1)}\}$ is orthonormal, and using $\Sigma\beta_{i\xi} * \beta_{k\xi} = \delta_{ik}$, we obtain $W_j^{(2)}|_i = W_j^{(2)}|_\xi$ and no signaling is possible, because the result is the same no matter the measuring basis Alice chooses to use. When $\varepsilon \neq 0$ instead, the second basis is not orthonormal and the proof is not valid. To see the changes explicitly, the coefficients b_{o+} and b_{e+} can be calculated, as:

$$\begin{aligned} b_{o+} &= a_{x+}\beta_{xo} + a_{y+}\beta_{yo} = (1/\sqrt{2})\cos\theta\cdot 1 + (1/\sqrt{2})\sin\theta\cdot 0 \\ &= (1/\sqrt{2})\cos\theta \qquad\qquad (A.6) \\ &= a_{x+}\beta_{xe} + a_{y+}\beta_{ye} = (1/\sqrt{2})\cos\theta\cdot 0 + (1/\sqrt{2})\sin\theta\cdot 1 \\ &= (1/\sqrt{2})\sin\theta \qquad\qquad (A.7) \end{aligned}$$

Then, the probability that Bob observes "+" is:

$$\begin{aligned} W_+^{(\mathrm{Bob})}|_{\text{real polarizer}} &= (b_{o+}^*\langle o| + b_{e+}^*\langle e|)(b_{o+}|o\rangle + b_{e+}|e\rangle) \\ &= b_{o+}^2 + b_{e+}^2 + 2\varepsilon b_{o+}b_{e+} \end{aligned} \tag{A.8}$$

Because now $\langle\xi'|\xi\rangle \neq \delta_{\xi'\xi}(\langle o|e\rangle = \varepsilon)$. This expression (and a similar one for W_-) leads to signaling in the same way as described in the main text.

The proof by Cantrell and Scully [5] is based on the description (of a single particle in a multiparticle state) with the reduced density matrix. They demonstrate that the reduced matrix of Bob's particle is the same (and proportional to the identity matrix, i.e. maximally mixed), regardless the operation Alice may perform on her particle. Once again, the argument does not hold here because of the non-orthogonality of the transformed basis. This can be shown from the calculation of Bob's particle reduced matrix (from Eq. (2)):

$$\begin{aligned} \rho_{\mathrm{Bob}} &= \langle o|\varphi^+_{\text{output}}\rangle\langle\varphi^+_{\text{output}}|o\rangle + \langle e|\varphi^+_{\text{output}}\rangle\langle\varphi^+_{\text{output}}|e\rangle \\ &= (1/2)\cdot(1+\varepsilon\sin 2\theta)|+\rangle\langle+| \\ &\quad + (1/2)\cdot\varepsilon\cos 2\theta\{|+\rangle\langle-| + |-\rangle\langle+|\} \\ &\quad + (1/2)\cdot(1-\varepsilon\sin 2\theta)|-\rangle\langle-| \end{aligned} \tag{A.9}$$

Then $P^+ = \langle+|\rho_{\mathrm{Bob}}|+\rangle$, $P^- = \langle-|\rho_{\mathrm{Bob}}|-\rangle$ and this leads to signaling again.

On the other hand, if the state $\langle+|\varphi^+_{\text{output}}\rangle_{\text{corrected}}$ (see Eq. (9)) is used in (A.9), the extra terms arising from the unfolding exactly cancel the signaling terms. The same happens when it is used in (A.6–A.7).

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