Extreme events in the Ti:sapphire laser

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We report experimental and theoretical evidence of the existence of extreme value events in the form of scarce and randomly emerging giant pulses in the femtosecond (self-pulsing or Kerr-lens mode-locked) Ti:sapphire laser. This laser displays complex dynamical behavior, including deterministic chaos, in two different regimes. The extreme value pulses are observed in the chaotic state of only one of these two regimes. The observations agree with the predictions of a well-tested theoretical model that does not include noise or self-*Q*-switching into its framework. This implies that, in this laser, the extreme effects have a nontrivial dynamical origin. The Ti:sapphire laser is hence revealed as a new and convenient system for the study of these effects.

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Extreme amplitude events, appearing rarely but still with a frequency much larger than expected from a Gaussian distribution, were originally reported in oceanic dynamics, where they received the name of "rogue" or "freak" waves [1–4]. As opposed to tsunamis or solitons, these phenomena do not necessarily propagate for large distances, but they can disappear in short spatial length, so that they are difficult to observe. A few years ago, an analogous phenomenon, i.e., the sporadic occurrence of pulses of light of extraordinary intensity, was found in a system based on a microstructured optical fiber pumped by an ultrashort pulse laser near the threshold of supercontinuum generation [5,6]. These sometimes called "optical rogue waves" have been described and interpreted as the collision of breathers [7–9], and conditions for their formation have been revealed from experiments on optical setups [10,11]. They have been observed in a VCSEL with a cw signal injected from a master laser oscillator [12]. Numerical simulations and theoretical studies predict that they should exist also in mode-locked fiber lasers [13,14]. There is, hence, a widespread and rising interest in the observation of extreme optical events.

In this Letter, we report what we believe is the first observation of extreme value optical events in the Kerr-lens-mode-locked (KLM) Ti:sapphire laser, which is the widespread source of ultrashort light pulses (<100 fs) nowadays. The nonlinear Schrödinger equation is one of the usual theoretical approaches to describe this laser [15]. This provides the first hint to anticipate the existence of extreme value effects here, because this equation is also a classical approach to describe them in ocean dynamics [2–4].

Our laser is the standard seven-element design, which uses a pair of prisms into the optical cavity to introduce negative group-velocity dispersion (GVD). It emits trains of light pulses of duration between 20 and 200 fs at a rate of 87 MHz. The output is observed with a fast silicon photodiode (100 ps rise time) and recorded in a 350 MHz, 5 Gs/s digital oscilloscope with memory of 16 MB. In this way we can save, for further numerical elaboration, signals proportional to the energy values of a series of about 10^4 mode-locking pulses [16].

The dynamics of the KLM Ti:sapphire laser includes multistability and deterministic chaos [17]. Two self-pulsing regimes with different properties are observed: one (named P1) involves transform-limited pulses; the other (named P2) involves pulses whose carrier optical frequency varies with time ("chirp"). The laser wanders spontaneously from one regime to the other in a time scale of several minutes. The two regimes can be distinguished by the pulse duration, the spectrum (see Fig. 1) and, even by the naked eye, as a change in the size of the laser spot. Applying a mechanical perturbation can induce a transition from one regime to the other.

One of the important parameters of this laser is the net value of the cavity's GVD. It is easily controllable by mechanically displacing one of the intracavity prisms. As the GVD is tuned negative and close to zero, the uniform train of pulses destabilizes into a chaotic state, which is observed as an intricate pulse-to-pulse change of some or all of the five variables that describe a pulse (energy, duration, chirp, spot size, and beam curvature). Each of the self-pulsing regimes (P1 and P2) has its own, different route to chaos. Once in the chaotic state, regime P2 displays spontaneous large fluctuations. Sometimes they "propagate" through some cavity round trips with an amplitude well above the average. Occasionally, they appear isolated. These fluctuations are recognized here as

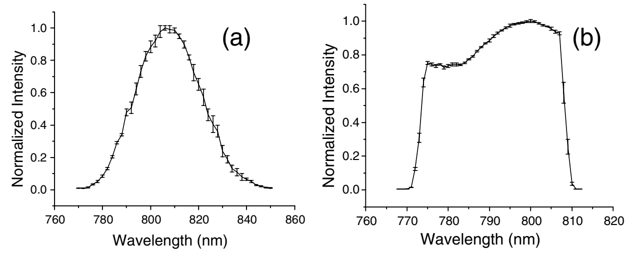

Fig. 1. Spectra obtained in the stable state of the two dynamical regimes: (a) P1, transform-limited pulses—extreme events are not observed and (b) P2, chirped pulses—extreme events are observed, note the similarity with the spectrum in microstructured fibers [5,6]. In the chaotic states of P1 and P2, the spectra are unstable but still clearly distinguishable.

optical extreme events. In what follows, we support this claim with several arguments.

As a first step, we show that the fluctuations have the statistical properties usually alleged to extreme value events. These events are characterized by an L-shaped distribution. In a simplified description, an *abnormality index* $\mathrm{AI} = H_w/H_s$ is often employed, where H_w is the amplitude of the event and H_s is the average amplitude among the one-third of the events with the largest amplitude in a time series. Events having $\mathrm{AI} > 2$ (or $H_w > 2 \times H_s$) are routinely considered extreme [1]. An alternative definition is in terms of the standard deviation σ of the distribution. In this case, the boundary is $H_w > 4 \times \sigma$. The two boundaries may be coincident or not, depending on the functional form of the statistical distribution.

In Fig. 2(a), we show the experimentally recorded distribution of pulse energies of a train of 9978 pulses in the chaotic state of the P2 regime (average pulse duration: 80 fs). The pulse energies are indicated in the horizontal axis in arbitrary units. They are scaled so that the highest pulses have a value of 100 (this scaling is applied to all the figures). The L shape, characteristic of extreme events, is clearly seen (note the logarithmic scale). Here $2\times H_s = 86.2$, and 237 pulses exceed it, $4\times\sigma = 87.1$, and 206 pulses exceed it (warning: in the figures, the two boundary values are indicated with a single vertical dotted line, for they are practically coincident). Therefore, pulses with the statistical signature of extreme value events, according to the two definitions in use, are observed in the chaotic state of regime P2.

A key experimental result is that the extreme events appear only in regime P2. The distribution in Fig. 2(b) is obtained for the same setup alignment and parameter values than Fig. 2(a), but with the laser operating in the chaotic state of regime P1 (average pulse duration: 40 fs). Note that the distribution is not L-shaped now. Here $2\times H_s = 178.4$ and $4\times\sigma = 165.7$—both boundaries are well outside of the figure. No extreme events are hence observed in regime P1. This is a key result, for, if the fluctuations in regime P2 were mere noise or caused by self-Q-switching (*see below*), there is no imaginable reason why they are not observed in the coexisting regime P1, too.

The main question to answer now is whether the fluctuations have a nontrivial dynamical origin, or not. It might be argued that they are self-Q-switching. A decisive argument is that a well-tested theoretical model, that does not allow Q-switching as a possible solution, correctly explains the observations.

The theoretical models for mode-locked lasers are based on, at least, three alternative approaches. The first one uses, as was said, the nonlinear Schrödinger equation [15]. A second approach derives Maxwell–Bloch equations from fundamental principles in the multimode laser case [18]. Both approaches require relatively complex numerical simulations, because they involve partial differential equations. The third approach reduces the continuous time dynamics to an associated discrete system: the iterative or Poincaré map. The description with maps is alternative to differential equations—no information is gained or lost. Writing the map equation can be as difficult as solving the differential equation, but the task is greatly simplified if the system has some "internal clock" defining the position of the adequate times to build up the map. In the case of KLM lasers, the clock is provided by the cavity round trip time. It is immediate then to obtain recursive relations linking the pulse variables in the $(n + 1)$ round trip with the ones at the n round trip by using 4×4 spatiotemporal matrices. Adding an equation for the pulse energy, a five-variable map is obtained [17]. This approach explains not only the stable mode-locking operation, but also the coexistence of several regimes, the statistics of the jumps among them, the different routes to chaos, the dimensions of embedding and fractals of the strange attractors, and the magnitudes of the Lyapunov exponents [19]. For reasons of space, we cannot review the five-variable map in this Letter. For our purposes here, it suffices to say that, in this approach, the available gain is a fixed parameter. In consequence, this approach is unable to produce a Q-switching state as a solution.

Yet, when applied to the description of the chaotic state of regime P2, the five-variable map model does predict the existence of extreme events. A numerically generated series of the energies of a train of 3×10^4 pulses has the distribution of Fig. 3(a). It is L shaped. Here $2\times H_s = 92.6$ and 140 pulses exceed it, and $4\times\sigma = 93.1$ and 138 pulses exceed it. Extreme events are predicted in this way for broad ranges of the parameters' values. We stress that no fine-tuning of the many parameters of this laser is performed; only the value of the GVD is adjusted, to fit the observed average pulse duration (80 fs). The fact that the five-variable map (which cannot display self-Q-switching solutions) correctly predicts the

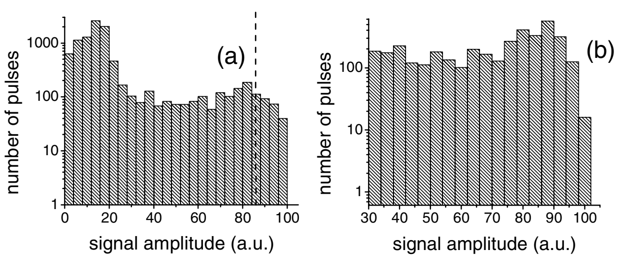

Fig. 2. Distributions of the pulse energies in the chaotic states, experimental results: (a) regime P2—the dotted line indicates the standard boundary values for extreme events and (b) regime P1—the boundary values ($2\times H_s = 178.4$ and $4\times\sigma = 165.7$) are rightward and out of the figure.

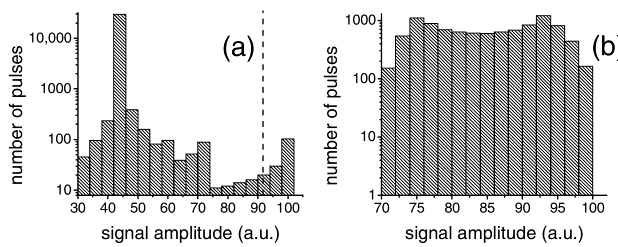

Fig. 3. Distributions of the pulse energies in the chaotic states, numerical results according to the five-variables map: (a) regime P2—the vertical dotted line indicates the standard boundary values for extreme events and (b) regime P1—the boundary values ($2\times H_s = 187.8$ and $4\times\sigma = 180.4$) are out of the figure. Therefore, extreme events are predicted to exist in regime P2 only, in agreement with the observations.

existence of extreme events for the chaotic state of regime P2 and discards self-Q-switching as their cause.

When applied to the description of the coexisting regime P1, the five-variables map does *not* predict extreme events, in agreement with the observations once again. A numerically generated series of the energies in the chaotic state of regime P1 has the distribution of Fig. 3(b), which is clearly not L shaped. The laser parameter values are the same as in Fig. 3(a). Here $2 \times H_s = 187.8$ and $4 \times \sigma = 180.4$, so that no extreme events are predicted in regime P1. They are exclusive of regime P2, both experimentally and theoretically.

An additional argument supporting the extreme nature of the fluctuations in regime P2 comes from the optical spectra. The spectrum of regime P1 has a nearly Gaussian shape [Fig. 1(a)]. The spectrum of regime P2 has a square shape instead [Fig. 1(b)], similar to the one obtained in the generation of supercontinuum light in photonic fibers [5,6], which is the recognized source of optical extreme events. This coincidence in two rather different physical situations (i.e., the passive, ultrashort pulse pumped, microstructured optical fiber, on the one hand, and, on the other hand, the active KLM laser in only one of its self-pulsing chaotic states) indicates that there is a close relationship between both phenomena. In the case of the KLM laser, the spectral shape is caused by the enhanced chirp due to the larger average self-phase-modulation suffered by the pulses in regime P2. It is also similar to the spectra predicted for highly chirped dissipative soliton solutions of the cubic-quintic complex Ginzburg–Landau equation [20].

It has been claimed that *granularity* and *inhomogeneity* are essential ingredients for the formation of extreme optical events [11]. A mode-locked laser has a huge number of oscillating cavity modes. It can be then considered as a (time) 1D version of the (spatial) 2D experiment described in [11]. The granularity is provided by mode interference, while the spatial inhomogeneity is provided by the clustering caused by the (inhomogeneous) spatial hole burning of the gain into the active volume, typical of solid-state lasers. The average values of the nonlinear terms due to the Kerr effect are larger in regime P2 than in P1, so that we conjecture that a minimum value of these nonlinearities is an additional condition for the formation of extreme events, at least in the KLM laser.

In summary: we present, for the first time to our best knowledge, experimental and theoretical evidence of the existence of optical extreme value events in a mode-locked laser. We claim the observed fluctuations are extreme events because (i) they have the distinctive statistical properties (L-shaped distribution and violation of the $2 \times H_s$ and $4 \times \sigma$ boundaries), (ii) they are correctly predicted by a theoretical model, and (iii) they display the spectral shape of the extreme events observed in optical fibers. We have confirmed that they are ruled by deterministic dynamics [12] and that they do not require noise to appear; although the precise role played by noise in the real system remains to be determined. The presence of extreme events in the chaotic state of a complex system is not obvious, as is demonstrated by the fact that they exist in regime P2 and not in P1. The coexistence of the two regimes makes the Ti:sapphire laser most attractive as a bench work for the study of optical extreme events, for one can turn them "on" and "off" at will, without disturbing the values of the control parameters. The two regimes are immediately distinguishable, which is an important practical advantage. Besides, a simple and well-tested theoretical model is at hand. The notice that the KLM Ti:sapphire laser displays extreme events has a large impact, for this laser is available in numerous laboratories. It means that experimental work on optical extreme events becomes accessible to many researchers worldwide.

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