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Range of interaction in an opinion evolution model of ideological self-positioning: Contagion, hesitance and polarization

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HIGHLIGHTS

- A new model for the study of the ideological self-positioning was developed.
- Our results were compared with real data from opinion polls in some countries.
- The presence of undecided agents is crucial to reproduce data from polls.

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ABSTRACT

The evolution of public opinion using tools and concepts borrowed from Statistical Physics is an emerging area within the field of Sociophysics. In the present paper, a Statistical Physics model was developed to study the evolution of the ideological self-positioning of an ensemble of agents. The model consists of an array of L components, each one of which represents the ideology of an agent. The proposed mechanism is based on the "voter model", in which one agent can adopt the opinion of another one if the difference of their opinions lies within a certain range. The existence of "undecided" agents (i.e. agents with no definite opinion) was implemented in the model. The possibility of radicalization of an agent's opinion upon interaction with another one was also implemented. The results of our simulations are compared to statistical data taken from the Latinobarómetro databank for the cases of Argentina, Chile, Brazil and Uruguay in the last decade. Among other results, the effect of taking into account the undecided agents is the formation of a single peak at the middle of the ideological spectrum (which corresponds to a centrist ideological position), in agreement with the real cases studied.

1. Introduction

In recent times, Statistical Physics has been extended beyond its usual limits to tackle Sociology problems, leading to the so-called *Sociophysics* [1–6]. In such a context, one of the most studied topics is the evolution of public opinion through simple mathematical models. Some of these models involve a population where, in a simple way, each member of the group can adopt an "opinion value" among two or several possible ones, according to specific interaction rules.

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One of the more studied opinion models is the voter model [7–10], which studies the evolution of opinion in a group of agents. Even though it is mainly applied to two possible opinions, it can be generalized to several opinion values. This model consists on the selection of an agent and one of his neighbors at random (it can be developed in a particular lattice or without structure, like a mean field). Afterwards, the agent will copy the neighbor's opinion and the procedure is repeated until convergence is reached.

Another model employed for the study of opinion evolution with two possible values is the Sznajd model [11,12]. Within this general model, the presence of "contrarians" (i.e. agents that change opinion in a direction opposite to the opinion of the agent with which they interact) has also been taken into account in some studies [13–16].

For the case of several possible opinion states, one possible model that was mainly studied in the continuous range is the "Bounded Confidence" [17–22], where agents have continuous opinions and the interactions are non-linear. The agents influence each other only if the distance between their opinions is below a threshold (i.e. it is assumed that people do not take into account opinions that are very different from their own).

In the present work, the value of the "opinion" represents the ideological self-placement of a citizen within the full range of possible positions, from left to right, with 0 being the extreme left, 10 the extreme right and 5 the center or middle point. This classical scale is derived from a way to understand the identity of human political behavior, as a cognitive metaphor used by both citizens and social scientists. As a part of Political Sciences' research, it finds its roots on the theory of electoral competition and its local applications to the study of contemporary massive democracies dynamics [23,24]. In our model, we also consider the possibility of an agent being undecided (i.e. that particular agent has no definite opinion). The role of undecided agents has also been studied by Balenzuela et al. [25], Vazquez et al. [26,27], and de la Lama et al. [28]. Another effect taken into account in our work is that of polarization, which takes place when two agents with opposite ideology interact and the position of one of them becomes more extreme as a result of the encounter. A similar effect has been studied by La Rocca et al. [29].

The opinion of each individual agent inside the model can evolve according to a simple set of rules governing the interaction between said agents. Although several two-dimensional lattice models and even small world networks have been studied [30–32], in our model the agents are disposed in a one dimensional array of L components. However, since in our model the interacting agents are selected at random, this spatial arrangement is of no consequence.

The simulation results are compared with real statistical data about political–ideological self-placement of citizens from Argentina, Brazil, Chile and Uruguay within the period ranging from 2000 to 2010 provided by Latinobarómetro [33], a Chilean consulting group specialized in comparative indicators of public opinion evolution on the region. The comparison is realized in order to introduce a sociological and contextual perspective about the opinion phenomenon, in the frame of a transdisciplinary reflection favored by the current Sociophysics field.

Nowadays, a renewed interest about the "ideology" is noticed as a transversal topic in the study of identification and political positioning processes. The ideology articulates a "coherent and relatively stable set of beliefs or values" [34] but also realizes – within every historic moment – a series of antagonisms that represent each society. In this regard, not only consistency but also stability and contrast are significant analytical dimensions in the study of the effects of political ideology.

According to this approach the present work is organized as follows: in Section 2 we introduce the model and simulation method, in Section 3 we present the main results and discussions, and in Section 4 we present the final conclusions.

2. Model and simulation method

The present model consists on a one-dimensional array of L components. Each component represents the ideological position of an individual agent and can take any integer value between 0 and 10, with 0 being the extreme left and 10 being the extreme right. There is also the possibility for an agent to be "undecided" (i.e. a person with no definite opinion), in which case the opinion value is taken as -1. It must be stated, though, that this value of -1 for the undecided agent must be thought of as a label, i.e., its numerical value has no significance in the context of the 0–10 scale.

For most of our simulations we consider a set of 1200 agents ($L = 1200$), and use a uniform distribution of opinions (i.e. 100 agents with each opinion value from -1 to 10) as a starting configuration.

The evolution mechanism is basically the Voter Model [5], in which two agents (say, agent i and agent j) are selected at random.

2.1. Contagion within range

If none of the selected agents is undecided and the difference between their two opinion values is less or equal than the "contagion range" (which is varied between 1 and 10), agent j takes the same opinion as agent i, with probability p_1.

2.2. Undecided agents

In the case that agent j is undecided and agent i has a definite opinion, agent j takes the opinion of agent i with probability p_2.

On the other hand, if none of the selected agents is undecided (and there was no contagion as explained in the preceding subsection) and their opinions are on opposite sides of the spectrum (i.e. one of them has an opinion value greater than 5 and the other one has an opinion value of less than 5), agent *j* becomes "undecided", with probability p_2. This implies that when a person with "left wing ideology" interacts with a person with "right wing ideology", there is the possibility of one of them becoming "undecided".

2.3. Polarization

If there was no opinion contagion nor generation of undecided agents, the model also contemplates the possibility that, when two agents with opposite ideology interact, the position of one of them becomes more extreme, as a consequence of the encounter, according to the following.

If agent *i* has an opinion value of 9 or 10 and agent *j* has an opinion value between 1 and 4, subtract 1 to the opinion value of agent *j* with probability p_3. On the other hand, if agent *i* has an opinion value of 0 or 1 and agent *j* has an opinion value between 6 and 9, add 1 to the opinion of agent *j* with probability p_3.

This implies that, when agent *i* has a radical position and agent *j* has a moderate but opposite position, the opinion of agent *j* has a probability p_3 of becoming more radical in the opposite direction.

2.4. General observations of the model

A general observation is that, in the cases of probability $p_2 = 1$, the value of the probability p_3 is not relevant. The explanation for this is that the conditions for the polarization are a particular case of the conditions for the generation of "undecided" agents. If the conditions for the generation of "undecided" agents are fulfilled and the probability of generating them (p_2) is equal to 1, the conditions for polarization will never get evaluated. On the other hand, if the conditions for the generation of "undecided" agents are not fulfilled, the conditions for the polarization will not be fulfilled either.

For the case in which $p_2 = 0$, "undecided" agents are not taken into account. Even though they might be present, they do not play any role in the contagion process.

2.5. Rate equations

A set of differential equations for the time evolution of the population of each opinion value x_i $(-1 < i < 10)$ are presented for interaction range equal to 1.

$$
\begin{aligned}
\frac{dx_5}{dt} &= p_1x_5x_4 - p_1x_4x_5 + p_1x_5x_6 - p_1x_6x_5 + p_2x_5x_{-1} \\
\frac{dx_4}{dt} &= p_1x_4x_3 - p_1x_3x_4 + p_1x_4x_5 - p_1x_5x_4 + p_2x_4x_{-1} - p_2x_4\sigma_{>5} - (1-p_2)p_3(x_9+x_{10})x_4 \\
\frac{dx_3}{dt} &= p_1x_3x_2 - p_1x_2x_3 + p_1x_3x_4 - p_1x_4x_3 + p_2x_3x_{-1} - p_2x_3\sigma_{>5} \\
&\quad + (1-p_2)p_3x_4(x_9+x_{10}) - (1-p_2)p_3x_3(x_9+x_{10}) \\
\frac{dx_2}{dt} &= p_1x_2x_1 - p_1x_1x_2 + p_1x_2x_3 - p_1x_3x_2 + p_2x_2x_{-1} - p_2x_2\sigma_{>5} \\
&\quad + (1-p_2)p_3x_3(x_9+x_{10}) - (1-p_2)p_3x_2(x_9+x_{10}) \\
\frac{dx_1}{dt} &= p_1x_1x_0 - p_1x_0x_1 + p_1x_1x_2 - p_1x_2x_1 + p_2x_1x_{-1} - p_2x_1\sigma_{>5} \\
&\quad + (1-p_2)p_3x_2(x_9+x_{10}) - (1-p_2)p_3x_1(x_9+x_{10}) \\
\frac{dx_0}{dt} &= p_1x_0x_1 - p_1x_1x_0 + p_2x_0x_{-1} - p_2x_0\sigma_{>5} + (1-p_2)p_3x_1(x_9+x_{10}) \\
\frac{dx_{-1}}{dt} &= 2p_2\sigma_{<5}\sigma_{>5} - p_2x_{-1}(1-x_{-1}).
\end{aligned}
$$

As the model is symmetric, only the equations for $i \leq 5$ are shown, where $\sigma_{<5} = \sum_{i=0}^{4} x_i$ and $\sigma_{>5} = \sum_{i=6}^{10} x_i$.

The equations show explicitly the terms representing all the contributions that add or remove agents from a particular opinion value. A general inspection shows that all the terms involving p_1 will cancel each other in the equation for each x_i. Because of this, it could be stated that the final distributions will mainly depend on the different values of p_2 and p_3 and how these probabilities operate on each x_i.

In the particular case of the equation for x_5, only one positive term remains, which explains the tendency to increase the population of agents at this opinion value.

In the particular case of the equation for x_0, for $p_2 = 0$, only one positive term associated with p_3 survives, which explains the peaks obtained at an opinion value of 0 (and 10 due to the symmetry of the model), when polarization is present (i.e. $p_3 > 0$).

An analytical solution of the equation system is under study and will be presented in a future contribution.

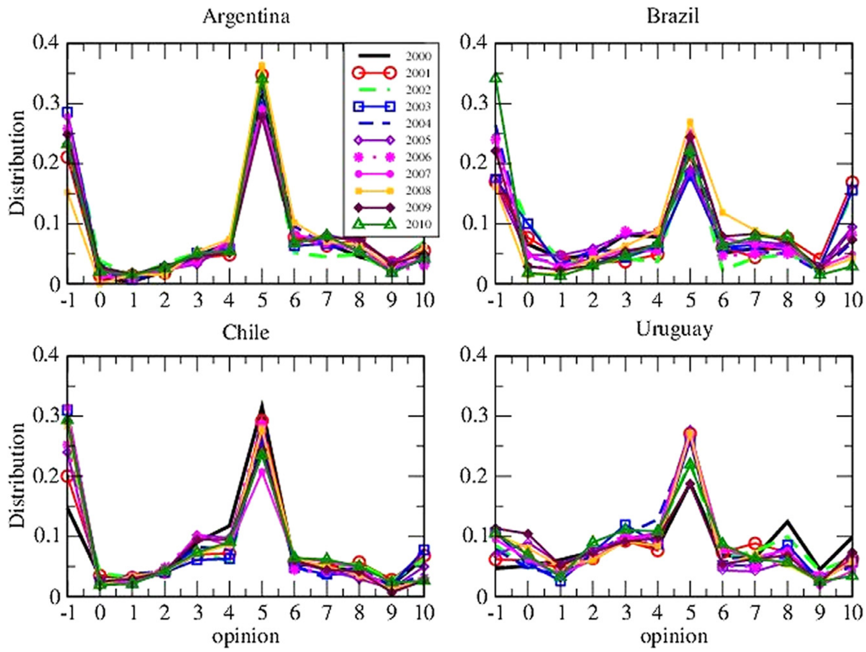

Fig. 1. Real distribution of opinions from 2000 to 2010 in Argentina, Brazil, Chile and Uruguay, according to Latinobarómetro.

3. Results and discussion

3.1. Real data from surveys and sociological clues for its analysis

In order to take into account real data for comparison, we have analyzed the temporal evolution of opinion from years 2000 to 2010 for the case of Argentina, Chile, Brazil and Uruguay, according to a poll conducted by Latinobarómetro, as it can be noticed in Fig. 1, for all years and all countries, there is a pronounced peak at opinion value 5. That is, most people consider themselves to be exactly in the middle (or neutral position) of the "left–right" ideology continuum. It is also noteworthy that, for three of the studied countries, the fraction of people that can be labeled as "undecided" constitutes a significant fraction of the total. In order to be able to compare the poll results with the results obtained from our model, we associated the undecided agents of our model with any people whose answer to the question of their self-placement within the "left–right" spectrum was one of the following options of the poll: "none", "does not know" or "does not respond".

Considering the relationship between ideology, party system and political alternation in the Southern Cone region, the cases of Brazil, Chile, Uruguay and Argentina stand out as governments on which the existence of an ideological leftist "spin" is discussed, and which together constitute a "partisan political region of relative high institutionalization in the context of Latin America" [35]. These are countries whose ideological dynamics are interesting to compare, given that they shared certain political itineraries during the second half of twentieth century: political radicalization and armed struggle in the '60s, coups during the '70s and democratic reopenings from the '80s onward.

Although at the level of the political system interesting contrasts are noticed between these countries in the context of a shared regional history, the evolution of ideological self-positioning of their populations analyzed in the present work demonstrates a high degree of homogeneity in statistical terms: over the decade studied through Latinobarómetro polls, the views of citizens in the four countries tend to focus on the "center" as a political alternative that involves high social desirability, as it is positioned away from the ideological extremes. In this sense, it is worth to consider the connection between ideological "centrism" and political "indecision", since both categories allude to contemporary patterns of mass political behavior. In the present study, these categories emerge as important keys for interpretation of the results obtained according to real statistical information, as detailed in the following section.

All of the above said, it must be stated that, for these countries, the relationship between ideology and voting intention remains a complex one, and that a thorough analysis of the topic is not the main goal and is beyond the scope of the present work.

3.2. Centrism and indecision as socio-political phenomena

In the field of political sociology, connecting lines are observed between research about center as category of mass ideological positioning and studies about undecided behavior as engine of electoral dynamics inside contemporary democracies.

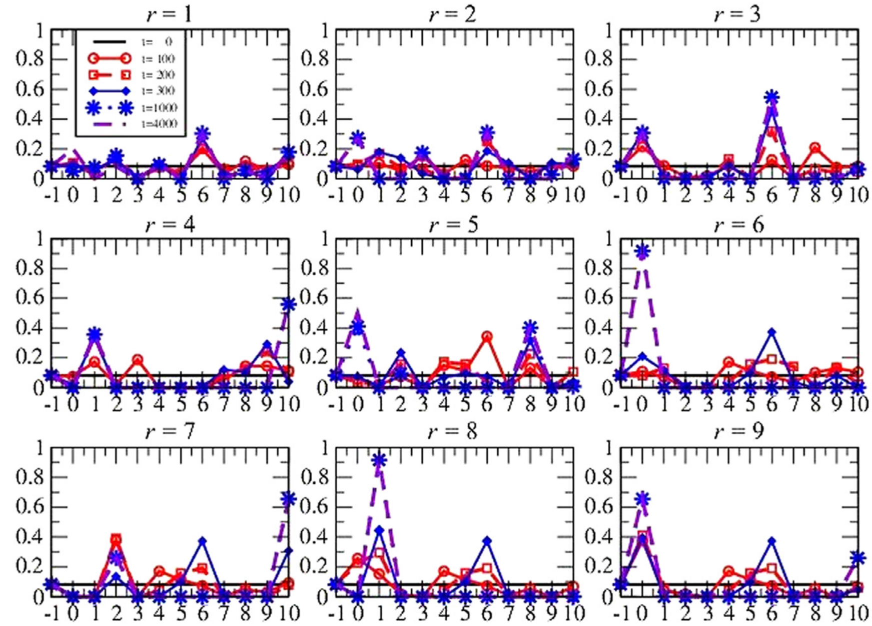

Fig. 2. Simulated temporal evolution of opinions, for $p_1 = 1$, $p_2 = 0$ and $p_3 = 0$, and contagion ranges from 1 to 9.

On one hand, moderate or intermediate ideological positions represented by the political center receive differential evaluation in analytical and axiological terms: they symbolize both an opportunist position of political autism marked by skepticism, low cognitive engagement and tendency to passivity [36], and also the democratic maturity of a cautious stance that, with tolerance and rationality, avoids the fanaticism of the ideological extremes [37].

On the other hand, indecision is a current and rising phenomenon which affects both recent and consolidated democracies. The comparative analysis of election statistics shows that, within the segment of undecided people, we may find "peripheral" or "marginal" individuals in terms of social resources and political attitudes as well as politically sophisticated people motivated and interested in participating or opining. Thus, social conditions and implicit attitudes [38] on individual level become significant variables for the prediction of fluctuating behavior. Inside the ideologically "neutral" electorate, people who are positioned in the center as well as those who cannot self-position on this scale, often are more impressionable and less determined subjects susceptible to change their positions during a campaign. This situation consequently validates the idea of a greater "porosity" of this group in terms of political marketing [39].

3.3. Computer simulations

In a first stage, we performed a series of computer simulations with different values of the interaction range, and with $p_1 = 1$, $p_2 = 0$ and $p_3 = 0$. This means that, in a first stage, we did not take into account undecided agents (as pointed out in Section 2.4, their number remains constant during the simulation) neither polarization.

Fig. 2 shows the temporal evolution of the opinion distribution for this first example, for contagion ranges from 1 to 9. It can be observed that, in most cases, the final state does not consist of a single peak, but several peaks that are separated by a distance larger than the considered contagion range. That means that the final opinion values of any two individual agents either is the same or differ by a number larger than the interaction range. This is in concordance with the theoretical results found in Ref. [22].

In several of these cases, most agents converge to a single opinion (i.e. the final distribution of opinions for a simulation consists of a single peak of one opinion value). In order to quantify the probability of convergence to any opinion value for each case, 20 000 simulations of 5000 steps were performed, and the average number of simulations that end with a given final value of opinion was calculated. The criterion used was to add 1 to the number of simulations ended with a given value every time that the fraction of agents with that opinion value exceeds 0.5 at the end of the simulation.

Fig. 3 shows the distribution of the probability of obtaining a single peak for each opinion value, with probabilities $p_1 = 1$, $p_2 = 0$ and $p_3 = 0$ (i.e. for the same cases analyzed in Fig. 2). The 10 possible values of contagion range are analyzed. In this case, only direct contagion of opinion within the contagion range is taken into account, and undecided agents do not play any role. From the figure, it can be seen that for the case of contagion ranges 1 and 2, convergence to a single peak is not very probable, this tendency being worse in the case of range 1. In the case of contagion range 2 this probability tends to zero for extreme opinion values. For contagion range 3, the opinion distribution is almost constant within the 3 to 7 interval of

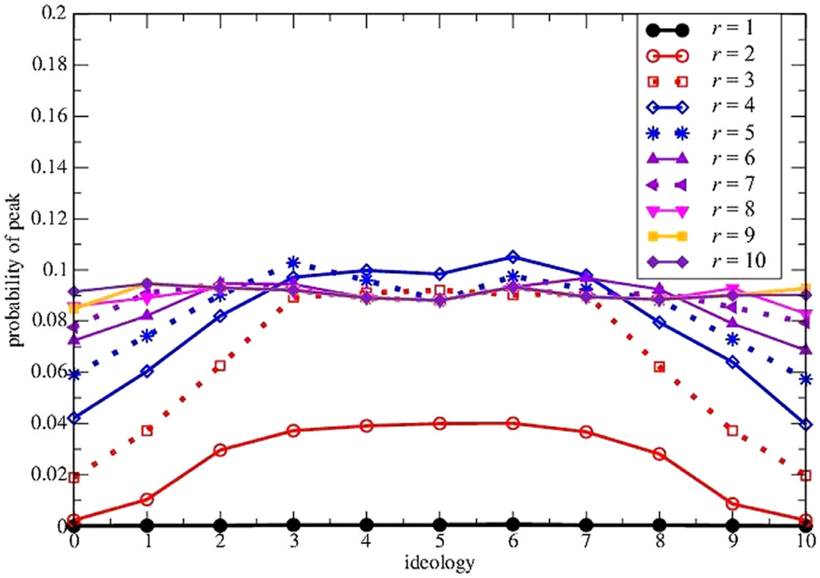

Fig. 3. Probability distribution of obtaining a single peak in every opinion value, averaged over 20 000 simulations, for the 10 possible range values, for $p_1 = 1$, $p_2 = 0$ and $p_3 = 0$.

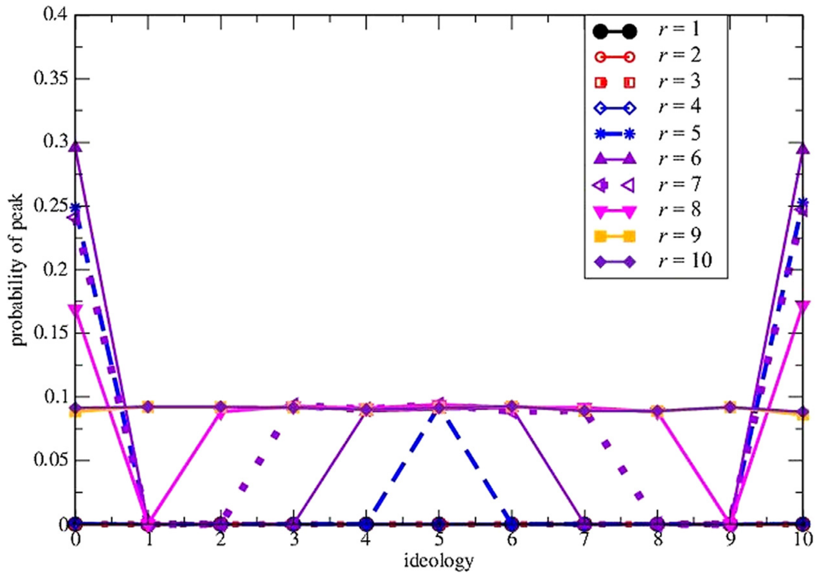

Fig. 4. Probability distribution of obtaining a single peak in every opinion value, averaged over 20 000 simulations, for the 10 possible range values, for $p_1 = 1$, $p_2 = 0$ and $p_3 = 1$, with polarization.

opinions, and decays for the first and final three values. As the contagion range increases, the distribution takes the shape of a wide plateau, and gets constant for the case of contagion range = 10, for which a single peak is always reached.

Fig. 4 analyzes the case that includes contagion and polarization. The probability of obtaining a single peak is trivially equal to 0 for contagion ranges 1–4, for all opinion values. For contagion range = 5 there is a peak at opinion value = 5 and two other peaks at 0 and 10. For contagion range 6, the probability of having a single peak is approximately 0.09 for opinion values from 4 to 6, has a value of around 0.3 for opinion values 0 and 10, and is zero otherwise. In general, it can be seen that for contagion ranges with values from $r = 5$ to $r = 8$, there is a probability of obtaining a single peak with a value approximately equal to 0.09 (1/11, according to the initial distribution) for opinions from $10 - r$ to r, and a probability with a higher value for opinions 0 and 10. For ranges $r = 9$ and 10, the distribution is almost uniform.

Fig. 5 shows the temporal evolution of the opinion distribution for $p_1 = 1$, $p_2 = 1$ and contagion ranges from 1 to 9. It can be observed that for small interaction ranges, the opinion evolves towards a single peak centered in 5, in analogy with the real cases studied in the surveys.

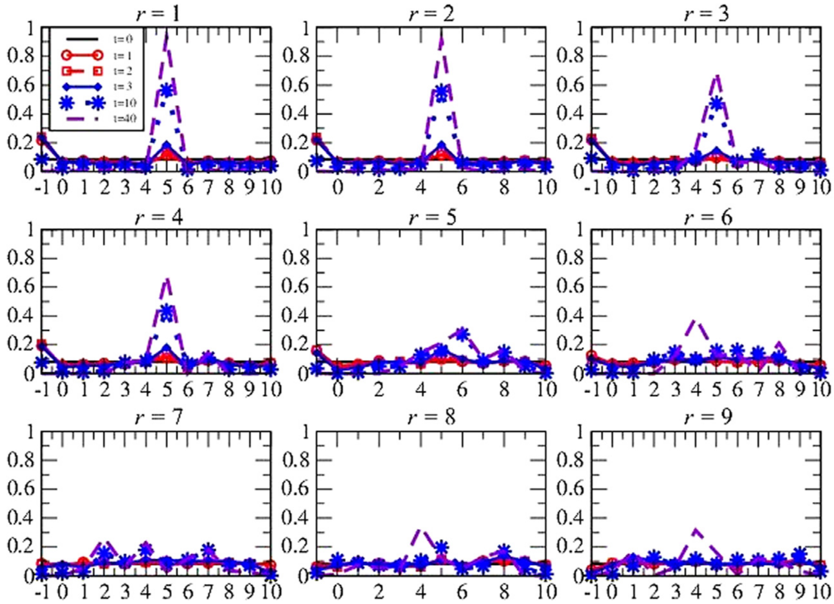

Fig. 5. Simulated temporal evolution of opinions, for $p_1 = 1$, $p_2 = 1$ and contagion ranges from 1 to 9.

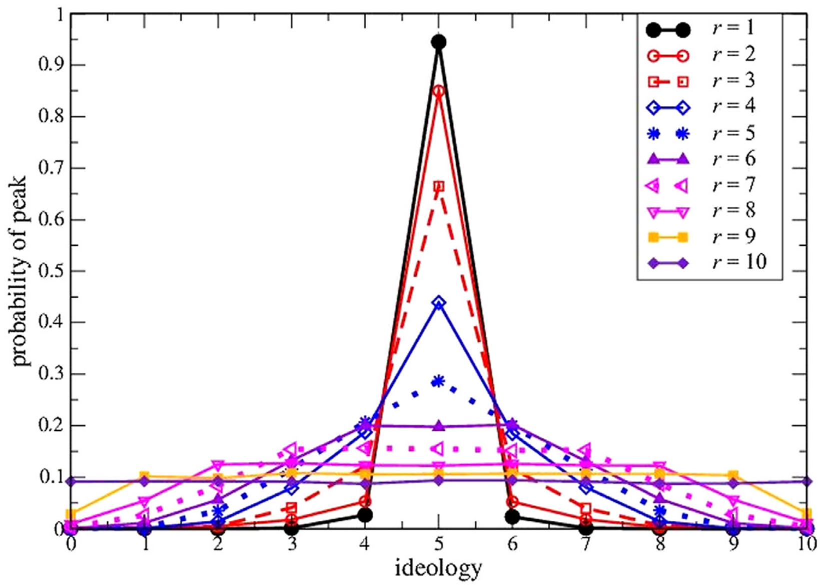

Fig. 6. Probability distribution of obtaining a single peak in every opinion value, averaged over 20 000 simulations, for the 10 possible range values, for $p_1 = 1$ and $p_2 = 1$.

Fig. 6 shows the distribution of peaks with probabilities $p_1 = 1$ and $p_2 = 1$ (p_3 is not relevant). It can be observed that a final peak for opinion 5 is highly favored, especially for small ranges of interaction. For r ranging from 1 to 5, there is a peak with a maximum at an opinion value of 5 whose height decreases with increasing r. For ranges from $r = 6$ to 10, the probability reaches a plateau at opinion values ranging from $10 - r$ to r and decays towards the extremes. For $r = 10$, the distribution is uniform.

Fig. 7 shows the distribution of peaks with probabilities $p_1 = 0.1$, $p_2 = 1$ and $p_3 = 0$. Here we can observe, for all of the cases, a high peak at an opinion value of 5. For interaction ranges from $r = 1$ to 4 there is also a small but non-zero value of probability for opinion values ranging from $5 - r$ to $5 + r$; for the rest of opinion values the probability is 0. For ranges from $r = 5$ to 10 we find a high peak at an opinion value of 5 and a uniform small non-zero probability for the rest of the opinion values.

In order to analyze the convergence of opinion and its dependence with the interaction range, the total probability of obtention of a single peak (regardless of the particular opinion value at which it was obtained) was calculated for different

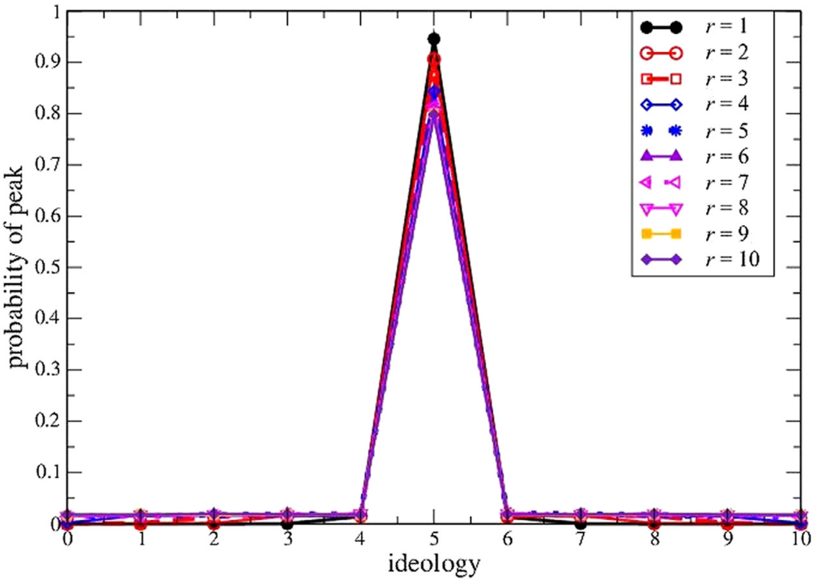

Fig. 7. Probability distribution of obtaining a single peak in every opinion value, averaged over 20 000 simulations, for the 10 possible range values, for $p_1 = 0.1$ and $p_2 = 1$.

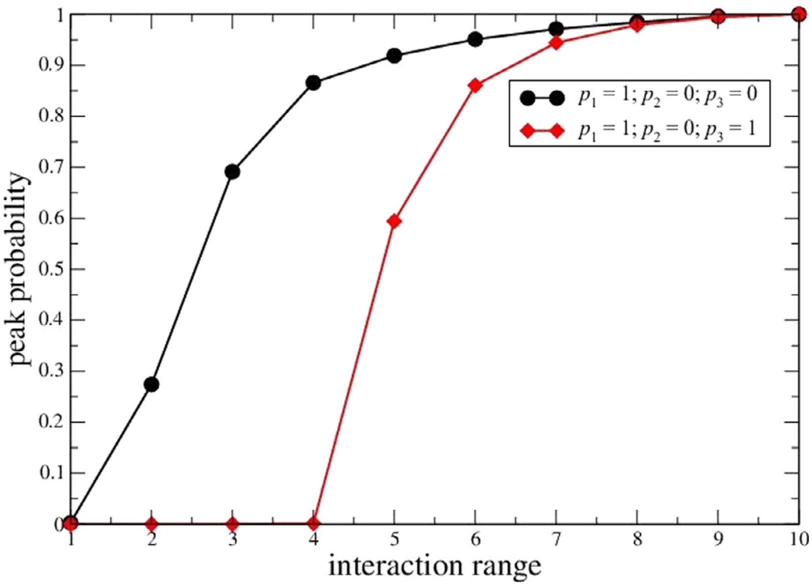

Fig. 8. Probability of obtention of a single final peak (at any opinion value), for $p_1 = 1$, $p_2 = 0$, and two values of p_3 (0 and 1).

interaction ranges. For most combinations of p_1, p_2 and p_3, in particular with the presence of undecided agents taken into account, the results were trivial, with a total probability equal to 1. That means that, in the presence of undecided agents, convergence to a single peak always occurs. Noteworthy results were obtained for the particular case of $p_2 = 0$, and are shown in Fig. 8. Two cases are considered: $p_1 = 1$, $p_2 = 0$ and $p_3 = 0$, that is, taking into account only the possibility of direct contagion within the interaction range; and $p_1 = 1$, $p_2 = 0$ and $p_3 = 1$, which has the added effect of polarization taken into account. For the first case, the total probability is zero for interaction range equal to 1 and then starts to grow smoothly. In the second case, the total probability is zero for ranges from 0 to 4 and then starts to grow.

3.4. Phase diagrams

Some phase diagrams were constructed in order to analyze systematically the effect of the values that the parameters can take on the final distribution of the peaks.

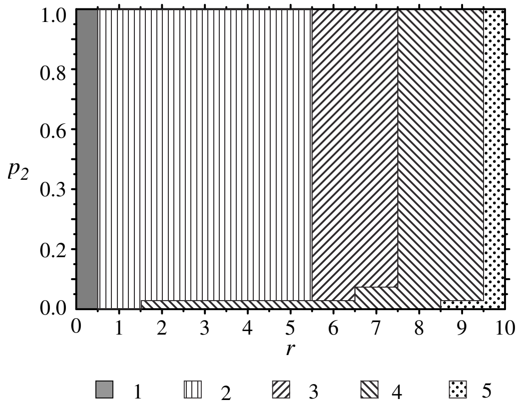

Fig. 9. Phase diagram p_2 vs. r for $p_1 = 1$ and $p_3 = 0$. Region 1: Peak at 5 and zero otherwise. Region 2: Peak at 5 with non-zero values for other opinions. Region 3: Smooth distribution without a definite maximum. Region 4: Almost uniform distribution decreasing towards the extremes. Region 5: uniform distribution.

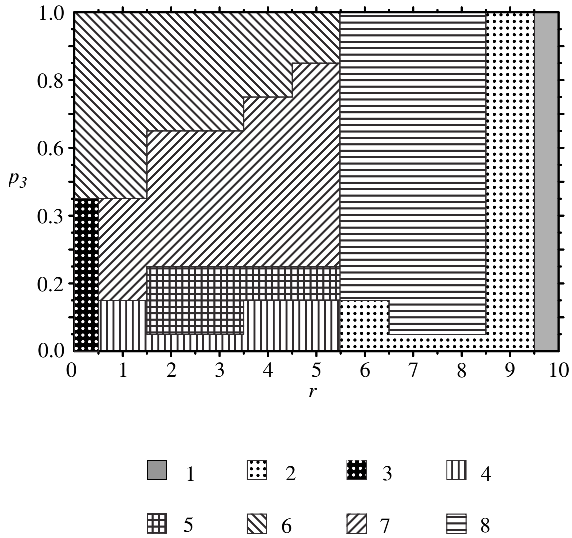

Fig. 10. Phase diagram p_3 vs. r for $p_1 = 1$ and $p_2 = 0.1$. Region 1: Uniform distribution. Region 2: Smooth distribution decreasing towards the extremes. Region 3: Peak at 5 and zero otherwise. Region 4: Peak at 5 with non-zero values for other opinions. Region 5: Maximum at 5 with smaller peaks for other opinion values. Region 6: Peaks at 5, 0 and 10 and zero otherwise. Region 7: Peaks at 5, 0 and 10 with non-zero values for other opinions. Region 8: Plateau between $10 - r$ and r with additional peaks at 0 and 10.

Fig. 9 explores the variation of p_2 and the range r for fixed values of p_1 and p_3. In these cases we have taken $p_1 = 1$ and $p_3 = 0$. In the resulting phase diagram 5 distinct regions were identified. For the case of range $r = 0$ (that is, in absence of direct contagion), we have a unique peak at 5, with zero probability for other values, which means that for all cases, the final state of the simulation is a single peak at opinion 5. This is observed for all possible values of p_2. The maximum at 5 is also observed for ranges 1–5 with an smooth distribution, decreasing towards the extremes, for all values of p_2, except the very small ones. For ranges 6–7 there is an smooth distribution centered at 5 and decreasing towards the extremes (except for very small values of p_2). For ranges 8–9 there is an almost uniform distribution, but decreasing at the extremes. Finally, for $r = 10$ we can observe a completely uniform distribution.

Fig. 10 shows a more complex phase diagram. In this case the values of p_1 and p_2 were fixed at $p_1 = 1$ and $p_2 = 0.1$ and all the possible values of r and p_3 were explored. For $r = 10$ we have a totally uniform distribution, while for $r = 9$ we have a smooth distribution decreasing towards the extremes. This behavior is also observed for ranges 6–8 and small values of p_3. For higher values of p_3 and ranges $r =$ 6–8 the following situation is observed: a plateau at the interval $[10 - r, r]$, then a decrease and finally two peaks at the extremes (0 and 10). On the other hand, for $r = 0$ and small values of (p_3), we have only a maximum at 5. For range 0–5 and high values of p_3 we can observe three peaks positioned at opinion values 0, 5 and 10. This is also observed for intermediate values of p_3, but with non-zero values for the rest of opinions. Finally, for ranges form 1 to 5 and small values of p_3 a maximum at 5 is also observed with non-zero values for the rest of opinions. In some cases, subpeaks at other opinion values are also observed.

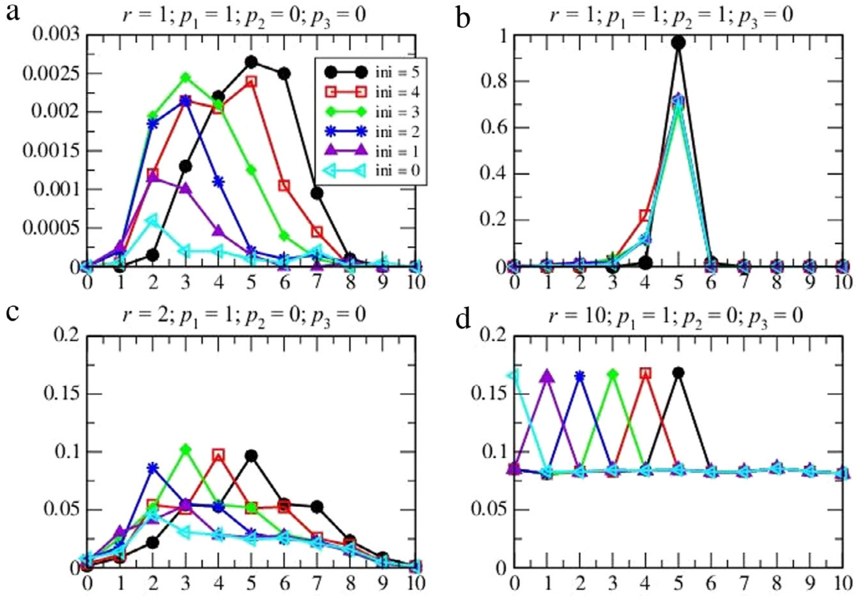

Fig. 11. Final distribution of the peaks for different initial conditions and four combinations of parameters: (a) range $r = 1, p_1 = 1, p_2 = 0$ and $p_3 = 0$; (b) range $r = 1, p_1 = 1, p_2 = 1$ and $p_3 = 0$; (c) range $r = 2, p_1 = 1, p_2 = 0$ and $p_3 = 0$; (d) range $r = 10, p_1 = 1, p_2 = 0$ and $p_3 = 0$.

Two more phase diagram were performed, but they are not shown because the results are trivial. We have explored the different combinations of p_1 and p_2 for the following cases: $r = 1, p_3 = 0$ and $r = 10, p_3 = 0$.

For the first case ($r = 1, p_3 = 0$) it was observed a maximum at 5 with a sharp decrease towards the extremes for most cases. One of the exceptions is the observation of one peak at 5, but with a smooth distribution towards the extremes, for the case of $p_1 = 1$ or for low values of p_2. The extreme case of $p_2 = 0$ gives as a result total absence of peaks. On the other hand, the extreme case of $p_1 = 0$ gives a sharp peak at 5.

For the second case ($r = 10, p_3 = 0$) and for most cases it was observed a maximum at 5 with uniform distribution for the rest of opinion values. One of the exceptions is the observation of uniform distribution for the extreme cases of $p_1 = 1$ and for $p_2 = 0$. The other exception is the observation of a single peak at 5 with a value of zero for all other opinions, which is observed for $p_1 = 0$ and all values of p_2.

3.5. Initial conditions

In order to analyze the effect of the initial conditions on the final distribution of peaks, a series of simulations were performed with 6 different initial conditions. In these cases, a total number of $L = 1300$ agents were considered with the following distribution: 100 agents for most of the opinion values and 200 agents for one particular opinion value, ranging from 0 to 5. The cases 6–10 were not considered due to the symmetry of the problem.

Fig. 11 shows the final distribution of the peaks for four combinations of parameters: (a) range $r = 1, p_1 = 1, p_2 = 0$ and $p_3 = 0$; (b) range $r = 1, p_1 = 1, p_2 = 1$ and $p_3 = 0$; (c) range $r = 2, p_1 = 1, p_2 = 0$ and $p_3 = 0$; (d) range $r = 10$, $p_1 = 1, p_2 = 0$ and $p_3 = 0$;

For the case (a), the main observation is the low fraction of results ending at one unique peak. That is the reason for the noisy results. We have a smooth distribution centered mainly around the initial peak. When we move the initial condition from 5 to 0, there is a displacement towards the left, but at the extremes, the maximum does not coincide exactly with the initial peak. For the initial peaks at 0 and 1, the final distribution has a maximum at 2.

For the case (b), that is with the contribution of undecided, there is always a maximum at 5. When we consider the initial peak at a certain value on the left, there is a tendency to have some ended at the opinion value 4.

For the case (c), that is with $r = 2$ and without the influence of undecided, the final distribution is centered in coincidence with the initial peak, except for the extreme cases of 0 and 1, where the final distribution is centered at 2.

For the case (d), that is with $r = 10$, the final distribution of peaks coincide exactly with the initial conditions. That means a uniform distribution for all values, except the value corresponding to the initial peak, where we have a maximum.

4. Summary and conclusions

In the present work, the evolution of the ideological self-positioning of an ensemble of agents was studied using a statistical physics model, and the outcomes were compared to real data opinion polls of some South American countries.

The opinion polls distributions exhibit a single peak centered at 5 for all countries and all years studied. On the other hand the simulations indicate that when undecided agents are not taken into account, the probability of obtaining a single peak depends on the contagion range. Nevertheless, when undecided agents are included in the simulation, it converges to a single peak of opinion. It is observed that the probability to find that peak at 5 increases when the range of contagion is small.

Based on sociological grounds, the observed connection within the scientific literature about collective behavior of undecided people and centrist sectors under the political conditions of contemporary mass democracies allows to assume – at least provisionally – the explanatory validity of the opinion dynamics studied in our model.

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References

[1] W. Weidlich, Sociodynamics—A Systematic Approach to Mathematical Modelling in Social Sciences, Taylor and Francis, London, 2002.
[2] D. Stauffer, Introduction to statistical physics outside physics, Physica A 336 (2004) 1.
[3] S. Galam, Application of statistical physics to politics, Physica A 274 (1999) 132–139.
[4] S. Galam, Sociophysics: a personal testimony, Physica A 336 (2004) 49.
[5] S. Galam, Sociophysics: A review of Galam models 2008, Internat. J. Modern Phys. C 19 (2008) 409–440.
[6] S. Galam, Contrarian deterministic effects on opinion dynamics: "The hung elections scenario", Physica A 333 (2004) 453.
[7] P. Clifford, A. Sudbury, A model for spatial conflict, Biometrika 60 (3) (1973) 581.
[8] R. Holley, T.M. Ligget, Ergodic theorem for weakly interacting infinite systems and the voter model, Ann. Probab. 3 (4) (1975) 643–663.
[9] K. Suchecki, V.M. Eguiluz, M. San Miguel, Voter model dynamics in complex networks: Role of dimensionality, disorder and degree distribution, Phys. Rev. E 72 (2005) 036132.
[10] F. Vazquez, V. Eguiluz, Analytical solution of the voter model on uncorrelated networks, New J. Phys. 10 (2008) 063011.
[11] K. Sznajd-Weron, J. Sznajd, Opinion evolution in closed community, Internat. J. Modern Phys. C 11 (2000) 1157–1165.
[12] J.R. Sánchez, A modified one-dimensional Sznajd model, 2004. cond-mat/0408518.
[13] M.S. de la Lama, J.M. López, H.S. Wio, Spontaneous emergence of contrarian-like behaviour in an opinion spreading model, Europhys. Lett. 72 (2005) 851.
[14] H.S. Wio, M.S. de la Lama, J.M. López, Contrarian-like behaviour and system size stochastic resonance in an opinion spreading model, Physica A 371 (2006) 108–111.
[15] M.C. Gimenez, J.A. Revelli, M.S. de la Lama, J.M. Lopez, H.S. Wio, Interplay between social debate and propaganda in an opinion formation model, Physica A 392 (2013) 278–286.
[16] M. Cecilia Gimenez, J.A. Revelli, H.S. Wio, Non local effects in the Sznajd Model: Stochastic resonance aspects, ICST Trans. Complex Syst. 12 (10–12) (2012) e3.
[17] R. Hegselmann, U. Krause, Opinion dynamics and bounded confidence models, analysis and simulation, J. Artif. Soc. Soc. Simul. 5 (2002) 3.
[18] G. Deffuant, D. Neau, F. Amblard, G. Weisbuch, Mixing beliefs among interacting agents, Adv. Complex Syst. 3 (2000) 87–98.
[19] G. Deffuant, F. Amblard, G. Weisbuch, T. Faure, How can extremism prevail? A study based on the relative agreement interaction model, J. Artif. Soc. Soc. Simul. 5 (4) (2002).
[20] J.C. Dittmer, Consensus formation under bounded confidence, Nonlinear Anal. 47 (2001) 4615–4621.
[21] M.F. Laguna, G. Abramson, D.H. Zanette, Minorities in a model for opinion formation, Complexity 9 (4) (2004) 31.
[22] J. Zhang, Y. Hong, Opinion evolution analysis for short-range and long-range Deffuant–Weisbuch models, Physica A 392 (2013) 5289–5297.
[23] L.E.L. Ganatios, La teoría espacial del voto: una propuesta de aplicación en Latinoamérica, Rev. Mex. Anál. Polít. Adm. Públ. II (2) (2013) 57–79.
[24] P.W. Thurner, The empirical application of the spatial theory of voting in multiparty systems with random utility models, Electoral Stud. 19 (2000) 493–517.
[25] P. Balenzuela, J.P. Pinasco, V. Semeshenko, The undecided have the key: Interaction-driven opinion dynamics in a three state model, PLoS One 10 (10) (2015) e01339572.
[26] F. Vazquez, P.L. Krapivsky, S. Redner, Constrained opinion dynamics: freezing and slow evolution, J. Phys. A: Math. Gen. 36 (2003) L61–L68.
[27] F. Vazquez, S. Redner, Ultimate fate of constrained voters, J. Phys. A: Math. Gen. 37 (2004) 8479–8494.
[28] M.S. de la Lama, I.G. Szendro, J.R. Iglesias, H.S. Wio, Van Kampen's expansion approach in an opinion formation model, 2006, Eur. Phys. J. B 51 (2006) 435–442.
[29] C.E. La Rocca, L.A. Braunstein, F. Vazquez, The influence of persuasion in opinion formation and polarization, Europhys. Lett. 106 (2014) 40004.
[30] A.O. Sousa, T. Yu-Song, M. Ausloos, Effects of agents' mobility on opinion spreading in Sznajd model, Eur. Phys. J. B 66 (2008) 115–124.
[31] M. Kuperman, D. Zanette, Stochastic resonance in a model of opinion formation on small world networks, Eur. Phys. J. B 26 (2002) 387–391.
[32] M.C. González, A.O. Sousa, H.J. Hermann, Renormalizing Sznajd model on complex networks taking into account the effects of growth mechanisms, Eur. Phys. J. B 49 (2006) 253–257.
[33] Latinobarometro Data Bank: http://www.latinobarometro.org.
[34] K. Knight, Transformations of the concept of ideology in the twentieth century, Amer. Polit. Sci. Rev. 100 (4) (2006) 619–626.
[35] C. Moreira, Sistemas de partidos, alternancia política e ideología en el Cono Sur, Rev. Urug. Cienc. Polít. 15 (2006) 31–56.
[36] A. Rodríguez Kauth, El 'centro' en política, Fundam. Humanid. IV (I–II) (2003) 19–28.
[37] R. Montoro Romero, Voto, ideología y centro político, Cuad. Pensam. Polít. abril-junio (14) (2007) 49–80.
[38] K.B. Lundberg, B.K. Payne, Decisions among the Undecided: implicit attitudes predict future voting behavior of undecided voters, PLoS One 9 (1) (2014) 1–11. e85680. http://dx.doi.org/10.1371/journal.pone.0085680.
[39] M. Lisi, O voto dos indecisos nas democracias recentes: um estudo comparado, Anál. Soc. XLV (194) (2012) 29–61.