INTERNATIONAL JOURNAL OF HYDROGEN ENERGY 37 (2012) 10371–10375

Available online at www.sciencedirect.com

SciVerse ScienceDirect

journal homepage: www.elsevier.com/locate/he

Robust polynomial approach for state of charge estimation in NiMH batteries

R.H. Milocco [a,*], J.E. Thomas [b], B.E. Castro [b]

[a] Grupo Control Automático y Sistemas (GCAyS), Depto. Electrotecnia, Facultad de Ingeniería, Universidad Nacional del Comahue, Buenos Aires 1400, 8300 Neuquén, Argentina

[b] Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas (INIFTA) Universidad Nacional de La Plata. Suc 4, CC16 (1900), La Plata, Argentina

ARTICLE INFO

Article history:
Received 26 August 2011
Received in revised form
20 December 2011
Accepted 31 January 2012
Available online 7 April 2012

Keywords:
State of charge
Metal Hydride battery
Wiener model
Robust filtering
Kalman filtering

ABSTRACT

In this work we estimate the state of charge (SOC) of NiMH rechargeable batteries using a robust optimal filter based on a simplified electrochemical model. The robust filter guarantees that the supremum of the error variance - difference between real and estimated SOC - with respect to all admissible uncertainties be minimum. The results are compared with those obtained using the linear Kalman filter. We conclude that both estimations have similar performance, although the robust filter is easier to tune. Experimental results with commercial batteries are provided to illustrate the estimation procedure and it performance.

1. Introduction

For good quality of service and to sustain a longer life of NiMH rechargeable battery it is needed to develop a battery management system (BMS). For an efficient BMS it is extremely important to have an accurate estimation of SOC in real-time and this topic is the main purpose of this paper.

By assuming the charge current is equal to the current through the battery terminals I_{bat}, the SOC can be obtained by performing the integral of such current, known as *coulomb-counting* technique. However, due to the presence of secondary reactions in the battery, the charge current could be different to the battery current and, even being slightly different, due to the integration process the differences between real and estimated SOC can be unacceptably large over time. By considering the case where the battery current is zero, $I_{bat} = 0$ – open circuit- and after reaching the steady-state, the measured potential at the battery terminals, E_{bat}, is related with SOC through the electro-motive force (EMF) curve $E_{bat}(\infty) = f_E(\text{SOC})$. Then, a simple and effective strategy to correct the errors of *coulomb-counting* consists in opening the battery circuit and waiting until the system relaxes to $E_{bat}(\infty)$ known as open circuit voltage (OCV). The procedure is well described in [1] and the references therein. The main disadvantage of this method is, in addition to the accumulative error of integration, the long time needed for the rest period which hinders their use in real-time.

In order to overcome this difficulties model based observers methods become an interesting strategy to estimate SOC. The idea consists in using the measured current I_{bat} as

* Corresponding author.
E-mail addresses: ruben.milocco@fain.uncoma.edu.ar (R.H. Milocco), jthomas@inifta.unlp.edu.ar (J.E. Thomas), bcastro@inifta.unlp.edu.ar (B.E. Castro).
0360-3199/$ – see front matter
doi:10.1016/j.ijhydene.2012.01.164

input of the battery model to obtain, by simulation, the potential $\hat{E}_{bat}$. The error between both the real and the estimated potential $E_{bat} - \hat{E}_{bat}$ is used to correct the internal model variables (states). Thus, if the model is exact and the correction is optimal, in the sense that the states of both coincide, the SOC can be optimally estimated.

The battery can be modeled as a cascade of a linear system with a static nonlinearity. This kind of structure is called *Wiener model* [5]. A convenient approach is to linearize the static nonlinearity at every sample time around the estimated state variables and using a linear Kalman filter. In this way the nonlinear model is transformed into a linear one with time-varying parameters. When the Kalman filter is used with this kind of model, the type of observer is called Extended Kalman Filter (EKF). In [2] (Part 1, 2, and 3) this approach was used in lithium-ion polymer battery packs for hybrid vehicles. In [3] an observer for linear time-variant systems was used to estimate the open circuit voltage (OCV) from an empirical battery model. Assuming observability of an extended model formulation, both parameters and state of charge are estimated at once. One drawback of theses approaches is that they require an explicit formulation of the static nonlinear structure, equations and parameters.

Instead of using the electrochemical model, which is very complex and difficult to obtain, in this paper we assume the battery as a simple integrator affected by dynamic uncertainties in series with a static nonlinearity. Since the inverse function exists it can be used as a software sensor. With this approach, instead of linearizing the system equations and dealing with nonlinear observers, it is possible to use linear estimators. However, the precision of the measurement depends on the misidentification of the static nonlinearity, which increases the uncertainties. In this paper, we propose a simple integrator with additive dynamical uncertainty as a battery model. Thus, we will derive a robust liner filter that, using the signal obtained from the software sensor, allows to obtain the SOC of the battery with very acceptable accuracy.

2. Theory

2.1. Model formulation and robust filter design

The SOC in the battery is determined by integrating the current as follows:

$$SOC(t) = \frac{1}{Q_{max}}\left(Q_0 + \int_0^t I_{bat}(\tau)d\tau\right), \tag{1}$$

where Q_0, and Q_{max} are the initial and maximum electrode charge. The quantity Q_{max} is called *capacity* of the battery and it is defined as the product of the constant current and the time required to reach the maximum charge, starting with battery completely discharged, $Q_0 = 0$. Thus, SOC is the unity for battery completely charged and zero for completely discharged. In order to be able to estimate the SOC, we need first to model the battery behavior.

A battery is basically a system conformed by two electrodes immersed in an electrolytic media with an adequate porous separator. During discharge, the negative electrode is oxidized, while the positive electrode is reduced, being this global process responsible for delivering energy. The process is reversed during charge. The goal in the present section is to derive a simple, yet general model to describe the charge/discharge processes of different batteries in order to be used in the derivation of a robust SOC estimator algorithm. For this purpose we assume that secondary reactions, which usually take place at extreme values of SOC, shall be disregarded in the model, these, shall be considered as disturbances as explained below.

In the energy accumulation process, two important stages can be distinguished, one corresponding to the charge transfer processes at the electrochemical interface of the electrodes, including electrochemical reactions and double layer charging, and another one corresponding to mass transfer, either in the electrolyte or in the electrode active material. Thus, the NiMH battery can be modeled as a cascade of a linear dynamic system followed by a static nonlinearity which it is know as *Wiener model* [4,5]. The linear dynamic system is related to the transport of the reacting substances in the active material, theses processes are governed by Fick's law. The static nonlinearity is due to the electrochemical reactions at the electrode interfaces and it is governed by a Buttler–Volmer type equation. In [5] is demonstrate that the model equations can be written as follows:

$$\dot{X}(t) = AX(t) + BI_{bat}(t) \tag{2}$$

$$x_0(t) = CX(t) \tag{3}$$

$$E_{bat}(t) = g_E(x_0, I_{bat}) \tag{4}$$

$$A = \alpha\begin{bmatrix} -d_0 & d_0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -(1+d_1) & d_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -(1+d_{N-1}) & d_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix},$$

$$B = [\beta, 0, \cdots, 0]^T; C = [10\cdots0],$$

$$X(t) = [x(z_0,t), x(z_1,t), \cdots, x(z_N,t)]^T,$$

where α a constant, β is the inverse of the battery capacity, $x(z_i,t)$ means concentration of hydrogen at distance z_i from the electrode surface, $x_0(t)$ is a short notation of $x(z_0,t)$, and $d_i = z_{i+1}/z_i$, for $1 \le i \le N$. N in the number of slides of the spatial discretization of the electrode. $g_E(x_0,I_{bat})$ is the static nonlinearity which is a nonlinear function of the battery current I_{bat} and concentration at the surface $x_0(t)$.

An important property is that the static nonlinearity has inverse, ie it is possible to obtain the concentration from a nonlinear function $x_0 = g_x(E_{bat},I_{bat})$, where E_{bat} is the potential of the battery. For the static nonlinearity we need an explicit expression that relates the concentration x_0 with the current and potential of the battery. Instead of that, we propose to represent the static nonlinearity as an interpolating function based on experimental data. The static nonlinearity is a soft function that can be well approximated by using different approaches. Basically, any 2-D interpolation method could be suitable. In [5] it is detailed a procedure to identify this nonlinear function using a Taylor series representation. However, any 2-D interpolation procedure of the experimental data can be used to estimate the concentration

x_0, that we call x_0^m, given pairs (E_{bat}, I_{bat}). It is interpreted as the true value, x_0, corrupted by measurement, errors due to the imperfect interpolation, and possible unmodeled dynamics.

Using the measured concentration x_0^m, and the linear set of equations (2) and (3), linear observers can be used to estimate the SOC. However, the linear model has two main drawbacks, first the dimension N which can be extremely high, and second the parameters d_i are difficult to know. Then, instead of considering all the equations, we assume the battery can be reduced to an imperfect accumulator, ie a system integrator plus a relaxation system [2]. Then, we can consider a simple linear equation to use in estimating the SOC. The model consists of an integrator but with additive uncertainty containing secondary reactions and relaxing dynamics not considered. The system equation, in discrete-time, is determined as follows:

$$s(k+1) = s(k) + \beta I_{bat}(k) \quad (5)$$

$$x_0^m(k) = \hat{g}_x(E_{bat}(k), I_{bat}(k)) \quad (6)$$

$$x_0^m(k) = S(k) + \Delta_r(k) + e(E_{bat}(k), I_{bat}(k)) \quad (7)$$

where s stand for SOC, Δ_r is the relaxing dynamics, $\hat{g}_x(E_{bat}, I_{bat})$ is the measured quantity by the software sensor, $e = g_x - \hat{g}_x$ is the error in the interpolation. In Fig. 1 the problem is schematically depicted where the system S represent the integrator given by equation (5).

2.2. Identification of the nonlinearity

The objective now is to estimate the nonlinear function $x_0 = g_x(E,I)$ of equation (6) where, for easy notation, I, E stand by I_{bat} and E_{bat}. Taking into account that when the current is constant and after the transient relaxation satisfies $x_0 = s$, we propose to identify the function using N -triplets of experimental data (E,I,s) which is equal to (E,I,x_0) in such conditions. The variable s is obtained by *coulomb-counting* of constant current I starting from a known initial charge and knowing the capacity of the battery. The objective is to estimate $x_0 = g_x(E,I)$ at arbitrary pair of values (I,E) by interpolation of the N experimental data set. The procedure employed is known as Kriging interpolation. For description and properties of the method see [6]. There are several software package that perform efficiently theses estimations, see for example [7].

In the next section we present a robust design to solve the estimation problem with the reduced and uncertain model equations (5–7).

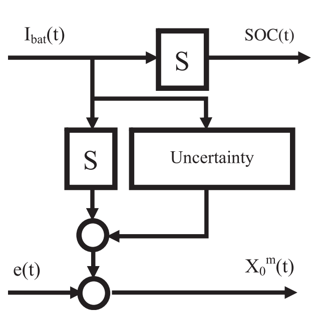

Fig. 1 – Uncertain battery model.

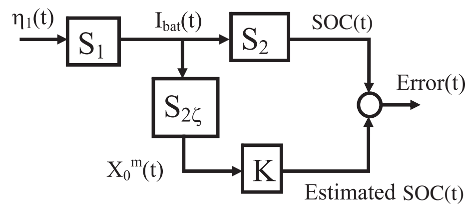

Fig. 2 – Robust filter problem.

2.3. Robust estimation

In this section we propose a polynomial approach to designing a H_2 – optimal robust SOC estimator based on measurements of the software sensor $x_0^m(k) = \hat{g}_x(E_{bat}(k), I_{bat}(k))$. In the present context, optimality refers to the filter ability to achieve an estimation error variance below a certain upper bound, which is the minimum over all admissible system uncertainties.

The SOC estimation problem, using the reduced and uncertain model equations (5–7), can be possed in a general structure shown in Fig. 2. In the setup of Fig. 2 we consider the signal η_1 a zero mean Gaussian stochastic white input with unit power spectral density. The system $S_{1\varsigma}$ represents the dynamics of current I_{bat} which is assumed to be stable and causal uncertain system. The system $S_{2\varsigma}$ represents the integrator and the uncertainties. Consider the error signal $e(k) = z(k) - \hat{z}(k)$ and its variance $\mathcal{E}\{e(k)^2\}$, where $\mathcal{E}$ denotes expectation. The robust design can be formally written as

$$\min_K J(K), \quad (8)$$

where

$$\sup_{S_{i\varsigma}} \left\{ \mathcal{E}\left\{ e(k)^2 \right\} \right\} \leq J(K) \quad (9)$$

The supremum is taken with respect to all admissible uncertain systems $S_{i\varsigma}$ for i = 1,2. In the appendix the stable and causal filter $K(\delta)$ that minimizes the equation (9) is derived.

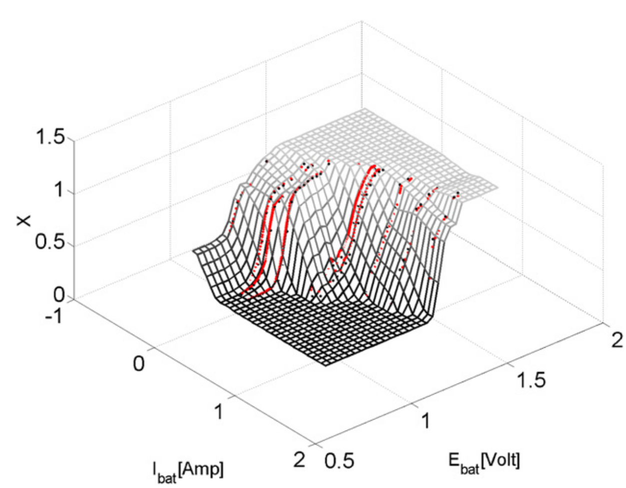

Fig. 3 – Static nonlinearity by interpolation using Kriging.

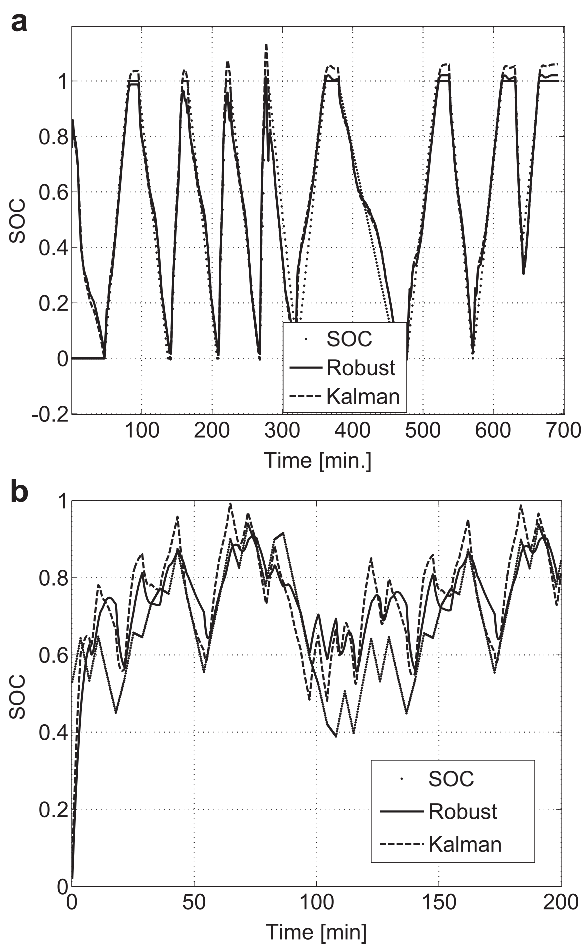

Fig. 4 – a) SOC estimations for experiment 1. b) SOC estimation for experiment 2.

3. Results and discussion

First, a set of constant current register were used to obtain the static nonlinearity by Kriging interpolation shown in Fig. 3. In the Fig. 4 the performance obtained from two experiments, based on measurements made in rechargeable batteries Duracell AAA/HR03/DX2400, NiMH/1,2/800 mAh, respectively, are shown. In both cases the Kalman filter [11] and the robust design are compared. The three parameters of the Kalman filter (variance of disturbances, covariance between current and measurement noise, and variance of the measurement error) and the only one parameter of the robust design (gain of the uncertainty) were adjust in the first experiment. It can be noticed that both estimators behave with similar performances. However, the Kalman filter requires the setting of three parameters. In contrast, the robust filter requires only the adjustment of the uncertainty gain w.

4. Conclusions

Based on a simple battery model and a software sensor, we analyzed a robust SOC estimator. The strategy of using the software sensor allows to fix the problem of estimating SOC in a framework of linear estimation which is important for it simplicity. The robust estimation method only requires knowing the value of gain uncertainty. The upper bound of errors was minimized and a bound of the error was derived. The robust filter was used to estimate the SOC of commercial NiMH battery showing a very acceptable performance.

Acknowledgments

This work was supported by the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Agencia Nacional de Promoción Científica y Técnica, Universidad Nacional del Comahue and Universidad Nacional de La Plata; all of República Argentina.

Appendix

In the polynomial notation used, the complex variable $\delta = z^{-1}$ for dt lti systems, where z denotes the variable of the z-transform, see [8]. For simplicity, the argument of polynomial matrices is sometimes omitted. The conjugate transpose of the polynomial $G(\delta)$ is denoted by $G^*(\delta) = G(\delta^{-1})$.

Let A and B polynomials and consider the following spectral factorization problem $CC^* = AA^* + BB^*$, then the polynomial solution C with stable inverse is called the spectral factor C and can be obtained with algorithms as those shown in [9]. The H_2 norm of a dt lti and causal matrix operator $\psi(\delta)$ with impulse response $\varphi(k)$ is

$$\|\psi\|_2^2 \triangleq \frac{1}{2\pi j}\oint \psi\psi^* \frac{d\delta}{\delta} = \sum_{k=0}^{\infty} \varphi(k)\varphi^T(k). \tag{10}$$

where $\oint$ means integration around the unit circle $|\delta| = 1$.

The SOC estimation problem, using the reduced and uncertain model equations (5–7), can be possed in a general structure shown in Fig. 2. In the setup of Fig. 2 we consider the signal η_1 a zero mean Gaussian stochastic white input with unit power spectral density. The system $S_{1\varsigma}$ represents the dynamics of current I_{bat} which is assumed to be stable and causal uncertain system that can be bounded by

$$S_{1\varsigma}S_{1\varsigma}^* \le S_1 S_1^* \quad \forall\omega \tag{11}$$

where S_1 is a causal, stable, and known transfer function. The uncertain system $S_{2\varsigma}$ is modeled by

$$S_{2\varsigma} = S_2 + W\Delta, \tag{12}$$

where S_2 and W are known stable transfer functions representing the nominal system and the weight of the uncertainty Δ respectively. The stable transfer function Δ is unknown and satisfies $|\Delta(j\omega)| \le 1, \quad \forall\omega$. Notice that S_2 is the integrator with transfer function $S_2 = \beta\delta/(1-\delta)$, that means it has a pole on the unit circle.

Consider the following polynomial representation of systems:

$$\begin{aligned} S_1(\delta) &= C_1(\delta)/D_1(\delta) \\ W(\delta)S_1(\delta) &= C_2(\delta)/D_2(\delta) \end{aligned} \tag{13}$$

where C_1, C_2, D_1, and D_2 are given polynomials. Let the polynomial Π be the stable, spectral factor obtained as solution of the following spectral factorization problem:

$$\Pi\Pi^* = \beta^2 D_2 C_1 C_1^* D_2^* (1+\alpha) D_1 C_2 C_2^* D_1^* (1-\delta)(1-\delta^*)\left(1+\alpha^{-1}\right) \quad (14)$$

where α is, for the moment, a scalar positive constant. The solution to this problem is given in the following theorem:

Theorem: The stable, causal and minimal degree discrete-time filter $K(\delta)$ that minimizes the cost $J(K)$ is given by

$$K(\delta) = \frac{(1-\delta)D_1 N - \pi}{\pi} \quad (15)$$

where N,M are the minimum degree solution with respect to N of the polynomial equation

$$\delta^* M^* D_2 + C_2 C_2^* D_1^* (1-\delta^*)\left(1+\alpha^{-1}\right) = N\Pi^* \quad (16)$$

The minimum cost is given by

$$J(\alpha) = \left\| \frac{C_1 N}{D_1 \pi} \right\|_2^2 (1+\alpha) + \left\| \frac{C_2 (D_1 (1-\delta) N - \pi)}{D_2 \pi} \right\|_2^2 \left(1+\alpha^{-1}\right) \quad (17)$$

The optimal robust filter $K(\delta)$ is the one that minimize of $J(\alpha)$ with respect to the parameter α. We only need a search procedure with respect to α based on some 1D optimization method.

Proof: follows from the procedure shown in [10]

Let us now apply the robust design of SOC estimation to the case of Fig. 1. Let's make the following assumptions: *i*). The dynamic of S_1 is given by a constant for all frequencies equal to the unity, which corresponds to $C_1 = D_1 = 1$. *ii*) The uncertainty weight W is a scalar w, which corresponds to $C_2 = w$ and $D_2 = 1$. The first step is to obtain the spectral factor of equation (14) which gives

$$\beta^2(1+\alpha) + w^2\left(1+\alpha^{-1}\right)\left(-\delta^{-1} + 2 - \delta\right)$$

The spectral factor is first order with structure $\pi = \pi_0 + \pi_1\delta$. The second step is to compute the diophantine equation (16) as follows:

$$m\delta^* + w^2(1-\delta^*)\left(1+\alpha^{-1}\right) = n(\pi_0 + \pi_1\delta^*)$$

where m and n are scalars. The solution for n is obtained by equating coefficients of same order which gives $n\pi_0 = w^2(1+\alpha^{-1})$. The filter finally can be written as

$$K = \frac{(n-\pi_0) - (n+\pi_1)\delta}{\pi_0 + \pi_1\delta} \quad (18)$$

The final step is, from the equation (17), to minimize numerically the following cost with respect to α:

$$J(\alpha) = \left\| \frac{\beta n}{\pi} \right\|_2^2 (1+\alpha) + \left\| \frac{((1-\delta)n - \pi)}{\pi} \right\|_2^2 \left(1+\alpha^{-1}\right)$$

REFERENCES

[1] Pop V, Bergveld H, Danilov D, Regtien P, Notten P. Battery management systems- accurate state of charge indication for battery powered applications. B.V: Springler Science+Business Media; 2008.

[2] Plett G. Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs, part 1. Background. J Power Sources 2004;134:252–61. Part 2, Modeling and identification, 2004, 262-276, Part 3, State and parameter estimation, 2004, 277–292.

[3] Chiasson J, Vairamohan B. Estimating the state of charge of a battery. Mayo. IEEE Trans Control Syst Technol 2005;13(3): 465–70.

[4] Billings SA, Fakhouri SY. Identification of a class of nonlinear systems using correlation analysis. Proc IEE 1978;125:691–7.

[5] Milocco R, Castro B. State of charge estimation in Ni-MH rechargeable batteries. J Power Sources 2009;194:558–67.

[6] Olea RA. Geostatistics for engineers and earth scientists. Norwell, Massachusetts: Kluwer Academic Publishers; 1999.

[7] Lophaven SN, Nielsen HB. Informatics and mathematical modelling. Technical University of Denmark; 2002. A MATLAB Kriging Toolbox, DACE, Version 2.0.

[8] Roberts A, Newmann M. Polynomial approach to Wiener filtering. Int J Contr 1987;45:1953–9.

[9] Ježek J, Kŭcera V. Efficient algorithm for matrix spectral factorization. Automatica 1985;21:663–9.

[10] Milocco R, Muravchik C. H_2 Optimal linear robust sampled-data filtering design using polynomial approach. IEEE Trans Signal Proc 2003;51:1816–24.

[11] Åström KJ, Wittenmark B. Computer-controlled systems: theory and design. 3rd ed. Englewood Cliffs, N.J.: Prentice Hall; 1997. 0-13-314899-8.