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Coupled axial/torsional vibrations of drill-strings by means of non-linear model

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Abstract

In the present work a geometrically non-linear model is presented to study the coupling of axial and torsional vibrations on a drill-string, which is described as a vertical slender beam under axial rotation. It is known that the geometrical non-linearities play an important role in the stiffening of a beam. Here, the geometrical stiffening is analyzed using a non-linear finite element approximation, in which large rotations and non-linear strain-displacements are taken into account. The effect of structural damping is also included in the model. To help to understand these effects comparisons of the present model with linear ones were simulated. The preliminary analysis shows that linear and non-linear models differ considerably after the first periods of stick-slip. The behavior is more evident with the increase of the friction in the lower part of the drill.

Keywords: Drill-string; Non-linear dynamics; Axial/torsional coupling; Finite elements; Stick-slip

1. Introduction

It is well known that flexible beams subjected to axial loads manifest stiffness variations due to the presence of the geometric non-linearities. The problem of geometric non-linearities in the context of dynamics of structures was analyzed by means of different schemes such as those reported in the works of Banerjee and Dickens (1990) and Trindade and Sampaio (2002). The non-linear effect is relevant in the case of drill-strings vibrations and it deserves some attention in order to develop suitable models, as it was advanced by Trindade et al. (2005). Vibrations of drill-strings are often analyzed by means of discrete or lumped parameter models (Yigit and Christoforou, 2003; Richard et al., 2004), with certain non-linear expressions to represent the forces/torques interactions with the rock formation. These models allow the study of a complex problem by connecting lumped masses, springs, etc., in a conceptually simplified fashion which also facilitate the implementation of

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control schemes. Yigit and Christoforou (2003) extended their previous work to analyze the coupled axial/torsional/flexural vibrations of drill-strings by means of a simplified lumped parameter differential system. Richard et al. (2004) analyzed the self-excited stick-slip vibrations of drill-strings by means of a simplified lumped parameter model, which accounts for torsional and extensional motions coupled in the boundary. Recently, Trindade et al. (2005) introduced a non-linear continuous beam model to study the influence of geometrical non-linearities in coupled axial/transversal vibrations of drill-strings. In this work, it is shown that the non-linear model has strong quantitative and qualitative discrepancies with respect to a linear one, and on the other hand an effort was made to show the importance of the use of a continuous model that, by discretization, gives a scheme of approximation; that is, given the error allowed the number of degrees of freedom of the discrete approximation is computed. In the present article, the coupled axial/torsional vibrations of drill-strings are studied by means of a non-linear beam model. The drill-string is subjected to distributed loads due to its own weight, leading to geometrical softening of its lower part due to compression. The finite element method is employed to analyze the vibration patterns of both the non-linear and the linear models in different operative conditions. The linear model can be obtained from the non-linear neglecting the geometrical stiffening. In this study it is possible to see the qualitative and quantitative differences between linear and non-linear models, especially when the drill-string undergoes stick-slip patterns. These qualitative differences are remarkable in the calculation of reactive forces and torques. Whereas in linear models there is no geometrical coupling between extensional and torsional vibrations, in the non-linear this kind of geometrical coupling has shown a remarkable effect, specially in long-time stick-slip simulation.

2. Theory for the non-linear model

2.1. Variational formulation

Let us consider an initially straight slender beam with annular cross-section (R_o and R_i are the outer and inner radii), and of length L in the undeformed state, which undergoes large displacements and small deformations as shown in the following Fig. 1. In this beam model only the coupling between axial and torsional deformations in the dynamics of drill-strings is analyzed.

The variational form of the strain energy accounting only for axial and torsional effects can be expressed by the following linear (δH_L) and non-linear (δH_{NL}) terms (Sampaio et al., 2005a):

$$\delta H_{\mathrm{L}} = \int_0^L [\delta u'(EAu') + \delta\theta'(GI_0\theta')]\mathrm{d}x \tag{1}$$

$$\delta H_{\mathrm{NL}} = \int_0^L \delta u' \left[\frac{EA}{2}(3u'^2 + u'^3) + \frac{EI_0}{2}(\theta'^2 + u'\theta'^2)\right]\mathrm{d}x + \int_0^L \delta\theta' \left[\frac{EI_0}{2}(2u' + u'^2)\theta' + \frac{EI_{02}}{2}\theta'^3\right]\mathrm{d}x \tag{2}$$

where E is the longitudinal modulus of elasticity and G is the transverse modulus of elasticity. A and I_0 stand for the cross sectional area and polar moment of inertia, whereas I_{02} is a generalized cross-sectional constant (Sampaio et al., 2005a).

The virtual work done by inertial forces, damping forces and own weight can be written in the following form (Sampaio et al., 2005a):

$$\{\delta T, \delta D, \delta W\} = \int_0^L \{\delta u \rho A \ddot{u} + \delta\theta\rho I_0\ddot{\theta}, -\delta u C_u \dot{u} - \delta\theta C_\theta \dot{\theta}, \delta u \rho g A\}\mathrm{d}x \tag{3}$$

The damping forces are taking into account as a Rayleigh damping proportional to the mass, where C_u and C_θ are the axial and torsional damping constants calculated from the considerations of Spanos et al. (1995).

2.2. Non-linear finite element formulation

A Finite Element model can be constructed through discretization of virtual work components of strain, inertia, damping and applied forces. The discretization is carried out using linear shape functions for both axial displacements and torsional rotations. Then, substituting the discrete expressions of displacements into

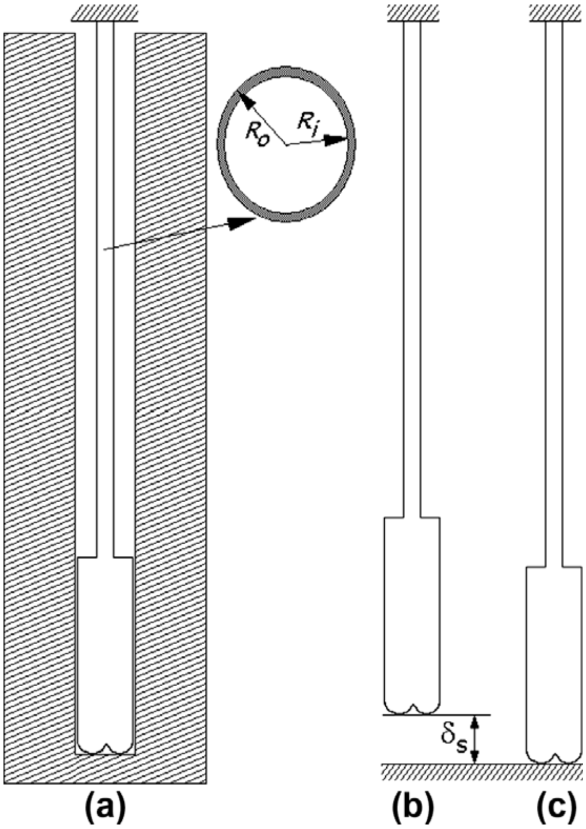

Fig. 1. Drill-string scheme: (a) Description; (b) Undeformed configuration and (c) Deformed configuration.

the virtual work expressions, and, operating and assembling in the usual way, one gets the discretized equations of motion:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + [\mathbf{K}_e + \mathbf{K}_g(\mathbf{q})]\mathbf{q} = \mathbf{F}_g \tag{4}$$

where $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}_e$ and $\mathbf{K}_g$ are the global matrices of mass, damping, elastic stiffness and geometric stiffness, respectively, whereas $\mathbf{F}_g$ is the global vector of gravity forces. The force vector in the discrete Eq. (4) can be extended to account for other force contributions, like impacts, etc.

2.3. Analysis about an initially deformed configuration

In order to analyze the dynamics of the coupled axial/torsional vibrations of the drill-strings, it is important to consider previously some aspects of the drilling process with the scope to characterize the FEM procedure. Drill-strings, such as the ones employed in oil well drilling, can be represented by a vertical cylinder with fixed axial motion at the top position and sliding down due to own weight at the bottom side (i.e. the drill bit). When the drill bit reaches the static equilibrium configuration (before the actual drilling process) acts a reaction, which is considered time-invariant in this work. At this stage the drill-string starts its rotational motion. Fig. 1b and c represent, respectively, the idealized undeformed and deformed configurations of the drill-string. In these circumstances two "a posteriori" forces are included in the finite element model. Then, in addition to the gravity force vector $\mathbf{F}_g$ present in Eq. (4), in the bottom node a time-independent force $\mathbf{F}_f$ is applied to simulate the axial reaction due to rock formation. In addition a reactive torque T_{bit} is applied through the external generalized force vector $\mathbf{F}_T$. This reactive torque is applied at the bottom node N, i.e. in the $(2N)$th degree of freedom, and it can be defined combining different interaction models (Kreuzer and Kust, 1996; Yigit and Christoforou, 2003), in the following form:

$$T_{\text{bit}} = \mathbf{F}_{\text{T}_{2N}} = \mu W_{\text{ob}} f_{\text{i}}(\theta_{\text{bn}}) \left[\text{Tanh}[\dot{\theta}_{\text{bn}}] + \frac{\alpha_1 \dot{\theta}_{\text{bn}}}{1 + \alpha_2 \dot{\theta}_{\text{bn}}^2} \right]$$
$$\text{with } f_{\text{i}}(\theta_{\text{bn}}) = \begin{cases} f_1(\theta_{\text{bn}}) = \frac{1}{2}(1 + \text{Cos}[\theta_{\text{bn}}]) \\ f_2(\theta_{\text{bn}}) = 1 \end{cases} \tag{5}$$

where W_{ob} is the axial reaction of the rock formation, μ is a factor depending on the drill cutter characteristics, α_1 and α_2 are constants depending on rock properties, $f_{\text{i}}(\theta_{\text{bn}})$ is introduced to exploit different modeling options and θ_{bn} and $\dot{\theta}_{\text{bn}}$ are the rotational angle and speed at the drill bit, respectively.

Therefore, considering the aforementioned background, Eq. (4) can be rewritten in the following form:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + [\mathbf{K}_{\text{e}} + \mathbf{K}_{\text{g}}(\mathbf{q})]\mathbf{q} = \mathbf{F}_{\text{g}} + \mathbf{F}_{\text{f}} + \mathbf{F}_{\text{T}} \tag{6}$$

In this work, it is supposed that after the quasi-static lowering and when the reaction force reaches a prescribed value the axial displacement of the drill bit is locked as it is suggested in Fig. 1c. Then further motions take place around this initial deformed configuration, which is obtained from the following equation:

$$\mathbf{q}_{\text{s}} = \mathbf{K}_{\text{e}}^{-1}(\mathbf{F}_{\text{g}} + \mathbf{F}_{\text{f}}) \tag{7}$$

It has to be pointed out that Eq. (7) was obtained assuming that the geometric stiffness is negligible compared to the elastic stiffness for the initial axial loading, as it was explained in Trindade et al. (2005). Then, defining a new displacement vector $\bar{\mathbf{q}}$ relative to the static $\mathbf{q}_{\text{s}}$, i.e. $\bar{\mathbf{q}} = \mathbf{q} - \mathbf{q}_{\text{s}}$, and, substituting $\mathbf{q}$ into Eq. (4), it is possible to obtain the following equations of motion (8) in terms of $\bar{\mathbf{q}}$, i.e. in terms of the relative displacement vector:

$$\mathbf{M}\ddot{\bar{\mathbf{q}}} + \mathbf{D}\dot{\bar{\mathbf{q}}} + [\mathbf{K}_{\text{e}} + \mathbf{K}_{\text{g}}(\bar{\mathbf{q}} + \mathbf{q}_{\text{s}})]\bar{\mathbf{q}} = \mathbf{F}_{\text{T}} \tag{8}$$

Then, the axial displacement of the drill bit it locked into its static value, that is: $\bar{u}^L = 0$ or $u = u_{\text{s}}^L$. On the other hand the top position of the drill-string is subjected to a constant rotary speed ω.

3. Preliminary numerical results

In the present section, the dynamics of typical drill-string configuration is simulated in order to identify the influence of the geometric axial/torsional coupling. The drill-string consists of two parts, the upper portion is composed of slender drill pipes normally subjected to large traction forces; on the other hand, the lower portion is subjected to compressive forces due to the action of own weight of the upper part and the reactive forces, consequently the lower part has larger diameters. The geometric properties of the upper segment are: length 2250 m, internal diameter 0.09718 m and external diameter 0.1143 m. The geometric properties of the lower segment are: length 250 m, internal diameter 0.05715 m and external diameter 0.1651 m. The longitudinal elastic modulus, Transversal elastic modulus and Mass density are the same for both segments: $E = 210$ GPa, $G = 80$ GPa and $\rho = 7850$ kg/m^3.

Since, in the present study the axial displacements are supposed to be initially at their static configuration, they can be excited by means of the coupling with the torsional vibrations. In these circumstances while the rock reaction is supposed constant (the drill-string is lowered until it reaches a decided configuration), the only axial/torsional interaction comes from the non-linear strain-displacement relations, not present in the linear model. In order to understand the stiffening/softening effects and axial/torsional interactions in the drilling process, a set of comparisons between linear and non-linear models are performed. The following rock-bit interaction parameters $\alpha_1 = \alpha_2 = 1$ and $f_{\text{i}}(\theta_{\text{bn}}) = f_1(\theta_{\text{bn}})$ are employed in Eq. (5). The drill-bit parameter has the value $\mu = 0.04$. The drill-string is subjected to a forcing rotary speed of 10 rad/seg at the top. The drill-string was modeled with 24 finite elements (5 in the lower segment and 19 in the upper segment) that proved to offer acceptable accuracy (Sampaio et al., 2005b). As explained in the previous section, the axial displacement of the drill bit is locked after a reactive axial force of 2.55×10^5 N due to the rock formation is reached (i.e. seventy percent of the BHA weight). The equations of motion (8) for the finite element models were numerically integrated with the aid of Matlab ODE algorithms based on implicit schemes (Ode15s).

Fig. 2 depicts the rotary speed at bottom position (in the figure $\partial_t\theta(t)$ means the rotary speed, i.e. derivation with respect to the time) under a stick-slip condition. The forcing rotary speed at the top position is also

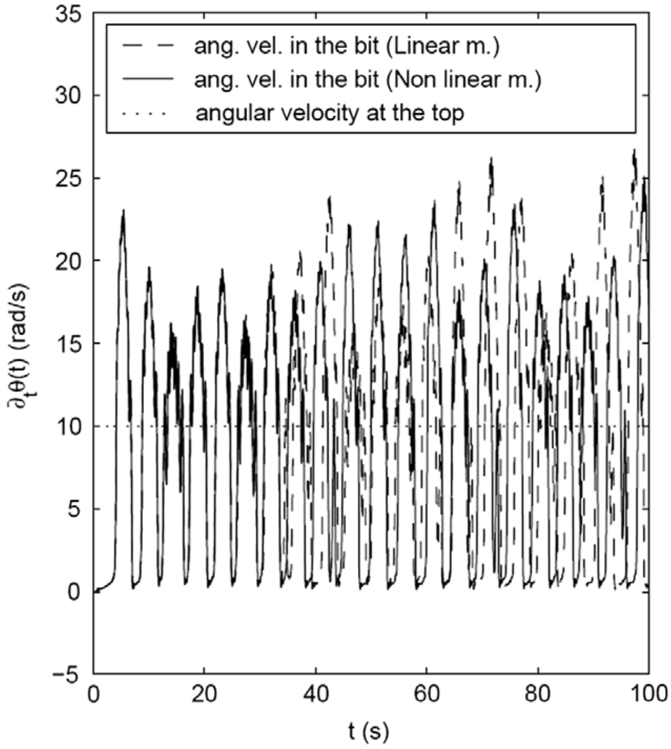

Fig. 2. Difference of non-linear and linear responses for the angular speed at bit. $\mu = 0.04$.

depicted for comparison purposes. One can see a divergence of both models, characterized by the difference between their corresponding rotary speeds. This difference starts to be sensible after the first six periods of stick-slip. Also one can observe that the non-linear model predicts higher speed-peaks in comparison with the linear model.

Fig. 3 shows the axial reaction at the top position for both linear and non-linear models. In this figure one can see that the qualitative differences of both models are sensible from the initial instant. In fact, the

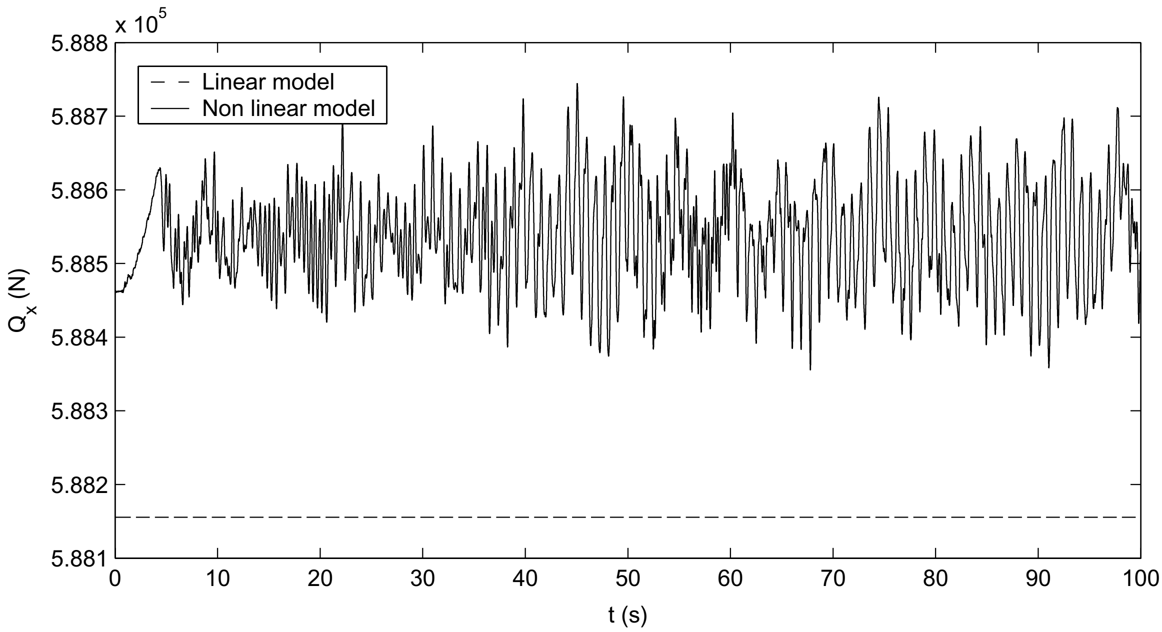

Fig. 3. Reaction forces at the top position using linear and non-linear models.

employment of a non-linear model leads to a modification in the axial force behavior due to the torsional vibration. In other words, the presence of torsional displacements geometrically coupled to axial displacements, induce an increase of the last ones, leading to an qualitative variation of the axial deformation.

4. Conclusions

In this article a non-linear model for simulation of the axial/torsional interactions in drill-strings dynamics was introduced. The axial/torsional interactions were analyzed by means of a comparison between the responses of linear and non-linear models, in operative conditions. Normally, the axial/torsional interaction of a linear model is only related to the bit torque, which has a non-linear form depending on rotation speed and rotation angle at the bit. However, the non-linear model has, in addition to the non-linear bit torque, the consideration of a geometric coupling due to non-linear strain-displacements relations. The non-linear beam model can be reduced to a linear one leaving out the non-linear strain-displacements effects. The linear and non-linear models differ after the first periods of stick-slip not only in the drill-bit rotary speed, but the predictions of forces are different since the beginning. This observations are important in order to simulate a long-time analysis of drilling process, as well as to consider feasible control methodologies. However, the consideration of control methodologies based on the present non-linear model as well as an extensive parametric analysis are the matter of a future research.

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References

Banerjee, A.K., Dickens, J.M., 1990. Dynamics of an arbitrary flexible body in large rotation and translation. J. Guid. Control Dynam. 13 (2), 221–227.

Kreuzer, E., Kust, O., 1996. Analysis of torsional strings by proper orthogonal decomposition. Arch. Appl. Mech. 67, 68–80.

Richard, T., Germay, C., Detournay, E., 2004. Self-excited stick-slip oscillations of drill bits. Comptes Rendus Mecanique 332, 619–626.

Sampaio, R., Piovan, M.T., Venero Lozano, G. 2005a. Non-linear model for coupled axial/torsional vibrations of drill-strings. In: Proceedings of the COBEM, 2005.

Sampaio, R., Piovan, M.T., Venero Lozano, G. 2005b. Qualitative modeling of non-linear coupled axial/torsional vibrations of drill-strings. In: Proceedings of the COBEM, 2005.

Spanos, P.D., Sengupta, A.K., Cunningham, R.A., Paslay, P.R., 1995. Modelling of roller cone bit lift-off dynamics in rotary drilling. J. Energ. Resour. Technol. 117, 197–207.

Trindade, M.A., Sampaio, R., 2002. Dynamics of beams undergoing large rotations accounting for arbitrary axial rotations. J. Guid. Control Dynam. 25 (4), 634–643.

Trindade, M.A., Wolter, C., Sampaio, R., 2005. Karhunen-Loève descomposition of coupled axial/bending vibrations of beams subjected to impact. J. Sound Vib. 279, 1015–1036.

Yigit, A.S., Christoforou, P., 2003. Fully coupled vibrations of actively controlled drill-strings. J. Sound Vib. 267, 1029–1045.