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Generation and correlation of orthogonal complementary pairs of sequences of lengths $2^N 10^M 26^P$ based on an improved multilevel complementary sequences approach

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SUMMARY

Binary complementary pairs of sequences of lengths $2^N 10^M 26^P$ posses some interesting properties for their application in signal detection in noisy channels. Nonetheless, they have not been broadly used because of their greater processing requirements as compared with binary complementary sequences with $L = 2^N$. The present work introduces a new approach for generation and correlation of binary complementary pairs of sequences of length $2^N 10^M 26^P$ that reduces the number of required operations. The proposed algorithm allows not only to optimize the number of arithmetic operations but also to correlate two orthogonal complementary pairs simultaneously with the use of a single architecture. Due to the fact that multilevel complementary sequences algorithms are used to generate and correlate binary complementary sequences of length $2^N 10^M 26^P$, the aforementioned algorithms are also improved. The proposal is theoretically proved and its arithmetic efficiency is assessed by comparing the number of operations with that of previously published architectures.

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KEY WORDS: complementary sequences; efficient algorithms; generation; correlation; multilevel

1. INTRODUCTION

Research about complementary sequences (CS) [1], along with research on other sequences with good correlation properties, is in constant evolution. The main topics being currently analyzed are their applications, such as in Ultra Wideband communications [2], active sensing [3], and multicarrier code-division multiple-access (CDMA) [4]; their mathematical properties [5]; and the search for new sequences [6, 7]. For the particular case of CS, in the last years, the focus has shifted from the well-known binary CS with lengths 2^N to the less used complementary sequences of length $2^N 10^M 26^P$[8], multilevel CS [9], and quadrature amplitude modulation (QAM)-CS [10] that provide more lengths and process gain flexibility than the aforementioned sequences. These sequences have not been extensively employed because of the complexity of the algorithms associated with their generation and correlation. However, several works devoted to the simplification of said algorithms, rendering them more efficient, have been recently published [11–14].

This paper proposes improved algorithms for the generation and the correlation of orthogonal pairs of binary complementary sequences of length $2^N 10^M 26^P$, which notably reduce the algorithms' complexity and allows to correlate simultaneously two orthogonal complementary pairs using a single architecture. This proposal is based on the generation algorithms for CS of length $2^N 10^M 26^P$ of [15] and [8], which, in turn, are based on the processing algorithms for multilevel CS. The improvement is obtained by a reformulation of the multilevel CS generation algorithm of [16] and the analysis of

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the correlation operation to obtain an efficient algorithm for the simultaneous correlation of orthogonal multilevel CS. The number of operations required to compute the algorithm is analytically calculated and compared with those in the literature, to show the improvement. Field programmable gate array (FPGA) implementation examples are provided to illustrate the amount of the logic resources needed for the correlation algorithm.

The paper is structured as follows. Section 2 deals with the definition and properties of CS. Section 3 develops the generation algorithm of non-power-of-2 CS. Sections 4 and 5 show the proposed generation and correlation of multilevel CS, respectively. The proposed generator and correlator of non-power-of-2 CS are described in Section 6. These algorithms are compared with those in the literature in Section 7. Section 8 provides an example of how the proposed architectures may be used in a multi-user system; and finally, Section 9 draws the conclusions of the paper.

2. COMPLEMENTARY PAIRS OF SEQUENCES

Binary complementary pairs of sequences were defined by Golay [1] as two sequences (S_1, S_2) of the same length (L), composed by binary elements (± 1), with the property that the number of pairs of like elements with any given separation in one sequence is exactly equal to the number of pairs of unlike elements with the same separation in the other sequence. The main property of these pairs is that the sum of the autocorrelation functions of the two sequences results in a Kronecker delta of amplitude proportional to the length, without the presence of sidelobes.

$$C_{S_1,S_1}[k] + C_{S_2,S_2}[k] = 2L\delta[k] \tag{1}$$

Where: C_{S_1,S_1} and C_{S_2,S_2} are the autocorrelation functions of the sequences S_1 and S_2, respectively.

Taking this property as an alternative definition of complementary sequences, the concept was extended to complementary sets of sequences [17], increasing the number of sequences from 2 to M and multilevel complementary pairs of sequences [9, 18, 19], which are composed of elements with amplitudes different to a binary set of symbols. The other property that characterizes complementary sequences is the existence of orthogonal pairs or sets. For every pair (or set), there is another pair (or set) of sequences that is orthogonal. Two pairs (or sets) are orthogonal if the sum of cross correlations between the sequences of the pairs (or sets) is zero for every time displacement [17].

3. GENERATION OF COMPLEMENTARY PAIRS OF SEQUENCES OF LENGTH 10 AND 26

Complementary sequences of length $L = 2^N 10^M 26^P$, with $M, P \neq 0$, have been traditionally left out due to the fact that kernels 10 and 26 are not easily generated. In [15] and [8], these kernels for binary complementary sequences were synthesized using the multilevel complementary sequences generation algorithm as follows:

$$\mathbf{G}_{10_a} = \mathbf{B}_{1,1}(z^{-3}) \times \mathbf{B}_{1,-1}(z^{-1}) \times \mathbf{B}_{1/2,-1}(z^{-1}) \times \mathbf{B}_{1,1}(z^{-4}) \tag{2}$$

$$\mathbf{G}_{10_b} = \mathbf{B}_{1,1}(z^{-1}) \times \mathbf{B}_{1,1}(z^{-3}) \times \mathbf{B}_{1/2,-1}(z^{-3}) \times \mathbf{B}_{1,1}(z^{-2}) \tag{3}$$

$$\begin{aligned}\mathbf{G}_{26} =& \mathbf{B}_{1,-1}(z^{-12}) \times \mathbf{B}_{1,1}(z^{-1}) \times \mathbf{B}_{1/2,1}(z^{-1}) \times \mathbf{B}_{1/5,-1}(z^{-1}) \\ &\times \mathbf{B}_{4/13,-1}(z^{-1}) \times \mathbf{B}_{3/37,1}(z^{-1}) \times \mathbf{B}_{81/106,-1}(z^{-1}) \times \mathbf{B}_{3/37,1}(z^{-1}) \\ &\times \mathbf{B}_{4/13,1}(z^{-1}) \times \mathbf{B}_{1/5,-1}(z^{-1}) \times \mathbf{B}_{1/2,1}(z^{-1}) \times \mathbf{B}_{1,1}(z^{-3})\end{aligned} \tag{4}$$

Where:

$$\mathbf{B}_{a_n,w_n^j}(z^{-d_n}) = \begin{bmatrix} 1 & a_n \cdot w_n^j \cdot z^{-d_n} \\ a_n & -w_n^j \cdot z^{-d_n} \end{bmatrix} \tag{5}$$

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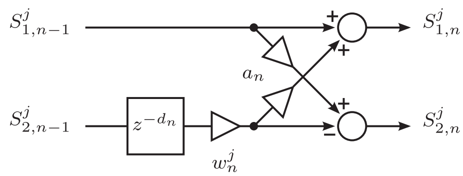

Figure 1. Block diagram of an iteration of the generation algorithm [8].

is the iteration n of the generation algorithm for multilevel complementary pairs of sequences ($\mathbf{S}^j(z)$) proposed by García *et al.* [16]. This algorithm is defined as follows:

$$\mathbf{S}^j(z) = \begin{bmatrix} S_1^j(z) \\ S_2^j(z) \end{bmatrix} = \prod_{n=1}^{N} \mathbf{B}_{a_n, w_n^j}(z^{-d_n}) \times \begin{bmatrix} 1 \\ a_0 \end{bmatrix} \tag{6}$$

Where:

- $\mathbf{B}_{a_n, w_n^j}(z^{-d_n})$ is the generation matrix (5),
- $j = \{0, 1\}$ is the index of the pair in an orthogonal set,
- $w_n^j = \pm 1$ is the generation seed of the iteration n,
- $a_n \in \mathbb{R}$ is the multilevel gain of the iteration n,
- d_n is the delay of the iteration n.

Figure 1 shows a block diagram of an iteration of this algorithm.

The use of different generation seeds w_n^j allows for the generation of different pairs of sequences with the same multilevel gains. To generate a pair (S_1^1, S_2^1) that is orthogonal to another pair of sequences, (S_1^0, S_2^0), w_n^0 must be kept equal to w_n^1 for the iterations 1 to N and of opposite sign for iteration 0 [20]:

$$\begin{cases} w_n^1 = w_n^0 & 1 < n < N \\ w_n^1 = -w_n^0 & n = 0 \end{cases} \tag{7}$$

The pairs of sequences generated with these seeds ($\mathbf{S}^0(z)$ and $\mathbf{S}^1(z)$) are orthogonal, so the sum of cross correlation functions between S^0 and S^1 is null for every k:

$$C_{S_1^0, S_1^1}[k] + C_{S_2^0, S_2^1}[k] = 0 \tag{8}$$

Where: $C_{S_1^0, S_1^1}[k]$ and $+C_{S_2^0, S_2^1}[k]$ are the cross correlation of the sequences S_1^0, S_1^1 and S_2^0, S_2^1, respectively.

4. NEW MULTILEVEL COMPLEMENTARY PAIRS OF SEQUENCES GENERATION APPROACH

The generation approach described in the previous section (6) depends on a multilevel processing algorithm. To improve the binary sequences generation, that multilevel algorithm can be reformulated in a more efficient way by analyzing the product operator. Considering the last two stages, N and $N-1$, without the inclusion of superscript j to simplify the notation:

$$\begin{aligned} &\mathbf{B}_{a_N, w_N}(z^{-d_N}) \times \mathbf{B}_{a_{N-1} w_{N-1}}(z^{-d_{N-1}}) = \\ &= \begin{bmatrix} 1 & a_N \cdot w_N \cdot z^{-d_N} \\ a_N & -w_N \cdot z^{-d_N} \end{bmatrix} \times \mathbf{B}_{a_{N-1} w_{N-1}}(z^{-d_{N-1}}) \\ &= \begin{bmatrix} 1 & a_N \cdot z^{-d_N} \\ a_N & -z^{-d_N} \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & w_N \end{bmatrix} \times \mathbf{B}_{a_{N-1} w_{N-1}}(z^{-d_{N-1}}) \end{aligned} \tag{9}$$

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Defining $\mathbf{\Psi}_{a_N}(z^{-d_N})$ as

$$\mathbf{\Psi}_{a_N}(z^{-d_N}) = \begin{bmatrix} 1 & a_N \cdot z^{-d_N} \\ a_N & -z^{-d_N} \end{bmatrix} \tag{10}$$

Equation (9) results are

$$\begin{aligned} &\mathbf{B}_{a_N,w_N}(z^{-d_N}) \times \mathbf{B}_{a_{N-1}w_{N-1}}(z^{-d_{N-1}}) = \\ &= \mathbf{\Psi}_{a_N}(z^{-d_N}) \times \begin{bmatrix} 1 & 0 \\ 0 & w_N \end{bmatrix} \times \mathbf{B}_{a_{N-1}w_{N-1}}(z^{-d_{N-1}}) \\ &= \mathbf{\Psi}_{a_N}(z^{-d_N}) \times \begin{bmatrix} 1 & a_{N-1} \cdot w_{N-1} \cdot z^{-d_{N-1}} \\ a_{N-1} \cdot w_N & -w_{N-1} \cdot w_N \cdot z^{-d_{N-1}} \end{bmatrix} \end{aligned} \tag{11}$$

Considering that w_n is ± 1 and the product $w_n \cdot w_n = 1$, the matrix on the right can be split as follows:

$$\begin{aligned} &\mathbf{B}_{a_N,w_N}(z^{-d_N}) \times \mathbf{B}_{a_{N-1}w_{N-1}}(z^{-d_{N-1}}) = \\ &= \mathbf{\Psi}_{a_N}(z^{-d_N}) \times \begin{bmatrix} 1 & a_{N-1} \cdot w_N \cdot z^{-d_{N-1}} \\ a_{N-1} \cdot w_N & -z^{-d_{N-1}} \end{bmatrix} \\ &\qquad \times \begin{bmatrix} 1 & 0 \\ 0 & w_N \cdot w_{N-1} \end{bmatrix} \\ &= \mathbf{\Psi}_{a_N}(z^{-d_N}) \times \mathbf{\Psi}_{a_{N-1} \cdot w_N}(z^{-d_{N-1}}) \\ &\qquad \times \begin{bmatrix} 1 & 0 \\ 0 & w_N \cdot w_{N-1} \end{bmatrix} \end{aligned} \tag{12}$$

Generalizing this process for all stages, the new generation algorithm is obtained:

$$\mathbf{S}(z) = \prod_{n=1}^{N} \mathbf{\Psi}_{\alpha_n}(z^{-d_n}) \times \begin{bmatrix} 1 \\ \alpha_0 \end{bmatrix} \tag{13}$$

with

$$\begin{aligned} \alpha_n &= a_n \cdot \prod_{i=n+1}^{N} w_i \\ \alpha_N &= a_N \end{aligned} \tag{14}$$

Figure 2 depicts a block diagram of an iteration of this algorithm.

In order to generate orthogonal pairs, the sign of w_0 must be changed. In (14), it can be seen that α_0 is the only α_n affected by w_0. Based on that, the generation of the orthogonal pairs depends only on the input signal of the generator, yielding the proposed generation algorithm:

$$\mathbf{S}^j(z) = \prod_{n=1}^{N} \mathbf{\Psi}_{\alpha_n}(z^{-d_n}) \times \begin{bmatrix} 1 \\ \alpha_0 \cdot (-1)^j \end{bmatrix} \tag{15}$$

Figure 3 illustrates the block diagram of the proposed multilevel complementary sequences generator.

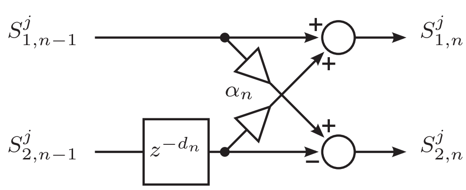

Figure 2. Block diagram of an iteration of the proposed generator.

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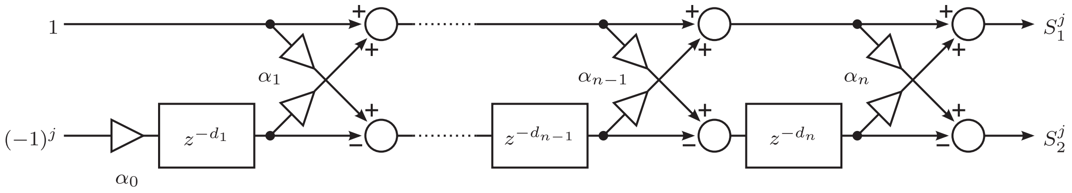

Figure 3. Block diagram of the proposed multilevel generator.

5. SIMULTANEOUS CORRELATION OF MULTILEVEL COMPLEMENTARY PAIRS OF SEQUENCES

Several applications (e.g., multi-user communications systems) require the simultaneous processing of two orthogonal pairs of sequences. In the said cases, the conventional correlation approach entails the use of four different correlators, each one configured with the corresponding sequence and seed. However, this solution leads to an inefficient use of the resources, because there are many common operations in both correlators. In [12], a more efficient correlator was proposed that uses a single architecture to correlate with one pair, thereby needing only two correlators. By analyzing it, it can be improved even further.

In that manuscript, it was shown that the sum of correlations of two inputs $(R_1(z), R_2(z))$ with respect to a pair of sequences $(Y^j(z))$ can be written in matrix form as

$$\begin{aligned} Y^j(z) &= \left[\, \overline{S_1^j}(1/z)\ \overline{S_2^j}(1/z) \,\right] \times \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \\ &= \mathbf{S}^j(1/z)^H \times \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \end{aligned} \tag{16}$$

Where: $\overline{S_1}(1/z)$ and $\overline{S_2}(1/z)$ are the complex conjugates of $S_1(1/z)$ and $S_2(1/z)$, respectively, and $(.)^H$ is the Hermitian operation.

Working with this expression and using (15), the following equation is obtained:

$$\begin{aligned} \mathbf{S}^j(1/z)^H &= \left(\prod_{n=1}^{N} \mathbf{\Psi}_{\alpha_n}(z^{d_n}) \times \begin{bmatrix} 1 \\ \alpha_0 \cdot (-1)^j \end{bmatrix} \right)^H \\ &= \begin{bmatrix} 1 \\ \alpha_0 \cdot (-1)^j \end{bmatrix}^H \times \left(\prod_{n=1}^{N} \mathbf{\Psi}_{\alpha_n}(z^{d_n}) \right)^H \\ &= \left[\, 1\ \ \alpha_0 \cdot (-1)^j \,\right] \times \prod_{n=N}^{1} \mathbf{\Psi}_{\alpha_n}(z^{d_n})^H \end{aligned} \tag{17}$$

To make a practically feasible system, all the powers of z on the equation describing its behavior must be non-positive, but as it can be observed in (17), the correlation equation obtained has positive powers of z. To solve this, $\mathbf{\Psi}^*_{\alpha_n}(z^{-d_n})$ is defined:

$$\mathbf{\Psi}^*_{\alpha_n}(z^{-d_n}) = z^{-d_n} \cdot \mathbf{\Psi}_{\alpha_n}(z^{d_n})^H = \begin{bmatrix} z^{-d_n} & \alpha_N \cdot z^{-d_n} \\ \alpha_n & 1 \end{bmatrix} \tag{18}$$

Replacing (18) in (17), the final sum of correlations results in

$$Y^j(z) = \left[\, 1\ \ \alpha_0 \cdot (-1)^j \,\right] \times \prod_{n=N}^{1} \mathbf{\Psi}^*_{\alpha_n}(z^{-d_n}) \times \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \tag{19}$$

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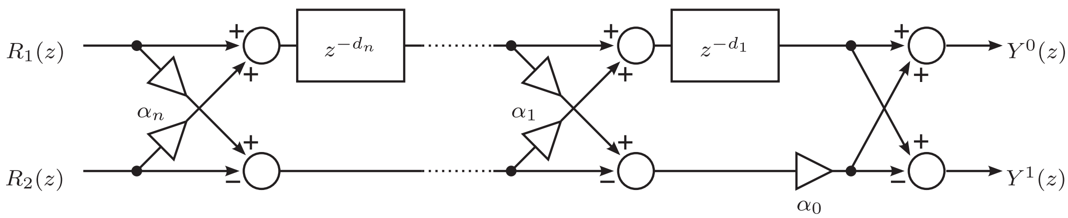

Figure 4. Block diagram of the proposed multilevel correlator.

Equation (19) shows an improved correlation algorithm for one pair of sequences. To obtain the sum of correlations with respect to both orthogonal pairs, $\mathbf{Y}(z)$ is defined:

$$\mathbf{Y}(z) = \begin{bmatrix} Y^0(z) \\ Y^1(z) \end{bmatrix} \tag{20}$$

Reordering the terms of $\mathbf{Y}(z)$, the proposed simultaneous correlation algorithm for orthogonal multi-level sequences is obtained.

$$\mathbf{Y}(z) = \begin{bmatrix} \begin{bmatrix} 1 & \alpha_0 \end{bmatrix} \times \prod_{n=N}^{1} \mathbf{\Psi}^*_{\alpha_n}(z^{-d_n})^H \\ \begin{bmatrix} 1 & -\alpha_0 \end{bmatrix} \times \prod_{n=N}^{1} \mathbf{\Psi}^*_{\alpha_n}(z^{-d_n})^H \end{bmatrix} \times \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \tag{21}$$

$$\mathbf{Y}(z) = \begin{bmatrix} 1 & \alpha_0 \\ 1 & -\alpha_0 \end{bmatrix} \times \prod_{n=N}^{1} \mathbf{\Psi}^*_{\alpha_n}(z^{-d_n})^H \times \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \tag{22}$$

Figure 4 shows a block diagram of the simultaneous correlation algorithm described in (22) obtained using a reasoning similar to that in [12].

6. PROPOSED GENERATOR AND CORRELATOR OF BINARY COMPLEMENTARY SEQUENCES OF LENGTH $2^N 10^M 26^P$

Using the reasoning in Section 4, the kernels shown in Section 3 can be generated using the modified architecture with the following equations:

$$\mathbf{G}_{10_a} = \mathbf{\Psi}_1(z^{-3}) \times \mathbf{\Psi}_1(z^{-1}) \times \mathbf{\Psi}_{-1/2}(z^{-1}) \times \mathbf{\Psi}_1(z^{-4}); \quad j = 0 \tag{23}$$

$$\mathbf{G}_{10_b} = \mathbf{\Psi}_1(z^{-1}) \times \mathbf{\Psi}_1(z^{-3}) \times \mathbf{\Psi}_{1/2}(z^{-3}) \times \mathbf{\Psi}_{-1}(z^{-2}); \quad j = 1 \tag{24}$$

$$\begin{aligned} \mathbf{G}_{26} = {} & \mathbf{\Psi}_1(z^{-12}) \times \mathbf{\Psi}_{-1}(z^{-1}) \times \mathbf{\Psi}_{-1/2}(z^{-1}) \times \mathbf{\Psi}_{-1/5}(z^{-1}) \\ & \times \mathbf{\Psi}_{4/13}(z^{-1}) \times \mathbf{\Psi}_{-3/37}(z^{-1}) \times \mathbf{\Psi}_{-81/106}(z^{-1}) \times \mathbf{\Psi}_{3/37}(z^{-1}) \\ & \times \mathbf{\Psi}_{4/13}(z^{-1}) \times \mathbf{\Psi}_{1/5}(z^{-1}) \times \mathbf{\Psi}_{-1/2}(z^{-1}) \times \mathbf{\Psi}_{-1}(z^{-3}); \quad j = 1 \end{aligned} \tag{25}$$

Regardless of the approach applied to generate the sequences, they can be correlated using the architecture described in (22). To obtain the correlation, the order of the stages of (23)–(25) must be reversed.

$$\mathbf{C}_{10_a} = \mathbf{\Psi}^*_1(z^{-4}) \times \mathbf{\Psi}^*_{-1/2}(z^{-1}) \times \mathbf{\Psi}^*_1(z^{-1}) \times \mathbf{\Psi}^*_1(z^{-3}); \quad j = 0 \tag{26}$$

$$\mathbf{C}_{10_b} = \mathbf{\Psi}^*_{-1}(z^{-2}) \times \mathbf{\Psi}^*_{1/2}(z^{-3}) \times \mathbf{\Psi}^*_1(z^{-3}) \times \mathbf{\Psi}^*_1(z^{-1}); \quad j = 1 \tag{27}$$

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$$\begin{aligned}\mathbf{C}_{26} = {} & \mathbf{\Psi}^*_{-1}(z^{-3}) \times \mathbf{\Psi}^*_{-1/2}(z^{-1}) \times \mathbf{\Psi}^*_{1/5}(z^{-1}) \times \mathbf{\Psi}^*_{4/13}(z^{-1}) \\ & \times \mathbf{\Psi}^*_{3/37}(z^{-1}) \times \mathbf{\Psi}^*_{-81/106}(z^{-1}) \times \mathbf{\Psi}^*_{-3/37}(z^{-1}) \times \mathbf{\Psi}^*_{4/13}(z^{-1}) \\ & \times \mathbf{\Psi}^*_{-1/5}(z^{-1}) \times \mathbf{\Psi}^*_{-1/2}(z^{-1}) \times \mathbf{\Psi}^*_{-1}(z^{-1}) \times \mathbf{\Psi}^*_{1}(z^{-12}); \quad j = 1\end{aligned} \tag{28}$$

To further demonstrate the applicability of the proposed algorithm, two examples of its use are shown below.

6.1. *Example of correlation of kernel 10*

Golay [1] found two kernels for length 10 complementary pairs of sequences. The first kernel is composed by the following sequences:

$$\begin{aligned} S_1 &= \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \end{bmatrix} \\ S_2 &= \begin{bmatrix} 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix} \end{aligned} \tag{29}$$

These sequences can be correlated with the proposed architecture using (26) as follows. In the first iteration, they should be affected by $\mathbf{\Psi}^*_1(z^{-3})$:

$$\begin{bmatrix} C_{3,1}(z) \\ C_{3,2}(z) \end{bmatrix} = \mathbf{\Psi}^*_1(z^{-3}) \times \begin{bmatrix} S_1(z) \\ S_2(z) \end{bmatrix} \tag{30}$$

$$\begin{aligned} C_{3,1} &= [\ 0\ 0\ 0\ 2\ {-2}\ {-2}\ 0\ 2\ 0\ \ 2\ 0\ 0\ 0\ ] \\ C_{3,2} &= [\ 0\ 0\ 0\ 2\ \ 0\ \ 2\ 0\ 2\ 2\ {-2}\ 0\ 0\ 0\ ] \end{aligned} \tag{31}$$

The resulting sequences are affected by $\mathbf{\Psi}^*_1(z^{-1})$ in the following iteration:

$$\begin{bmatrix} C_{2,1}(z) \\ C_{2,2}(z) \end{bmatrix} = \mathbf{\Psi}^*_1(z^{-1}) \times \begin{bmatrix} C_{3,1}(z) \\ C_{3,2}(z) \end{bmatrix} \tag{32}$$

$$\begin{aligned} C_{2,1} &= [\ 0\ 0\ 0\ 0\ \ 4\ {-2}\ 0\ 0\ \ 4\ 2\ 0\ 0\ 0\ 0\ ] \\ C_{2,2} &= [\ 0\ 0\ 0\ 0\ {-2}\ {-4}\ 0\ 0\ {-2}\ 4\ 0\ 0\ 0\ 0\ ] \end{aligned} \tag{33}$$

The obtained sequences are processed by the last two stages, $\mathbf{\Psi}^*_{-1/2}(z^{-1})$ and $\mathbf{\Psi}^*_1(z^{-4})$, respectively:

$$\begin{bmatrix} C_{1,1}(z) \\ C_{1,2}(z) \end{bmatrix} = \mathbf{\Psi}^*_{-1/2}(z^{-1}) \times \begin{bmatrix} C_{2,1}(z) \\ C_{2,2}(z) \end{bmatrix} \tag{34}$$

$$\begin{aligned} C_{1,1} &= [\ 0\ 0\ 0\ 0\ 0\ 5\ 0\ 0\ 0\ \ \ 5\ 0\ 0\ 0\ 0\ 0\ ] \\ C_{1,2} &= [\ 0\ 0\ 0\ 0\ 0\ 5\ 0\ 0\ 0\ {-5}\ 0\ 0\ 0\ 0\ 0\ ] \end{aligned} \tag{35}$$

$$\begin{bmatrix} C_{0,1}(z) \\ C_{0,2}(z) \end{bmatrix} = \mathbf{\Psi}^*_1(z^{-4}) \times \begin{bmatrix} C_{1,1}(z) \\ C_{1,2}(z) \end{bmatrix} \tag{36}$$

$$\begin{aligned} C_{0,1} &= [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 10\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \\ C_{0,1} &= [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 10\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \end{aligned} \tag{37}$$

The last step for the correlation is the product with α_0 and the addition, resulting in

$$Y = [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 20\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ] \tag{38}$$

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These equations demonstrate that the correlator works as expected, yielding as result of the correlation a single Kronecker delta of amplitude equal to $2L$, without sidelobes.

6.2. Example of correlation of kernel 26

In [8], a kernel 26, based on a pair of sequences found in [21], is generated:

$$\begin{aligned} S_1 = [\ & 1\ \ 1\ \ 1\ \ 1\ -1\ \ 1\ \ 1\ -1\ -1\ \ 1\ -1\ \ 1\ -1 \\ & 1\ -1\ -1\ \ 1\ -1\ \ 1\ \ 1\ \ 1\ -1\ -1\ \ 1\ \ 1\ \ 1\ ] \\ S_2 = [\ & 1\ \ 1\ \ 1\ \ 1\ -1\ \ 1\ \ 1\ -1\ -1\ \ 1\ -1\ \ 1\ \ 1 \\ & 1\ \ 1\ \ 1\ -1\ \ 1\ -1\ -1\ -1\ \ 1\ \ 1\ -1\ -1\ -1\ ] \end{aligned} \tag{39}$$

These sequences can be correlated using the proposed algorithm and (28), following a process analogous to the previous example, with 12 iterations rather than 4. Starting with $C_{12,1} = S_1$ and $C_{12,2} = S_2$, the partial results of the correlation (C_{11} to C_0) are shown in the following equations. For notation purposes, the leading and trailing zeros of the vectors are represented as $\mathbf{0}^k$, where k represent the amount of zeros.

$$\begin{aligned} C_{11,1} &= [\ \mathbf{0}^{12}\ \ 2\ \ 2\ \ 2\ \ 2\ -2\ \ 2\ \ 2\ -2\ -2\ \ 2\ -2\ \ 2\ \ 0\ \ 2\ \ \mathbf{0}^{12}\ ] \\ C_{11,2} &= [\ \mathbf{0}^{12}\ -2\ \ 0\ -2\ -2\ \ 2\ -2\ \ 2\ \ 2\ \ 2\ -2\ -2\ \ 2\ \ 2\ \ 2\ \ \mathbf{0}^{12}\ ] \end{aligned} \tag{40}$$

$$\begin{aligned} C_{10,1} &= [\ \mathbf{0}^{13}\ \ 4\ \ 2\ \ 4\ \ 4\ -4\ \ 4\ \ 0\ -4\ -4\ \ 4\ \ 0\ \ 0\ -2\ \ \mathbf{0}^{13}\ ] \\ C_{10,2} &= [\ \mathbf{0}^{13}\ -2\ \ 0\ \ 0\ \ 0\ \ 0\ -4\ \ 0\ \ 0\ \ 0\ \ 4\ -4\ -2\ -4\ \ \mathbf{0}^{13}\ ] \end{aligned} \tag{41}$$

$$\begin{aligned} C_{9,1} &= [\ \mathbf{0}^{14}\ \ 5\ \ 2\ \ 4\ \ 4\ -4\ \ 6\ \ 0\ -4\ -4\ \ 2\ \ 2\ \ 1\ \ \mathbf{0}^{14}\ ] \\ C_{9,2} &= [\ \mathbf{0}^{14}\ -1\ -2\ -2\ \ 2\ \ 2\ \ 0\ \ 2\ \ 2\ -6\ \ 4\ \ 2\ \ 5\ \ \mathbf{0}^{14}\ ] \end{aligned} \tag{42}$$

$$\begin{aligned} C_{8,1} &= [\ \mathbf{0}^{15}\ \ 26\ \ 12\ \ 22\ \ 18\ -22\ \ 30\ -2\ -22\ -14\ \ 6\ \ 8\ \ \mathbf{0}^{15}\ ]/5 \\ C_{8,2} &= [\ \mathbf{0}^{15}\ \ 8\ \ 6\ -14\ -6\ -6\ -10\ -6\ \ 34\ -22\ -12\ -26\ \ \mathbf{0}^{15}\ ]/5 \end{aligned} \tag{43}$$

$$\begin{aligned} C_{7,1} &= [\ \mathbf{0}^{16}\ \ 74\ \ 36\ \ 46\ \ 42\ -62\ \ 70\ -10\ -30\ -54\ \ 6\ \ \mathbf{0}^{16}\ ]/13 \\ C_{7,2} &= [\ \mathbf{0}^{16}\ -6\ \ 54\ \ 30\ -2\ \ 50\ \ 14\ -106\ \ 46\ \ 36\ \ 74\ \ \mathbf{0}^{16}\ ]/13 \end{aligned} \tag{44}$$

$$\begin{aligned} C_{6,1} &= [\ \mathbf{0}^{17}\ \ 212\ \ 90\ \ 124\ \ 120\ -188\ \ 196\ -4\ -96\ -162\ \ \mathbf{0}^{17}\ ]/37 \\ C_{6,2} &= [\ \mathbf{0}^{17}\ -162\ -96\ -4\ -128\ -56\ \ 304\ -124\ -90\ -212\ \ \mathbf{0}^{17}\ ]/37 \end{aligned} \tag{45}$$

$$\begin{aligned} C_{5,1} &= [\ \mathbf{0}^{18}\ \ 481\ \ 234\ \ 182\ \ 312\ -208\ -52\ \ 130\ -39\ \ \mathbf{0}^{18}\ ]/53 \\ C_{5,2} &= [\ \mathbf{0}^{18}\ \ 39\ -130\ \ 52\ \ 286\ -650\ \ 182\ \ 234\ \ 481\ \ \mathbf{0}^{18}\ ]/53 \end{aligned} \tag{46}$$

$$\begin{aligned} C_{4,1} &= [\ \mathbf{0}^{19}\ \ 338\ \ 156\ \ 130\ \ 234\ -182\ -26\ \ 104\ \ \mathbf{0}^{19}\ ]/37 \\ C_{4,2} &= [\ \mathbf{0}^{19}\ \ 104\ -26\ -182\ \ 442\ -130\ -156\ -338\ \ \mathbf{0}^{19}\ ]/37 \end{aligned} \tag{47}$$

$$\begin{aligned} C_{3,1} &= [\ \mathbf{0}^{20}\ \ 10\ \ 4\ \ 2\ \ 10\ -6\ -2\ \ \mathbf{0}^{20}\ ] \\ C_{3,2} &= [\ \mathbf{0}^{20}\ \ 2\ \ 6\ -10\ \ 2\ \ 4\ \ 10\ \ \mathbf{0}^{20}\ ] \end{aligned} \tag{48}$$

$$\begin{aligned} C_{2,1} &= [\ \mathbf{0}^{21}\ \ 52\ \ 26\ \ 0\ \ 52\ -26\ \ \mathbf{0}^{21}\ ]/5 \\ C_{2,2} &= [\ \mathbf{0}^{21}\ -26\ \ 52\ \ 0\ -26\ -52\ \ \mathbf{0}^{21}\ ]/5 \end{aligned} \tag{49}$$

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$$\begin{aligned} C_{1,1} &= [\ \mathbf{0}^{22}\quad 13\ 0\ 0\ 13\ \mathbf{0}^{22}\ ] \\ C_{1,2} &= [\ \mathbf{0}^{22}\ -13\ 0\ 0\ 13\ \mathbf{0}^{22}\ ] \end{aligned} \tag{50}$$

$$\begin{aligned} C_{0,1} &= [\ \mathbf{0}^{25}\quad 26\ \mathbf{0}^{25}\ ] \\ C_{0,2} &= [\ \mathbf{0}^{25}\ -26\ \mathbf{0}^{25}\ ] \end{aligned} \tag{51}$$

The last correlation step is the product with α_0 and the final addition, yielding the final correlation result.

$$Y = [\ \mathbf{0}^{25}\ \ 52\ \ \mathbf{0}^{25}\ ] \tag{52}$$

Equations (39) and (52) show the expected correlation results, demonstrating that the proposed algorithm also works in binary complementary sequences of length $2^N10^M26^P$ with $M, P \neq 0$.

7. RESULTS

In order to evaluate the proposal, a comparison of the required amount of operations between the different approaches was conducted. In the work by García *et al.* [8], the number of operations is presented after an additional optimization of the algorithm's iterations. In such optimization, products by 1 are removed and products by -1 are implemented as a sign change in the additions/subtractions of the iteration, which makes the algorithm lose regularity. Even though this optimization can also be applied to the proposed algorithm, for the sake of simplicity and to keep the structure regular, this optimization is disregarded for the calculations of the number of operation

Table I. Operations required for the generation of complementary sequences.

Additions	
Efficient [8]	$2 \cdot (N + 4M + 12P)$
Proposed	$2 \cdot (N + 4M + 12P)$
Products	
Efficient [8]	$3 \cdot (N + 4M + 12P)$
Proposed	$2 \cdot (N + 4M + 12P)$
Delays	
Efficient [8]	$2^N10^M26^P - 1$
Proposed	$2^N10^M26^P - 1$

Table II. Operations required for the correlation of complementary sequences.

Additions	
Efficient [8]	$8 \cdot (N + 4M + 12P) - 2$
Optimized [12]	$4 \cdot (N + 4M + 12P) + 2$
Proposed	$2 \cdot (N + 4M + 12P) + 2$
Products	
Efficient [8]	$12 \cdot (N + 4M + 12P)$
Optimized [12]	$6 \cdot (N + 4M + 12P)$
Proposed	$2 \cdot (N + 4M + 12P) + 1$
Delays	
Efficient [8]	$4 \cdot (2^N10^M26^P - 1)$
Optimized [12]	$2 \cdot (2^N10^M26^P - 1)$
Proposed	$2^N10^M26^P - 1$

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Table III. FPGA correlator implementation resources and speed.

L	Stages	Slices	Max frequency
4	2	21	266.52 MHz
8	3	39	152.07 MHz
10	4	61	118.96 MHz
16	4	53	125.61 MHz
20	5	73	107.34 MHz
26	12	164	18.54 MHz
32	5	66	107.91 MHz
40	6	90	102.26 MHz
64	6	82	105.45 MHz

Table I lists the required number of additions, products, and delays to generate a pair of binary complementary sequences of length $2^N 10^M 26^P$, as a function of their length, through the values of N, M, and P. The values in the table are calculated as follows:

- Each iteration of the algorithm in [8] uses 2 additions, 3 products, and d_n delays.
- Each iteration of the proposed algorithm uses 2 additions, 2 products, and d_n delays.
- Power of 2 sequences use 1 algorithm iteration, with a total delay of 2.
- Power of 10 sequences use 4 algorithm iterations, with a total delay of 10.
- Power of 26 sequences use 12 algorithm iterations, with a total delay of 26.

It can be seen from this table that the proposed generation architecture uses two thirds of the products of the algorithm of [8], which represents a 33% reduction in the total number of products. It is also noteworthy that, with the proposed generator, it is possible to generate both orthogonal pairs by changing its input, while with the generator in [8], this is achieved by modifying the coefficients of the algorithm.

Table II lists the number of additions, delays, and products needed for the simultaneous correlation of orthogonal binary complementary pairs of sequences of length $2^N 10^M 26^P$ in the same way as in Table I. The values of the table are calculated as follows:

- Each iteration of the algorithm in [8] and [12] uses 2 additions, 3 products, and d_n delays.
- Four instances of the algorithm in [8] are required, each with one of its outputs unused.
- Two instances of the algorithm in [12] are required.
- Each iteration of the proposed algorithm uses 2 additions, 2 products, and d_n delays.
- Power of 2 sequences use 1 algorithm iteration, with a total delay of 2.
- Power of 10 sequences use 4 algorithm iterations, with a total delay of 10.
- Power of 26 sequences use 12 algorithm iterations, with a total delay of 26.
- Two final additions are required to obtain the results.

This table reflects that the reduction obtained with respect to the efficient approach of [8] is four times in the number of additions and delays and almost six times in the products. With respect to the approach of [12], the proposed correlator needs half as many additions and delays and a three times less products that represents a reduction of 50% in the additions and delays and a 66% in the number of products. This significant reduction in the number of operations is of great relevance for the practical use of these sequences.

To do a further analysis, the proposed correlation algorithm was implemented in a Xilinx Spartan 6 FPGA using several lengths of sequences. The products were implemented using Look-up tables (LUTS). Their input width depends on the required number of input bits, with a maximum of 18 bits per input, and have a maximum of 36 bits of output. Each algorithm stage output is trimmed to a width of 18 bits. Table III lists the number of used slices and the maximum frequency obtained in those implementations.

From this table, there are results that are worthy to analyze. Correlations with sequences with $L = 10$ need more resources than $L = 16$ even though both lengths require the same number of stages. The cause of this is that one stage of $L = 10$ has a product with a constant different to ± 1, .5 that requires an additional bit in the multiplication. The same happens for sequences with $L = 20$ and $L = 40$. For $L = 26$ (and every product of 26), there are nine correlation stages with products with a constant

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different to ± 1, and many of them require the full 18 bits to have an acceptable error. This directly impacts in the maximum frequency obtained for that length, being 5.8 times slower than for a similar $2^N 10^M$ length.

8. APPLICATION IN A NOISY MULTI-USER SYSTEM

The previous section dealt with the mathematical aspects of the proposed architectures. This section provides a particular example of the use of binary sequences in multi-user systems and the usefulness of a simultaneous correlator of orthogonal pairs of complementary sequences. The system under study is composed of two remote units that asynchronously send an alarm signal to the central unit over the same physical medium. The alarm signal of unit 1 (U_0) is encoded with a binary complementary pair of sequences of length 40, while the alarm signal of unit 2 (U_2) is encoded with a binary complementary pair of sequences of the same length, orthogonal to the first one. Figure 5 depicts a block diagram of the system, where the blocks identified as generators are implemented with the architectures shown in Figure 3.

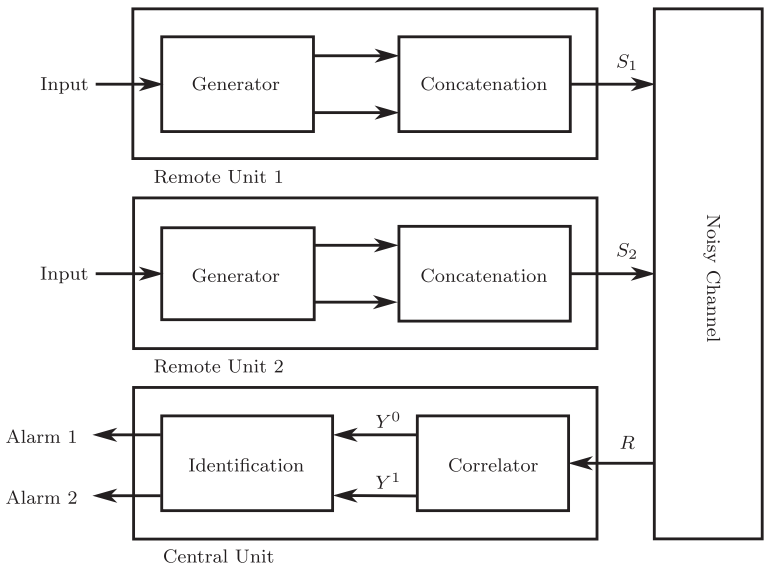

Figure 5. Example multi-user system operating with a multilevel complementary sequences' coding.

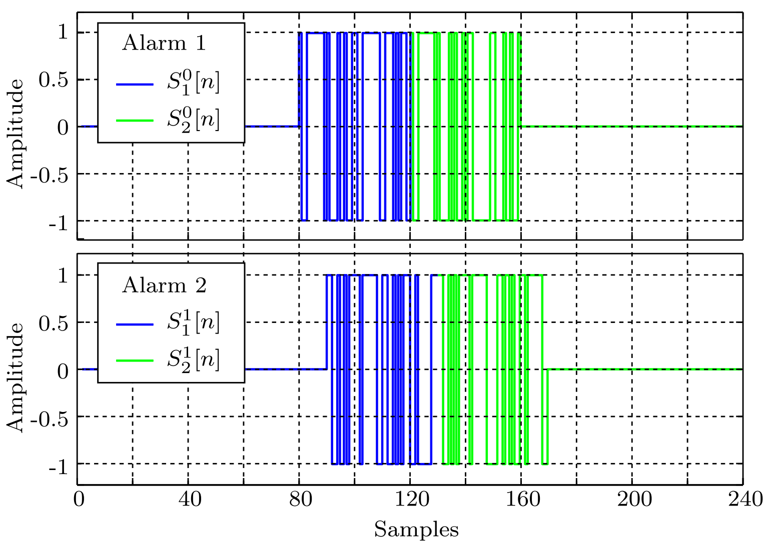

Figure 6. Both alarms without noise. [Colour figure can be viewed at wileyonlinelibrary.com]

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$$U_0 = [S_1^0, S_2^0]$$
$$U_1 = [S_1^1, S_2^1] \tag{53}$$

The signals are sent in baseband without modulation, sending the second sequence (S_2^j) after the first sequence of the pair (S_1^j). Assuming a situation in which the two remote units transmit an alarm with a delay of 10 samples between them (U_1 starts 10 samples after U_0). Figure 6 shows the alarm signals and how they are composed. Figure 7 illustrates the received signal in the central unit without the presence of noise, where it can be seen how the alarm signals overlap in time.

Considering a channel Signal to noise ratio (SNR) of 0dB, Figure 8 shows the received signal plus noise. From this figure, it can be seen that the transmitted signals are completely immersed in noise and cannot be easily discerned from it.

In the central unit, the proposed correlator is used to process the received signals. Figure 9 shows the processed signals. The first output is shown in blue and the second output in green. As it can be observed, the received alarms can now be easily identified overcoming not only the noise influence but also the superposition in time of the signals. All signal processing was carried out with the optimized architectures proposed in this work, reducing the amount of mathematical calculations, which is a key issue for the hardware implementation of the system.

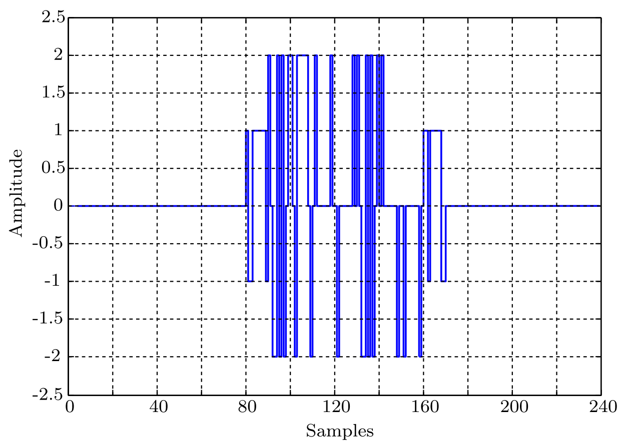

Figure 7. Both transmitted alarm signals overlapped in time without noise. [Colour figure can be viewed at wileyonlinelibrary.com]

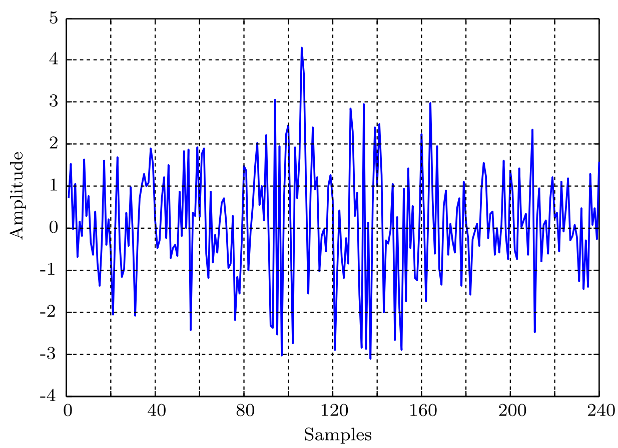

Figure 8. Signal at the input of the central unit (R) in the case of SNR=0dB. [Colour figure can be viewed at wileyonlinelibrary.com]

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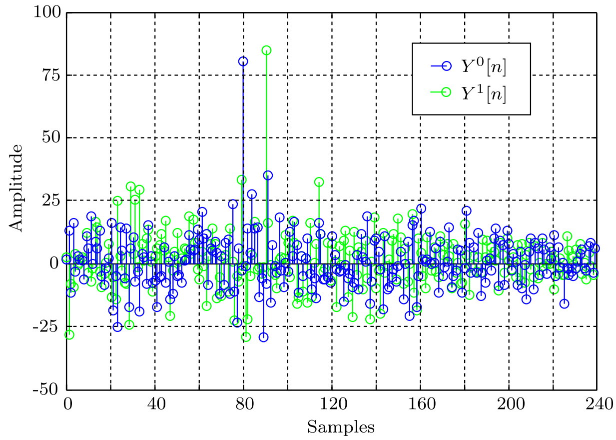

Figure 9. Processed signals after correlation. [Colour figure can be viewed at wileyonlinelibrary.com]

9. CONCLUSIONS

This paper presented a new approach for the generation of binary complementary pairs of sequences of lengths $2^N 10^M 26^P$, which enables the generation of orthogonal pairs by changing only the input of the generator. This work also presented a new approach for the correlation that simultaneously computes the correlation with respect to two orthogonal pairs of complementary sequences of lengths $2^N 10^M 26^P$. Given the fact that multilevel sequences' algorithms are used in the generation and correlation of these sequences, the generation and correlation of multilevel complementary sequences were also improved. The proposed algorithms were compared with the algorithms described in the literature, yielding a 33% reduction in the number of products required by the proposed generation algorithms, a 50% of the required additions and delays, and 66% of the products for the correlation algorithm, which is an important achievement for the practical implementation of these sequences.

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