Searching for criticality in nuclear fragmentation

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ABSTRACT: Experiments providing a better identification of particles produced in heavy ion collisions make now possible the search of critical phenomena in these reactions. In this work a molecular dynamics model (*Latino*) is used to study collisions of $Ni + Ni$ at different impact parameters and as a function of the beam energy. After identifying the participants using a kinematical prescription, we apply tests for criticality (Fisher's power law mass spectra, the normalized variance of the maximum fragment, and Campi's scatter plot) and conclude that critical phenomena has occurred in these collisions.

RESUMEN: Los experimentos con buena identificación de partículas producidas en choques de iones pesados, hacen ahora posible la búsqueda de fenómenos críticos en estas reacciones. En este trabajo se usa un modelo de dinámica molecular (*Latino*) para estudiar choques de $Ni+Ni$ a diferentes parámetros de impacto y en función de la energía del haz. Después de identificar a los participantes usando una regla cinemática, aplicamos pruebas de criticalidad (ley de Fisher, varianza del fragmento mayor, y la gráfica de Campi) y concluimos que han ocurrido fenómenos críticos en estas colisiones.

I. INTRODUCTION

Several experiments [1–5] have shown that achieving critical phenomena in heavy ion reactions at energies between $10 - 100\ MeV/A$ is indeed a real possibility. Although there are serious questions [6, 7] about this conclusion, these investigations appear to prove the existence of a phase transition by obtaining the nuclear caloric curve and the "conditional moments" [8, 9] from the experimental mass spectra.

These approaches, however, are based on an analysis of the final mass spectra produced in the reaction, and are limited by the experimental detection of particles and by a restrictive identification of central and peripheral events. Since the reaction dynamics plays an important role in the breakup occurring in these collisions [10–18], the further advance of this type of studies depends on the refining of the particle detection techniques. This is specially true for the separation of the participant nucleons from the spectators.

A key element to extract possible signals of critical phenomena is the determination of the amount of matter and energy participating in the collision, as well as the identification of the participating nucleons. These aspects are directly related to the centrality of the reaction and to the beam energy. While at low energies the excitation leads to some particle evaporation, at high energies and non-central impact parameters, the total number of nucleons is divided into participants and spectators. Once the participants are identified, tests for criticality can be applied.

The present work abounds on these type of studies by using a microscopic model, which allows the identification of the participant nucleons, and applies criticality tests *only to the participant nucleons.* To obtain illustrative results, the analysis is performed only in one specific reaction ($Ni + Ni$) at different impact parameters and as a function of the beam energy. Criticality tests stemming from Fisher's nucleation theory and percolation are used, and the results obtained appear to confirm the possibility of critical behavior in heavy ion reactions. Other proposed tests for criticality were tested (*eg.* Campi's scatter plot) and found to be less conclusive as Fisher's power law in identifying critical phenomena.

II. THE MODEL

To study the problem at hand, namely the finding of signatures of critical phenomena, a model capable of reproducing a phase transition is needed. However, the popular models used to study heavy-ion collisions at intermediate energies show difficulties in producing fragmentation. The Boltzmann-Uehling-Uhlenbeck (BUU) [19], for instance, has an *ab initio* impediment as it does not incorporate higher-order correlations. Likewise, Quantum Molecular Dynamics (QMD) [20] has never produced fragment patterns truly comparable with the asymptotic Configurational clusters. [Additional problems of QMD arise from their fragment recognition algorithm, SACA, which is based on ECRA, developed by one of the authors, but used incorrectly.]

To induce a phase transition, a model should include all-order nucleon-nucleon correlations and should be able to include all of the collision-induced correlations, as well as the nucleon dynamics at the proper kinetic energies (to reproduce the collision realistically). The only model that can describe changes of phase, hydrodynamic flow, and non-equilibrium dynamics, without adjustable parameters is molecular dynamics (MD).

In the present work, we use a MD model with a two-body potential developed to reproduce nucleon-nucleon cross sections realistically, based on the model developed by [21], with a fragment-recognition algorithm added. The model, nicknamed $LATINO$ was described recently [22]. In particular this model yields correct empirical energies and densities of nuclear matter with realistic effective scattering cross sections. For our analysis of finite nuclei, the Coulomb term is also included.

It is worth mentioning that the classical potential we use does not explicitly include a Pauli term in phase space. These terms, perhaps due to the time scales involved, tend to limit the applicability of the models up to about 100 MeV/nucleon [23]. For a complete description of the potential see reference [21] and for a proper understanding of the fragment-recognition algorithm see [24].

In this study, "nuclei" of different sizes were constructed and used as targets and projectiles while their trajectories of motion were calculated using the method of molecular dynamics with a standard Verlet algorithm ensuring an energy conservation of the order of 0.01%. These nuclei successfully reproduced the binding energy and radii of real nuclei and were found to be stable for times longer than the reaction time. In the collisions, the center-of-mass velocity of the projectile is boosted to the desired energy. Every time a collision is performed, the relative orientations of the projectile and target are randomly selected by rotations with random values of the corresponding Euler angles.

To compare to experiments, the information in terms of nucleons must be transformed into fragment information. To detect fragments at the times in which they are formed or emitted we use the Early Cluster Formation Model ($ECRA$) of [24]. Thanks to this method it is now known that the final asymptotic fragments are conceived very early in the evolution when the system is still dense in configurational space.

The sum of these two algorithms, MD and $ECRA$, allows us to extract valuable fragment information as a function of time. We now turn to a description of the analysis performed on these collisions.

III. SEARCH FOR CRITICALITY

Collisions were performed for the reaction $Ni + Ni$ at beam energies ranging from $E =$ 800 to 2000 MeV and impact parameters $b = 0$ and 3 fm, with 200 collision for each combination of energy and b. Upon identification, the fragments produced in each collision were used to analyze mass and velocity spectra, to produce Fisher's power law fits, study the normalized variance of the size of the maximum fragment (NVM) and Campi's scatter plots.

The tests for criticality we use, namely Fisher's power law and the behavior of NVM, can be perceived as being based on premise of equilibrium. Independent of whether this is true or not, here we apply them based on previous findings which indicate that *local* equilibrium is well defined during fragmentation time[25]. Other concerns related to the existence or lack of compression during breakup are irrelevant when microscopic models, such as MD, are used.

A Identification of participants

The complexity of heavy ion reactions turns the identification of the matter participating in a possible phase transition into an extreme sport. To begin with, the initial impact between the colliding nuclei has the potential of emitting nucleons which come out with a range of velocities not representative of the excitation energy that will be achieved by the rest of the system.

A second complication is the division of the system into spectators and participant. Although at central collisions all nucleons can be said to be participants of the reaction, this is not the case at larger impact parameters. At middle to large b's, part of the projectile does not get into direct contact with part of the target and remain relatively less excited than the overlapping sections.

This creates a three-region picture each of which absorbs varying amounts of energy and momentum and decay by different mechanisms. While the overlap region receives the maximum amount of energy, the other two pieces remain as spectators mildly excited. The hotter region can, thus, undergo a phase transition, while the spectators most likely evaporate lighter isotopes.

Figure 1 shows the correlation between fragment mass and their velocities (in units of c) in the beam direction for the fragments resulting in $b = 3$ fm collisions at (a) $E = 1000$ MeV, (b) $E = 1200$ MeV, (c) $E = 1500$ MeV, and

(d) $E = 2000\ MeV$. Clearly seen is the existence of a single velocity group which moves at the center of mass (*com*) velocity at low energies (*i.e.* at 1000 and 1200 MeV) which becomes a three-group distribution at higher energies.

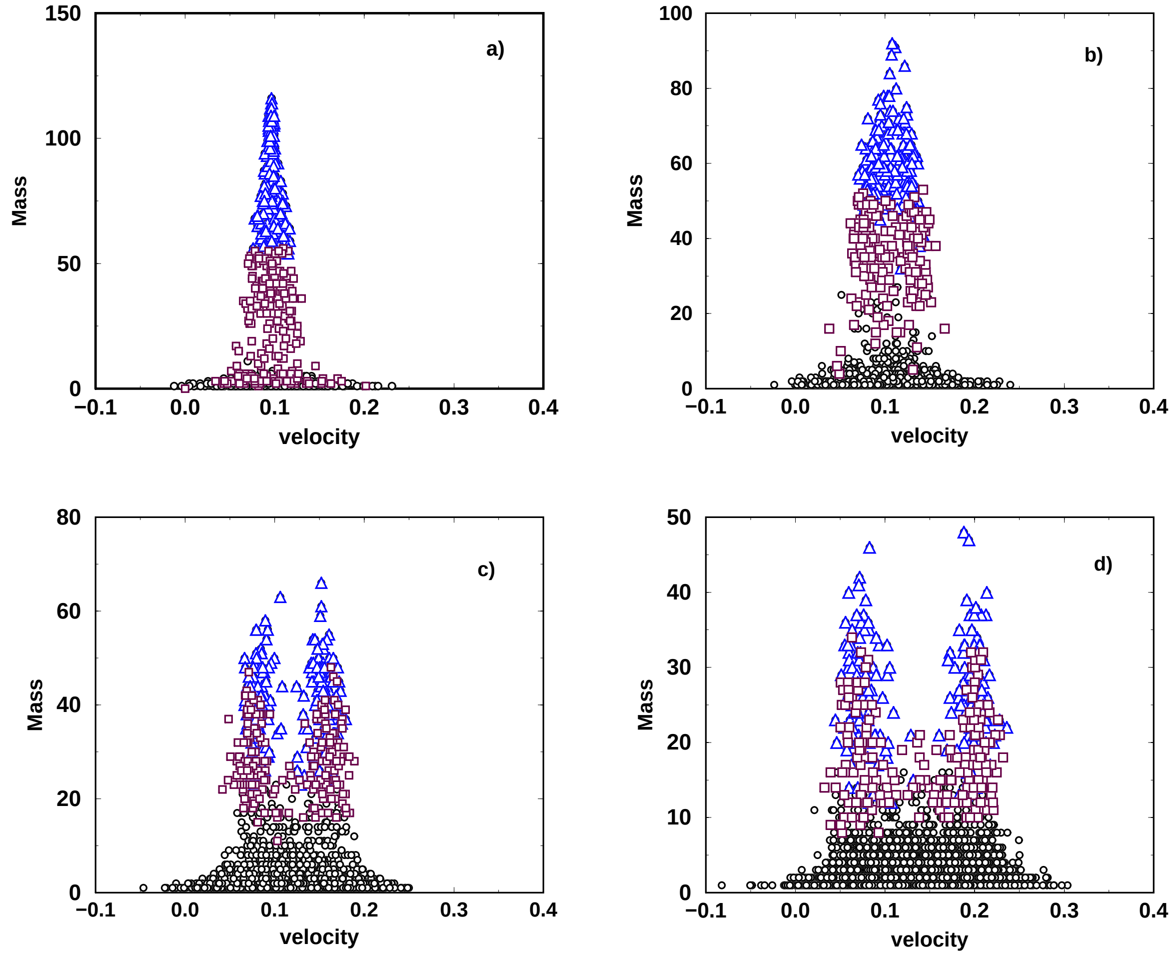

FIG. 1: Fragment velocities in the beam direction versus their mass. Obtained from the reaction $Ni+Ni$ at $b = 3\ fm$ and energies (a) $E = 1000\ MeV$, (b) $E = 1200\ MeV$, (c) $E = 1500\ MeV$, and (d) $E = 2000\ MeV$. Triangles and squares denote the largest and second largest fragments, and circles the rest of the mass sizes.

In the $E = 1500\ MeV$ plot, the groups that peak at the beam velocity ($0.16c$), at the center of mass velocity ($0.08c$) and at a slower velocity, correspond to the projectile spectators, participants, and target spectators, respectively. This is remarked by the fact that the heavy pieces (triangles and squares higher in the y axis) are around the *com* velocity at low energies and split into groups around the beam and the low velocities at higher energies. Figure 2 shows a plot similar to figure 1 for $b = 0\ fm$ and $E = 1000$ (circles) and $E = 1600\ MeV$ (triangles). Since at this centrality all nucleons participate, all of the resulting fragments come out with velocities around that of their *com* velocities.

Of course that this transition from a one-zone to a three-zone situation creates problems when trying to extract only those particles emitted by the *com* zone. In the end, one is presented with a superposition of particles produced by these sources, out of which one needs to extract a possible signature for criticality.

A possible way to separate the *com* particles is by looking at the mass spectra. Figure 3 shows the mass spectra obtained from these collisions at several energies and $b = 3\ fm$ (left) and $b = 0\ fm$ (right). Clearly seen in the left panel is the growth of the population of mid-size particles ($A = 20$ to 70) at large energies, while the spectra produced at 1000 MeV produces mostly light particles and a heavy residue.

Then, if target and projectile spectator zones get produced, one can assume that they get only mildly excited during the reaction and can be expected to decay mainly by light particle evaporation leaving a heavy piece behind. [Something similar to this effect is seen in the $E = 1000\ MeV$ (triangles) curve of the left panel in figure 3.] Thus one can think that the medium-size fragments are produced by the decay of the participant, or *com*, zone when the

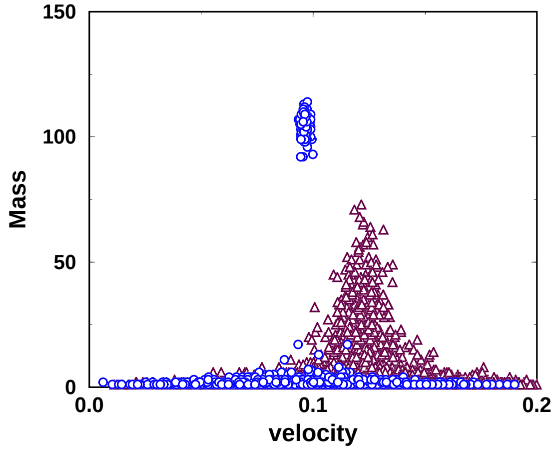

FIG. 2: Fragment velocities (in units of c) in the beam direction versus their mass. Obtained from the reaction $Ni + Ni$ at $b = 0\ fm$ and energies $E = 1000\ MeV$ (circles) and $E = 1600\ MeV$ (triangles).

excitation energy is large enough. These medium-size fragments are, thus, the particles that must be used in our search for criticality.

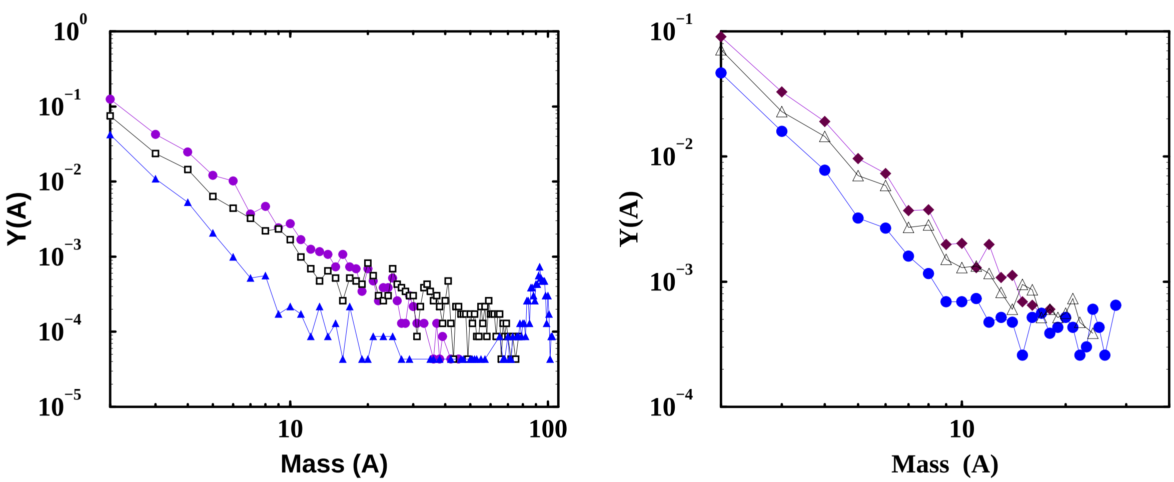

FIG. 3: (Left) Mass spectra from the reaction $Ni + Ni$ at $b = 3\ fm$ at $E = 1000\ MeV$ (triangles), $E = 1500\ MeV$ (squares), and $E = 2000\ MeV$ (circles). (Right) Similar results for $b = 0$ at $E = 1000\ MeV$ (circles), $E = 1500\ MeV$ (triangles), and $E = 2000\ MeV$ (diamonds).

B Power law behavior

Fluctuation theory [26, 27] states that for systems in thermodynamic equilibrium, the probability of having a drop in a vapor is given by $P_r(A) \propto e^{-\Delta G/T}$ where r, G, and T stand for the radius of the drop, the Gibbs free energy of the mixture, and the temperature. Now, depending on the "position" on the coexistence curve, $P_r(A)$ will adopt different functional forms. For instance, in the supersaturated region, $P_r(A)$ displays the U-shaped behavior:

$$P_r(A) = Y_0 A^{-\tau} e^{(\mu_g - \mu_l)A/T} e^{-\left(4\pi r_0^2 \sigma(T) A^{2/3}/T\right)} \tag{1}$$

where μ_g and μ_l are the chemical potentials of the gas and liquid phases, σ is the nuclear surface energy, r_0 is the drop radius, and Y_0 an overall normalization factor. In the coexistence region, where the equilibrium condition $(\mu_l - \mu_g) = 0$ is satisfied, the following power law decay plus an exponential fall-off dominating for large masses is found:

$$P_r(A) = Y_0 A^{-\tau} e^{-4\pi r_0^2 \sigma(T) A^{2/3}/T} \ . \tag{2}$$

And at the critical point, where once again $(\mu_l - \mu_g) = 0$ and $\sigma(T_c) = 0$ since the liquid and vapor are indistinguishable, the yield distribution is

$$P_r(A) = Y_0 A^{-\tau} \ , \tag{3}$$

which is a pure power law characterized by the "critical exponent" τ. It is important to mention that the appearance of a power law is not a sufficient condition but a necessary condition for the existence critical phenomena. It is under this scheme that we now proceed to analyze our results.

Using only masses in the range of $A = 2$ to 20 nucleons of figure 3 (*i.e.* eliminating the target and projectile residues and most of their evaporation products), a set of τs can be obtained. Under the light of the fluctuation theory presented before, pre- and post-critical events should result in "apparent" τ larger than the critical one due to the exponential factors. Only at the critical point the power τ can be considered as the critical exponent. Since the exponential factors tend to enlarge the yield, the minimum value of τ should correspond to the critical one.

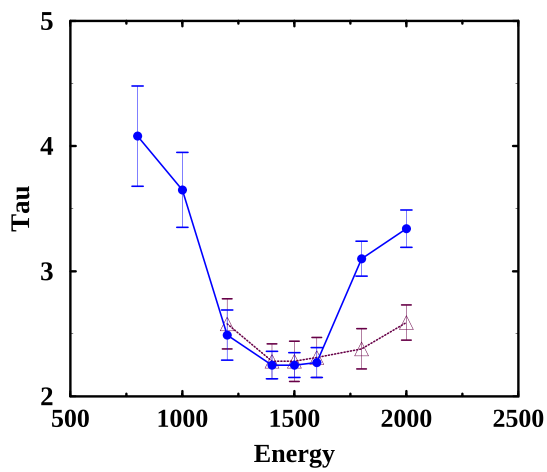

FIG. 4: Variation of the "apparent" exponent τ as a function of the beam energy obtained for the reaction $Ni + Ni$ at $b = 0$ (circles) and $b = 3\ fm$ (triangles).

Figure 4 shows the variation of the "apparent" exponent τ as a function of the beam energy for the cases $b = 0$ and $b = 3\ fm$. In both cases a minimum appears around $E = 1500\ MeV$. These minima can be taken as a signature of critical behavior and are in agreement to the value of 2.14 ± 0.06 obtained by the EOS collaboration [28]) for Au fragmentation.

C Normalized variance of the maximum fragment

A second piece of evidence for critical behavior can be obtained from the normalized variance of the size of the maximum fragment (NVM) [25]. This quantity is given by the standard deviation of the size of the maximum fragment normalized by the average value:

$$NVM = \frac{\sigma_{A_{\max}}}{< A_{\max} >}. \tag{4}$$

Percolation data suggests that NVM should peak at the critical point [29]. This measure of the fluctuations is quite robust against fluctuations introduced by the finite size of the system. Readers Interested in the connection between the normalized variance in the size of the maximum fragment and the power law parameter are referred to [25].

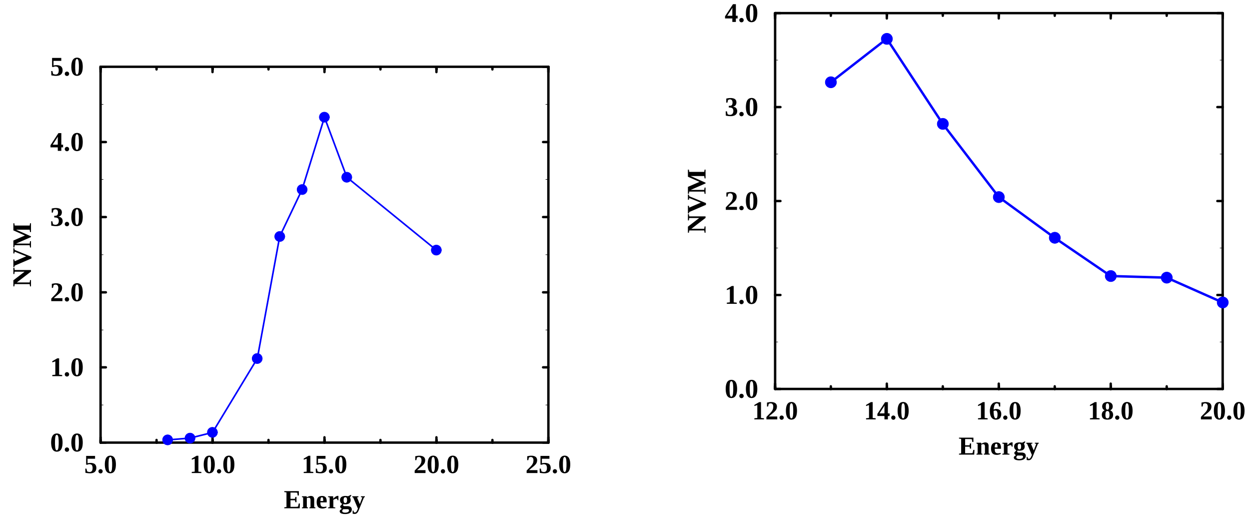

FIG. 5: Normalized variance of the maximum fragment size as a function of beam energy obtained in $Ni + Ni$ at $b = 0$ (left) and $b = 3\ fm$ (right).

Figure 5 shows the values of NVM obtained for our cases. Clearly seen are the peaks occurring at roughly the same energies at which the power laws exponents reach their minima, namely $E = 1400$ and $E = 1500\ MeV$. Taking into consideration the independence between the τ and NVM tests for criticality, these finding are extremely reassuring.

D Campi scatter plots

A common tool for identifying critical behavior is the Campi scatter plot which analyzes the final mass spectra using the so-called method of "conditional moments" [8, 9]. This technique has been used, for instance, by Belkacem *et al.* [4], Mastinu *et al.* [5] and others.

The Campi scatter plot is defined as the relationship between the $\ln(A_{Max})$ versus $\ln(m_2)$, where A_{Max} is the mass number of the largest asymptotic fragments, and the second moment is defined as $m_2 = \sum_A A^2 n(Z)/A_{Tot}$ where $n(A)$ is the multiplicity of fragments with a mass A, and the sum is over all fragments excluding A_{Max}. Typically these plots exhibit a "boomerang" shape and it is widely believed that the central part corresponds to critical events [9, 30].

One problem in the application of this method is, again, the required discrimination of the spectators. Here we use our velocity-inspired selection criterion and apply the technique to the data discussed before. Since the number of participants varies from collision to collision, instead of using the total mass of the system for A_{Tot}, we use a reduced mass eliminating the heaviest two fragments to avoid using those pieces that did not participate in the collision.

Figure 6 shows the scatter plots obtained for the case of $Ni + Ni$ at impact parameter $b = 0\ fm$ and energies in the range $E = 900$ to $2000\ MeV$ (left), and $b = 3\ fm$ and energies in the range $E = 1300$ to $2000\ MeV$ (right). Each point corresponds to one collision. [Notice that this superposition of events with beam energies is not realistic as real experiments occur always at single energies.] In the $b = 0$ case (left), it is easy to observe the existence of a trend where different parts of the Campi "boomerang" get populated at different energies.

Although the $b = 0$ plot in figure 6 resembles the percolation results, this is not the case for the $b = 3$ plot. In this case the boomerang shape is lost and its meaning is not clear at all. In view of the previous tests (Fisher's power law and NVM) which indicate the possible existence of critical phenomena, the validity of Campi's plot to establish criticality in reactions, thus, remain unsubstantiated.

IV. CONCLUSIONS AND PERSPECTIVES

The extraction of signatures of critical phenomena from the richer experiments is complicated by the entangling of geometrical aspects of the reaction, which depend on the impact parameter and the beam energy. The present molecular dynamics study of $Ni + Ni$ makes this problem apparent by studying the distribution of velocities of the fragments produced in the breakup of the colliding nuclei. The observed distributions reflect the existence of either one or three velocity groups, depending on the energy and impact parameter.

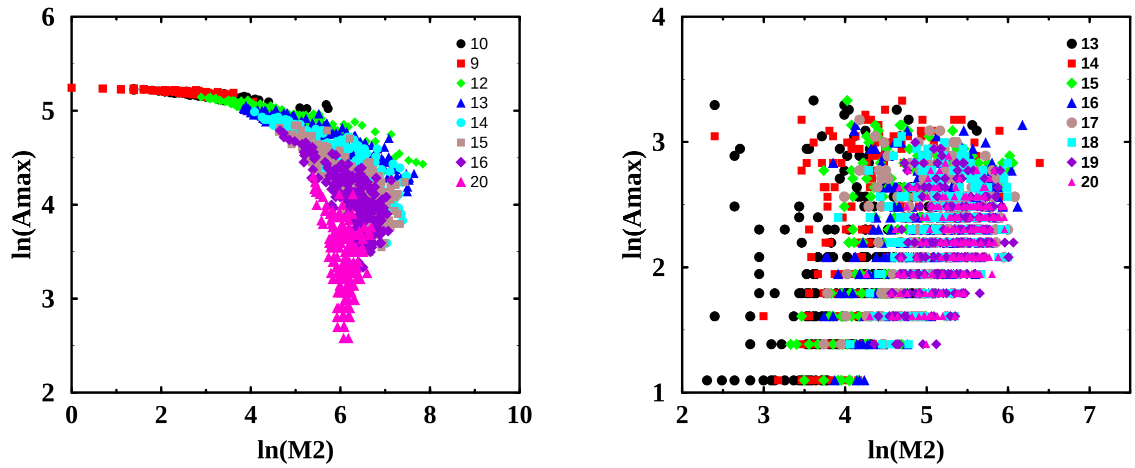

FIG. 6: (Left) Campi scatter plot obtained for $Ni + Ni$ at impact parameter $b = 0\ fm$ and energies in $E =$ 900-2000 MeV. (Right) similar plot for $b = 3\ fm$ and energies in $E =$ 1300-2000 MeV. The labels represent the energy in hundreds of MeV.

To search for signs of criticality, the mass spectra were reduced according to the distribution of velocities leaving only those particles believed to have been emitted from the participant zone. The mass distribution of these particles was then analyzed using Fisher's power law mass spectra, NVM, and Campi's scatter plot. The first two tools indicate that critical phenomena occurred in these collisions at specific energies, which vary slightly for different impact parameters. The Campi plot gave little information when used with the reduced data set. This study will be extended in the future to other reactions in which a more robust data reduction criterion will be applied [31].

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