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On the net radiation method for heat transfer

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Abstract

A simplifying approach for calculating the radiant energy is achieved by introducing the concept of net transmittance, resulting in a novel variation of the net radiation method that provides an easy way for solving a variety of situations. In particular, a closed form for the net radiation between two grey plates through a radiation shield formed by a series of partially transparent partially reflecting partially absorbing plates is found. In addition, the method is generalized to cylindrical and spherical geometries.

1. Introduction

Heat transport by radiation is not just a theoretical problem, since understanding and predicting the radiant energy becomes crucial in many practical situations. Our goal consists in showing how apparently intractable problems in heat transport by radiation can be easily solved using the concept of net transmittance. This subject is also pertinent to the design of multi-coverplate solar collectors. Indeed, in [1] the solar-radiation transmittance through a multi-plate planar window is calculated and a matrix-method derivation of the formulae is presented. In the present work, transmittance is obtained with a much simpler algebraic method. The present paper also deals with some complications regarding energy absorption and re-emission. Specifically, a closed form for the net radiation between two grey planes through a radiation shield formed of partially transparent partially reflecting partially absorbing plates is found.

The basic concepts related to heat transport by radiation are very well known. For an ideal grey surface the emitted thermal radiation leaving a surface, per unit time and unit area, is given by

$$S = \varepsilon\sigma T_s^4 \tag{1}$$

where ε is the emissivity, σ the Stefan–Boltzmann constant and T_s the absolute temperature. Most real surfaces exhibit a selective emission, in the sense that the emissivity is different for different wavelengths. In general ε can be a function of the wavelength and the surface temperature, i.e. $\varepsilon = \varepsilon(\lambda, T_s)$. A special type of non-black surface, called a grey body, is defined as one for which the emissivity is independent of the wavelength. For simplicity we will restrict our study to grey bodies. In addition, we will consider that emission is diffuse, so the intensity leaving a surface is independent of direction.

When radiation falls on a surface, part of it may be absorbed by the body, part may be reflected away from the surface, and part may be transmitted through the body. Accordingly we can define

absorptivity = fraction of radiation absorbed,

reflectivity = fraction of radiation reflected, and

transmissivity = fraction of radiation transmitted.

Obviously these three fractions must have sum equal to 1 and the absorptivity must be equal to the emissivity in steady state, a condition that will be assumed to hold in this work. We consider that bodies are thick in comparison with the wavelengths involved, and so it is not necessary to take into account wave interference effects.

These concepts provide sufficient background to determine the net radiant exchange between arbitrarily located surfaces of any shape in steady state. Although the approach presented here, known as the net radiation method, can be applied to any geometrical arrangement, we will restrict our discussion to the radiation exchange between opaque parallel infinite planes and show how to apply it to problems with spherical and cylindrical symmetry. We do not claim to be original since the net radiation method can be found in the literature [2]. It is our goal to present it in a didactic way and to show how to use it in a series of problems of increasing complexity. We will start by finding the radiant exchange between two black or grey opposed plates of null transmissivity to finally determine the effect of increasing complex shields. We will introduce the novel concept of net transmittance that simplifies calculations and solve some problems not found in the literature. In addition, the generalization to cylindrical and spherical geometries will be discussed.

2. Black and grey opposed planes

If two opposing planes are black, i.e. $\varepsilon_1 = \varepsilon_2 = 1$, the net exchange becomes

$$S_{12} = S_1 - S_2 = \sigma(T_1^4 - T_2^4). \tag{2}$$

This is simply solved because all the radiation emitted by each plane is absorbed by the other one. For grey surfaces this is not the case and the problem becomes more complex than might be anticipated.

Consider now the same case as above, except that the planes are grey instead of black. In figure 1 we follow the usual method of solving this problem by showing what happens with the radiation leaving one of the surfaces. The fact that the radiant flux of one of the planes is only partially absorbed by the second means that the rest is reflected. This radiation flux is again partially absorbed and reflected by the first plane. The process of absorption and reflection goes on an infinite number of times and to determine the net radiation flux we need to calculate the following sum

$$S_{12} = \varepsilon_1\varepsilon_2\sigma(T_1^4 - T_2^4)\sum_{i=1}^{\infty}[(1-\varepsilon_1)(1-\varepsilon_2)]^i, \tag{3}$$

which can be evaluated as

$$S_{12} = \frac{\varepsilon_1\varepsilon_2\sigma(T_1^4 - T_2^4)}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1\varepsilon_2}. \tag{4}$$

This series approach, known as the ray-tracing method, is regularly used in textbooks [3].

There exists a second way of obtaining equation (4) which circumvents the infinite sum. Consider S_1 and S_2 as the total radiant energy leaving planes 1 and 2, i.e. S_1 and S_2 now include the reflected radiation as well as the original emission. Then, the total radiant energies for each plane can be expressed as

$$S_1 = \varepsilon_1\sigma T_1^4 + (1 - \varepsilon_1)S_2 \tag{5}$$

$$S_2 = \varepsilon_2\sigma T_2^4 + (1 - \varepsilon_2)S_1. \tag{6}$$

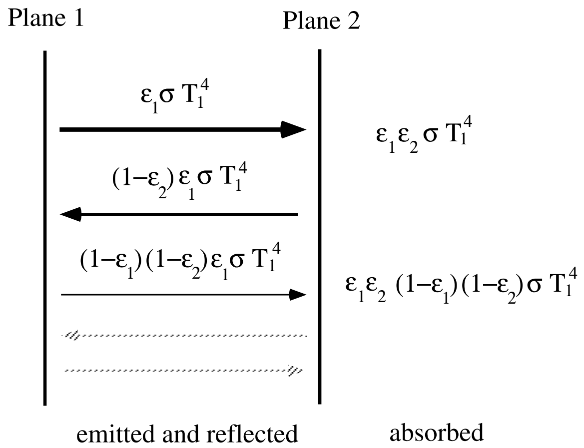

Figure 1. Schematic drawing showing how radiation is reflected and absorbed between two parallel infinite planes. In particular, it is shown what happens with the radiation leaving one plane (plane 1) after two reflections.

Equations (5) and (6) have two unknowns, S_1 and S_2, which can be easily evaluated to calculate the difference $S_1 - S_2$ representing the net radiation fluxes between the planes. In this way equation (4) is obtained without resorting to an infinite sum. We need to solve a geometrical series or a system of two equations with two unknowns. We believe that the second approach is simpler and more elegant, but this is a matter of preference. This net radiation method or *radiosity* approach is not new and can be found in some textbooks [4]; in particular, this method is discussed and exploited well by Seagel and Howell in [2]. However, in our work a system of two equations must be solved, whereas the analysis proposed in [2] leads to a system of four equations.

3. Radiation shields

At a first glance, the problem depicted in figure 2 does not look very challenging. We only have inserted a plate, known as a radiation shield, that can be partially transparent to radiation. This problem may be solved by an infinite series; however, the following approach is much simpler.

We define the net transmittance T as the fraction of the incident radiation that in the steady state reaches the other side of the shield due to the partial transparency of the body and the re-emission of the absorbed energy. (T will be calculated in the next section.) The net radiation flux between the surface at T_1 and the surface at T_2 can be calculated as depicted in figure 2, where S_i^+ is the net radiation flux leaving a plane i to the right and S_i^- is the net radiation flux sent back to the left; then

$$S_1^+ = \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_1) S_2^- \tag{7}$$

$$S_2^- = (1 - T) S_1^+ + T S_3^- \tag{8}$$

$$S_2^+ = T S_1^+ + (1 - T) S_3^- \tag{9}$$

$$S_3^- = \varepsilon_2 \sigma T_2^4 + (1 - \varepsilon_2) S_2^+. \tag{10}$$

The net radiation flux between planes 1 and 2, S_{12}, can be obtained by solving equations (7)–(10) as $S_2^+ - S_3^-$ or $S_1^+ - S_2^-$

$$S_{12} = \frac{\sigma (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 + 1/T - 2}. \tag{11}$$

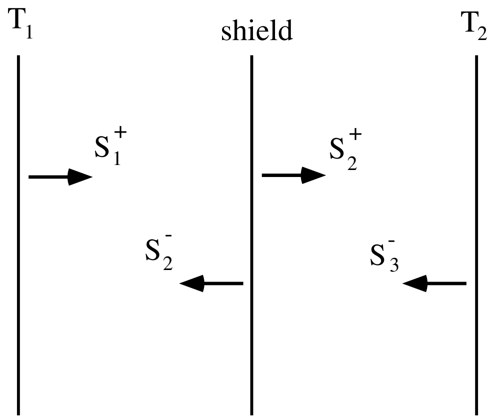

Figure 2. Radiant energy exchange between two grey parallel infinite planes with a third plane, the radiation shield, placed in between.

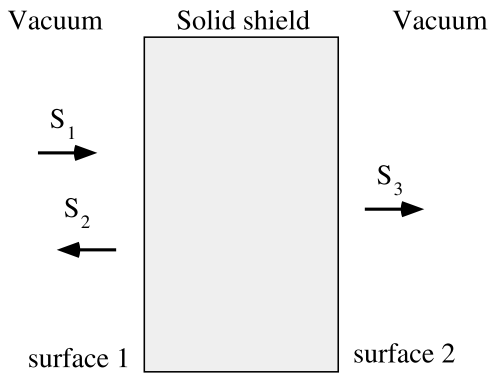

Figure 3. Scheme of radiant energy through an opaque solid shield.

This is the key equation to solve this kind of problem. Next, the net transmittance T must be calculated in each case and then equation (11) can be applied. Equivalent equations can be found in the literature. In [5], for example, the problem depicted in figure 2 is solved resorting to electrical analogies considering that the shield is opaque. We stress here that equation (11) can be applied to any type of shield, including shields formed by any number of plates that can also be partially transparent. To do so, the net transmittance T must be calculated.

4. The net transmittance

In this section we address the calculation of the net transmittance through radiation shields of increasing complexity. For a shield of null transmissivity and different emissivities on the front and back sides the net transmittance can be calculated with the aid of figure 3. We assume that there is an incident flux, S_1, from the left. S_2 and S_3 are the total radiant energies leaving the left and right surfaces of the shield, respectively. The incident flux, S_1, must be equal to the radiant energies leaving the shield, $S_2 + S_3$. If the temperature of the shield, T_s, is uniform we can write

$$S_1 = S_2 + S_3 \tag{12}$$

$$S_2 = (1 - \varepsilon_1)S_1 + \varepsilon_1 \sigma T_s^4 \tag{13}$$

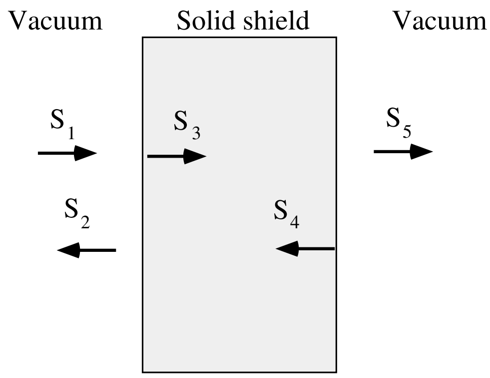

Figure 4. Scheme of radiant energy through a partially transparent solid shield.

$$S_3 = \varepsilon_2 \sigma T_s^4. \tag{14}$$

With equations (12)–(14) the net transmittance can be calculated to be

$$T = \frac{S_3}{S_1} = \varepsilon_1 \varepsilon_2 / (\varepsilon_1 + \varepsilon_2). \tag{15}$$

It should be noted that although the transmissivity may be zero the net transmittance T need not be null in general. For example, taking for simplicity $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $T = \varepsilon/2$ in equation (15), then for black surfaces the net transmittance is 50% meaning that half of the incident flux is sent forward even though we are dealing with an opaque body. With this novel idea of including the re-emission in a transmittance coefficient, investigating the effect of inserting radiation shields is greatly simplified, as seen in what follows.

The situation for a partially transparent shield is depicted in figure 4. An incident flux, S_1, from the left is assumed. We will also assume that a fraction r of the energy incident upon each vacuum-solid interface is reflected. Finally, we will assume that from the energy radiated through the material, only a fraction t reaches the second interface. Then, by disregarding the heat re-emitted by the shield we can write

$$S_2 = S_1 r + S_4 t(1 - r) \tag{16}$$

$$S_3 = S_1(1 - r) + S_4 tr \tag{17}$$

$$S_4 = S_3 tr \tag{18}$$

$$S_5 = S_3 t(1 - r). \tag{19}$$

S_2 is the total radiant energy leaving surface 1. S_3 is the total radiant energy from left to right at the left end of the shield. S_4 is the total radiant energy from right to left at the right end of the shield. S_5 is the total radiant energy that leaves surface 2. For example, S_2 includes the reflected fraction r of S_1 and the fraction of S_4 that reaches surface 1 after being transmitted through the shield and that is not reflected in surface 1. The total energy fraction transmitted through the solid can be obtained from equations (16)–(19) and then the transmissivity can be calculated to be

$$t_N = \frac{S_5}{S_1} = \frac{t(1 - r)^2}{(1 - r^2 t^2)}. \tag{20}$$

The total fraction that is absorbed is

$$a_N = \frac{(1 - t)(S_3 + S_4)}{S_1} = \frac{(1 - r)(1 - t)}{(1 - rt)}. \tag{21}$$

In a steady state, the absorbed energy is re-emitted. Assuming that the temperature in the shield is uniform, the emission is the same in both directions. Finally, the net fraction of radiant energy sent forward by the radiation shield is given by

$$T = t_N + \frac{a_N}{2} = \frac{(1-r)(t+1)}{2(1+rt)}. \tag{22}$$

If the reflected fractions of the incident energies upon each vacuum–solid interface are different, say r_1 and r_2, equations (16)–(19) can be generalized to

$$S_2 = S_1 r_1 + S_4 t(1-r_1) \tag{23}$$

$$S_3 = S_1(1-r_1) + S_4 t r_1 \tag{24}$$

$$S_4 = S_3 t r_2 \tag{25}$$

$$S_5 = S_3 t(1-r_2). \tag{26}$$

Hence, the transmissivity can be calculated to be

$$t_N = \frac{S_5}{S_1} = \frac{t(1-r_1)(1-r_2)}{(1-r_1 r_2 t^2)}. \tag{27}$$

The total fraction that is absorbed is now

$$a_N = \frac{(1-t)(S_3+S_4)}{S_1} = \frac{(1-r_1)(1-t)(1+tr_2)}{(1-r_1 r_2 t^2)}, \tag{28}$$

and the net fraction of radiant energy sent forward by the radiation shield is given by

$$T = \frac{(1-r_1)(1+t)(1-r_2 t)}{2(1-r_1 r_2 t^2)}. \tag{29}$$

Now, with these results, equation (11) can be directly used and the radiation heat transfer determined.

Finally, we will apply the present approach to a problem that would be intractable using the ray-tracing method. A similar problem is solved in [6]; however, re-emission is not included and a final closed form is not offered. A specific application to thermal conductivity of plastic foams can be found in [7]. The situation is depicted in figure 5. S_i^+ is the total radiation flux that leaves plane i to the right and S_i^- the total radiation flux sent back to the left. Then

$$S_i^+ = TS_{i-1}^+ + (1-T)S_{i+1}^-, \tag{30}$$

$$S_{i+1}^- = (1-T)S_i^+ + TS_{i+2}^-, \tag{31}$$

$$S_{i+1}^+ = TS_i^+ + (1-T)S_{i+2}^-. \tag{32}$$

From equations (30)–(32) the following recurrent relation can be obtained

$$\frac{S_i^+}{S_{i-1}^+} = \frac{1}{2 - S_{i+1}^+/S_i^+}. \tag{33}$$

This equation is valid for $i < n$, for the last layer we can write

$$\frac{S_n^+}{S_{n-1}^+} = T. \tag{34}$$

With equations (33) and (34) the ratio between the total fluxes to the right can be figured out

$$\frac{S_{n-1}^+}{S_{n-2}^+} = \frac{1}{2-T} \tag{35}$$

$$\frac{S_{n-2}^+}{S_{n-3}^+} = \frac{2-T}{3-2T} \tag{36}$$

$$\frac{S_{n-3}^+}{S_{n-4}^+} = \frac{3-2T}{4-3T} \tag{37}$$

$$\frac{S_1^+}{S_0^+} = \frac{(n-1)-(n-2)T}{n-(n-1)T}. \tag{38}$$

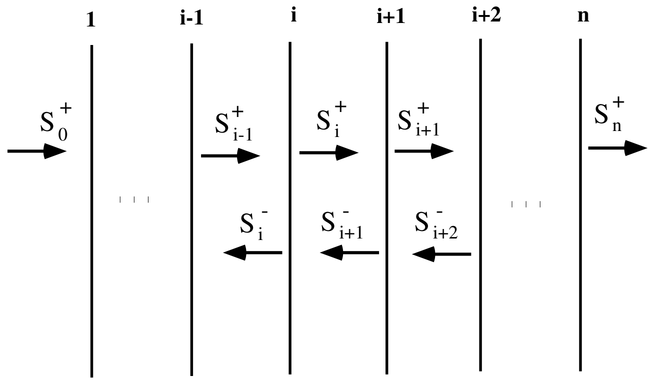

Figure 5. Scheme of radiant energy through a set of n solid layers.

From the radiation impinging on the first layer, S_0^+, S_n^+ will pass through the whole set of layers. With the ratios found between total fluxes, equations (35)–(38), the net transmissivity is

$$T_{set} = \frac{S_n^+}{S_0^+} = \frac{1}{1 + n\left(\frac{1}{T} - 1\right)}. \tag{39}$$

Now the net radiation between two grey planes with a set of plates in between can be calculated by means of equation (11). This final result is surprisingly simple but, to our knowledge, it cannot be found even in the specialized literature.

5. Concentric spheres and co-axial cylinders

This situation is more complicated than the parallel geometry since surfaces have unequal areas and larger surfaces can partially intercept their own radiation. The determination of radiation exchange is facilitated by the introduction of the view factor. The fraction of the total radiation emitted by surface 1 that is intercepted by surface 2 is given by the view factor F_{12}. It can be shown that $A_1 F_{12} = A_2 F_{21}$, which is called the reciprocity relation (a direct derivation can be found in [5]). Figure 6 is a scheme of two concentric spheres or co-axial cylinders. The inner sphere or cylinder with surface A_1 is a convex surface, it does not intercept its own radiation and hence $F_{11} = 0$. All the radiation emitted by surface 1 is intercepted by surface 2, then $F_{12} = 1$. Now, taking into account the reciprocity relation it is found that $F_{21} = A_1/A_2$, which is the fraction of the radiation originated in surface 2 that intercepts surface 1. The radiation originated in surface 2 not intercepted by surface 1 must be intercepted by surface 2 itself, hence $F_{22} = 1 - A_1/A_2$.

We will first apply the net radiation method as done in deriving equations (5) and (6). Since the areas are now different we will deal with the radiant energy per unit time. Having in mind figure 6, that shows two concentric spheres or cylinders, the total radiant energies can be expressed as

$$Q_1 = A_1 \varepsilon_1 \sigma T_1^4 + (1 - \varepsilon_1)(A_1/A_2) Q_2 \tag{40}$$

$$Q_2 = A_2 \varepsilon_2 \sigma T_2^4 + (1 - \varepsilon_2) Q_1 + (1 - \varepsilon_2)\left(1 - \frac{A_1}{A_2}\right) Q_2. \tag{41}$$

The second term in equation (40) is the radiant energy leaving surface 2 that is reflected in surface 1. The second term in equation (41) is the radiant energy leaving surface 1 reflected in surface 2 and the third term corresponds to the radiant energy leaving surface 2 intercepted

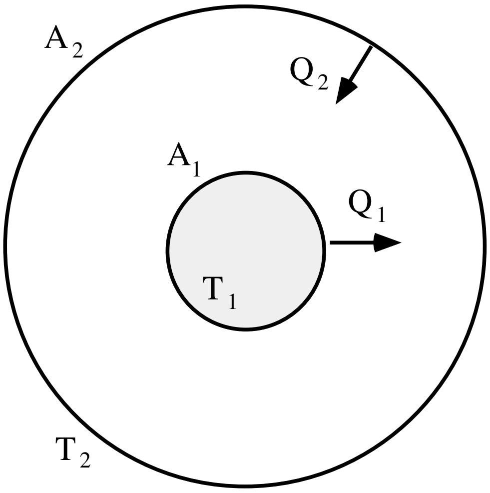

Figure 6. Radiant energy exchange between two concentric spheres or co-axial cylinders.

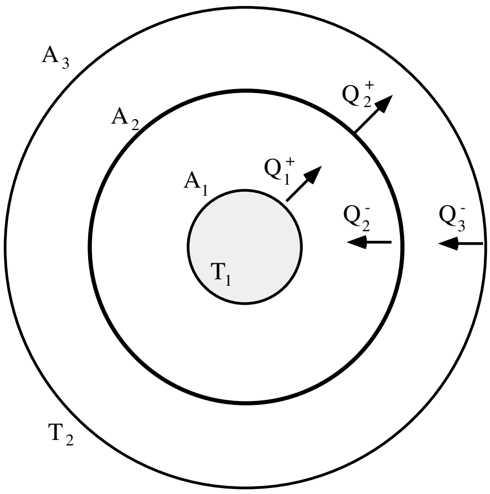

Figure 7. Radiant energy exchange between two concentric spheres or co-axial cylinders with a radiation shield placed in between.

by itself and then reflected. Equations (40) and (41) can be solved for the two unknowns Q_1 and Q_2. The net heat transfer is given by the difference $Q_1 - (A_1/A_2)Q_2$

$$Q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + (A_1/A_2)(1/\varepsilon_2 - 1)}. \tag{42}$$

Equation (42) is derived in [2] using a similar approach but with a system of four equations.

In figure 7 we show two concentric spheres or cylinders with a radiation shield inserted in between. Taking into account the view factors, the relevant equations are

$$Q_1^+ = A_1\varepsilon_1\sigma T_1^4 + (1 - \varepsilon_1)(A_1/A_2)Q_2^- \tag{43}$$

$$Q_2^- = T(A_2/A_3)Q_3^- + (1 - T)Q_1^+ + (1 - T)(1 - A_1/A_2)Q_2^- \tag{44}$$

$$Q_2^+ = (1 - T)(A_2/A_3)Q_3^- + TQ_1^+ + T(1 - A_1/A_2)Q_2^- \tag{45}$$

$$Q_3^- = A_3\varepsilon_2\sigma T_2^4 + (1-\varepsilon_2)Q_2^+ + (1-\varepsilon_2)(1-A_2/A_3)Q_3^-. \tag{46}$$

These equations can be solved for the four unknowns Q_1^+, Q_2^+, Q_2^-, and Q_3^-. The net heat transfer is given by the difference $Q_1^+ - (A_1/A_2)Q_2^-$ or $Q_2^+ - (A_2/A_3)Q_3^-$,

$$Q_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + (A_1/A_3)/\varepsilon_2 + (A_1/A_2)/T - (A_1/A_2 + A_1/A_3)}. \tag{47}$$

As with equation (11) for the parallel-plate geometry, equation (47) is the key equation for solving problems with spherical and cylindrical symmetry. To our knowledge, this generalization is not found in the literature.

6. Conclusions

An easy and systematic procedure for calculating the net radiation between parallel infinite planes was presented. The net radiation method can be readily applied to obtain an expression for the net transmittance of a shield. In particular, the problem containing a series of partially transparent shields between two parallel infinite planes illustrates the procedure and yields a closed form that is not found in the literature. As described in section 5, the procedure can be readily extended to cylindrical and spherical geometries.

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