# Scattering of light from metamaterial gratings with finite length

Vivian Grünhut, Mauro Cuevas, and Ricardo A. Depine*

Grupo de Electromagnetismo Aplicado, Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria Pabellón I, C1428EHA; Buenos Aires, Argentina

*Corresponding author: rdep@df.uba.ar

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Using an integral equation approach based on the Rayleigh hypothesis, we investigate the scattering of a plane wave at the rough surface of a metamaterial with a finite number of sinusoidal grooves. To show the adequacy of the model, we present results that are in agreement with the predictions of physical optics and that quantitatively reproduce the polarization and angular dependences predicted by the C-formalism for metamaterial gratings with an infinite number of grooves. © 2012 Optical Society of America

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## 1. Introduction

Among the unique magnetic properties of metamaterials, the one that has perhaps attracted more attention from the scientific community is the possibility of having negative refractive index. Many well-known phenomena associated with the interaction between electromagnetic radiation and material surfaces, such as the Doppler effect, the law of refraction, or the Cerenkov radiation, change dramatically when the relative refractive index of the surface changes sign, even in the geometrically simple case of flat geometries [1].

In the ideal case of a lossless isotropic metamaterial, a negative refractive index occurs when there is a range of frequencies over which both the electric permittivity and the magnetic permeability are simultaneously negative. Therefore, to investigate theoretically the new features that the seemingly simple change of sign of the refractive index produces in the scattering properties of a nonflat surface, existent scattering formalisms that usually have been developed for the conventional optical case of nonmagnetic (magnetic permeability equal to one) materials, need to be extended to magnetic media with negative permeability. A short account of these extensions for different surface shapes can be found in [2].

Restricting ourselves to the paradigmatic case of two isotropic half-spaces separated by a nonperiodic rough surface, this kind of extension has been performed, very recently and almost simultaneously, for two different scattering formalisms in [2] and [3]. The extension presented in [3] is based on Green's theorem surface integral equations [4] and leads to the numerical resolution of a system of integral equations, whereas the extension presented in [2] is based on the Rayleigh hypothesis [5] and leads to two methods of resolution: a direct numerical method and a perturbative method, valid when the height of the corrugation is small compared to the wavelength of the incident radiation.

Because of the lack of experimental data in the related literature, the validity of the results obtained with the magnetic extensions presented in [2] and [3] has been tested in examples with different geometries by verifying the fulfillment of theoretical criteria. In [3], the scattering from random gratings with subwavelength roughness is considered and the angular distributions of intensities for the reflected fields are shown to exhibit the symmetries that the phenomenon of backscattering enhancement, due to

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surface plasmon polaritons, is expected to fulfill. In [2], on the other hand, the scattering from a surface with a single sinusoidal protuberance is considered, very good agreement is obtained between the results predicted by the direct and the perturbative methods, the results are shown to fulfill the power conservation criterion, and the main features of the diffraction patterns are shown to coincide with the physical optics predictions.

Although the fulfillment of expected criteria can provide confidence on the validity of the results obtained for magnetic media with negative permeability in the geometries considered in [2] and [3], it seems quite natural and desirable to quantitatively compare the results obtained with the scattering formalisms developed for nonperiodic surfaces with those obtained for perfectly periodic surfaces, i.e., metamaterial diffraction gratings [6–8]. The comparison can be done by considering a grating with a finite number of grooves, in other words, a surface that is mostly plane but for a limited area that is periodically corrugated. These kinds of structures have been considered extensively in the case of nonmagnetic media [9–14], not only because of their varied and important applications but also because they have provided quantitative tests [10–12,14] for a diverse variety of theories of scattering at random rough surfaces. The purpose of the present paper is to perform similar quantitative tests for the theoretical formalism presented in [2]. To do so, we use this formalism to obtain the angular distribution of power scattered from metamaterial surfaces with a finite number of sinusoidal grooves and compare the results with those obtained with our numerical implementation of the formalism presented in [8] for perfectly periodic diffraction gratings of arbitrary permittivity and permeability. The perfectly periodic grating formalism is an extension to metamaterials of the very well known C-method (see [15] and references therein). Originally developed for gratings of conventional (nonmagnetic) materials [16], the most distinctive feature of this formalism is its virtually uniform convergence, regardless of the incident polarization state and the permittivity of the refracting material [17]. As shown in [8], these features are valid even for diffraction gratings with a negative refractive index. Our examples show that the angular distribution of power obtained with the scattering formalism [2] applied to metamaterial gratings with finite length are in good agreement with the diffracted efficiencies obtained with the C-method for perfectly periodic gratings and that the infinite-grating predictions are obtained with a relatively small number of grooves. These features, already observed for conventional gratings [10–12,14], provide new evidence on the validity of the Rayleigh methods for metamaterial rough surfaces.

The plan of the paper is as follows. In Section 2 we give a brief description of the boundary value problem for the scattering of a plane wave at a locally periodic metamaterial surface and outline the method used for the calculation of the scattered fields. The comparison between the numerical results obtained for finite [2] and infinite gratings [8] is presented in Section 3 and finally, concluding remarks are given in Section 4. We use a time dependence of the type $\exp(-i\omega t)$ where $\omega$ is the angular frequency of the incident radiation, $t$ the time, and $i = \sqrt{-1}$.

## 2. Theory

We consider a rough surface represented by the function $y = g(x)$ [see Fig. 1]. This surface separates two homogeneous and isotropic materials characterized by the constitutive parameters $\epsilon_i$ (electric permittivity) and $\mu_i$ (magnetic permeability), $i = 1, 2$. Medium 1 ($y > g(x)$, medium of incidence, usually air) is a conventional material with positive refractive index $\nu_1 = \sqrt{\epsilon_1\mu_1}$, $\epsilon_1 > 0$, $\mu_1 > 0$, while medium 2($y < g(x)$) is a metamaterial with frequency-dependent constitutive parameters $\epsilon_2 = \epsilon_{2R} + i\epsilon_{2I}$ and $\mu_2 = \mu_{2R} + i\mu_{2I}$, real parts $\epsilon_{2R}$ and $\mu_{2R}$ of arbitrary sign and positive imaginary parts $\epsilon_{2I} > 0$ and $\mu_{2I} > 0$. The conditions for the refractive index of the metamaterial $\nu_2 = \pm\sqrt{\epsilon_2\mu_2}$ being positive or negative are given in [18].

The rough surface is illuminated by an electromagnetic, linearly polarized plane wave that propagates along the $(x, y)$ plane (incidence plane) and forms an angle $\theta_0$ with the $y$ axis. We analyze two independent polarization cases separately: $s$ (electric field in the $z$ direction) and $p$ (magnetic field in the $z$ direction) polarization. In both cases, the scattered fields conserve the polarization of the incident wave.

We denote by $\Psi(x, y)$ the $z$-directed component either of the total electric field ($s$ polarization) or the total magnetic field ($p$ polarization). Outside the corrugated region ($\min g(x) \le y \le \max g(x)$), $\Psi(x, y)$ can be rigorously represented by superpositions of plane waves. If $y > \max g(x)$,

$$\Psi_1(x, y) = \exp[i(\alpha_0 x - \beta_0^{(1)} y)] + \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(\alpha)\,\exp[i(\alpha x + \beta_\alpha^{(1)} y)]\mathrm{d}\alpha \qquad (1)$$

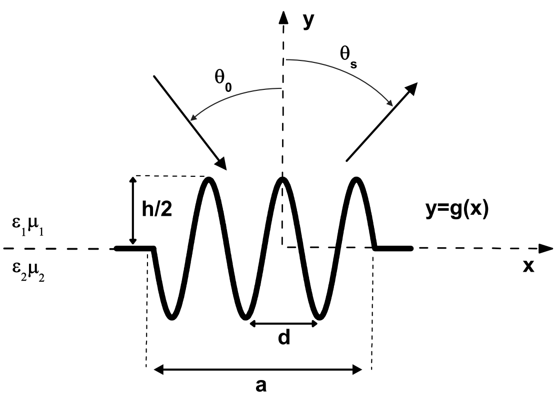


Fig. 1. Scattering of a plane wave from a metamaterial surface with a finite number of equally spaced identical grooves. $\theta_0$ is the angle of incidence and $\theta_s$ is the observation angle.

represents the incident plane wave (first term, with unit amplitude) and the scattered fields in medium 1 (second term, reflected field), while if $y < \min g(x)$,

$$\Psi_2(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} T(\alpha)\,\exp[i(\alpha x - \beta_\alpha^{(2)} y)]\mathrm{d}\alpha \qquad (2)$$

represents the scattered fields in medium 2 (transmitted field). The quantity $\alpha_0 = k_0\nu_1 \sin\theta_0$, $k_0 = \omega/c$, represents the $x$ component of the incident wave vector. Note that the integrand in (1) represents a plane wave with amplitude $R(\alpha)$ and wave vector $\vec{k}^{(1r)}(\alpha) = \alpha\hat{x} + \beta_\alpha^{(1)}\hat{y}$, while the integrand in (2) represents a plane wave with amplitude $T(\alpha)$ and wave vector $\vec{k}^{(2t)}(\alpha) = \alpha\hat{x} - \beta_\alpha^{(2)}\hat{y}$. The components along $y$ of the wave vectors $\vec{k}^{(1r)}$ and $\vec{k}^{(2t)}$ are

$$\beta_\alpha^{(j)} = \beta^{(j)}(\alpha) = (k_0^2\epsilon_j\mu_j - \alpha^2)^{1/2}, \qquad j = 1, 2, \qquad (3)$$

and we define $\beta_0^{(j)} = \beta^{(j)}(\alpha_0)$. Also, note that the quantities $\beta_\alpha^{(1)}$ are real or purely imaginary. In the first case, which occurs in the so-called radiative zone $|\alpha/k_0| < \nu_1$, we must fulfill the condition $\mathrm{Re}\,\beta_\alpha^{(1)} \geq 0$, so that the fields in Eq. (1) represent propagating plane waves that move away from the surface into the half-space $y > g(x)$. In the second case, which occurs in the so-called non radiative zone $|\alpha/k_0| \geq \nu_1$, we must fulfill the condition $\mathrm{Im}\,\beta_\alpha^{(1)} \geq 0$, so that these fields represent evanescent waves that attenuate for $y \to +\infty$. Taking into account that in this paper we are considering real (lossy, $\mathrm{Im}\,\epsilon_2 > 0$, $\mathrm{Im}\,\mu_2 > 0$) metamaterials, the quantities $\beta_\alpha^{(2)}$ are always complex and, in order that the fields in Eq. (2) attenuate for $y \to -\infty$, their nonzero imaginary part must fulfill the condition $\mathrm{Im}\,\beta_\alpha^{(2)} > 0$.

The Rayleigh methods presented in [2] are based on the assumption that the equations (1) and (2) can be used to satisfy the boundary conditions at $y = g(x)$ (Rayleigh hypothesis [5]). After some manipulations, the problem can be reduced to the solution of a Fredholm integral equation of the first kind, with the complex amplitudes $R(\alpha)$ as the unknown function (see [2] for details).

From the definition of the outgoing, time-averaged scattered flux it is easy to show that the fraction of the incident power that is scattered (reflected) into the incident medium is given by

$$P_r = \frac{\mathrm{Re}}{2\pi a}\int_{-\infty}^{+\infty} \frac{\beta_\alpha^{(1)}}{\beta_0^{(1)}}|R(\alpha)|^2\mathrm{d}\alpha\,, \qquad (4)$$

which shows that only the values of $R(\alpha)$ in the radiative zone, $|\alpha/k_0| < \nu_1$, contribute to the scattered power. In this spectral zone the integrand in Eq. (1) represents plane waves propagating away from the surface along a direction that forms an angle $\theta_s$, ($|\theta_s| < \pi/2$, see Fig. 1) with the $+y$ axis. Thus, the integrand in Eq. (1) gives the normalized angular distribution of power scattered into medium 1. To better visualize the finite-length effect, in the examples below we redefine $R(\alpha) = \tilde{R}(\alpha) + R^{(0)}\delta(\alpha - \alpha_0)$, with $R^{(0)}$ the Fresnel coefficient of the infinite plane and $\delta()$ the Dirac delta distribution, and we plot the quantity

$$\frac{dP}{d\alpha} = \frac{\mathrm{Re}}{2\pi a}\frac{\beta_\alpha^{(1)}}{\beta_0^{(1)}}|\tilde{R}(\alpha)|^2, \qquad (5)$$

which represents the contribution of the limited area $a$ where the surface is periodically corrugated.

The C-method [8,15–17] for perfectly periodic diffraction gratings avoids the use of the Rayleigh hypothesis and is based in the introduction of a new coordinate system that not only maps corrugated grating surfaces to planar surfaces, making the matching of boundary conditions easy, but also transforms Maxwells equations into a matrix eigenvalue problem and thus makes the numerical solution of the grating problem straightforward [15]. Traditionally [16,17], Maxwells equations written in covariant form and tensor theory have been used to formulate the C-method. For details, we refer the interested reader to [15], where the C-method has been reformulated without using any tensor notation or tensor concepts.

## 3. Results

In order to compare the scattering formalism for nonperiodic rough surfaces with the C-method for perfectly periodic gratings, we consider finite sinusoidal gratings with a finite number $N$ of grooves, represented by a roughness function $g(x) = \frac{h}{2}\sin(\frac{2\pi}{d}x)\mathrm{rec}(x/a)$, where $h$ is the groove height, $d$ is the period of the grating, $a = Nd$, and $\mathrm{rect}(u)$ is the rectangular function centered at the origin with unit width and height. The medium of incidence is vacuum ($\epsilon_1 = 1$, $\mu_1 = 1$) and for the magnetic metamaterial we have chosen constitutive parameters $\epsilon_2 = -4 + 0.001i$ and $\mu_2 = -1.5 + 0.001i$, that is, a lossy metamaterial with negative refractive index [18] $\nu_2 = -2.449 + 0.001i$.

To show the evolution of the results from the locally periodic to the infinitely periodic case, in Fig. 2 we have plotted the angular distribution of power scattered into medium 1 [Eq. (5)] for similar sinusoidal gratings ($h/\lambda = 0.02$, $d/\lambda = 2$) with an increasing number of grooves ($N = 3$, 9 and 15), illuminated by a plane wave at an angle of incidence $\theta_0 = 20°$. The results in the left-hand column correspond to $s$-polarized incident waves, whereas the results in the right-hand column correspond to $p$-polarized incident waves. We observe that all the scattering patterns shown in this figure exhibit maxima at the angular positions $\theta_s$ predicted by the grating equation,

$$\sin\theta_s = n\frac{\lambda}{d} + \sin\theta_0\,, \qquad (6)$$

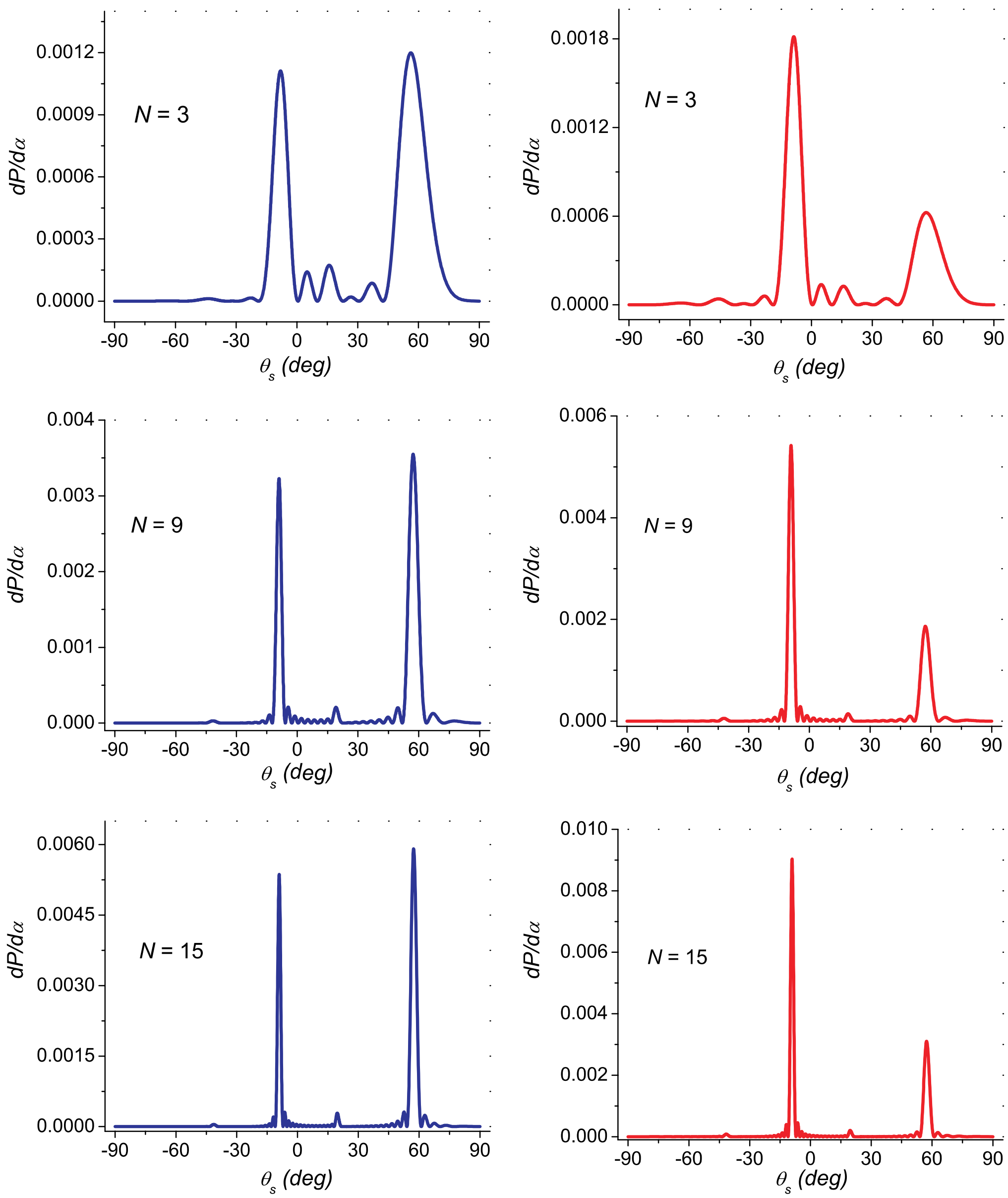


Fig. 2. (Color online) Angular distribution of power scattered into medium 1 for three sinusoidal finite gratings with identical geometrical ($h/\lambda = 0.02, d/\lambda = 2$) and constitutive parameters ($\epsilon_2 = -4 + 0.001i$ and $\mu_2 = -1.5 + 0.001i$) but different number of grooves ($N = 3$, 9 and 15), illuminated by a linearly polarized plane wave at an angle of incidence $\theta_0 = 20°$. Left-hand column: $s$ polarization; right-hand column: $p$ polarization.

with $n$ an integer. Taking into account that $\tilde{R}(\alpha)$ in Eq. (5) is associated with the difference between the total field scattered by the finite grating and the field reflected by an infinite plane, the strongest maxima in Fig. 2 occur for $n = -1(\theta_s \simeq -9.09°)$ and $n = +1(\theta_s \simeq 57.35°)$, but not for $n = 0$ ($\theta_s = \theta_0$, specular direction). In other words, the relatively low intensities observed in the specular direction $\theta_s = \theta_0$ indicate that for these geometrical parameters, the finite gratings and the flat surface scatter almost the same power into this direction. The results in Fig. 2 also show that when we increase the number of grooves, the width of the interference maxima located at the angular positions predicted by the grating equation decreases, as expected for infinitely periodic gratings, where each scattering pattern becomes a set of Dirac's delta functions located at these angular positions.

In full analogy with the results presented in [2] for the case of magnetic surfaces with a single corrugation, the results in Fig. 2 clearly evidence that, when magnetic gratings with a finite number of grooves are considered, the angular distributions of intensities obtained with the scattering formalism sketched in Section 2 exhibit cinematic features –such as position and width of the peaks—that are in perfect agreement with the results predicted by physical optics. However, what physical optics fails to predict is the dynamical or energetic features, such as the right distribution of power between peaks corresponding to a given set of constitutive parameters, roughness function $g(x)$, incident polarization, etc. This statement is particularly true for structures such as the finite gratings considered here, with $d/\lambda = 2$, where a typical dimension of the scatterer is comparable to the wavelength of the incident radiation, as can be clearly seen in Fig. 2 by noting the different relative intensity between the height of the $n = \pm 1$ peaks obtained at this angle of incidence for identical structures with the same value of $N$ in the cases of $s$ or $p$ polarized incident waves: whereas for $s$ polarization the height of the $n = -1$ peak is always lower than the height of the $n = +1$ peak, for $p$ polarization the opposite relation holds.

To involve energetic features, we first use the reciprocity theorem [19], which states that the response is not different when source and detector are interchanged. In Figure [3] we have plotted the angular distribution of power scattered into medium 1 for a finite grating with $N = 15$ sinusoidal grooves, illuminated by $s$ and $p$-polarized plane waves at angles of incidence $\theta_0 = 9.09°$ and $-57.35°$. Using Eq. (6) and the sign convention for the angles $\theta_0$ and $\theta_s$ shown in Fig. 1, it is easy to show that an angle of incidence $\theta_0 = 9.09°$ is used when illuminating the surface along a direction opposite to the propagation direction of the $n = -1$ peak in Fig. 2. We observe that for this new angle of incidence a strong maximum occurs for $\theta_s = -20°$. The value of this maximum for $s$ polarization (≈0.00537) agrees quite well with the value of the $n = -1$ peak in Fig. 2 (≈0.00535). A similar agreement is observed for $p$ polarization, but now both maxima take the value ≈0.00901. In a similar manner, it can be shown that the angle of incidence $\theta_0 = -57.35°$ is used when illuminating the surface along a direction opposite to the propagation direction of the $n = +1$ peak in Fig. 2 and that for this new angle of incidence a strong maximum occurs again for $\theta_s = -20°$. The value of this maximum for $s$ polarization is ≈0.00585 whereas the value of the $n = -1$ peak in Fig. 2 is ≈0.00591. A similar agreement is observed for $p$ polarization but now the values are ≈0.00317 and ≈0.00311.

The comparison between the scattering patterns in Figs. 2 and 3 indicates that in these cases the results obtained with the scattering formalism developed in [2] are in good agreement with the reciprocity theorem. To perform quantitative tests more demanding than those provided by physical optics and the reciprocity theorem, in what follows we compare these results with those obtained in the limit $N \to \infty$ with the magnetic extension of the C-method [8], an

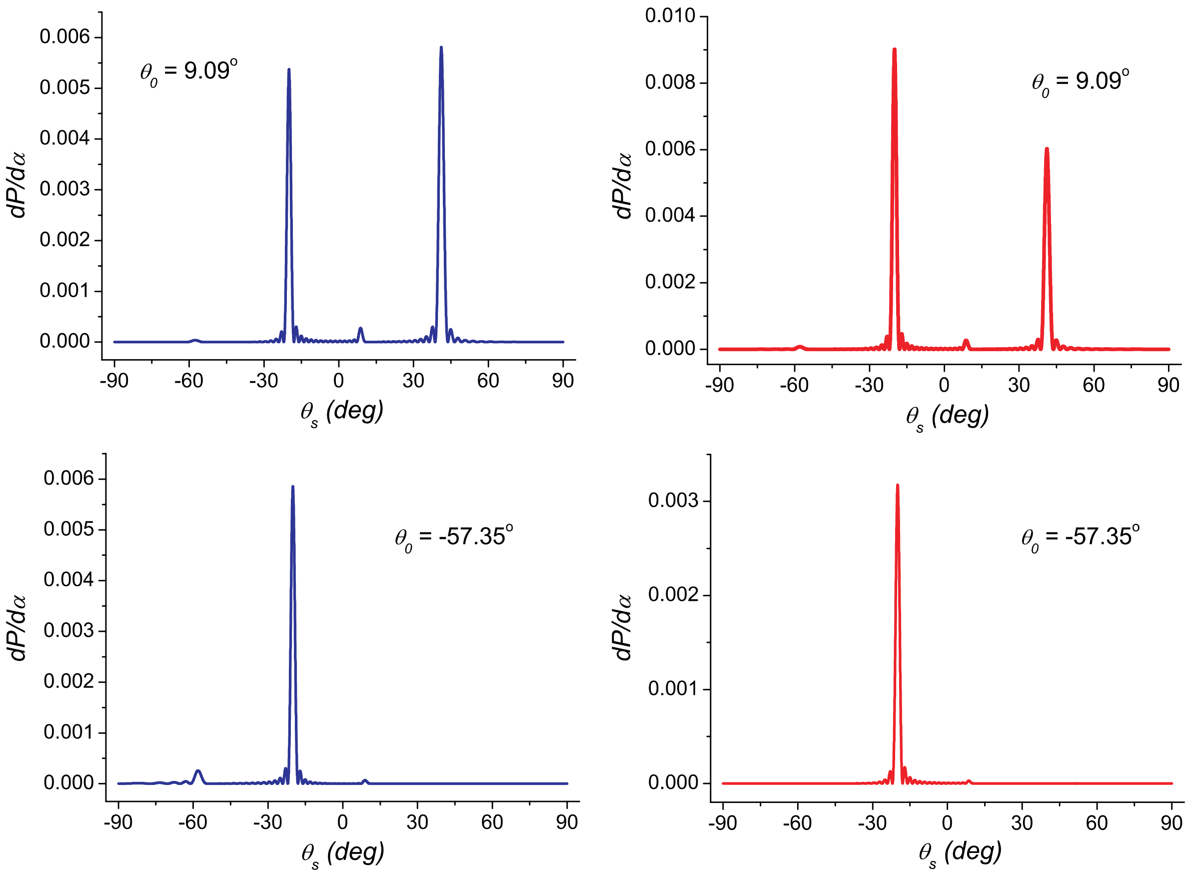


Fig. 3. (Color online) Angular distribution of power scattered into medium 1 for a finite grating with $N = 15$ sinusoidal grooves, illuminated by linearly polarized plane waves at angles of incidence $\theta_0 = -57.35°$ and 9.09°. The geometrical and constitutive parameters are as in Fig. 2. Left-hand column: $s$ polarization; right-hand column: $p$ polarization.

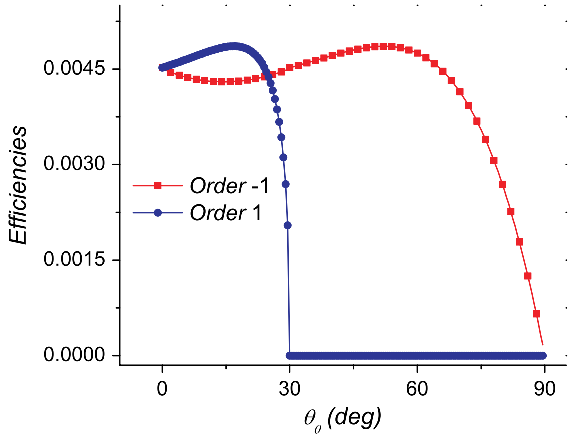


Fig. 4. (Color online) Efficiency of the $n = -1$ and $n = +1$ diffracted orders as functions of the angle of incidence $\theta_0$ for a perfectly periodic grating with the same geometrical and constitutive parameters as those in Fig. 2 ($s$ polarization).

electromagnetically rigorous and a completely different procedure that has shown to provide reliable results for perfectly periodic, magnetic diffraction gratings with a negative index of refraction [20–23]. The efficiencies of the $n = -1$ and $n = +1$ diffracted orders calculated with the C-method for a perfectly periodic grating with the same parameters as the finite gratings used in Fig. 2 ($\epsilon_2 = -4 + 0.001i$, $\mu_2 = -1.5 + 0.001i$, $h/\lambda = 0.02$ and $d/\lambda = 2$) are shown in Fig. 4 ($s$ polarization) and Fig. 5 ($p$ polarization) as functions of the angle of incidence $\theta_0$. Regarding the secondary peak observed in Fig. 2 in the specular direction, for the same parameters we have verified with the C-method that in the limit $N \to \infty$ the value of this peak must take very small values for all angles of incidence. In this limit the peak should behave as the squared modulus of the difference between the amplitude of the field diffracted by the infinite grating in the specular direction (obtained with the C-method) and the amplitude of the field reflected by the surface without corrugation (obtained with the Fresnel coefficient).

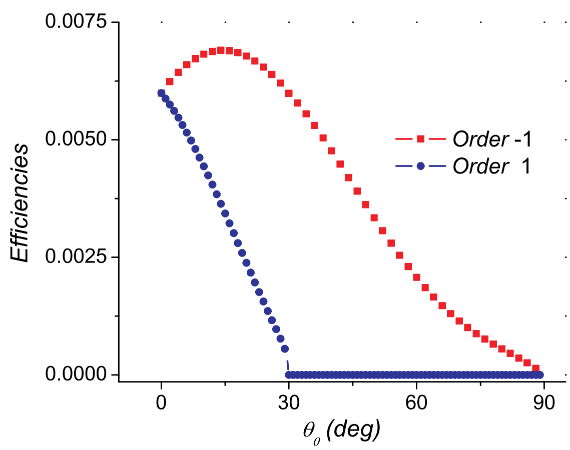


Fig. 5. (Color online) Efficiency of the $n = -1$ and $n = +1$ diffracted orders as functions of the angle of incidence $\theta_0$ for a perfectly periodic grating with the same geometrical and constitutive parameters as those in Fig. 2 ($p$ polarization).

The curves in Fig. 4 for $s$ polarization show that at the angle of incidence $\theta_0 = 20°$, the efficiency of the $n = -1$ diffracted order is lower than the efficiency of the $n = +1$ diffracted order, in total agreement with the results shown in the left-hand column in Fig. 2, even for the finite grating with the lowest number of grooves ($N = 3$). Moreover, the ratio $e_{-1}/e_{+1}$ between the efficiencies of the perfectly periodic grating at this angle of incidence is $e_{-1}/e_{+1} \approx 0.9$, while the corresponding ratio between the heights of the $n = -1$ and the $n = +1$ peaks in the scattering patterns in the left-hand column in Fig. 2 takes the values $\approx 0.92$ ($N = 3$), $\approx 0.916$ ($N = 9$), and $\approx 0.903$ ($N = 15$). We observe that the values of this ratio obtained for the finite gratings not only agree well with the value predicted by the perfectly periodic grating formalism, but are also almost independent of the number of grooves. This rather curious fact (considering the relatively small number of grooves considered in our example), has already been observed for conventional gratings [10–12].

Analogously, the curves in Fig. 5 for $p$ polarization show that at the angle of incidence $\theta_0 = 20°$, the efficiency of the $n = -1$ diffracted order is greater than the efficiency of the $n = +1$ diffracted order, in total agreement with the results shown in the right-hand column in Fig. 2, even for the finite grating with the lowest number of grooves ($N = 3$). Moreover, the ratio $e_{-1}/e_{+1}$ between the efficiencies of the perfectly periodic grating at this angle of incidence is $e_{-1}/e_{+1} \approx 2.85$, while the corresponding ratio between the heights of the $n = -1$ and the $n = +1$ peaks in the scattering patterns in the right-hand column in Fig. 2 takes the values $\approx 2.91$ ($N = 3$), $\approx 2.897$ ($N = 9$), and $\approx 2.87$ ($N = 15$). As obtained in the other polarization case, we observe that the values of this ratio for the finite gratings agree well with the value predicted by the perfectly periodic grating formalism, even for the rather small number of grooves considered in these examples.

As shown in Figs. 4 and 5, the ratio $e_{-1}/e_{+1}$ between the efficiencies of the perfectly periodic grating depends on the angle of incidence. Figure 4 shows that when the angle of incidence is changed from $\theta_0 = 20°$ to $\theta_0 = 28°$, the ratio $e_{-1}/e_{+1}$ changes from the value $\approx 0.9$ to the value $\approx 1.293$, that is, the efficiency of the $n = -1$ diffracted order is greater (and not lower, as it was for $\theta_0 = 20°$) than the efficiency of the $n = +1$ diffracted order. To see if the formalism for nonperiodic magnetic surfaces exhibits this angle of incidence behavior, in Fig. 6 we have plotted the angular distribution of power scattered into medium 1 [Eq. (5)] for a finite sinusoidal grating with $N = 15$ grooves illuminated by an $s$-polarized wave at an angle of incidence $\theta_0 = 28°$, with the remaining parameters considered in Fig. 2. The scattering pattern in this figure shows that the ratio between the heights of the $n = -1$ and the $n = +1$ peaks takes the value $\approx 1.294$, in total agreement

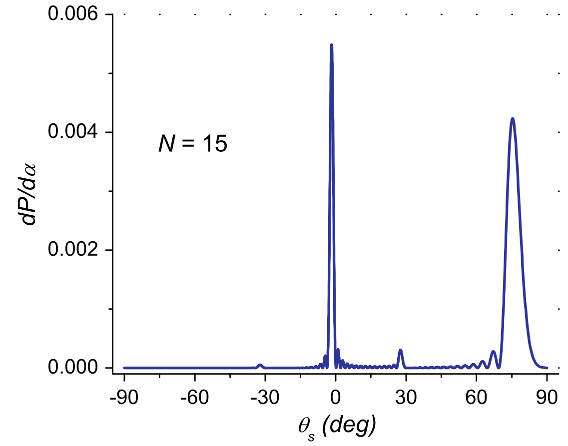


Fig. 6. (Color online) Angular distribution of power scattered into medium 1 for a sinusoidal finite grating with $N = 15$ grooves illuminated by an $s$-polarized wave at an angle of incidence $\theta_0 = 28°$, the other parameters as in Fig. 2.

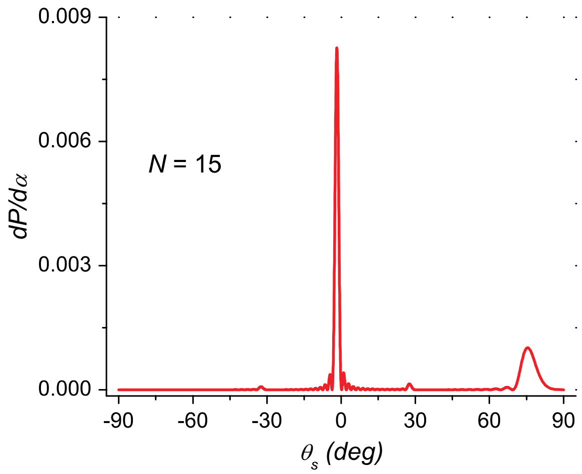


Fig. 7. (Color online) Angular distribution of power scattered into medium 1 for a sinusoidal finite grating with $N = 15$ grooves illuminated by a $p$-polarized wave at an angle of incidence $\theta_0 = 28°$, the other parameters as in Fig. 2.

with the angle of incidence behavior predicted in $s$ polarization by the C-method for a similar magnetic grating with an infinite number of grooves. Similarly, the scattering pattern in Fig. 7, obtained for the same finite grating and the same angle of incidence considered in Fig. 6 but now illuminated by a $p$-polarized incident wave, also exhibits the angle of incidence behavior predicted by the C-method in the $N \to \infty$ case, as can be seen from the fact that in Fig. 5 the ratio $e_{-1}/e_{+1}$ between the efficiencies for $\theta_0 = 28°$ takes the value ≈8.0, while the ratio between the heights of the corresponding $n = -1$ and $n = +1$ peaks in Fig. 7 takes the value ≈7.9.

## 4. Conclusions

In this paper we have provided qualitative and quantitative evidence proving the validity of the theoretical method presented in [2] for the study of the scattering of electromagnetic waves at the nonperiodic rough surface of a metamaterial with arbitrary values (positive or negative) of its magnetic permeability and electric permittivity. By considering metamaterials with a negative refractive index and locally periodic gratings with a finite number of grooves, we have shown that the angular distributions of scattered power obtained with the method presented in [2] exhibit all the characteristics predicted by physical optics. Moreover, we have shown that the polarization and the angle of incidence dependence of the results obtained with this Rayleigh method are in excellent quantitative agreement with the diffracted efficiencies obtained with the C-method for perfectly periodic gratings. As a byproduct of this study, and in complete agreement with the results known in the grating literature for conventional materials, we have shown that in surfaces with a negative refractive index, the infinite-grating predictions are obtained with a relatively small number of grooves. As a final remark, it should be noted that the quantitative agreement between the Rayleigh method of 2 and the C-method (not invoking the Rayleigh hypothesis) provides new evidence on the validity of the Rayleigh hypothesis –one of Lord Rayleigh's profound intuitions [5]—*for negatively refracting rough surfaces*.

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