The use of π*N*Δ gauge couplings in elastic π*N* scattering

The use of $\pi N \Delta$ gauge couplings in elastic πN scattering

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Abstract

We perform a phenomenological analysis on the use of $\pi N \Delta(1232)$ alternative couplings in πN scattering, within a single and well-controlled dynamical model. We compare numerical results for the elastic cross section obtained using the conventional couplings, already adopted in several reaction calculations, with those obtained with the so-called 'spin-3/2 gauge-invariant' vertexes suggested recently. Confronting with experimental cross-sectional data in the region around the resonance, we see that these results are by no means equivalent and the differences between them cannot be eliminated by a readjustment of free parameters of the meson-exchange contributions. We find that the use of the conventional couplings leads to better fits.

1. Introduction

The study of the spin-3/2 fields in hadron physics begins very early with the pioneering work of Rarita Schwinger (RS) [1]. This theory has shown several difficulties along the time, when interactions were introduced. In fact, when the RS field propagates in an external electromagnetic field, being the coupling obtained from the minimal substitution in the free Lagrangian, two problems are reported in the literature. One is that, while the free and electromagnetic Lagrangians are fully covariant, the second quantization is not realizable in all reference frames [2]. The other one is the apparition of acausal all order solutions of the equation of motion coming from these Lagrangians [3]. Posteriorly, using the RS field to describe the Δ(1232 MeV) resonance in πN scattering, Nath *et al* [4] proposed a consistent $\pi N \Delta$ vertex invariant from the point of view of the contact transformations of the spin-3/2 field and its quantization. Soon, similar problems as those mentioned previously were found, but now with the hadronic $\pi N \Delta$ interaction [5, 6]. The RS equation of motion describes a 'constrained' dynamical system, and for this reason is supplemented by certain primary and secondary constraints or subsidiary conditions that eliminate the redundant degrees of

freedom (DOFs). The Lagrangian constraint analysis can be achieved without problems for
the free case as we will show below. When the interaction is included, the same analysis can be
done. However, for 'certain' values of the external interacting fields the secondary constraints
can degenerate, being necessary to generate a set of tertiary constraints, this leading to a
loss of DOFs [7, 8]. The values of the fields at the onset of previously mentioned acausal and
quantization problems that appear are the same at which we have the loss of DOF, this situation
pre-emptying the problems. This, of course, offers no resolution of the mentioned paradoxes
present in the coupled RS fields, but gives an understanding of their deeper structure. On the
other hand, one can see that these problems appear at the level of the representation space
without invoking any interaction [9].

One of the proposals to avoid these problems in the $\pi N\Delta$ coupling case is to replace
the 'conventional' Nath's Lagrangian [4] by the one containing spin-3/2 'gauge-invariant
couplings' (to be defined below), i.e. with the same spin-3/2 gauge symmetry as the free
Lagrangian [10]. Nevertheless, when one intends to introduce the electromagnetic interaction
through a minimal substitution in the spin-3/2 gauge Lagrangian, one finds that this symmetry
and the electromagnetic gauge one have coexistence problems [11]. In view of the evident
complexity of the problem, and because of the mentioned problems with the usual pion
derivative vertex that will not appear in a perturbative calculation [12], we adopt here a
phenomenological point of view comparing those usual and gauge $\pi N\Delta$ interactions through
the evaluation of πN elastic scattering cross section. The paper is organized as follows: in
section 2, we review the properties of the free RS field, while the $\pi N\Delta$ interactions are
introduced in section 3; in section 4, we show our numerical results and our conclusions are
summarized in section 5.

2. The free Rarita-Schwinger field

The RS spinor, ψ_μ, is an element of the non-unitary representation of the Lorentz group

$$[(1/2, 0) \oplus (0, 1/2)] \otimes (1/2, 1/2). \tag{1}$$

That is, $\psi_\mu \equiv \Psi \otimes W_\mu$, where Ψ is a Dirac spinor field, while W_μ is a Dirac 4-vector [9] and
satisfies the Dirac equation

$$(i\slashed{\partial} - m)\psi_\mu(x) = 0. \tag{2}$$

One can build by contraction the Dirac spinors $\partial^\mu\psi_\mu$ and $\gamma_5\gamma^\mu\psi_\mu$ that satisfy the Dirac
equation[1], viewing these contractions as 'projecting' [13] ψ_μ on spin-1/2 representation
subspaces, with opposite parities. From the sixteen $(4 \otimes 4)$ constructed states, only eight (four
particle + four antiparticle) satisfy the subsidiary conditions (see [9])

$$\partial^\mu\psi_\mu = \gamma^\mu\psi_\mu = 0, \tag{3}$$

and we say that in the free case, we have the right DOF counting. Mathematically, we are
equating to zero all tensors of the lower rank (Dirac spinors) that can be formed with ψ_μ,
avoiding the spontaneous transitions to a 1/2 spinor (Ψ) in amplitudes such as $\overline{\Psi}\partial^\mu\psi_\mu$ and
$\overline{\Psi}\gamma^\mu\psi_\mu$. We consider that these eight '3/2 fields' represent physical on-shell $\Delta(1232)$ states.
On the other hand, we have eight more 1/2 additional states in the space (1), split in two
subsectors. In the first, the states satisfy only the condition $\partial^\mu\psi_\mu = 0$ and are called subsector
1. In the second one, the states do not satisfy any of the subsidiary conditions and are indicated
by subsector 2. It is possible to define projectors on these 3/2 and 1/2 sectors [14], respectively,

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[1] Taking the partial derivative of (2), one sees that $\partial^\mu\psi_\mu$ satisfies it too, and that if $\partial^\mu\psi_\mu \equiv 0$ also $\gamma_5\gamma^\mu\psi_\mu$ does it.

and to build up a Lagrangian for the RS field from which both equations (2) and (3) can be generated [15]:

$$\mathcal{L}_{\text{free}} = \overline{\psi}^{\mu}(x)\left\{i\partial_{\alpha}\Gamma^{\alpha}_{\mu\nu} - mB_{\mu\nu}\right\}\psi^{\nu}(x), \tag{4}$$

where

$$\Gamma^{\alpha}_{\mu\nu} = g_{\mu\nu}\gamma^{\alpha} + \tfrac{1}{3}\gamma_{\mu}\gamma^{\alpha}\gamma_{\nu} - \tfrac{1}{3}\left(\gamma_{\mu}g^{\alpha}_{\nu} + g^{\alpha}_{\mu}\gamma_{\nu}\right) \quad \text{and}$$
$$B_{\mu\nu} = g_{\mu\nu} - \tfrac{1}{3}\gamma_{\mu}\gamma_{\nu} \tag{5}$$

do not depend on ∂_{μ} and do not mix the 3/2 with the 1/2 states in the space (1). This Lagrangian was originally proposed by RS [1]. Since (4) only fix the 3/2 component of the states (through the constraints (3)), being certainly arbitrary as regards the 1/2 ones, it should be invariant under the contact transformation:

$$\psi^{\mu} \to \psi'^{\mu} = R(a)^{\mu\nu}\psi_{\nu} \equiv (g^{\mu\nu} + a\gamma^{\mu}\gamma^{\nu})\psi_{\nu}. \tag{6}$$

If we write $a = (1 + 3A)/2$, $A \neq -1/2$ (we will see why later) and apply the transformation on (4), we get the most general one-parameter Lagrangian (found through other procedure in [13])

$$\mathcal{L}_{\text{free}}(A) = \overline{\psi}_{\mu}(x)\mathcal{K}(\partial, A)^{\mu\nu}\psi_{\nu}(x), \tag{7}$$

where

$$\mathcal{K}(\partial, A)^{\mu\nu} = R\left(\tfrac{1}{2}(1 + 3A)\right)^{\mu\mu'}\left\{i\partial_{\alpha}\Gamma^{\alpha}_{\mu'\nu'} - mB_{\mu'\nu'}\right\}R\left(\tfrac{1}{2}(1 + 3A)\right)^{\nu'\nu}$$
$$= R\left(\tfrac{1}{2}(1 + 3A)\right)^{\mu\mu'}\mathcal{K}\left(\partial, -\tfrac{1}{3}\right)_{\mu'\nu'}R\left(\tfrac{1}{2}(1 + 3A)\right)^{\nu'\nu}. \tag{8}$$

$\mathcal{K}(\partial, A)$ could also be alternatively expressed as

$$\mathcal{K}(\partial, A)^{\mu\nu} = i\partial^{\alpha}\Gamma(A)^{\mu\nu}_{\alpha} - mB(A)^{\mu\nu}, \tag{9}$$

with

$$\Gamma(A)^{\alpha}_{\mu\nu} = g_{\mu\nu}\gamma^{\alpha} + \left(\tfrac{3}{2}A^2 + A + \tfrac{1}{2}\right)\gamma_{\mu}\gamma^{\alpha}\gamma_{\nu} + A\left(\gamma_{\mu}g^{\alpha}_{\nu} + g^{\alpha}_{\mu}\gamma_{\nu}\right),$$
$$B(A)_{\mu\nu} = g_{\mu\nu} - (3A^2 + 3A + 1)\gamma_{\mu}\gamma_{\nu}. \tag{10}$$

Substituting (9) into (7), the terms with Γ and B induce the decomposition

$$\mathcal{L}_{\text{free}}(A) \equiv \mathcal{L}_{\text{kin}}(A) + \mathcal{L}_{\text{mass}}(A). \tag{11}$$

Finally, using the properties of $R(a)$, it is easy to show that this Lagrangian is also invariant under the change

$$A \to A' = \frac{A - 2a}{1 + 4a}, \qquad a \neq -1/4, \; A \neq -1/2 \tag{12}$$

when the transformation (6) is done.

The spin-3/2 propagator $G(p, A)_{\beta\nu}$ should satisfy (in momentum space, we replace $-i\partial \to p$),

$$\mathcal{K}(p, A)^{\beta}_{\mu}G(p, A)_{\beta\nu} = g_{\mu\nu}, \tag{13}$$

for any value of A and to keep consistence with equation (8), it should be transformed as

$$G(p, A)^{\mu\nu} = \left[R^{-1}\left(\tfrac{1}{2}(1 + 3A)\right)^{\mu}_{\alpha}\right]G\left(p, -\tfrac{1}{3}\right)^{\alpha\beta}\left[R^{-1}\left(\tfrac{1}{2}(1 + 3A)\right)^{\nu}_{\beta}\right], \tag{14}$$

where $G(p, -1/3) = \mathcal{K}^{-1}(p, -1/3)$, being

$$G\left(p, -\frac{1}{3}\right)_{\mu\nu} = -\left[\frac{\not{p} + m}{p^2 - m^2}\hat{P}^{3/2}_{\mu\nu} + \frac{2}{m^2}(\not{p} + m)\left(\hat{P}^{1/2}_{11}\right)_{\mu\nu} + \frac{\sqrt{3}}{m}\left(\hat{P}^{1/2}_{12} + \hat{P}^{1/2}_{21}\right)_{\mu\nu}\right]. \tag{15}$$

We have introduced P^k_{ij} (defined in [14]) which projects on the $k = 3/2, 1/2$ sector of the space (1), with $i, j = 1, 2$ being the subsectors of the 1/2 subspace. This equation indicates that, at the difference of the on-shell case where the subsidiary conditions select only the 3/2 states, when the Δ propagates off-shell ($p^2 \neq m^2$), the 1/2 ones appear.

3. The $\pi N\Delta$ interaction

The invariance of the free Lagrangian (7) under the contact transformations means that the physical quantities (and thus the amplitudes) should be independent of A. Consequently, we demand the interaction Lagrangian for the 3/2 field coupled to a nucleon (Ψ) and a pseudoscalar meson (ϕ), as usually appear in a resonance decay, be invariant under (6) and (12). The most general interaction Lagrangian satisfying such requirement is

$$\mathcal{L}_{\text{int}}(A) = g\bar{\psi}_\mu R\left(\tfrac{1}{2}(1+3A)\right)^{\mu\nu} F_\nu(\psi, \Psi, \phi, \ldots) + \text{h.c.}, \tag{16}$$

where F_ν is a function of the fields and its derivatives, and g is the coupling constant. Note that the A-dependence introduced by the propagator (14) in the $\phi\Psi \to \Delta \to \phi\Psi$ amplitude is canceled by the $R(a)^{\mu\nu}$ in the vertex generated from (16). The next step is to define a group of A-independent 'reduced' Feynman rules [16] to evaluate the amplitude, they being the RS propagator $G(p, -\tfrac{1}{3})$ and the vertex $V(p, p_N, p_\pi, -1/3)$. However, from the property $R(a)_{\mu\nu}R(b)^\nu_\lambda = R(a+b+4ab)_{\mu\lambda}$ [15], we have the important relation

$$R\left(\frac{1}{2}(1+3A)\right)_{\alpha\beta} = R\left(\frac{1}{2}(2Z+(1+4Z)A)\right)_{\alpha\alpha'} R\left(\frac{1}{2}\left(1+3\frac{-2Z}{1+4Z}\right)\right)^{\alpha'}_\beta, \tag{17}$$

which introduces an additional arbitrary parameter Z [4]. Clearly, our previous choice $a = (1+3A)/2$ in $R(a)^{\mu\nu}$ corresponds to $Z = 1/2$. Now, the replacement of equation (17) in (14) and (16) shows that another choice of Z simply yields a change in reduced Feynman rules to $G\left(p, \frac{-2Z}{1+4Z}\right)$ and $V\left(p, p_N, p_\pi, \frac{-2Z}{1+4Z}\right)$, respectively. This means that an A-independent set of reduced Feynman rules can be built for different values of Z, but always leading to the same amplitudes. We feel it is important at this point to mention that in other approaches to include the Δ, as it is the chiral perturbation theory [17], the requirement of consistency from the point of view of the right number of DOFs leads to the same A-parameter structure for the πN vertex as that in equation (16). Also, analyzing the structure of the constraints on the Lagrangian, the authors can define the A-dependence of the $\pi\Delta$ interaction, where also the invariance under contact transformations comes up automatically and the use of any off-shell parameter (Z) is not necessary. We could relate this point with our previous discussion in this section, where we see that changing to another value of Z does not affect the amplitude.

In what follows, we develop two different models for the vertex function F^μ: the 'conventional' (C) and 'gauge-invariant' (G) couplings.

3.1. Conventional coupling

We begin with the so-called 'conventional coupling' which, based on the nonlinear realization of the chiral symmetry, predicts a derivative of the pion field (isospin omitted)

$$\mathcal{L}_{\text{int}_C} = \frac{f_{\pi N\Delta}}{m_\pi}\bar{\psi}^\mu R\left(\frac{1}{2}(1+3A)\right)_{\mu\nu}\Psi\partial^\nu\phi + \text{h.c.}, \tag{18}$$

that, in turn, leads to $V(p, p_N, p_\pi, -1/3) = -\frac{f_{\pi N\Delta}}{m_\pi}p_\pi^\alpha$. This interaction is considered 'inconsistent' advocating that it violates the free DOF counting, when some of the conditions (3) (extended for the interacting case) could fall for a determined fixed value of the pion field [6], being necessary to follow the constraint analysis and generate new subsidiary conditions. These lead to a reduction in the DOF due to the fixing of the pion field [8]. Note that because $p_\pi^\mu(\hat{P}_{ij}^{1/2})_{\mu\nu}(p) \neq 0$, there exists a coupling to the 1/2 component of the off-shell intermediate Δ, which rises to a 'lower spin background' contributions in observables. However, this situation is by no means exclusive of the RS field. In fact, let us consider, for example, the W boson contribution in the $\pi \to W \to \bar{\nu}\mu$ decay [18] or the $W \to \pi$ one in the pion–pole

terms in the $\nu N \to \mu \pi$ neutrino–nucleon scattering [19]. The W vector field φ_μ belongs to the spin-1 sector of the (1/2, 1/2) representation, satisfying the Proca equation (in the momentum space)

$$[(-p^2 + m^2)g^{\mu\nu} + p^\mu p^\nu]\varphi_\nu = 0, \tag{19}$$

with the subsidiary condition $p^\mu \varphi_\nu = 0$, which leads to the Klein–Gordon equation

$$(-p^2 + m^2)\varphi_\nu = 0. \tag{20}$$

There is also a spin-0 state that satisfies equation (20) but not the subsidiary condition. This is totally analogous to the spin-1/2 sector of the space (1) that satisfies the Dirac (2) equation but not the conditions (3). The W propagator looks

$$\Delta^{\mu\nu}(p) = -\left[\frac{P_1^{\mu\nu}(p)}{p^2 - m^2} + \frac{P_0^{\mu\nu}(p)}{m^2} \right], \tag{21}$$

being $P_0^{\mu\nu} = p^\mu p^\nu / p^2$ and $P_1^{\mu\nu} = g^{\mu\nu} - P_0^{\mu\nu}$ as the projectors on the 0 and 1 sectors, respectively, we also have an off-shell lower spin contribution. The $W \leftrightarrow \pi$ decay vertex goes as p_π^μ [18] or p^μ [19], being in both cases $p_{\pi\mu} P_0^{\mu\nu} \neq 0$ or $p_\mu P_0^{\mu\nu} \neq 0$, respectively. Still more, it would be impossible for the pion to decay without the off-shell spin-0 piece of the W propagator.

3.2. Gauge-invariant coupling

We start reviewing the relation of the DOF counting with gauge transformations. For simplicity, we take $A = -1$ in (7), being (with the help of (9) and (10))

$$\mathcal{L}_{\text{free}}(A = -1) = \overline{\psi}_\mu(x) \left(\epsilon^{\mu\nu\alpha\beta} \frac{\partial}{\partial x^\alpha} \gamma_\beta \gamma_5 + im\sigma^{\mu\nu} \right) \psi_\nu(x). \tag{22}$$

In the massless case ($\mathcal{L}_{\text{free}} \equiv \mathcal{L}_{\text{kin}}$), this is invariant under the spin-3/2 gauge transformation [11]

$$\psi_\mu(x) \to \psi_\mu(x) + \partial_\mu \chi(x), \tag{23}$$

(the analysis can be done for any A value, but under the transformation $\psi_\mu(x) \to \psi_\mu(x) + R^{-1}(\frac{1}{2}(1 + 3A))_{\mu\nu}R(-1)^{\nu\mu'}\partial_{\mu'}\chi(x)$), where χ is a spinor field, and we are left with DOF = 2 as in the photon case. The mass term breaks this symmetry and we have DOF = $2 \times 3/2 + 1 = 4$, corresponding to the spin-3/2 sector of the space (1). From these remarks, we follow the procedure of [20], where $\pi N \Delta$ interactions are introduced with the same type of gauge symmetry as $\mathcal{L}_{\text{kin}}$. That is, $F^\mu \partial_\mu \chi = 0$ (or $F^\mu p_\mu = 0$ in the momentum space) is satisfied. Because $G(p, -1)$ (incorrectly called the RS propagator) is built with $P_{22}^{\frac{1}{2}}, P_{12}^{\frac{1}{2}}, P_{21}^{\frac{1}{2}}$ [11], all satisfying $P_{ij}^{\frac{1}{2}\mu}F_\mu = 0$, the spin-1/2 sector is decoupled not contributing to the amplitude.

The gauge couplings, considered now as 'consistent', can be introduced from $\mathcal{L}(A) = \mathcal{L}_{\text{kin}}(A) + \mathcal{L}_{\text{mass}}(A) + \mathcal{L}_{\text{int}_C}(A)$ through a redefinition of the RS field

$$\psi_\mu \to \psi_\mu + \frac{f_{\pi N \Delta}}{m_\pi m} R^{-1} \left(-\frac{1}{2}(A + 1) \right)_\mu^\nu \Psi \partial_\nu \phi, \tag{24}$$

using the properties of the R matrices, they giving a net change $\mathcal{L}_{\text{int}_C}(A) \to \mathcal{L}_{\text{int}_G}(A) + \mathcal{L}_{\text{cont}}$ in $\mathcal{L}(A)$, with

$$\mathcal{L}_{\text{int}_G}(A) = \frac{f_{\pi N \Delta}}{m_\pi m} \bar{\Psi} \partial_\mu \phi^\dagger \epsilon^{\mu\nu\alpha\beta} \gamma_\beta \gamma_5 R \left(-\frac{1}{2}(A + 1) \right)_{\nu\sigma} \partial_\alpha \psi^\sigma + \text{h.c.},$$

$$\mathcal{L}_{\text{cont}} = \frac{f_{\pi N \Delta}^2}{m_\pi^2 m^2} \overline{\Psi}[\epsilon^{\mu\nu\alpha\beta} \gamma_\beta \gamma_5 \partial_\alpha + im\sigma^{\mu\nu}]\partial_\mu \phi^\dagger \Psi \partial_\nu \phi. \tag{25}$$
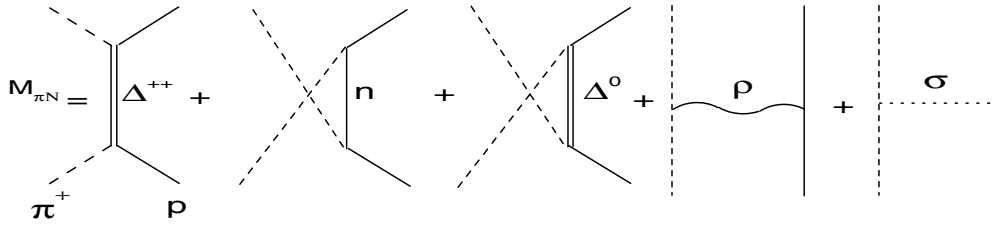
Figure 1. Feynman graphs corresponding to different contributions to the elastic $\pi^+ p$ scattering amplitude.

Here, $\mathcal{L}_{\text{cont}}$ contains contact terms involving only pion and nucleon fields. Both, the original and the new Lagrangian, should lead to the same *S*-matrix elements (observables) in concordance with the equivalence theorem [20, 21]. A generalization of the procedure developed above to go from C to G couplings for the case of bilinear couplings has been achieved in [22] within chiral effective field theory framework for the $\pi\Delta$ interaction. Here, pion couplings to off-mass-shell components of the $\Delta - 3/2$ field included in some terms of Lagrangian are absorbed in other Lagrangian's terms. Finally note that (24) eliminates $\mathcal{L}_{\text{int}_C}$ from the original Lagrangian for any value of *A*, at the difference of the transformation used in [20] where only the particular case $A = -1$ is presented. The new interaction Lagrangian, $\mathcal{L}_{\text{int}_G}(A)$, is invariant under the generalization of (23) for any *A* since it comes from transformations on the free one. This fact intuitively leads to suppose that we can extend to the interacting case constraints (3) without problems. This point and the coexistence of the electromagnetic and spin-3/2 gauge symmetries, within a dynamical model as we will develop in the following section, should be further analyzed in the future with more detail. Finally, we mention that the propagator to be used in this case to get A-independent amplitudes obtained from (14) using (17) (with $Z = -1/2$) is

$$G^{\mu\nu}(p, A) = R^{-1}\left(-\tfrac{1}{2}(A+1)\right)^{\mu}_{\alpha} G^{\alpha\beta}(p, -1) R^{-1}\left(-\tfrac{1}{2}(A+1)\right)^{\nu}_{\beta}.$$

4. $\pi^+ p$ elastic scattering calculation

In this section, C and G interactions are compared numerically within a simple effective isobar model for $\pi^+ p$ elastic scattering. Comparison with experimental data has been already done elsewhere [24] for the case of the C vertex and the Δ parameters (mass, width and coupling constants) were obtained consistently. We again perform the same task under the same standards, but now using the G interaction. Our minimum 'dynamical model' involves nucleons, $\Delta^{++,0}$ resonances and ρ and σ meson DOF, where throughout this paper we will assume isospin symmetry in the masses and strong couplings of hadrons. The elastic scattering amplitude is given by

$$\mathcal{M}(\pi^+ p \rightarrow \pi^+ p) = \sum_{i=\Delta^{++},\Delta^0, n, \rho, \sigma} \mathcal{M}_i(\pi^+ p \rightarrow \pi^+ p) , \qquad (26)$$

with each contribution shown in figure 1. The last four graphs are included at the *tree level* and they provide a smoothly varying background around the resonance region. The various Lagrangian densities necessary to build those terms are found in [24] and references therein. We will work with effective Lagrangian inspired models not representing elementary particles and form factors, which, taking into account their structure, should be important if we go away from the threshold. Because we will move with invariant masses between

$m_N + m_\pi \leqslant \sqrt{s} \leqslant m_\Delta + m_\pi$, we have to consider that they can be ignored to reproduce πN scattering in this region. Besides, the improvement of the model by the consideration of the effect of final state interactions into the vertexes and self-energies, and the lack of unitarity due to the fact that the background amplitudes are real, has been carefully analyzed in [24] and [25]. Nevertheless, since for $\pi^+ p$ scattering the Δ-pole term is dominant and the cross section depends on the modulus-squared T-matrix, the limitations of the present model have a minor impact on the cross-sectional fitting; we prefer to keep it at a more simple level.

We now focus on the dominant Δ^{++} contribution (first graph in figure 1) and compare the amplitude calculated using the $V_{\pi N \Delta_C}^\sigma = -\frac{f_{\pi N \Delta}}{m_\pi} p_\pi^\sigma$ vertex with that obtained employing $V_{\pi N \Delta_G}^\sigma = i \frac{f_{\pi N \Delta}}{m_\pi m} \gamma_5 \gamma_\beta p_\alpha p_{\pi \mu} \epsilon^{\alpha \mu \beta \sigma}$, coming from $\mathcal{L}_{int_C}$ and $\mathcal{L}_{int_G}$, respectively. They read

$$\mathcal{M}_{\Delta^{++}}^C(\pi^+ p \to \pi^+ p) = \frac{f_{\pi N \Delta}^2}{m_\pi^2} \bar{u}(p_p', m_s') p_\pi^\mu G_{\mu\nu}\left(p, -\frac{1}{3}\right) p_\pi^\nu u(p_p, m_s), \quad (27)$$

being $p = p_p + p_\pi$, and

$$\mathcal{M}_{\Delta^{++}}^G(\pi^+ p \to \pi^+ p) = \frac{f_{\pi N \Delta}^2 p^2}{m_\pi^2 m^2} \bar{u}(p_p', m_s') p_\pi^\mu (-) \frac{\slashed{p} + m}{p^2 - m^2} \hat{P}_{\mu\nu}^{\frac{3}{2}} p_\pi^\nu u(p_p, m_s), \quad (28)$$

where the fact that $V_{\pi N \Delta_G}^\mu (\hat{P}_{22}^{1/2})_{\mu\nu} = V_{\pi N \Delta_G}^\mu (\hat{P}_{21}^{1/2})_{\mu\nu} = (\hat{P}_{12}^{1/2})_{\mu\nu} V_{\pi N \Delta_G}^\nu = 0$ has been used. On the same footing, we can build the Δ^0 contribution which is part of the background. Note the relation

$$\mathcal{M}_{\Delta^{++,0}}^C = \mathcal{M}_{\Delta^{++,0}}^G + \mathcal{M}^{cont}, \quad (29)$$

with $\mathcal{M}^{cont}$ being the amplitude obtained with the contact Lagrangian from equation (25). However, it will not be included here since as a background, we expect that it could be absorbed by a readjustment of free parameters in the N, ρ and σ amplitudes [20]. Propagators in equations (27) and (28) blow up when $\sqrt{p^2}$ approaches m. The simplest solution to cure this bad behavior maintaining electromagnetic gauge invariance in the radiative $\pi^+ p$ scattering amplitude is to replace $m^2 \to m^2 - im\Gamma$ (Γ being the decay width of the Δ) in *all* the Feynman rules involving the Δ^{++} resonance [16]. This is the so-called complex mass scheme [16]. It can be shown that the dressed Δ propagator by the one-loop πN self-energy correction corresponds to replace in (15) $m \to m - i\Gamma(s)/2$, with $\Gamma(s)$ being usually obtained in terms of m and $f_{\Delta N \pi}$ [16]. When this variable width is used, a violation of the Ward identities at order Γ/m is produced, being necessary vertex corrections (a troublesome task) to restore gauge invariance. On the other side, the one-loop πN self-energy is the lowest order contribution and one should include πN rescattering into the bubble through the so-called non-pole T-matrix, which iterates all the background contributions in figure 1 to all orders [25]. Because we consider that to work within a gauge-invariant formalism is fundamental to have confidence in the results, we will adopt the complex mass scheme which has worked very well before in the description of several different processes as elastic and radiative πN scattering, π photoproduction and weak π production [24–26]. We have introduced an energy-dependent width at higher energies for neutral current π production, the difference with the complex mass scheme being not important [27]. In addition, to take into account at least effectively another contribution to the self-energy, we treat the constant width as an adjustable parameter. The finite widths of the Δ^0 baryon and the ρ, σ mesons do not play any role since these resonances do not appear in the s-channel.

Some of the couplings appearing into the Lagrangians used to build the background terms are taken from other low energy processes: the coupling constants $g_\rho^2/4\pi = 2.9$, $g_{\pi NN}^2/4\pi = 14$ from $\rho\pi\pi$ decays and the analysis of NN scattering data [28, 29], whereas the magnetic ρNN coupling, $\kappa_\rho = 3.7$, from the values of nucleon magnetic moments. The
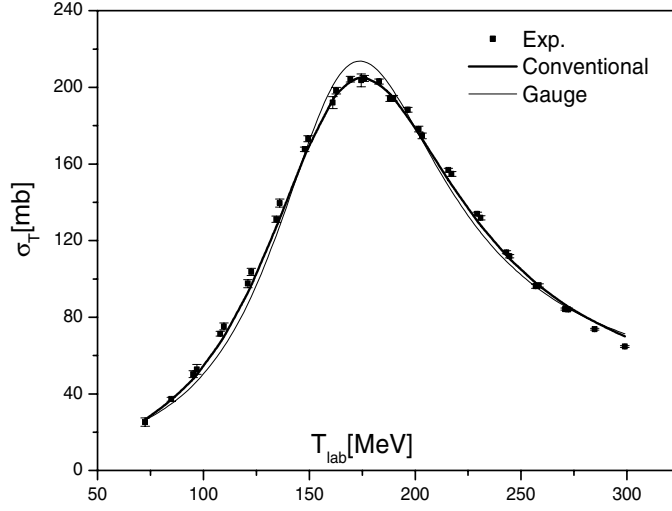
Figure 2. Elastic π^+p total cross section calculated with the amplitude (26) with $\mathcal{M}^C_{\Delta^{++}}$ from equation (27) (conventional) and $\mathcal{M}^G_{\Delta^{++}}$ from equation (28) (gauge). Data were taken from [31].

masses of the ρ meson and the nucleon were taken from [30], and the mass of the hypothetical σ meson (that runs on the range 400–1000 MeV region [30]) was set to 450 or 650 MeV [28, 29] depending on the model for $\mathcal{M}_{\Delta^{++}}$, analyzed and used previously [24]. The Δ mass, width and coupling constants $g_\sigma = g_{\sigma\pi\pi}g_{\sigma NN}$ and $f_{\Delta N\pi}$ are left as the only free parameters to be determined from the fitting of the total cross section of π^+p scattering to the data.

In figure 2, we compare results for elastic π^+p total cross section calculated with the amplitude (26) with $\mathcal{M}_{\Delta^{++}}$ taken from equation (27) and $\mathcal{M}^G_{\Delta^{++}}$ from equation (28). For the C coupling, $m_\sigma = 650$ MeV was used as before [24], and we get $f^2_{\Delta N\pi}/4\pi = 0.317 \pm 0.003$, $m_\Delta = 1211.2 \pm 0.4$ MeV, $\Gamma = 88.2 \pm 0.4$ MeV, $g_\sigma/4\pi = 1.50 \pm 0.12$ and $\chi^2/\text{dof} = 4.5$, being the last three points in the upper tail of the total cross section excluded. Our obtained values (or $m_\Delta = 1211.7 \pm 0.4$ MeV, $\Gamma = 92.2 \pm 0.4$, which could be obtained in an improved more evolved model [25]) are consistent with the complex pole parameters ($m_\Delta = 1211$ MeV, $\Gamma = 97.0 \pm 0.4$) obtained with the chiral perturbation calculations in [11] making the identification $h_A/2f_\pi = f_{\Delta N\pi}/m_\pi$.

On the other side, for the G coupling (where the best fits were obtained with $m_\sigma = 450$ MeV), we obtain 0.278 ± 0.002, 1211.6 ± 0.3 MeV, 76.62 ± 0.25 MeV, 1.00 ± 0.05 and 13.5, respectively. In figure 3 also, we show separately the Δ-pole (the first graph in figure 1), the background (the four last graphs in figure 1) and the sum amplitude contributions to the total cross section. This is done for both the conventional and gauge couplings. As can be seen, the background contribution is practically the same in both couplings, with the difference being due to the small contribution of the Δ-cross (the third graph in figure 1) term and the difference in the σ parameters for each coupling that is not very important. The Δ-pole contribution presents a more appreciable difference due to the fact that the virtual $\Delta - 1/2$ contribution is absent from the gauge coupling amplitude (also, the factor $\sqrt{p^2}/m$ is very important) which, however, is present in the background. This fact also affects the important interference contribution to the cross section. Our calculation differs from that in [23] at the tree level, where a minor difference between C and G results was reported, by three important points. Firstly, their κ_ρ was fixed to zero when the C coupling is used. Secondly, they have an opposite sign in the $V_{\sigma NN,\sigma\pi\pi}$ potential as regards to us, since we assume a two-pion-correlated
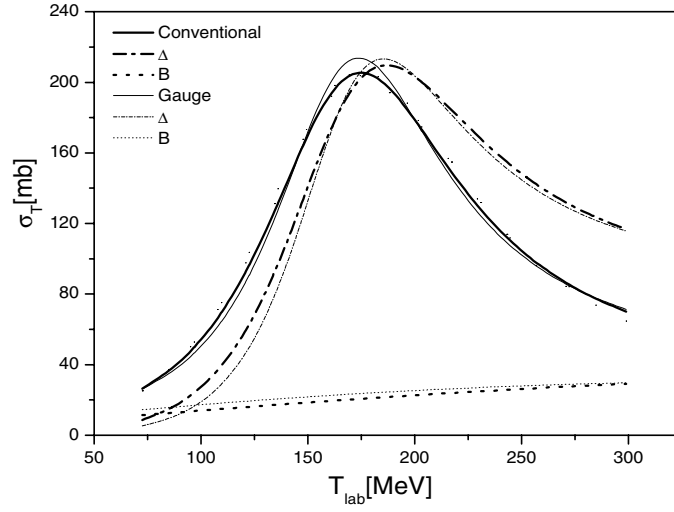
Figure 3. Δ-pole amplitude contribution (Δ), background (B) amplitude contribution and total amplitude (conventional or gauge) contribution to the elastic $\pi^+ p$ total cross section calculated both couplings. Line conventions are shown in the figure.
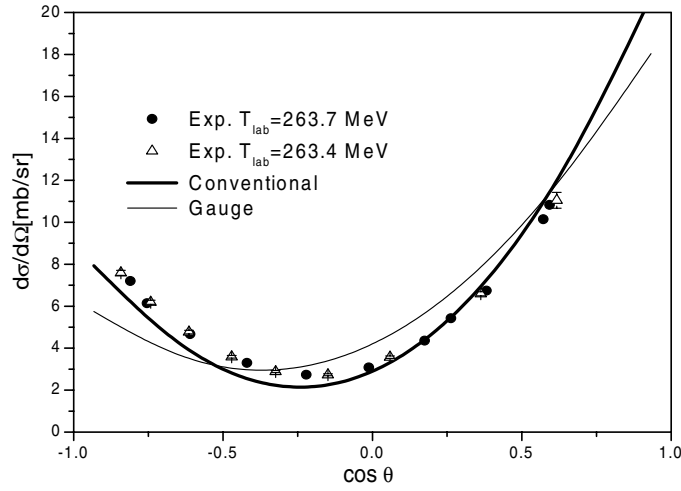

Figure 4. Differential $\pi^+ p$ cross section calculated with the conventional (C) and gauge (G) amplitudes for $T_{lab} = 263.7$ MeV. Circles and triangles indicate experimental data from [32] and [33], respectively.

model in spite of the fact that they are at the tree level. Finally, they avoid the contribution of the σ meson to S partial waves through an additional factor in the potential, with the S-wave lengths being then explained only by the nucleon and ρ terms.

With these obtained parameters, we also show the predicted differential cross section at two fixed T_{lab} energy values. Our results are compared with the available data for both the C and G couplings in figures 4 and 5. As can be seen from figure 2, the fitting to the total cross section is clearly better by using the C couplings than the G ones, this being due to the different behavior of the Δ-pole contribution and not due to the background that is almost the same in
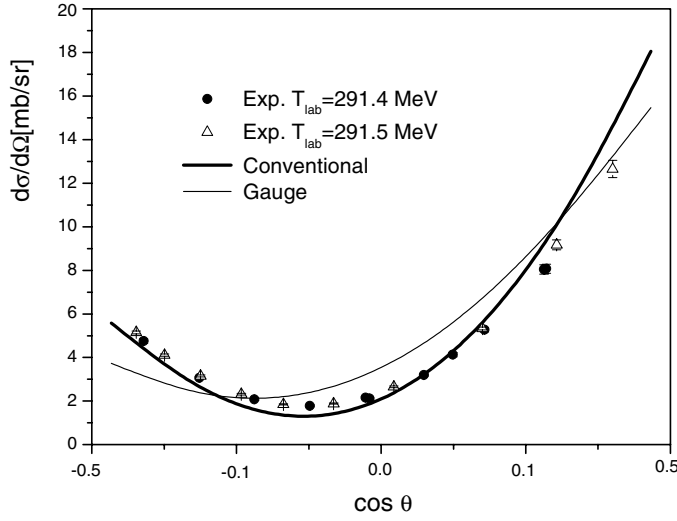
Figure 5. Same as figure 4 with predictions at $T_{\text{lab}} = 291.4$ MeV.

both the cases. This fact is better appreciated for the differential cross sections in figures 4 and 5, where eventual compensation of differences due to the angle integration is not present.

5. Conclusions

From the formal point of view, we have analyzed in detail the connection between different fixings for the Z parameter appearing at the moment of introducing the $\pi N \Delta$ interaction. Also, we analyze the generation of a set of reduced (A-independent) Feynman rules to compute contact-invariant amplitudes for different Z values. The $\pi N \Delta$ interaction has been included in a contact-invariant fashion through both the usual 'conventional' couplings based on the nonlinear realization of the chiral symmetry predicting a derivative of the pion field and the called 'gauge couplings' respecting the same spin-3/2 gauge symmetry as the massless Δ free Lagrangian. These last vertexes have the particularity to uncouple the off-shell 1/2 Δ components 'really' present in its propagator. Also, we show the transformation that enables to generate a gauge-invariant interacting Lagrangian from the conventional one for 'any' value of A. We obtain the relation $\mathcal{M}_{\Delta}^{C} = \mathcal{M}_{\Delta}^{G} + \mathcal{M}^{\text{cont}}$, where the last term includes certain background contact terms, different to that included by N, ρ and σ DOFs. It was shown that the so-called inconsistence of the off-shell propagation of 1/2 components for the Δ fields is clearly present, with a notable parallelism, in other cases as is the W boson off-shell propagation in $\pi \rightarrow W \rightarrow \bar{\nu}\mu$ decay or the $W \rightarrow \pi$ one, present in the pion–pole terms contributing to $\nu N \rightarrow \mu \pi'$ neutrino–nucleon scattering.

　　We have calculated within a 'minimum' dynamical model, without the inclusion of form factors due to the considered energy region, and where we have a good control on the majority of the background parameters, the total and differential cross section for the elastic πp scattering with both kinds of $\pi N \Delta$ couplings. From our numerical results, we arrive at the following conclusions.

(i) Within this simple model, the fittings achieved with the conventional couplings are clearly better than those obtained with the gauge ones.

(ii) It does not seem possible to accommodate the parameters of the σ meson (those of ρ are fixed in both approaches by low energy phenomenology) to obtain identical results with both types of couplings. In fact, the obtained value $g_\sigma/4\pi = 1.00 \pm 0.05$ for $m_\sigma = 450$ MeV using the gauge couplings is also consistent with the conventional resonant amplitudes [24]. Then, it does not seem possible to simulate the not included $\mathcal{M}^{\text{cont}}$ by changing consistently parameters in $\mathcal{M}_n + \mathcal{M}_\rho + \mathcal{M}_\sigma$. The contribution of $\mathcal{M}^{\text{cont}}$, automatically included in the conventional amplitude, is clearly necessary to reproduce the experimental results. Note also the presence of the p^2/m^2 factor in the resonant gauge amplitude, which could have an important effect on the cross section through its direct contribution and the interference with the background amplitude.

Acknowledgments

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